The Study of Students' thinking Mathematically in Classroom using Transformative Lesson Study Incorporated with Open Approach

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Abstract

This research aimed to study students' thinking mathematically in the classroom using transformative lesson study incorporated with open approach. It used a qualitative research methodology emphasizing participatory observation. The target group is 25 sixth-grade students of schools affiliated with the Phuket city municipality education office. The research instruments were lesson plans, field notes, and interview forms. Research data were collected during the second semester of the academic year 2024. The data were analyzed using protocol and descriptive analysis based on the conceptual framework established by Mason et al. (2010).

The research revealed that the students' thinking mathematically in the classroom using transformative lesson study incorporated with open approach including: 1) entry phases, students interpret the tasks or problem situations given by the teacher. They think of how to solve the problem situations and make it easier to solve the real problem. 2) attack phases, students solve the problem by conjecturing about how to solve the problem and justifying the solution. 3) review phases, students validate and reflect on their ideas, and extend their ideas to wider contexts or situations.

Keywords : Thinking Mathematically/ Lesson Study Incorporated with Open Approach

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Introduction

An essential objective of education is to develop the ability to think mathematically and apply mathematical thinking to solve problems (Stacey, 2007). Thinking Mathematically serves as a positive driving force, encouraging students to explore various strategies that can lead to success and enhance their confidence in achieving problem-solving goals (Tall, 2013). As Mason et al. (2010) stated, thinking mathematically is not just about studying numbers and mathematical formulas but also includes perceptions and emotions that play an important role in understanding and solving problems. Feeling stuck is not an obstacle but an opportunity to learn while expressing emotions when understanding occurs can release energy and lead to progress in the learning process. For example, happiness and excitement in discovery are important because they help build motivation and the power to learn. In addition, developing a questioning attitude allows us to see problems in both the real world and mathematics. Collaboration in the learning process and receiving support from others are essential because they help stimulate the expression of ideas and effective connections with others.

Thinking mathematically is central to teaching mathematics for several reasons, such as providing a foundation for problem-solving and a tool for students to develop skills in analyzing problems, identifying relevant information, and developing effective strategies for finding solutions to problems (Stacey, 2006; Inprasitha, 2023). We can therefore consider thinking mathematically as a life skill that extends beyond the classroom in terms of developing learners with the ability to analyze data, make decisions, and solve real-world problems that will lead to success in personal and professional endeavors (Inprasitha, 2023).

Problem-solving has been a central topic of mathematics education research since the 1980s and is central to thinking mathematically. The ability to think mathematically and use mathematical thinking to solve problems is therefore an important goal of mathematics at the school level. Creating a problem-solving-focused mathematics classroom is essential to begin developing thinking mathematically (Schoenfeld, 1992; Stacey, 2006). A problem-solving classroom begins with the teacher presenting a task or problem situation. Students then solve the problem using a variety of methods, compare and contrast the solutions, and draw conclusions. A key principle of problem-solving instruction is to foster students' ability to learn mathematics on their own and for themselves (Isoda & Katagiri, 2012). For Thailand, Inprasitha (2023) systematically adapted the concepts of Lesson Study and Open Approach from Japan to align with the educational context of Thailand, with the objectives of fostering independent learning and enhancing students' mathematical thinking. This effort culminated in the development of a model that integrates Lesson Study with Open Approach, emphasizing collaborative teaching practices to support and cultivate students' thinking mathematically within the classroom. However, the process of thinking mathematically, which is a process that students will explore by solving problems by themselves (Mason et al., 2010), is an issue that will help expand the perspectives of teachers and those involved in mathematics classes to see the details of thinking mathematically; it is still an issue that has not been studied. This makes researchers interested in the study of students' thinking mathematically in the classroom using transformative lesson study incorporated with open approach to help teachers know about the mathematical thinking process and realize the importance of students' thinking mathematically.

Objective

To study students' thinking mathematically in the classroom using transformative lesson study incorporated with open approach.

Research Methodology

The target group

The target group used in this research was 25 sixth-grade students at Wat Khacharangsan Municipality School, under the Phuket City Municipality Education Office, Phuket Province, in the second semester of the academic year 2024, using a specific selection method.

Research tools

This research is qualitative. The research tools are as follows:

1. Lesson plan: the process of creating and developing a learning management plan has the following details: 1) analyze standards and indicators of the core curriculum, 2) select activities from Japanese mathematics textbooks (Gakko Tosho Publishing), 3) design problem situations, and teaching materials. 4) anticipating students' mathematical ideas, 5) determining the teaching sequence of the open approach, 6) implementing the lesson plan in the classroom, and 7) reflecting on teaching results and improving the learning management for the next period.

2. Field Notes: used to record of student behavior and speech that occur in the classroom using lesson study and open approach classroom learning.

3. Video recorder: used to record the movement and sound of the lesson study team while planning the lesson plan and recording from the beginning of the teaching activity until the end of the summary by linking students' mathematical concepts that emerged in the classroom and the weekly teaching reflection period.

4. Voice Recorder: recorded the lesson study team's voices while they planned the lesson plans, the teacher and students as they delivered lessons, and the lesson study team's voices during weekly reflections.

5. Camera: used to record still images according to the steps of the lesson study and record events in the classroom according to the teaching steps of the open approach, including images of student work and images of the classroom board.

6. Interview form: used to interview students after the completion of the course. The questions asked students whether they felt difficulty solving the problem or what methods they used to solve the problem.

Data Collection

The researcher conducted data collection according to the steps of the lesson study incorporated with open approach: 1) collaborative learning planning stage, 2) open approach teaching and collaborative classroom observation stage, and 3) collaborative classroom observation reflection stage, using video, still images, and audio recordings, and field notes. In addition, after each teaching hour, the researcher conducted interviews with students about the concepts used in solving problems.

Data Analysis

The data obtained from this study were qualitative data, including student work, protocols, field notes, and interviews. The researcher used protocol analysis and descriptive analysis to demonstrate students' thinking mathematically using the theoretical framework of Mason et al. (2010).

Results

The analysis of students' thinking mathematically in the classroom using transformative lesson study incorporated with open approach. The researcher presented an example of data analysis as follows.

1. Results of the analysis of students' thinking mathematically in the activity of multiplying fractions with whole numbers

This activity aims to have students think about situations using fractional multiplication and how to calculate the answer. The problem situation is that "1 L of green paint can paint exactly $\frac{2}{5}$ m^2 of fence. If you have 3 L of paint, how many square meters of fence can you paint?" In the teaching of Lesson Plan 1, the teacher began presenting the problem situation by showing a picture of the problem situation on the board. Then, the teacher presented the problem situation. With the first command, “Draw a picture to show,” the teacher distributes the activity sheet to the students. Then, the teacher lets the students solve the problem by themselves by letting them think of a way to find the area to paint in the amount of 3 L. The teacher observes the students’ ideas and takes notes. Then, the ideas that arise in the classroom are presented for discussion and conclusion in the classroom. The results revealed as follows:

1. Entry Phases

Students read the problem and understand the problem situation and the instructions given by the teacher on what to do. They try to find out what the question is asking and think of a way to solve the problem to make it easier to solve the real problem.

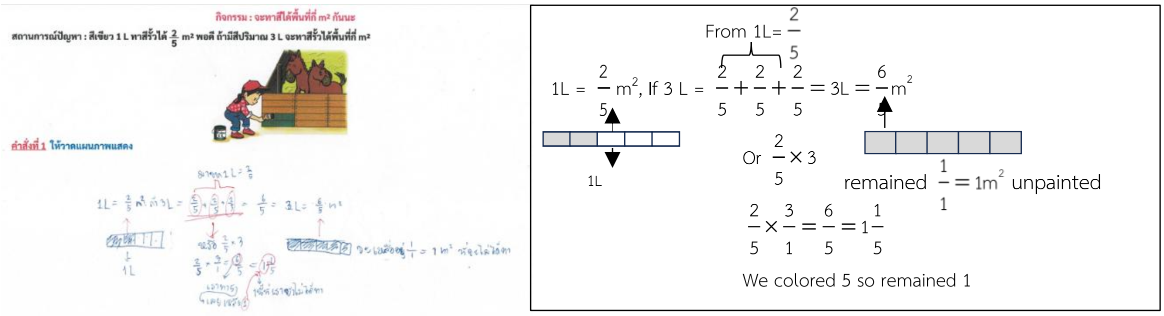


Figure 1. Students' thinking mathematically: Entry Phases

From Figure 1, when the teacher gave the problem situation to the students, the students understood the problem situation and the teacher's instructions that they had to find the area to paint the fence in the amount of 3 L of paint by drawing a picture to show it. The students tried to find the way before solving the problem by reviewing the given situation that if there was 1 L of green paint, they could paint $\frac{2}{5}$ m^2 of the fence. If there is a volume of 3 L, it will be possible to paint $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{6}{5}$ m^2 of fence before starting to paint the picture as shown in the activity sheet.

1. Attack Phases

Students solve the problem by themselves by having them think of a way to find the area to paint in 3 L. The teacher observes the students' ideas and makes notes, then presents the ideas in class for discussion and comparison in front of the class.

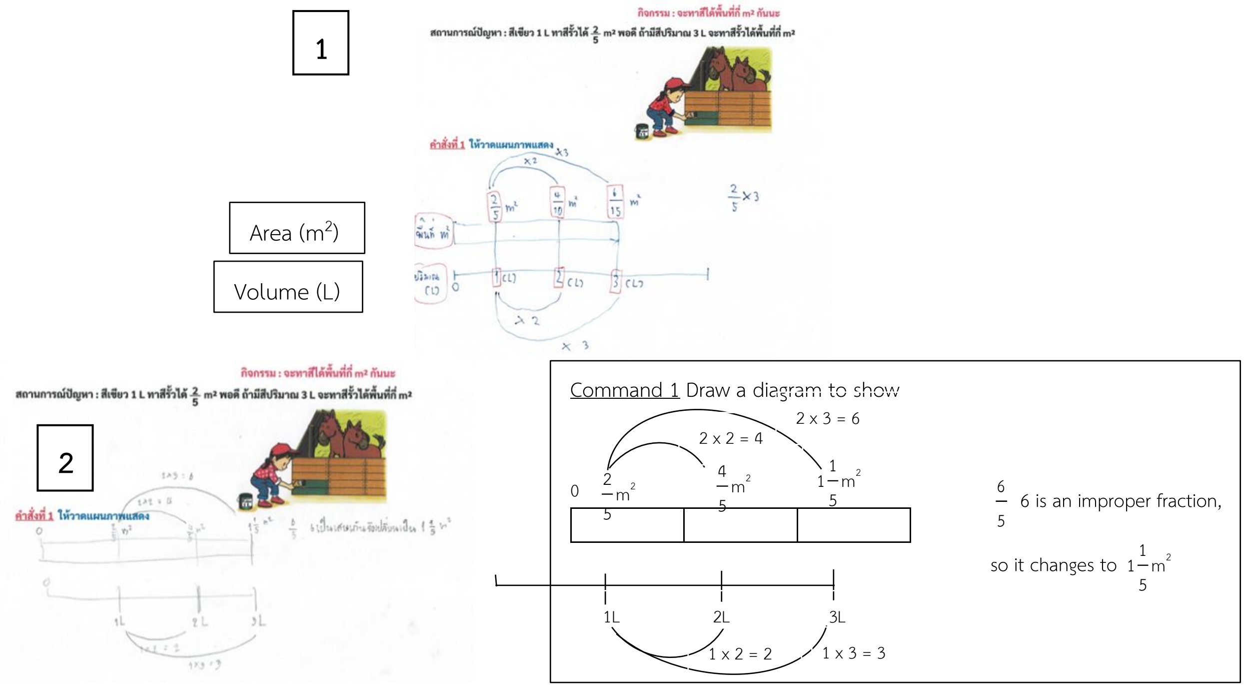


Figure 2. Students' thinking mathematically: Attack Phases

From Figure 2, after finding a way to solve the problem, the students started solving the problem in the activity sheet. It was found that in activity sheets 1 and 2, the students tried to draw pictures showing the amount of 1 L of green paint that would paint a fence with an area of $\frac{2}{5}$ m^2. If there were 3 L of paint, it would be possible to paint. The students saw the relationship between the amount of paint and the area. If the amount of paint increased, the area would also increase. They used a diagram showing the results with 2 lines and connected the relationship by multiplying by 3 in the amount of paint and the area. In Activity Sheet 1, students still felt stuck with the calculation method to solve this problem because students multiplied whole numbers with fractions by multiplying both the numerator and the area by 3, giving students the answer of $\frac{6}{15}$ m^2, which did not match what they had thought before solving the actual problem. This caused students to conjecture to find possible ways to solve the problem situation. From Activity Sheet 2, students multiplied fractions by whole numbers, multiplying only the whole numbers by the numerators. Students felt satisfied with their solution, confirming with their calculations that $\frac{2}{5}\times 3$ would yield $\frac{6}{5}$ m^2, as well as their predictions before solving the problem that 3 L would paint $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{6}{5}$ m^2 of the fence.

2. Review Phases

Students review and learn from what they have drawn on the diagram, to extend their understanding and thinking skills.

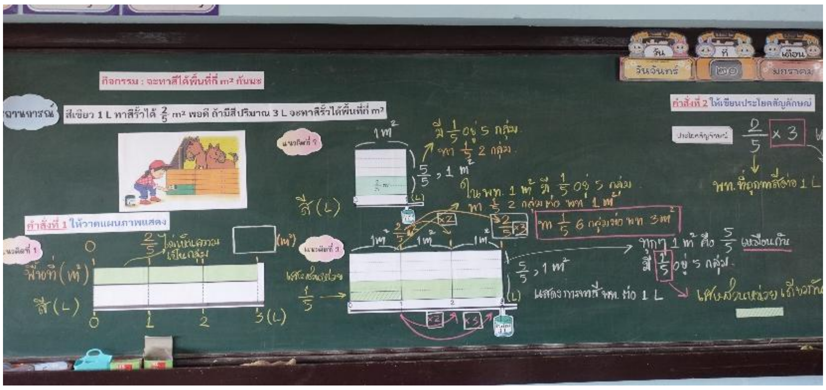


Figure 3. Students' thinking mathematically: Review Phases

From Figure 3, students try to validate their solutions and discuss their ideas together to ensure that their ideas are correct, as per the protocol Item 80 – Item 87.

Item 80	T	**:**	The painted area, you said was divided into $\frac{5}{5}$, which is 1. So how did they paint it? Where did they paint it?	
Item 81	S3	**:**	Paint where there is a grid.	
Item 82	T	**:**	Let me show you which grids your friend painted. How many grids are there now?	
Item 83	S3	**:**	Five grids.	
Item 84	T	**:**	There are five grids that your friend has divided. How did he paint them?	
Item 85	S3	**:**	Painted $\frac{1}{5}$ at a time because it has five parts. My friend said it's $\frac{1}{5}$, paint two grids.	
Item 86	T	**:**	Ah, did you hear that your friend said to apply for two of them $\frac{1}{5}$? What does your friend mean $\frac{1}{5}$?	
Item 87	S3	**:**	Unit fraction.	

From protocol item 80 to item 87, it was found that students engaged in thinking mathematically in the review phases, where students reviewed diagrams and discussed what they had done to solve the problem. The condition was “There are five grids that my friend has divided, how will he paint them?” Students demonstrate their understanding of the concept to the teacher and peers by explaining the source and pointing to the activity sheet on the board to help them understand what they have said, such as Item 85 “*Paint* $\frac{1}{5}$ *at a time because it's five parts. My friend said it's* $\frac{1}{5}$*, paint or color two grids.*” And from the question “Did you hear that my friend said to apply $\frac{1}{5}$, two of them? What does my friend mean $\frac{1}{5}$?”, in Item 87 words and behavior, said “*Unit fraction*” Make your friends understand that $\frac{1}{5}$ is a fraction of a unit and that 1 L of green paint can paint $\frac{1}{5}$ m^2 of the area twice, demonstrating thinking mathematically in review phases.

From the problem situation, “1 L of green paint can paint exactly $\frac{2}{5}$ m^2 of fence. If you have 3 L of paint, how many square meters of fence can you paint?” In the second command, “Write a symbolic sentence,” and the third command, “Show how to calculate the answer,” the teacher gave the activity sheet to the students. Then, the teacher let the students solve the problem by themselves by writing a symbolic sentence and showing how to calculate the answer. The teacher observed the students’ ideas and made notes. Then, the ideas that arose in the classroom were presented for discussion and conclusion in the classroom. The results of the thinking mathematical data analysis found that from the behavior and work of the students who let the teachers solve the problems, the discussion in the classroom can be analyzed by dividing it into 3 stages as follows:

1. **Entry Phases**

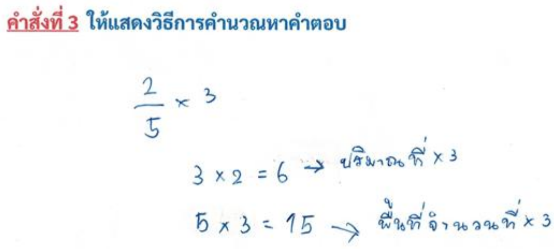
At this stage, students understand the instructions given by the teacher and try to find an easy way to solve the problem by taking what they have learned from solving the problem in Instruction 1 and then drawing a diagram to show it, writing it as a symbolic sentence, and showing the calculation method as per the protocol items 194–197.

Item 194	T	:	How do you write a symbolic sentence?
Item 195	Sa	:	$\frac{2}{5} \times 3$
Item 196	T	:	This symbolic sentence $\frac{2}{5} \times 3$, why is it like this?
Item 197	S5	:	*Because* $\frac{2}{5}$ *is equal to painted 1 L, if 3 comes from painted 3 L.*

From protocol, Item 194 – Item 197, it was found that students were able to predict plausible concepts for solving the problem. Students used $\frac{2}{5} \times 3$ observations from their previous problem-solving experience, which showed a relationship between the number of colors and the fenced area of the diagram increasing by a factor of 3, as in Item 197 "*Because* $\frac{2}{5}$ *is equal to painted 1 L, if 3 comes from painted 3 L.*" It showed that these ideas expressed mathematical thinking in the entry phases.

2. Attack Phases

In this phase, students solve problems and demonstrate how to calculate the answer from symbolic sentences. $\frac{2}{5} \times 3$



Command 3 Show how to calculate the answer.

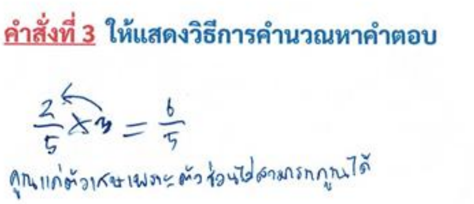
$\frac{2}{5} \times 3$

3 x 2 = 6 → Quantity multiplied by 3

5 x 3 = 15 → Area of numbers multiplied by 3

Figure 4. Students' thinking mathematically: Attack Phases (Stuck!)

From Figure 4, it can be seen that some students tried to solve the problem and still felt stuck in solving it by multiplying fractions with whole numbers. From Activity Sheet 1, students multiplied both the numerator and the denominator by 3.



Command 3 Show how to calculate the answer.

$\frac{2}{5} \times 3 = \frac{6}{5}$

Multiply only the numerators because the denominators cannot be multiplied.

Figure 5. Students' thinking mathematically: Attack Phases (AHA!)

From Figure 5, it can be seen that students solved the problem and felt AHA! with what happened during the problem-solving process. Students saw something from the diagram showing the unit fractions that they had solved earlier. Students justify through class discussion that when multiplying fractions by whole numbers, multiply only the whole numbers in the numerator, and if it is an improper fraction, it can be made into a mixed number.

3. Review Phases

Students review the problem situation provided by the teacher and look back to instructions 1-3 from drawing a diagram to showing how to calculate the answer. Students check their calculations by looking at the diagram to ensure they are correct, and students reflect on the concept and solution they have calculated, multiplying fractions by whole numbers and following protocol items 381–389.

Item 381 T : Okay, let's summarize.

Item 382 Ss : Multiplying fractions of whole numbers.

Item 383 T : How do you multiply fractions by whole numbers?

Item 384 Ss : *Multiply the numerator by the whole number.*

Item 385 T : Multiplying the numerator by a whole number. Oh, I would like to write *multiply the numerator by a whole number and zero*. Is this the end?

Item 386 Ss : *Multiply the denominator by 1*

Item 387 T : Multiply by 1; besides multiplying by 1, what other ways are there?

Item 388 S15 : No need to multiply

Item 389 T : I would like to write leave the denominator the same as you said we didn't need to multiply.

From the protocol Item 381 – Item 389, it was found that students were able to summarize the multiplication of fractions with whole numbers and zero, as in Item 384“*Multiply the numerator by the whole number*.” และ Item 386 *“Multiply the denominator by 1”* or the denominator does not need to be multiplied by anything, leaving the denominator the same. Students can apply the method of calculating fractions multiplied by whole numbers and zero to other contexts or situations that are wider.

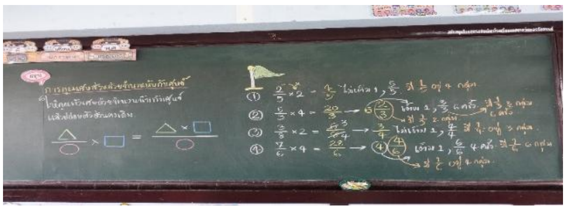


Figure 6. Students' thinking mathematically: Review Phases

From Figure 6, students have summarized the multiplication of fractions with whole numbers and zero in the form of a Squid Game by giving the numerator a triangle, the denominator a circle, and the whole numbers a square. It turns out that the triangle is multiplied by the square, and the denominator by the circle remains the same. It can be seen that students have seen a pattern and created a rule that allows them to apply their understanding of calculating fractions x whole numbers to solve new problems.

2. Results of the analysis of students' thinking mathematically in the activity of dividing fractions with whole numbers

This activity aims to get students thinking about situations where fractions are used and how to calculate the answer. The problem is "From the wall seen, 2 L of paint can cover exactly $\frac{5}{6}$ m^2 of wall. If there is 1 L of paint, how many square meters of fence can be painted?" In teaching the second learning management plan, the teacher started presenting the problem situation by showing a picture of the problem situation on the board. Then, the problem situation was presented with the first command, "Draw a picture to show"; the second command, "Write a symbolic sentence"; and the third command, "Show how to calculate the answer." The teacher distributed the activity sheet to the students. Then, the teacher let the students solve the problem by themselves by having them think of a way to find the area to paint the wall in the amount of 1 L. The teacher observed the students' ideas and recorded them. Then, the ideas that arose in the classroom were presented for discussion and conclusion in the classroom. The results found that from the behavior and students' work when the teacher lets the students solve the problems and discuss together in the classroom, it can be analyzed by dividing it into 3 stages as follows:

1. **Entry Phases**

At this stage, students review the problem situations given by the teacher and see the differences between the problem situations in the learning management plan that they had previously solved.

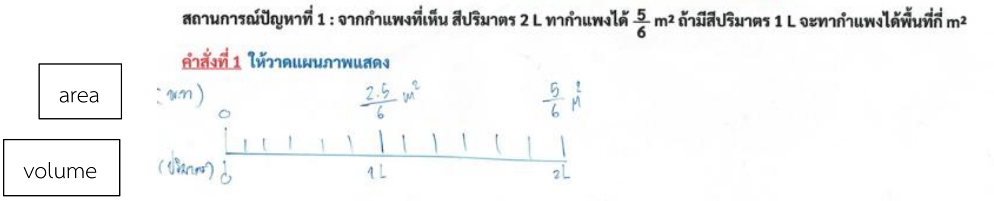


Figure 7. Students' thinking mathematically: Entry Phases

From Figure 7, students predicted the concept of solving the problem from observation from solving the problem previously. Observing the problem situation given by the teacher, it can be seen that the problem states that 2 L of paint can paint $\frac{5}{6}$ m^2 of wall, and what the problem asks is how many m^2 of fence can be painted with 1 L of paint. Students therefore got the concept for solving this problem situation by dividing the fraction by the whole number.

2. Attack Phases

At this stage, students start solving problems by understanding the problem situation and finding ways to solve it. It can be seen that most students will write symbolic sentences as $\frac{5}{6} \div 2$

$\frac{5 \div 2}{6} = 2.5$ there are $\frac{1}{6}$ of 2.5 groups

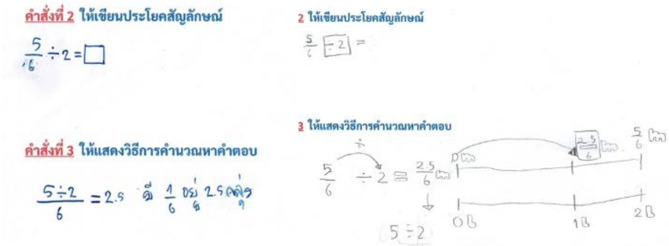


Figure 8. Students' thinking mathematically: Attack Phases (Stuck!)

From Figure 8, it can be seen that students attempted to demonstrate how to calculate dividing fractions by whole numbers by conjecturing from the steps of multiplying fractions by whole numbers that were summarized together in the previous period. This made students feel stuck with the method they chose to use to solve the problem by dividing the numerator by a whole number and leaving the denominator the same, which would answer $\frac{2.5}{6}$. This made students feel dissatisfied with the answer, so they justified drawing a diagram.

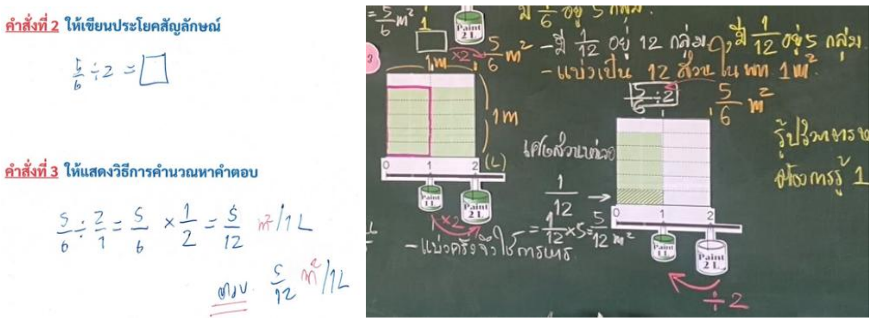


Figure 9. Students' thinking mathematically: Attack Phases (AHA!)

From Figure 9, it can be seen that students demonstrate how to calculate the answer from the symbolic sentence $\frac{5}{6} \div 2$. The students' calculation steps in this picture, from observing the diagram that solved the problem in command 1, made the students see the amount of the denominator increase. The students therefore changed the symbol from division to multiplication and reversed the numerator to the denominator and the denominator to the numerator. This made the students see that dividing a fraction by a whole number is multiplying the whole number by the denominator to make the denominator have an increasing amount, as shown in the diagram.

3. **Review Phases**

In this phase, students check their calculations by looking at the diagram to make sure their answers match the diagram and summarize their steps together as a class.

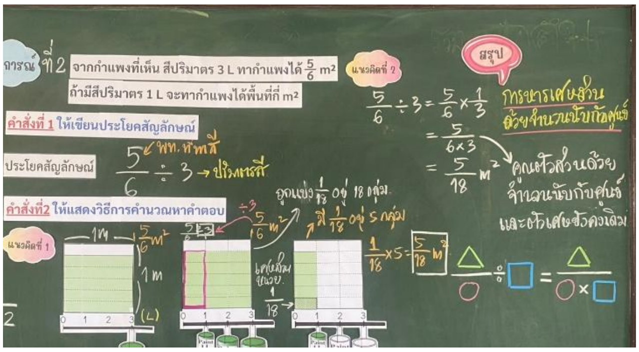


Figure 10. Students' thinking mathematically: Review Phases

From Figure 10, it can be seen that students jointly discussed and summarized the steps of dividing fractions by whole numbers and zeros from their shared understanding in front of the class, “multiplying the denominator by a whole number and zero and the numerator remains the same,” to apply it to other wider situations.

From the analysis of data in the activities of multiplying fractions with zero and dividing fractions with whole numbers, it was found that the processes leading to the thinking mathematically of students in the classroom were found in all 3 phases: entry phases, attack phases, and review phases.

Discussion

The analysis results found that students had been thinking mathematically in the classroom using transformative lesson study incorporated with an open approach as follows: 1) entry phases, students understood the tasks or problem situations given by the teacher and anticipated easy ways to solve problems based on their previous knowledge, 2) attack phases, students able to solve the problems by themselves and finding a way to solve the problem, along with discussing and sharing their problem-solving ideas, and 3) review phases, students verified their solutions and applied them to wider contexts or situations. The results are consistent with the work of Nasinsroy et al. (2015), who surveyed mathematical thinking in the classroom using lesson study and open approach, and the research results found mathematical thinking, which is mathematical thinking related to the content according to the conceptual framework of Isoda & Katagiri (2012). This conceptual framework describes mathematical thinking through three practical components: mathematical concepts, mathematical methods, and mathematical attitudes, which is consistent with the research of Jai-on & Bangtho (2020), who studied students' mathematical thinking using innovative lesson study and open approach**.** The research results found that students had 7 types of mathematical thinking: 1) concept of set, students identified groups with clear conditions, 2) concept of unit, students focused on size and relationship, 3) concept of representation, students used symbols to represent their ideas, 4) concept of operation: Students specified the method of solving problems by performing operations to find the area, such as considering the square used as a unit to find the area, 5) concept of algorithm, students tried to create a formula for finding the area, 6) concept of basic properties, students used related properties to explain, and 7) concept of expressing ideas, students used to speak to show the way to solve the problem and writing to explain the idea of solving the problem. It shows that the classroom developed according to the approach of lesson study incorporated with open approach is an area where teachers can organize teaching and learning that promotes students' thinking mathematically.

Suggestion

1. Suggestions for applying research results

This research is research on the transformative lesson study incorporated with open approach, demonstrating an open approach problem-solving and self-expression-based learning approach as one way to encourage students to demonstrate thinking mathematically.

2. Suggestions for future research

Factors affecting students' thinking mathematically should be studied in terms of teachers' roles, mathematical tasks, and the sequence of mathematical tasks.

Research Originality

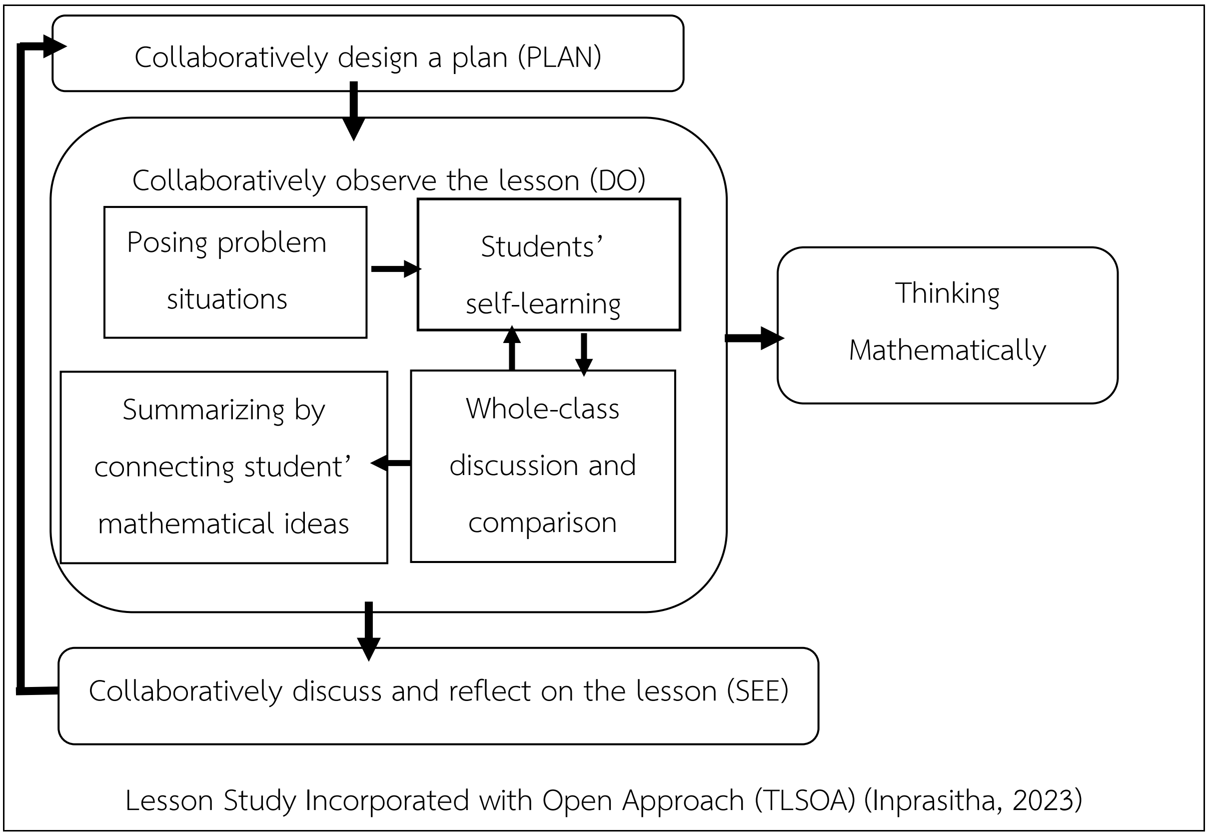


Figure 11 Thinking Mathematically in Classroom Using Transformative Lesson Study incorporated with Open Approach

Conclusion

Classrooms implementing the innovation of lesson study incorporated with open approach represent a distinctive model of learning management, utilizing a weekly cycle as a key mechanism for systematically improving instructional quality. Teachers collaborate through lesson planning, classroom observation, and reflective discussion to effectively promote students' thinking mathematically. The Open Approach plays a crucial role in developing students' abilities in analytical thinking, problem-solving, and mathematical reasoning, while also encouraging independent learning, offering diverse problem-solving strategies, and enhancing skills in mathematical communication and collaborative work. Furthermore, this innovation fosters a positive learning environment that supports the development of higher-order thinking skills and cultivates a sustainable positive attitude toward mathematics learning.

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