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Sensitivity analysis and global stability of epidemic between Thais and tourists for Covid -19

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This study employs a mathematical model to analyze and forecast the severe outbreak of SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus 2), focusing on the socio-economic ramifications within the Thai population and among foreign tourists. Specifically, the model examines the impact of the disease on various population groups, including susceptible (S), exposed (E), infected (I), quarantined (Q), and recovered (R) individuals among tourists visiting the country. The stability theory of differential equations is utilized to validate the mathematical model. This involves assessing the stability of both the disease-free equilibrium and the endemic equilibrium using the basic reproduction number. Emphasis is placed on local stability, the positivity of solutions, and the invariant regions of solutions. Additionally, a sensitivity analysis of the model is conducted. The computation of the basic reproduction number (R0) reveals that the disease-free equilibrium is locally asymptotically stable when R0 is less than 1, whereas the endemic equilibrium is locally asymptotically stable when R0 exceeds 1. Notably, both equilibriums are globally asymptotically stable under the same conditions. Through numerical simulations, the study concludes that the outcome of COVID-19 is most sensitive to reductions in transmission rates. Furthermore, the sensitivity of the model to all parameters is thoroughly considered, informing strategies for disease control through various intervention measures.

Coronavirus disease (COVID-19) is an infectious disease caused by the SARS-COV-2 virus which has spread throughout the world. The World Health Organization (WHO) has declared it a serious epidemic[1–5]. The World Health Organization (WHO) has coordinated and asked for international cooperation to stop the spread of the coronavirus -19, which the epidemic is continuously spreading. From reports around the world starting with H1N1 influenza infection, there was a clear outbreak in 2009, with a new outbreak starting on December 31, 2019. A group of cases of pneumonia of unknown ethology in Wuhan, Hubei Province in China. The outbreak was later reported to WHO in January 2022. An outbreak of a new virus[1–4] and[6–9] was identified, and the new virus was later named the 2019 novel coronavirus. By analyzing the genetics of viruses from personal illnesses, including Coronavirus Disease 2019 by WHO in February 2020 on behalf of the virus. This virus is called SARS-CoV-2 and a disease in the same family is COVID-19[3–11].

Coronaviruses are a set of viruses that cause sicknesses such as respiratory diseases or gastrointestinal diseases. Respiratory diseases can extend from the common cold to the more serious diseases e.g. Middle East Respiratory Syndrome (MERS-COV), Severe Acute Respiratory Syndrome (SARS-COV). The novel coronavirus (nCOV) is a new strain that has not been identified in humans. New diseases caused by viruses are named according to where they were first discovered, such as the Spanish flu and the Hong Kong flu. West Nile Flu, etc. The official name of the disease in this article is COVID-19, not Wuhan Flu (or Chinese flu) Coronaviruses are zoonotic[1–3] and[6–9] and[14–18], which means they are transmitted between animals and humans meaning that they are transmitted between animals and humans. It has been definite that MERS-COV was transmitted from dromedary camels to humans and SARS-COV from civet cats to humans[5–8]. While the original source of the COVID-19 virus has not been precisely determined, ongoing investigations point it to be zoonotic[15,16].

In a person infected with the COVID-19 virus, respiratory symptoms can appear almost immediately. In most cases, the person can exhibit no symptoms or mild symptoms. The symptoms of this disease are very similar to those of seasonal flu[17] and[20–23]. Laboratory and clinical signs of the COVID-19 infection can appear 2–14 days after exposure. The period between the initial exposure to the disease and the time when the symptoms first appear is called the incubation period. During the incubation of the disease, there is a probability of transmitting

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or spreading the COVID-19 virus. The clinical signs of COVID-19 infections are a fever, a cough, and a general tiredness. Other early symptoms of COVID-19 of the slight loss of taste or smell, shortness of breath or difficult breathing, muscle aches, chills, sore throat, runny nose, headache, chest pain, pink eye (conjunctivitis), nausea. While many of the other illnesses are caused by other viruses, the main cause at the early stages of the current pandemic was the COVID-19 and it seems to have targeted older people[21–25]. Older people (people over 70 years of age) often suffer from other serious chronic illnesses, such as diabetes, cardiovascular disease, chronic respiratory disease, cancer, hypertension, chronic liver disease and people who are physically inactive[1–4] and[13–16] have weaker immune symptoms and may succumb to the disease (COVID-19). The WHO reported cases of COVID-19 from January 2020 to the present. The number of cases and the number of deaths in Thailand are shown in the following in Fig. 1. The reasons for separating the populations into Thais and foreigners are that there is shortage of season labor (needed for the farming industry) and tourism is one of the top industries in Thailand. The spread of this disease to become a pandemic is due to the ease of moving from one country to another. The slowness of the great Spanish Flu was the difficulty of traveling from country to country or continent to continent.

WHO has issued guidelines for the treatment of COVID-19 in the high-risk groups (older people and people with serious chronic illness). These are the people who are the most susceptible to infection by the virus and who are in most danger of dying. The World Health Organization has issued guidelines for preventing COVID-19 infection. Not separated from each other, but able to live together with groups of people at risk. They will take care of their treatment and social care. In Thailand from January 2020 to October 2022, there were 4,689,897 confirmed cases of COVID-19 and 32,922 deaths, according to the WHO 2022 September report. A total of 142,635,014 vaccine doses have been administered.

To understand the nature and dynamics of the COVID-19 of epidemic (a pandemic in the larger scheme), mathematical modeling is used to forecast the transmission dynamics needed for controlling and planning strategies. Most epidemiological modelling studies of COVID-19 are based on WHO data. The studies on COVID-19 modelling done in Thailand[16] and[26]. The authors considered a mathematical model for the transmission dynamics of COVID-19. The data from Thailand, which considers the special features pertaining to Thailand and other neighboring countries[4–6] and[16–18], and[24]. From the information obtained, we estimate the values of unknown parameters by statistical and mathematical methods. It should be noted that the effective parameters for the spread of the virus differ from country to country and that the effective control over the rate of virus transmission from country to country will be different. In necessary to stop the spread of the virus. It was found in other studies, that the spread of COVID-19 be managed by minimizing the contact rate of infected and increasing the

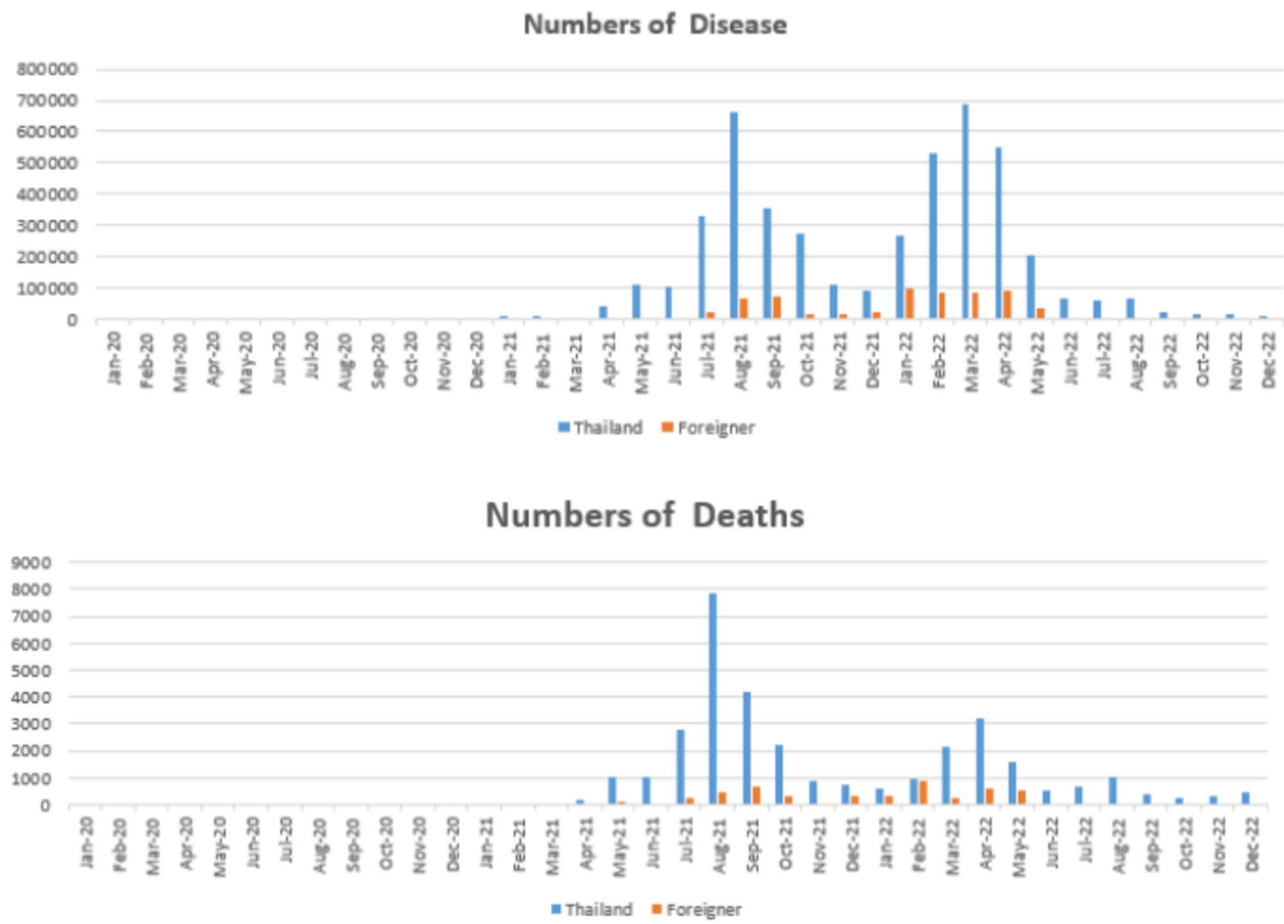

Fig. 1. Number of patients and deaths of COVID-19 cases per month around the world and in Thailand[1–4].

quarantine of exposed individuals[17–25]. This study examined a mathematical model of the COVID-19 transmission dynamics by dividing it into two groups of coronavirus transmission. The research was organized as follows: explanation of the mathematical models, formulation of the differential equations, mathematical analysis of models, followed by numerical solutions of the differential equations, summarization and discussion.

Materials and methods

In this study, a deterministic mathematical model was created. It covers the well-known SEIR epidemic model[20–26]. By adding people who are in quarantine and do not have symptoms of the disease. Symptomatic and asymptomatic infected people will be collected. The SEIQR epidemic model was thus obtained, which evolved with the following subpopulations: Susceptible (S), Exposed (E- (people not yet infectious)), Infectious (I), Quarantined (Q- (setting aside individuals who are exposed), and Recovered (R). This is because people in the Q (Quarantined) group, which represents people who are required to stay in the hospital and at home for a period of time due to the disease, are concerned about their illness. The COVID-19 pandemic in Thailand, we used a ten-dimensional SEIQR (Susceptible, Exposed, Infected, Quarantined and Recovered) containing two populations S_1, E_1, I_1, Q_1, R_1 are Thais population respectively and S_2, E_2, I_2, Q_2, R_2 are Foreign (tourist) or migrant workers) population respectively of COVID-19 transmission model[21–26].

The Recruitment term of the susceptible population in Thais and the rest if the Foreign (tourist) are given as μ and C respectively. Only exposed and infectious are considered, it is assumed that those infected show symptoms of Thais and Foreign (tourist). The natural death rate of Thais population and the natural death rate of Foreign (tourist) population is assumed to be the same across the world are given as δ_1 and δ_2. The force of infections in Thais population $\varphi_1 S_1(E_1 + I_1)$ (Transmission rate of virus between population from Thais population to Foreign (tourist) population (in Thais) and $\varphi_{12} S_1(E_2 + I_2)$ (When Foreign (tourist) are present, a susceptible Thais can also be infected by an infected or exposed Foreign (tourist) (in Thais)) are the new infections caused by other infected individuals in Thais. The force of infection in rest of Foreign (tourist) population $\varphi_2 S_2(E_2 + I_2)$ (Transmission rate of virus between population from Foreign (tourist) population to Thais population (in Foreign (tourist)) and $\varphi_{21} S_2(E_1 + I_1)$ (When Thais are present, the susceptible Thais can also be infected by an infected or exposed Thais (in Foreign (tourist)) are the new infections caused by other infected individuals in Foreign (tourist). Taking into consider the above discretion, the schematic flow diagram for COVID-19 model is appeared in Fig. 2.

The host population was divided into five compartments: S_1 number of Thais susceptible to COVID-19 infection at time t, E_1 number of Thais exposed to COVID-19 infection at time t, I_1 number of infectious Thais at time t, Q_1 number of Thais quarantined for COVID-19 at time t, R_1 number of recovered Thais at time t, S_2 number of Foreign (tourist) susceptible at time t, E_2 number of Foreign (tourist) exposed at time t, I_2 number of Foreign (tourist) infected at time t, Q_2 number of Foreign (tourist) quarantined at time t, R_2 number of Foreign (tourist) recovered at time t.

A system of ordinary differential equations can be used to model the influence of two populations on each other as a set nonlinear differential equation[20] and[24,25] as follows:

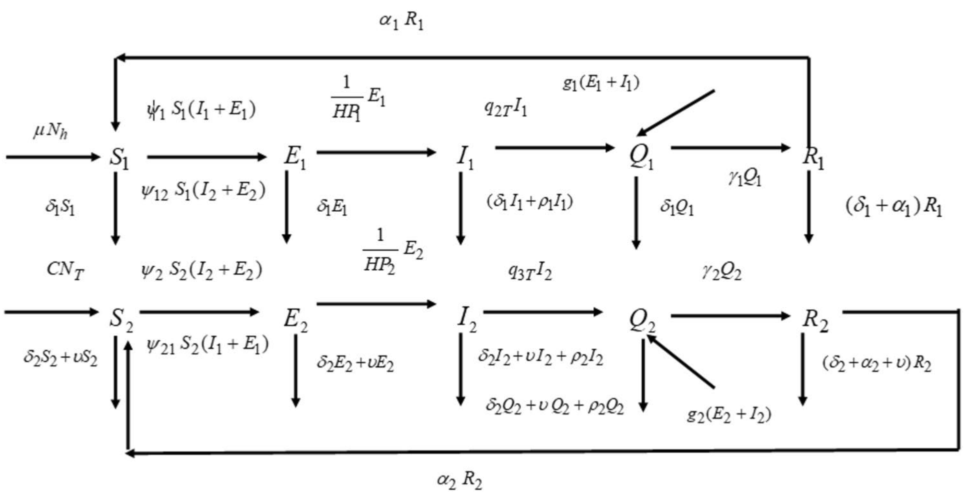

Fig. 2. The flowchart illustration the dynamics of the model.

$$
\begin{aligned}
\frac{dS_1}{dt} &= \mu N_h - \varphi_1 S_1(E_1+I_1) - \delta_1 S_1 + \alpha_1 R_1 - \varphi_{12} S_1(E_2+I_2),\\
\frac{dE_1}{dt} &= \varphi_1 S_1(E_1+I_1) + \varphi_{12} S_1(E_2+I_2) - \delta_1 E_1 - \frac{1}{IIP_1}E_1,\\
\frac{dI_1}{dt} &= \frac{1}{IIP_1}E_1 - q_{2T} I_1 - (\delta_1 I_1 + \rho_1 I_1),\\
\frac{dQ_1}{dt} &= q_{2T} I_1 - \gamma_1 Q_1 - \delta_1 Q_1 + g_1(E_1+I_1),\\
\frac{dR_1}{dt} &= \gamma_1 Q_1 - (\delta_1+\alpha_1)R_1,\\
\frac{dS_2}{dt} &= CN_T - \varphi_2 S_2(E_2+I_2) - \varphi_{21} S_2(E_1+I_1) - \delta_2 S_2 - \vartheta S_2 + \alpha_2 R_2,\\
\frac{dE_2}{dt} &= \varphi_2 S_2(E_2+I_2) + \varphi_{21} S_2(E_1+I_1) - \delta_2 E_2 - \vartheta E_2 - \frac{1}{IIP_2}E_2,\\
\frac{dI_2}{dt} &= \frac{1}{IIP_2}E_2 - q_{3T} I_2 - (\delta_2+\rho_2+\vartheta)I_2,\\
\frac{dQ_2}{dt} &= q_{3T} I_2 - \gamma_2 Q_2 - (\delta_2+\rho_2+\vartheta)Q_2 + g_2(E_2+I_2),\\
\frac{dR_2}{dt} &= \gamma_2 Q_2 - (\delta_2+\vartheta+\alpha_2)R_2.
\end{aligned}
\tag{1}
$$

with initial densities: $S_1 \geq 0, E_1 \geq 0, I_1 \geq 0, Q_1 \geq 0, R_1 \geq 0$ in Thais population and $S_2 \geq 0, E_2 \geq 0, I_2 \geq 0, Q_2 \geq 0, R_2 \geq 0$ in the Foreign (tourist) population.

All the parameters and corresponding biological meaning are defined in Table 1.

The total Thais population N_h is $S_1 + E_1 + I_1 + Q_1 + R_1$. The equations for the human compartment are the following Eq. (1) and the total Foreign (tourist) population is $N_T = S_2 + E_2 + I_2 + Q_2 + R_2$. We assume that there are constant total number of human Thais population and of Foreign (tourist) population. Therefore the rate of change for total number of human Thais population and of Foreign (tourist) population are equivalent to zero. Thus, the Recruitment term of human and death rate are equivalent. We defined the new state variables as follows: $\frac{S_1}{N_h} = S_1^{'}, \frac{E_1}{N_h} = E_1^{'}, \frac{I_1}{N_h} = I_1^{'}, \frac{Q_1}{N_h} = Q_1^{'}, \frac{R_1}{N_h} = R_1', \frac{S_2}{N_T} = S_2', \frac{E_2}{N_T} = E_2', \frac{I_2}{N_T} = I_2', \frac{Q_2}{N_T} = Q_2', \frac{R_2}{N_T} = R_2'$ Renormalizing model (1) we obtain the following:

Description	Symbol
Recruitment term of the susceptible population in Thais	μ
Total Thais population	N_h
Recruitment term of the susceptible population in Foreign (tourist)	C
Total Foreign (tourist) population	N_T
Transmission rate of virus between population from Thais population to Foreign (tourist) population (in Thai) $\varphi_1 S_1(E_1+I_1)$	φ_1
When Foreign (tourist) are present, a susceptible Thais can also be infected by an infected or exposed Foreign (tourist)(in Thais) $\varphi_{12} S_1(E_2+I_2)$	φ_{12}
Transmission rate of virus between population from Foreign (tourist) population to Thais population (in Foreign (tourist)) $\varphi_2 S_2(E_2+I_2)$	φ_2
When Thais are present, a susceptible Thais can also be infected by an infected or exposed Thais (in Foreign (tourist))$\varphi_{21} S_2(E_1+I_1)$	φ_{21}
Per capita rate of progression of Thais population from the exposed state to the infectious state	IIP_1
Per capita rate of progression of Foreign (tourist) population from the exposed state to the infectious state	IIP_2
The rate at which the exposed Thais are put into quarantine from the exposed and infected Thais	g_1
The rate at which the exposed Foreign (tourist) are put into quarantine from the exposed and infected Foreign (tourist)	g_2
The number of infected Thais that leave the quarantine period with the virus intact	q_{2T}
The number of infected Foreign (tourist) that leave the quarantine period with the virus intact	q_{3T}
Per capita recovery rate for population in Thais from the infectious state to the recovered state	γ_1
Per capita recovery rate for population in Foreign (tourist) from the infectious state to the recovered state	γ_2
Natural death rate of Thais population	δ_1
Natural death rate of Foreign (tourist) population	δ_2
Per capita rate of loss of immunity in Thais population	α_1
Per capita rate of loss of immunity in Foreign (tourist) population	α_2
Rate at which Foreign (tourist) population move out the country	ϑ
Death rate due to COVID-19 of Thais population	ρ_1
Death rate due to COVID-19 of Foreign (tourist) population	ρ_2

Table 1. The description of the state variables and parameters of the model.

$$
\begin{aligned}
\frac{dS_1'}{dt} &= \mu - \varphi_1 S_1'\left(E_1' + I_1'\right) - \delta_1 S_1' + \alpha_1 R_1' - \varphi_{12} S_1'\left(E_2' + I_2'\right),\\
\frac{dE_1'}{dt} &= \varphi_1 S_1'\left(E_1' + I_1'\right) + \varphi_{12} S_1'\left(E_2' + I_2'\right) - \delta_1 E_1' - \frac{1}{IIP_1} E_1',\\
\frac{dI_1'}{dt} &= \frac{1}{IIP_1} E_1' - q_{2T} I_1' - \left(\delta_1 I_1' + \rho_1 I_1'\right),\\
\frac{dQ_1'}{dt} &= q_{2T} I_1' - \gamma_1 Q_1' - \delta_1 Q_1' + g_1\left(E_1' + I_1'\right),\\
\frac{dR_1'}{dt} &= \gamma_1 Q_1' - (\delta_1 + \alpha_1) R_1',\\
\frac{dS_2'}{dt} &= C - \varphi_2 S_2'\left(E_2' + I_2'\right) - \varphi_{21} S_2'\left(E_1' + I_1'\right) - (\delta_2 + \vartheta) S_2' + \alpha_2 R_2',\\
\frac{dE_2'}{dt} &= \varphi_2 S_2'\left(E_2' + I_2'\right) + \varphi_{21} S_2'\left(E_1' + I_1'\right) - (\delta_2 + \vartheta) E_2' - \frac{1}{IIP_2} E_2'\\
\frac{dI_2'}{dt} &= \frac{1}{IIP_2} E_2' - q_{3T} I_2' - (\delta_2 + \rho_2 + \vartheta) I_2',\\
\frac{dQ_2'}{dt} &= q_{3T} I_2' - \gamma_2 Q_2' - (\delta_2 + \rho_2 + \vartheta) Q_2' + g_2\left(E_2' + I_2'\right),\\
\frac{dR_2'}{dt} &= \gamma_2 Q_2' - (\delta_2 + \vartheta + \alpha_2) R_2'.
\end{aligned} \tag{2}
$$

Positivity of solution

Model (2) must been found to be biologically and epidemiologically meaningful and well positioned. To do this, we needed to show that the solutions of all state variables were non-negative all the time. The following theorem[24–26] were required.

Theorem 1: The given solution $\{S_1, E_1, I_1, Q_1, R_1, S_2, E_2, I_2, Q_2, R_2\}$ of the epidemiological systems (2) with non-negative initial data when $S_1 \geq 0, E_1 \geq 0, I_1 \geq 0, Q_1 \geq 0, R_1 \geq 0$ and $S_2 \geq 0, E_2 \geq 0, I_2 \geq 0, Q_2 \geq 0$, $R_2 \geq 0$ stills non-negative for all time non-negative $t > 0$.

Proof of Theorem 1 Given the initial data $S_1(0), E_1(0), I_1(0), Q_1(0), R_1(0)$ and $S_2(0), E_2(0), I_2(0), Q_2(0), R_2(0)$ are non—negative. It is clear from the first sub-equation of the model (2) that

$$
\frac{dS_1'}{dt} = \left[\varphi_1 S_1'\left(E_1' + I_1'\right) + \delta_1 S_1' + \varphi_{12} S_1'\left(E_2' + I_2'\right)\right] \geq 0 \text{ so that}
$$

$$
\frac{d}{dt}\left[S_1' \exp(\delta_1 + \varphi_1 + \varphi_{12} \int_0^t E_1'(\zeta_1) + I_1'(\zeta_1) + E_2'(\zeta_1) + I_2'(\zeta_1)) d\zeta_1] \geq 0\right] \tag{3}
$$

Integrating (3) gives

$$
S_1'(t) \geq S_1'(0) \exp\left[-\left(\delta_1 + \varphi_1 + \varphi_{12} \int_0^t E_1'(\zeta_1) + I_1'(\zeta_1) + E_2'(\zeta_1) + I_2'(\zeta_1)\right) d\zeta_1\right] \geq 0, \forall t > 0 \tag{4}
$$

Further, one sees from the second sub-equation of the model (2) that

$$
\frac{dE_1'}{dt} = \varphi_1 S_1'\left(E_1' + I_1'\right) + \varphi_{12} S_1'\left(E_2' + I_2'\right) - \delta_1 E_1' - \frac{1}{IIP_1} E_1' \tag{5}
$$

$\frac{dE_1'}{dt}\left[\delta_1 E_1' + \frac{1}{IIP_1} E_1'\right] \geq 0$ implies $\frac{d}{dt}\left[E_1' \exp\,(\delta_1 + \frac{1}{IIP_1} \int_0^t E_1'(\zeta_1)) d\zeta_1\right] \geq 0$ which on integration yields

$$
E_1'(t) \geq E_1'(0) exp\left[-\left(\delta_1 + \frac{1}{IIP_1} \int_0^t E_1'(\zeta_1)\right) d\zeta_1\right] > 0, \forall t > 0. \tag{6}
$$

Further, one sees from the third sub-equation of the model (2) that

$$
\frac{dI_1'}{dt} = \frac{1}{IIP_1} E_1' - q_{2T} I_1' - \left(\delta_1 I_1' + \rho_1 I_1'\right)
$$

$$\frac{dI_1'}{dt}\left[q_{2T}I_1' + \delta_1 I_1' + \rho_1 I_1'\right] \geq 0 \text{ so that} \frac{dI_1'}{dt}\left[I_1' \exp q_{2T} + \delta_1 + \rho_1 \int_0^t I_1'(\zeta_1))d\zeta_1\right] \geq 0 \quad (7)$$

which upon integration yields

$$I_1'(t) \geq I_1'(0)exp\left[-\left(q_{2T} + \delta_1 + \rho_1 \int_0^t I_1'(\zeta_1)\right)d\zeta_1\right] > 0, \forall t > 0 \quad (8)$$

Further, one sees from the fourth sub-equation of the model (2) that

$$\frac{dQ_1'}{dt} = q_{2T}I_1' - \gamma_1 Q_1' - \delta_1 Q_1' + g_1\left(E_1' + I_1'\right),$$

$\frac{dQ_1'}{dt} = [\gamma_1 + \delta_1)Q_1'] \geq 0$ implies $\frac{dQ_1'}{dt}\left[Q_1' \exp(\gamma_1 + \delta_1 \int_0^t Q_1'(\zeta_1)\right))d\zeta_1] \geq 0$ which upon integration yields

$$Q_1'(t) \geq Q_1'(0) \exp\left[-(\gamma_1 + \delta_1 \int_0^t Q_1'(\zeta_1))d\zeta_1\right] \geq 0, \forall t > 0. \quad (9)$$

Further, one sees from the fifth sub-equation of the model (2) that $\frac{dR_1'}{dt} = \gamma_1 Q_1' - (\delta_1 + \alpha_1)R_1'$, $\frac{dR_1'}{dt}\left[(\delta_1 + \alpha_1)R_1'\right] \geq 0$ implies $\frac{dR_1'}{dt}\left[R_1' \exp\left(\delta_1 + \alpha_1 \int_0^t R_1'(\zeta_1)\right)d\zeta_1\right] \geq 0$ which upon integration yields.

$$R_1'(t) \geq R_1'(0)exp\left[-\left(\delta_1 + \alpha_1 \int_0^t R_1'(\zeta_1)\right)d\zeta_1\right] \geq 0, \forall t > 0 \quad (10)$$

In a similar model, it can be shown that $S_2'(t) \geq 0, E_2'(t) \geq 0, I_2' \geq 0, Q_2' \geq 0$ and $R_2'(t) \geq 0$ for all time $t > 0$. This completes the proof. It is important to note that the model (2) has be analyzed in the region β given by

$$\beta = \left\{\left(S_1, E_1, I_1, Q_1, R_1, S_2, E_2, I_2, Q_2, R_2\right) \in R_+^{10} : S_1 + E_1 + I_1 + Q_1 + R_1 + S_2 + E_2 + I_2 + Q_2 + R_2 = 1\right\}$$

Divided into two groups $S_1 + E_1 + I_1 + Q_1 + R_1 = 1$ and $S_2 + E_2 + I_2 + Q_2 + R_2 = 1$ which can easily be shown to be positively univariate according to model (2). In the following, model (2) is epidemiologically and mathematically well-positioned in β.

Theorem 2: *The solution of system (1) is possible for all if entering an invariant region.* $\Omega = \Omega_1 \times \Omega_2$, *where* $\Omega_1 = \{S_1, E_1, I_1, Q_1, R_1 \in R_+^5 : 0 < N_h(t) \leq \frac{\mu}{\delta_1}\}$ *as* $t \to \infty$, *when* $\theta = \min\{\delta_1, \delta_1 + \rho_1\}$ *and* $\Omega_2 = \left\{S_2, E_2, I_2, Q_2, R_2 \in R_+^5 : 0 < N_T(t) \leq \frac{C}{\delta_2}\right\}$ *as* $t \to \infty$, *when* $\theta_2 = \min\{\delta_2 + \vartheta, \delta_2 + \rho_2 + \vartheta\}$.

Proof of Theorem 2 The invariant region is received from the bounded situation of the system. Here, $N_h(t) = S_1(t) + E_1(t) + I_1(t) + Q_1(t) + R_1(t)$ and $N_T(t) = S_2(t) + E_2(t) + I_2(t) + Q_2(t) + R_2(t)$. It following that,

$$\begin{aligned}\frac{dN_h}{dt} &= \frac{dS_1}{dt} + \frac{dE_1}{dt} + \frac{dI_1}{dt} + \frac{dQ_1}{dt} + \frac{dR_1}{dt}\\ &= \mu - (\rho_1 + g_1)I_1 - g_1E_1 - \delta_1 N_h\\ &\leq \mu - \delta_1 N_h\end{aligned}$$

This inequality can be expressed in a general solutions as

$$N_h(\mathrm{t}) \leq \frac{\mu}{\delta_1} + \left(N_h(0) - \frac{\mu}{\delta_1}\right)e^{-\delta_1 t},$$

where $N_h(0)$ is the initial values, i.e., $N_h(t) = N_h(0)$ at $t = 0$.

In a similar model, it can be shown that $N_T(t) = S_2(t) + E_2(t) + I_2(t) + Q_2(t) + R_2(t)$ for the bounded situation of the system all time $t > 0$. Moreover, every solution for systems (1) with initial conditions in Ω remains in Ω for all $t > 0$. Therefore, the dynamics of our model will be poised in Ω.

Analysis of the model
Basic reproduction number

The next generation matrix method is used to calculate the basic reproduction number, R_0[27–36] the number of secondary infections caused by a single infected individual in a completely susceptible population (including of the local Thais and the Foreign (tourist)). The behavior of the disease in the total system defined by Eq. (2)[15–19] will be determined by R_0 which has the form

$$R_0 = \sqrt{R_{0T} R_{0F}} \tag{11}$$

where $R_{0T} = \frac{(q_{2T}+\gamma_1+\delta_1)\mu(1+IIP_1(\delta_1+\rho_1))\varphi_1}{\delta_1(1+IIP_1\delta_1)(g_1 q_{2T}+(q_{2T}+\gamma_1+\delta_1)(\delta_1+\rho_1))}$ is the basic reproduction number for Thais and $R_{0F} = \frac{\alpha_3 C(1+\alpha_2 IIP_2)\varphi_2}{\alpha_1(\alpha_2\alpha_3+g_2 q_{3T})IIP_2(\delta_2+\vartheta)}$ is the basic reproduction number for the Foreign (tourist) population only with $\alpha_1 = \delta_2 + \vartheta + \frac{1}{IIP_2}, \alpha_2 = \delta_2 + \vartheta + \rho_2, \alpha_3 = \delta_2 + \vartheta + \gamma_2 + q_{3T}$ and $\alpha_4 = \delta_2 + \vartheta + \alpha_2$, we have

Theorem 3: *To find the basic reproduction number of our proposed differential Eq. (2), using help of the next generation matrix formulas*[17–25]. *We initially define* $K = (E_1, I_1, Q_1)^T$ *and* $K_1 = (E_2, I_2, Q_2)^T$. *The model (2) is rewritten in the following form* $\frac{dy}{dt} = F(y) - V(y)$, *where* $F(y)$ *is the non-negative matrix of the newly infected (Thais and Foreign (tourist) populations) and* $V(y)$ *is the non-singular matrix for the transfers between the parts in the infective equations (Thais and Foreign (tourist) populations) (when y represents Thais populations and Foreign (tourist) populations) as follows*:

$$F(y) = \begin{bmatrix} \varphi_1 S_1\left(E_1 + I_1\right) + \varphi_{12} S_1\left(E_2 + I_2\right) \\ 0 \\ 0 \end{bmatrix} \text{ and } V(y) = \begin{bmatrix} \delta_1 E_1 + \frac{1}{IIP_1} E_1 \\ -\frac{1}{IIP_1} E_1 + \left(\delta_1 I_1 + \rho_1 I_1\right) + q_{2T} I_1 \\ -g_1\left(E_1 + I_1\right) + (\gamma_1 + \delta_1) Q_1 - q_{2T} I_1 \end{bmatrix}$$

for the Thais population.

$$F_1(y) = \begin{bmatrix} \varphi_2 S_2\left(E_2 + I_2\right) + \varphi_{21} S_2\left(E_1 + I_1\right) \\ 0 \\ 0 \end{bmatrix} \text{ and } V_1(y) = \begin{bmatrix} (\delta_2 + \vartheta) E_2 + \frac{1}{IIP_2} E_2 \\ -\frac{1}{IIP_2} E_2 + (\delta_2 + \vartheta + \rho_2) I_2 + q_{3T} I_2 \\ -g_2\left(E_2 + I_2\right) + (\gamma_2 + \delta_2 + \vartheta + \rho_2) Q_2 - q_{3T} I_2 \end{bmatrix}$$

for the Foreign (tourist) population.

The basic reproductive number (R_0) is the threshold for the stability of the disease-free equilibrium B_0. It can be calculated by $R_0 = \rho(FV^{-1})$ where, FV^{-1} is called the next generation matrix and $\rho(FV^{-1})$ is the spectral radius of the matrix FV^{-1}. Then we get reproduction number (R_0) where,

$$R_0 = \frac{\alpha_{3C(1+\alpha_2 IIP_2)\varphi_2(q_{2T}+\gamma_1+\delta_1)\mu(1+IIP_1(\delta_1+\rho_1))\varphi_1}}{\alpha_1(\alpha_2\alpha_3 + g_2 q_{3T})IIP_2(\delta_2 + \vartheta)\delta_1(1 + IIP_1\delta_1)(g_1 q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1))}. \tag{12}$$

Finally, the Routh–Hurwitz criteria is used for determining the stabilities of the model. If $R_0 > 1$, then the endemic equilibrium is local asymptotically stable, but if $R_0 < 1$, then the disease free equilibrium point is local asymptotically stable.

Equilibrium point

The standard method is used to analyze the model. The equilibrium points are found by setting the right-hand side of Eq. (2) to zero. By doing this, the equilibrium points are determined as follows[24–37].

(A) The COVID-19 free equilibrium of the Eq. (2) exists and then given by

$$B_0 = \left(S_1, E_1, I_1, Q_1, R_1, S_2, E_2, I_2, Q_2, R_2\right), B_0 = \left(\frac{\mu}{\delta_1}, 0, 0, 0, 0, \frac{C}{\delta_2}, 0, 0, 0, 0\right)$$

(B) The COVID-19 endemic equilibrium of the Eq. (2) exists with infection and then given by

$$
\begin{aligned}
&B_1 = \left(\underset{1}{\overset{*}{S}}, \underset{1}{\overset{*}{E}}, \underset{1}{\overset{*}{I}}, \underset{1}{\overset{*}{Q}}, \underset{1}{\overset{*}{R}}, \underset{2}{\overset{*}{S}}, \underset{2}{\overset{*}{E}}, \underset{2}{\overset{*}{I}}, \underset{2}{\overset{*}{Q}}, \underset{2}{\overset{*}{R}}\right)\\
&\underset{1}{\overset{*}{S}} = \frac{\underset{1}{\overset{*}{R}}\alpha_1 + \mu}{\varphi_1\left(\underset{1}{\overset{*}{E}}+\underset{1}{\overset{*}{I}}\right) + \delta_1 + \varphi_{12}\left(\underset{2}{\overset{*}{E}}+\underset{2}{\overset{*}{I}}\right)},\\
&\underset{1}{\overset{*}{E}} = \frac{IIP_1\,\underset{1}{\overset{*}{S}}\left(\varphi_1\,\underset{1}{\overset{*}{I}} + \varphi_{12}\left(\underset{2}{\overset{*}{E}}+\underset{2}{\overset{*}{I}}\right)\right)}{1 + IIP_1\left(\delta_1 - \underset{1}{\overset{*}{S}}\varphi_1\right)},\\
&\underset{1}{\overset{*}{I}} = \frac{\underset{1}{\overset{*}{E}}}{IIP_1\left(-q_{2T} + \delta_1 + \rho_1\right)},\\
&\underset{1}{\overset{*}{Q}} = \frac{g_1\left(\underset{1}{\overset{*}{E}}+\underset{1}{\overset{*}{I}}\right) - \underset{1}{\overset{*}{I}}\,q_{2T}}{\gamma_1 + \delta_1},\\
&\underset{1}{\overset{*}{R}} = \frac{\gamma_1\,\underset{1}{\overset{*}{Q}}}{\delta_1 + \alpha_1},\\
&\underset{2}{\overset{*}{S}} = \frac{C + \underset{2}{\overset{*}{R}}\alpha_2}{\varphi_2\left(\underset{2}{\overset{*}{E}}+\underset{2}{\overset{*}{I}}\right) - \varphi_{21}\left(\underset{1}{\overset{*}{E}}+\underset{1}{\overset{*}{I}}\right) + (\delta_2 + \vartheta)},\\
&\underset{2}{\overset{*}{E}} = \frac{IIP_2\,\underset{2}{\overset{*}{S}}\left(\varphi_2\,\underset{2}{\overset{*}{I}} + \varphi_{21}\left(\underset{1}{\overset{*}{E}}+\underset{1}{\overset{*}{I}}\right)\right)}{1 + IIP_2\left(\delta_2 + \vartheta - \underset{2}{\overset{*}{S}}\varphi_2\right)},\\
&\underset{2}{\overset{*}{I}} = \frac{\underset{2}{\overset{*}{E}}}{IIp_2\left(-q_{3T} + \delta_2 + \vartheta + \rho_2\right)},\\
&\underset{2}{\overset{*}{Q}} = \frac{g_2\left(\underset{2}{\overset{*}{E}}+\underset{2}{\overset{*}{I}}\right) - \underset{2}{\overset{*}{I}}\,q_{3T}}{\gamma_2 + \delta_2 + \vartheta + \rho_2},\\
&\underset{2}{\overset{*}{R}} = \frac{\gamma_2\,\underset{2}{\overset{*}{Q}}}{\delta_2 + \vartheta + \alpha_2}.
\end{aligned}
\tag{13}
$$

Local asymptotically stability of disease—free equilibrium point

Lemma 1: (The Generalized Routh–Hurwitz Criterion). Given the charactistic equation

$$\lambda^k + a_1\lambda^{k-1} + a_2\lambda^{k-2} + \cdots + a_k = 0$$

Define k matrices as follows:

$$H_1 = (a_1), H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix}, H_3 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix}, \ldots$$

$$H_J = \begin{pmatrix} a_1 & 1 & 0 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & 1 & \ldots & 0 \\ a_5 & a_4 & a_3 & a_2 & \ldots & 0 \\ a_{2j-1} & a_{2j-2} & a_{2j-3} & a_{2j-4} & \ldots & a_j \end{pmatrix} \cdots H_k = \begin{pmatrix} a_1 & 1 & 0 & \ldots & 0 \\ a_3 & a_2 & a_1 & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & & a_k \end{pmatrix}.$$

where the (l, m) term in the matrix H_j is a_{2l-m} for $0 < 2l - m < k, 1$ for $2l = m$.

0 for $0 < 2l$ or $2l < k + m$.

Then all eigenvalues have negative real parts; that is, the steady-state $\overline{N}$ is stable if and only if the determinants of all Hurwitz are positive:

$$detH_j > 0 \;\; j = (1, 2, \ldots, k).$$

When is $\overline{N} = \overline{N}_i + a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t} + \cdots + a_k e^{\lambda_k t}$.

Theorem 4: *The local stability of disease-free equilibrium point is determined from the Jacobian matrix of the model of Eq. (2) evaluated at the equilibrium points. If $R_0 > 1$, the point is stable and unstable otherwise.*[12–18], *and*[24–29], *and*[31].

Proof of Theorem 4 To determine the local stability of J_0, we evaluate the Jacobian matrix at the disease-free state to be

$$J_0 = \begin{vmatrix} \delta_1 & -\varphi_1 \underset{1}{S'} & -\varphi_1 \underset{1}{S'} & 0 & \alpha_1 & 0 & -\varphi_{12} \underset{1}{S'} & -\varphi_{12} \underset{1}{S'} & 0 & 0 \\ 0 & \theta_1 & \varphi_1 \underset{1}{S'} & 0 & 0 & 0 & \varphi_{12} \underset{1}{S'} & \varphi_{12} \underset{1}{S'} & 0 & 0 \\ 0 & \frac{1}{IIP_1} & -\left(\delta_1 + \rho_1 + q_{2T}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_1 & q_{2T} + g_1 & -(\gamma_1 + \delta_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1 & -(\delta_1 + \alpha_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varphi_{21} \underset{2}{S'} & -\varphi_{21} \underset{2}{S'} & 0 & 0 & -(\delta_2 + \vartheta) & -\varphi_2 \underset{2}{S'} & -\varphi_2 \underset{2}{S'} & 0 & \alpha_2 \\ 0 & \varphi_{21} \underset{2}{S'} & \varphi_{21} \underset{2}{S'} & 0 & 0 & 0 & \theta_2 & \varphi_2 \underset{2}{S'} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{IIP_2} & \theta_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_2 & q_{3T} + g_2 & \theta_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & \theta_5 \end{vmatrix} = 0 \tag{14}$$

where $\theta_1 = \varphi_1 \underset{1}{S} - \left(\delta_1 + \frac{1}{IIP_1}\right)$, $\theta_2 = \varphi_2 \underset{2}{S} - \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right)$, $\theta_3 = -(\delta_2 + \vartheta + q_{3T} + \rho_2)$, $\theta_4 = (\delta_2 + \vartheta + \gamma_2 + \rho_2)$ and $\theta_5 = -(\delta_2 + \vartheta + \alpha_2)$.

The eigenvalues of the J_0 are obtained by solving $Det(J_0 - \lambda I) = 0$. We obtain the characteristic equation, where λ is an eigenvalue of the matrix J_0. The, root of the model (2) i.e., eigenvalue of the matrix J_0 are

$$(\lambda + \delta_2 + \vartheta + \alpha_2)(\lambda + \delta_1 + \alpha_1)(\lambda + \delta_1)(\lambda^7 + A_1\lambda^6 + A_2\lambda^5 + A_3\lambda^4 + A_4\lambda^3 + A_5\lambda^2 + A_6\lambda + A_7 = 0 \tag{15}$$

The three eigenvalues from Eq. (15) were $\lambda_1 = -\delta_2 - \vartheta - \alpha_2$, $\lambda_2 = -\delta_1 - \alpha_1$ and $\lambda_3 = -\delta_1$ and all of them must have negative real parts. For the other seven eigenvalues, we examine the stability of disease-free equilibrium state by using the Routh Hurwitz principle (R-H criterion) to show that all eigenvalues given by Eq. (14) has a negative real part, i.e., coefficients of the seventh order the polynomial appearing in Eq. (14) satisfies all R-H conditions when $A_1, A_2, A_3, A_4, A_5, A_6, A_7 > 0$(The coefficients appearing in Eq. 15 from the Routh Hurwitz condition are plotted on the graph by the x axis being the coefficient A_2. and the Y-axis is the coefficient of $A_1, A_2, A_3, A_4, A_5, A_6$ and A_7 obtained by finding determinants from size nxn, parameter values from Table 1 by the use the Mathematica program.) This is displayed for $R_0 < 1$, disease-free equilibrium point will be stable as showed in Fig. 3.

When

$$\beta_1 = A_1A_2 - A_3, \;\; \beta_2 = -A_3^2 - A_1^2A_4 + A_1(A_2A_3 + A_5),$$

$$\beta_3 = -A_4\left(-A_1A_2A_3 + A_3^2 + A_1^2A_4\right) + \left(-A_1A_2^3 + A_2A_3 + 2A_1A_4\right)A_5 - A_5^2,$$

$$\begin{aligned}\beta_4 = &-A_5\left(A_4\left(-A_1A_2A_3 + A_3^2 + A_1^2A_4\right) + \left(-A_2A_3 + A_1\left(A_2^3 - 2A_4\right)\right)A_5 + A_5^2\right) \\ &+ \left(A_3^2 - A_1A_3(A_2A_3 + 2A_5) + A_1^2(A_3A_4 + A_2A_5)\right)A_6,\end{aligned}$$

$$\begin{aligned}\beta_5 = &A_6(-A_5\left(A_4\left(-A_1A_2A_3 + A_3^2 + A_1^2A_4\right) + \left(-A_2A_3 + A_1\left(A_2^3 - 2A_4\right)\right)A_5 + A_5^2\right) \\ &+ \left(A_3^3 - A_1A_3(A_2A_3 + 2A_5) + A_1^2(A_3A_4 + A_2A_5)\right)A_6,\end{aligned}$$

$$\begin{aligned}\beta_6 = &A_7(A_6\left(-A_5\left(A_4\left(-A_1A_2A_3 + A_3^2 + A_1^2A_4\right) + \left(-A_2A_3 + A_1\left(A_2^3 - 2A_4\right)\right)A_5 + A_5^2\right)\right. \\ &+\left(A_3^3 - A_1A_3(A_2A_3 + 3A_5) + A_1^2(A_3A_4 + 2A_2A_5)\right)A_6 - A_1^3A_6^2\Big) + \Big(A_1A_4\left(-A_2A_3A_4 + A_2^3A_5 - 2A_4A_5\right) \\ &+A_4\left(A_3^2A_4 - A_2A_3A_5 + A_5^2\right) + A_3(-A_2A_3 + 3A_5)A_6 + A_1\left(2A_2^3A_3 + A_3A_4 - A_2A_5\right)A_6 + A_1^2\left(A_4^3 - 3A_2A_4A_6 + 3A_6^2\right)\Big)A_7 \\ &- \left(-A_2^2A_3 + 2A_3A_4 + A_2A_5 + A_1\left(A_2^3 - A_2A_4 + 3A_6\right)\right)A_7^2 + A_7^3),\end{aligned}$$

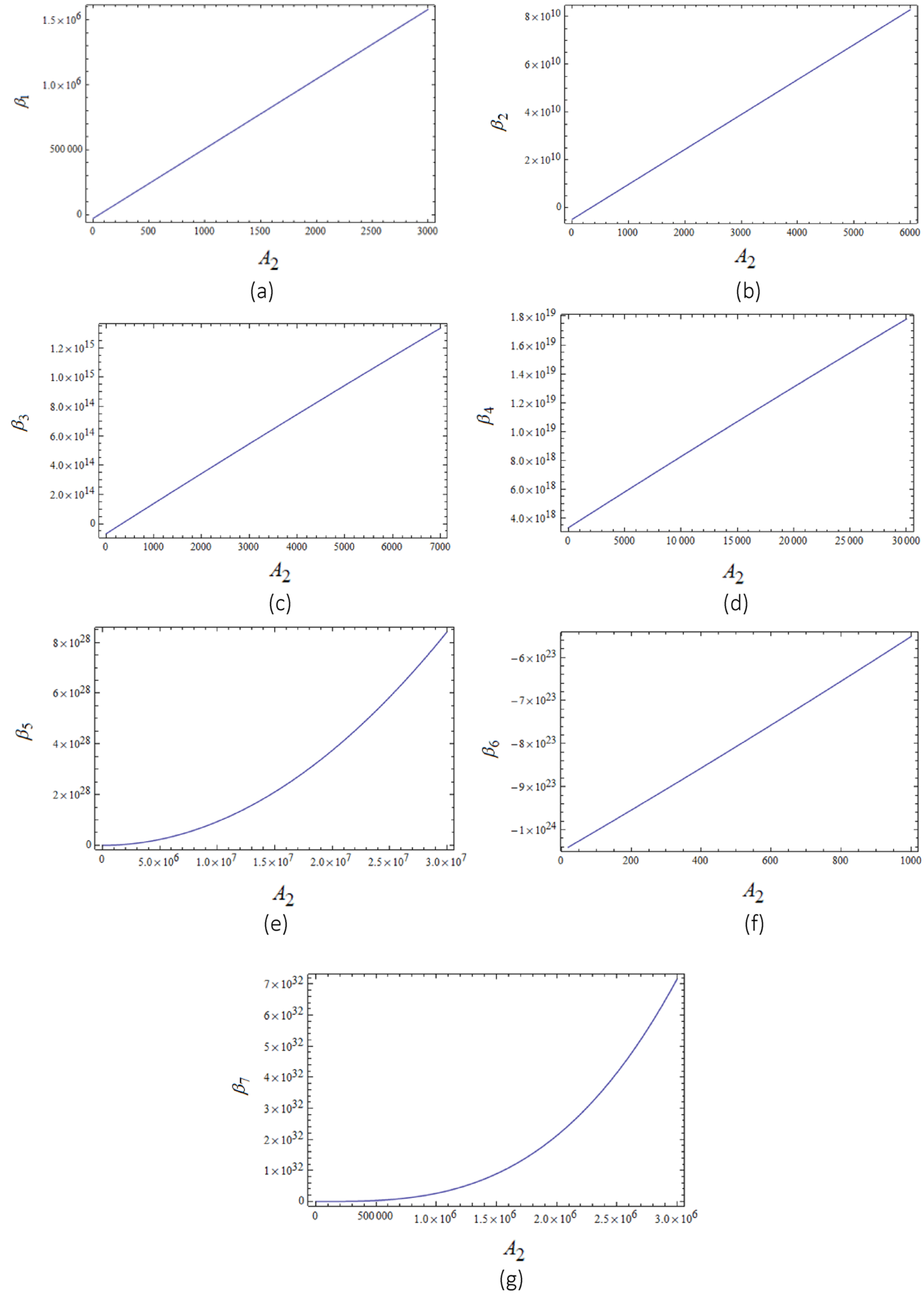

Fig. 3. The parameter areas for disease free equilibrium state which satisfies the Routh-Hurwitz criteria with the value of parameters: respectively, for with $(\lambda^7 + A_1\lambda^6 + A_2\lambda^5 + A_3\lambda^4 + A_4\lambda^3 + A_5\lambda^2 + A_6\lambda + A_7 = 0.$

a n d $\beta_7 = A_8(A_7\big(A_6\big(-A_5\big(A_3^2A_4 - A_2A_3A_5 + A_5^2 + A_3^3A_6\big) + \big(A_4\big(A_3^2A_4 - A_2A_3A_5 + A_5^2\big) + A_3(-2A_2A_3 + 3A_5)A_6\big)A_7$
$+\big(A_2^2A_3 - 2A_3A_4 - A_2A_5\big)A_7^2 + A_7^3\big) + \big(A_5^4 - A_3A_5^2(A_2A_5 + 4A_7) - \big(A_3^3(A_5A_6 + 2A_4A_7) + A_3^2A_4A_5^2$
$+3A_2A_5A_7 + 2A_7^2\big)\big)A_8 + A_3^4A_8^2 + A_1\big(A_7\big(-A_6\big(A_5\big(-A_2A_3A_4 + A_2^2 - 2A_4A_5\big)\big)A_7 - \big(A_3^2 - 3A_2A_4 + 3A_6\big)A_7^2\big)$
$+\Big(-2A_4A_5^3 + 3A_3A_5^2A_6 + 4A_3A_4A_5A_7 + A_3^2A_6A_7 + 4A_5A_7^2 + A_2^2\Big(A_5^3 - 3A_3A_5A_7\Big) + A_2(A_3A_5(-A_4A_5 + A_3A_6)+$
$+\Big(2A_3A_4^2 - A_2A_4A_6\Big) + A_2A_3A_6 + 5A_5A_6\Big)A_7 + 3(-A_2 + A_4)A_7^2\Big)A_8 + A_3^2A_4 + 3A_2A_3A_5 + 2A_5^2 + 4A_3A_7\Big)A_8^2\Big).$

Local asymptotically stability of disease endemic equilibrium point

Theorem 5: The disease endemic equilibrium point is set from the Jacobian matrix of the system of Eq. (2) evaluated at every equilibrium point. If $R_0 < 1$ the state is stable and unstable otherwise[12–18], and[24–26], and[30,31].

Proof of Theorem 5 The Jacobian matrix of the model (2) at pandemic equilibrium point is

$$J_1 = \begin{vmatrix} \eta_1 & -\varphi_1 S_1^{'} & -\varphi_1 S_1^{'} & 0 & \alpha_1 & 0 & -\varphi_{12} S_1^{'} & -\varphi_{12} S_1^{'} & 0 & 0 \\ \eta_2 & \eta_3 & \varphi_1 S_1^{'} & 0 & 0 & 0 & \varphi_{12} S_1^{'} & \varphi_{12} S_1^{'} & 0 & 0 \\ 0 & \frac{1}{IIP_1} & -(q_{2T} + \delta_1 + \rho_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_1 & g_1 + q_{2T} & -(\gamma_1 + \delta_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_1 & -(\alpha_1 + \delta_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & -\varphi_{21} S_2^{'} & -\varphi_{21} S_2^{'} & 0 & 0 & \eta_4 & -\varphi_2 S_2^{'} & -\varphi_2 S_2^{'} & 0 & \alpha_2 \\ 0 & \varphi_{21} S_2^{'} & \varphi_{21} S_2^{'} & 0 & 0 & \eta_5 & \eta_6 & \varphi_2 S_2^{'} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{IIP_2} & \eta_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_2 & g_2 + q_{3T} & \eta_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & \eta_9 \end{vmatrix} = 0 \quad (16)$$

Where $\eta_1 = -\varphi_1\left(\underset{1}{E} + \underset{1}{I}\right) - \varphi_{12}\left(\underset{2}{E} + \underset{2}{I}\right) - \delta_1$, $\eta_2 = \varphi_1\left(\underset{1}{E} + \underset{1}{I}\right) + \varphi_{12}\left(\underset{2}{E} + \underset{2}{I}\right)$
$\eta_3 = \varphi_1 \underset{1}{S} - \left(\delta_1 + \frac{1}{IIP_1}\right)$, $\eta_4 = -\varphi_2\left(\underset{2}{E} + \underset{2}{I}\right) - \varphi_{21}\left(\underset{1}{E} + \underset{1}{I}\right) - (\delta_2 + \vartheta)$, $\eta_5 = -\varphi_2\left(\underset{2}{E} + \underset{2}{I}\right) + \varphi_{21} \underset{2}{S}\left(\underset{1}{E} + \underset{1}{I}\right)$,
$\eta_6 = -\varphi_2 \underset{2}{S} - (\delta_2 + \vartheta + \frac{1}{IIP_2}$, $\eta_7 = -\left(q_{3T} + \delta_2 + \rho_2 + \vartheta\right)$, $\eta_8 = -(\gamma_2 + \delta_2 + \rho_2 + \vartheta)$ and $\eta_9 = (\delta_2 + \vartheta + \alpha_2)$.

The endemic equilibrium point $(B1)$ exists and is positive if $R_0 > 1$. The eigenvalues of J_1 are obtained by solving $Det(J_1 - \lambda I) = 0$. The characteristic equation is as follows; we obtain the characteristic equation $\lambda^{10} + W_1\lambda^9 + W_2\lambda^8 + W_3\lambda^7 + W_4\lambda^6 + W_5\lambda^5 + W_6\lambda^4 + W_7\lambda^3 + W_8\lambda^2 + W_9\lambda + W_{10} = 0$ where λ are eigenvalues of the matrix J_1. To consider the local stability of the endemic equilibrium state, we check the stability of endemic equilibrium state by using the Routh-Hurwitz criteria required for all the eigenvalues defined by Eq. (16) to have negative real parts. We find that the Routh-Hurwitz conditions for the above the all the eigenvalues of the above 10th order polynomial to have negative real parts when $W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10} > 0$ (The coefficients appearing in Eq. (15) from the Routh Hurwitz condition are plotted on a graph by the x axis being the coefficient W_2 and the Y-axis is the coefficient of $W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}$, obtained by finding determinants from size nxn, parameter values from Table 1 by the use the Mathematica program.) This is displayed for $R_0 > 1$, endemic equilibrium point will be stable as showed in Fig. 4.

$\lambda^{10} + W_1\lambda^9 + W_2\lambda^8 + W_3\lambda^7 + W_4\lambda^6 + W_5\lambda^5 + W_6\lambda^4 + W_7\lambda^3 + W_8\lambda^2 + W_9\lambda + W_{10} = 0$. When

$$\kappa_1 = W_1W_2 - W_3, \kappa_2 = -W_3^2 - W_1^2W_4 + W_1(W_3W_2 + W_5), \kappa_3 = -W_4\left(-W_1W_2W_3 + W_3^2 + W_1^2W_4\right) + \left(-W_1W_2^3 + W_2W_3 + 2W_1W_4\right)W_5 - W_5^2,$$

$$\kappa_4 = -W_5\left(W_4\left(-W_1W_2W_3 + W_3^2 + W_1^2W_4\right) + \left(-W_2W_3 + W_1\left(W_2^3 - 2W_4\right)\right)W_5 + W_5^2\right) + (W_3^2 - W_1W_3(W_2W_3 + 2W_5) + W_1^2(W_3W_4 + W_2W_5))W_6,$$

$$\kappa_5 = W_6(-W_5\left(W_4\left(-W_1W_2W_3 + W_3^2 - W_1^2W_4\right) + \left(-W_2W_3 + W_1\left(W_2^3 - 2W_4\right)\right)W_5 + W_5^2\right) + \left(W_3^3 - W_1W_3(W_2W_3 + 2W_5) + W_1^2(W_3W_4 + W_2W_5)\right)W_6),$$

$$\begin{aligned}\kappa_6 = &W_7(W_6\left(-W_5\left(W_4\left(-W_1W_2W_3 + W_3^2 + W_1^2W_4\right) + \left(-W_2W_3 + W_1\left(W_2^3 - 2W_4\right)\right)W_5 + W_5^2\right)\right.\\ &+\left.\left(W_3^3 - W_1W_3(W_2W_3 + 3W_5) + W_1^2(W_3W_4 + 2W_2W_5)\right)W_6 - W_1^3W_6^2\right)\\ &+ \left(W_1W_4\left(-W_2W_3W_4 + W_2^3W_5 - W_2W_5\right) + W_4\left(W_3^2W_4 - W_2W_3W_5 + W_5^2\right) + W_3(-W_2W_3 + 3W_5)W_6\right.\\ &+\left.W_1\left(2W_2^3W_3 + W_3W_4 - W_2W_5\right)W_6 + W_1^2\left(W_4^3 - 3W_2W_4W_6 + 3W_6^2\right)\right)\\ &W_7 - \left(-W_2^2W_3 + 2W_3W_4 + W_2W_5 + W_1\left(W_2^3 - W_2W_4 + 3W_6\right)\right)W_7^2 + W_7^3),\end{aligned}$$

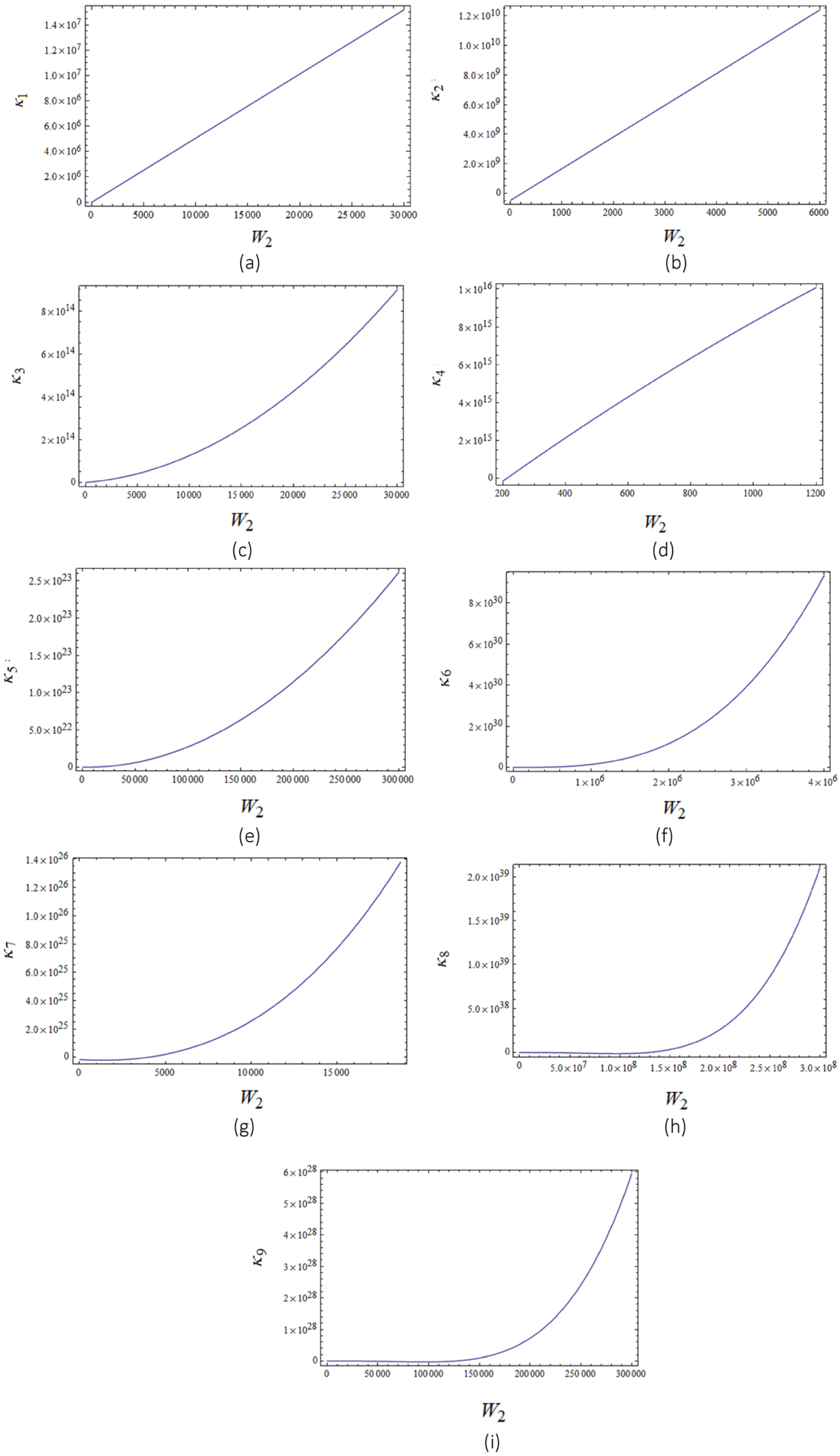

Fig. 4. The parameter areas for endemic equilibrium point which satisfies the Routh-Hurwitz criteria with the value of parameters: respectively, for with.

Parameter	Description	Value/range (Units)
δ_1	Natural death rate of Thais population	0.0000365
δ_2	Natural death rate of Foreign (tourist) population	0.000033
α_1	Per capita rate of loss of immunity in Thais population	0.045
α_2	Per capita rate of loss of immunity in Foreign (tourist) population	0.067
IIP_1	Per capita rate of progression of Thais population from the exposed state to the infectious state	0.13–0.785 Estimation[26–32]
IIP_2	Per capita rate of progression of Foreign (tourist) population from the exposed state to the infectious state	0.13–0.785 Estimation[27–30]
g_1	The rate at which the exposed Thais are put into quarantine from the exposed and infected Thais	0.341–0.854 Estimation[24–30]
g_2	The rate at which the exposed Foreign (tourist) are put into quarantine from the exposed and infected Foreign (tourist)	0.341–0.854 Estimation[23–29]
q_{2T}	The number of infected Thais that leave the quarantine period with the virus intact	0.08–0.099 Estimation[22–27]
q_{3T}	The number of infected Foreign (tourist) that leave the quarantine period with the virus intact	0.08–0.099 Estimation[26–32]
γ_1	Per capita recovery rate for population in Thais from the infectious state to the recovered state	0.0035–0.097
γ_2	Per capita recovery rate for population in Foreign (tourist) from the infectious state to the recovered state	0.0035–0.097
ρ_1	Death rate due to COVID-19 of Thais population	0.000067–0.003
ρ_2	Death rate due to COVID-19 of Foreign (tourist) population	0.000067–0.003
φ_1	Transmission rate of virus between population in Thais population	0.03125
φ_{12}	When Foreign are present, a susceptible Thais can also be infected by an infected or exposed Foreign (in Thais)	0.239–0.988
φ_2	Transmission rate of virus between population in Foreign (tourist) population	0.04167
φ_{21}	When Thais are present, a susceptible Thais can also be infected by an infected or exposed Thais (in Foreign (tourist)	0.239–0.988
ϑ	Rate at which Foreign (tourist) population move out the country	0.0078–0.5

Table 2. Values of the parameter of the model (2) on COVID-19 transmissions.

$$\begin{aligned}
\kappa_7 =& W_8 (W_7 (W_6 (-W_5 (W_3^2 W_4 - W_2 W_3 W_5 + W_5^2 + W_3^3 W_6) + W_4 (W_3^2 W_4 - W_2 W_3 W_5 + W_5^2) + W_3 (-2W_2 W_3 + 3W_5) W_6) W_7 \\
&+ (W_2^2 W_3 - 2W_3 W_4 - W_2 W_5) W_7^2 + W_7^3) + (W_5^4 - W_3 W_5^2 (W_2 W_5 + 4W_7) - W_3^3 (W_5 W_6 + 2W_4 W_7) + W_3^2 W_4 W_5^2 + 3W_2 W_5 W_7 + 2W_7^2)) W_8 \\
&+ W_3^4 W_8^2 + W_1^4 W_8^3 + W_1 (W_7 (-W_6 (W_5 (-W_2 W_3 W_4 + W_2^2 - 2W_4 W_5)) W_7 - (W_2^3 - 3W_2 W_4 + 3W_6) W_7^2) \\
&+ (-2W_4 W_5^3 + 3W_3 W_5^2 W_6 + 4W_3 W_4 W_5 W_7 + W_3^2 W_6 W_7 + 4W_5 W_7^2 + W_2^2 (W_5^3 - 3W_3 W_5 W_7) \\
&+ W_2 (W_3 W_5 (-W_4 W_5 + W_3 W_6) + (2W_3^2 W_4 + W_5^2) W_7 - 5W_3 W_7^2)) W_8 - W_3^2 (W_2 W_3 + 4W_5) W_8^2) \\
&+ (W_1^2 (W_7 (-W_4 W_5 W_6 + W_4^3 W_7 + W_6^2 (2W_2 W_5 + 3W_7) + W_4 W_6 (W_3 W_6 - 3W_2 W_7)) - (W_5 (-W_4^2 W_5 + W_3 W_4 W_6 + 2W_2 W_5 W_6) \\
&+ 2W_3 W_4^2 - W_2 W_4 W_6 + W_2 W_3 W_6 + 5W_5 W_6) W_7 + 3(-W_2 + W_4) W_7^2) W_8 + W_3^2 W_4 + 3W_2 W_3 W_5 + 2W_5^2 + 4W_3 W_7) W_8^2)),
\end{aligned}$$

$$\begin{aligned}
\kappa_8 =& W_1^5 W_{10}^4 - W_{10}^3 W_3^5 + W_{10}^2 (W_5^5 - W_3 W_5^3 (W_2 W_5 + 5W_7)) + W_3^4 (2W_6 W_7 + W_5 W_8 + 3W_4 W_9) + W_3^2 W_5 (W_4 W_5^2 + 4W_2 W_5 W_7 + 5W_7^2 + 5W_5 W_9) \\
&- W_3^3 (W_5^2 W_6 + 3W_4 W_5 W_7 + 2W_2 W_7^2 + 5W_7 W_9)) - W_{10} (W_7^5 - W_5 W_7^3 (W_2 W_7 + 5W_9) + W_5^4 (W_7 W_8 + 2W_3 W_9 + W_3^4 W_8 (W_7 W_8 + 3W_6 W_9) \\
&+ W_5^2 W_7 (W_4 W_7^2 + 4W_2 W_7 W_9 + 5W_9^2) - W_5^3 (W_6 W_7^2 + 3W_4 W_7 W_9 + 2W_2 W_9^2) + W_3^3 (W_7 (W_6^2 W_7 - W_5 W_6 W_8 - 2W_4 W_7 W_8) \\
&- (2W_5 W_6^2 - W_4 W_6 W_7 + W_4 W_5 W_8 + 5W_2 W_7 W_8) W_9 - 3 (-W_4^2 + W_2 W_6 + W_8) W_9) + W_3^2 (W_7 (W_4^2 W_7^2 + W_4 W_5 (-W_6 W_7 + W_5 W_8) \\
&+ W_7 (-2W_2 W_6 W_7 + 3W_2 W_5 W_8 + 2W_7 W_8)) + (-3W_4^2 W_5 W_7 + W_6 (4W_2 W_5 - 3W_7) + W_4 (2W_5^2 W_6 + W_2 W_7^2) + W_5 (W_2 W_5 + 7W_7) W_8) W_9 \\
&+ (-5W_2 W_4 W_5 + 7W_5 W_6 + 4W_3^2 W_7 - 7W_4 W_7) W_9 + 3W_2 W_9^2) + W_3 (W_7 (W_7 (W_2 W_5^2 W_6 + W_5 (-2W_5^3 W_6 + 3W_4 W_5^2 W_7 + W_7^2) \\
&- W_5 (6W_5 W_6 W_7 + 6W_4 W_7^2 + W_5^2 W_8)) W_9 + ((2W_2^3 + 3W_4) W_5 - 7W_2 W_5 W_7 + 5W_7^2) W_9 - 5W_5 W_9^3)) + W_9 (W_3^4 W_8^3 \\
&+ W_8 (-W_5^3 W_6 W_7 + W_4 W_5^2 W_7^2 - W_2 W_5 W_7^3 + W_7^4 + W_5^4 W_8) + (-W_6 W_7^3 \\
&+ W_5 W_7^2 (W_6 W_2 - 4W_8) + W_5^2 W_7 (-W_4 W_6 + 3W_2 W_8) + W_5^3 (W_6^2 - 2W_4 W_8)) W_9))
\end{aligned}$$

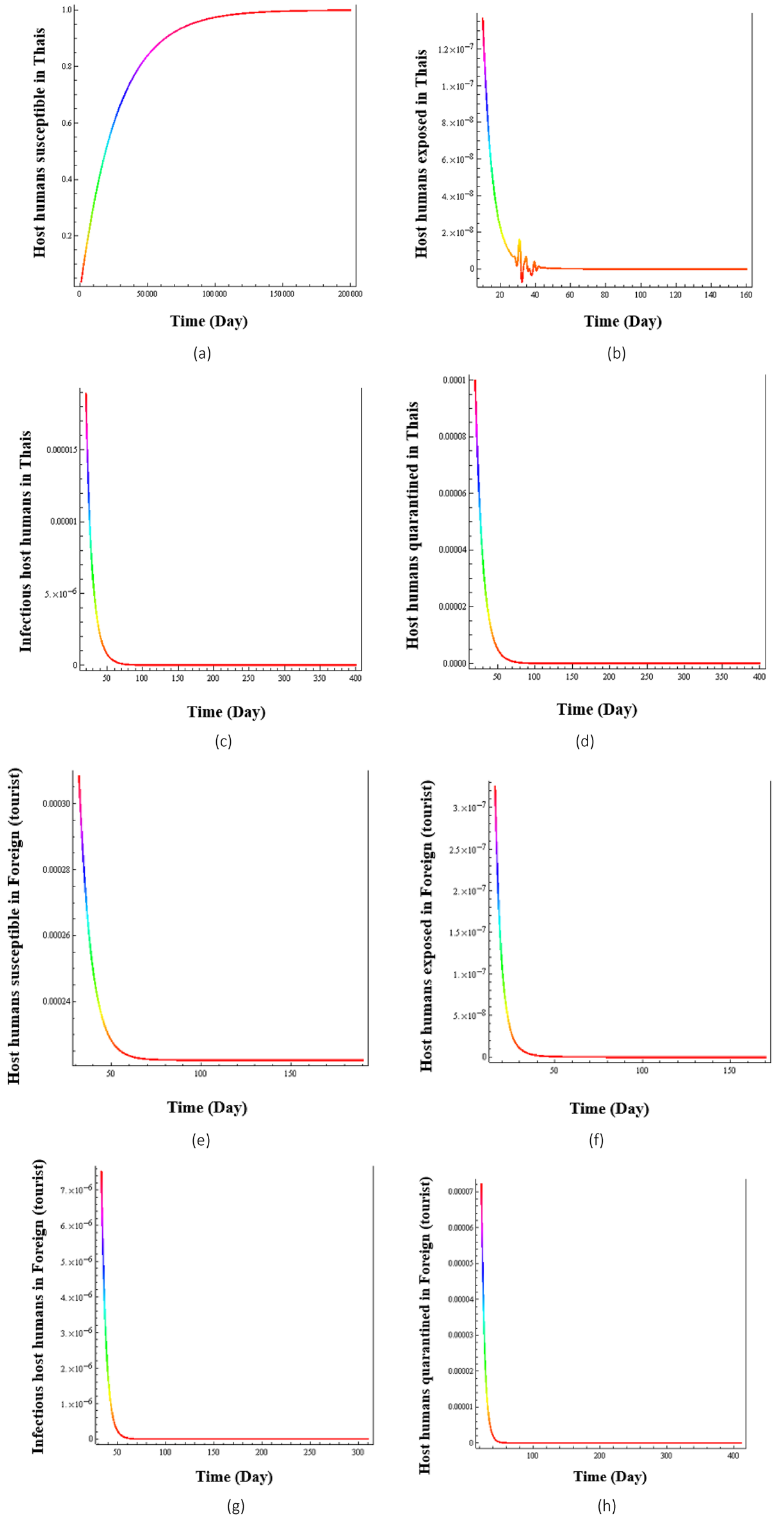

Fig. 5. Numerical simulations of each population for the disease free state. We will see that the solutions converge to the disease free state when it satisfy.

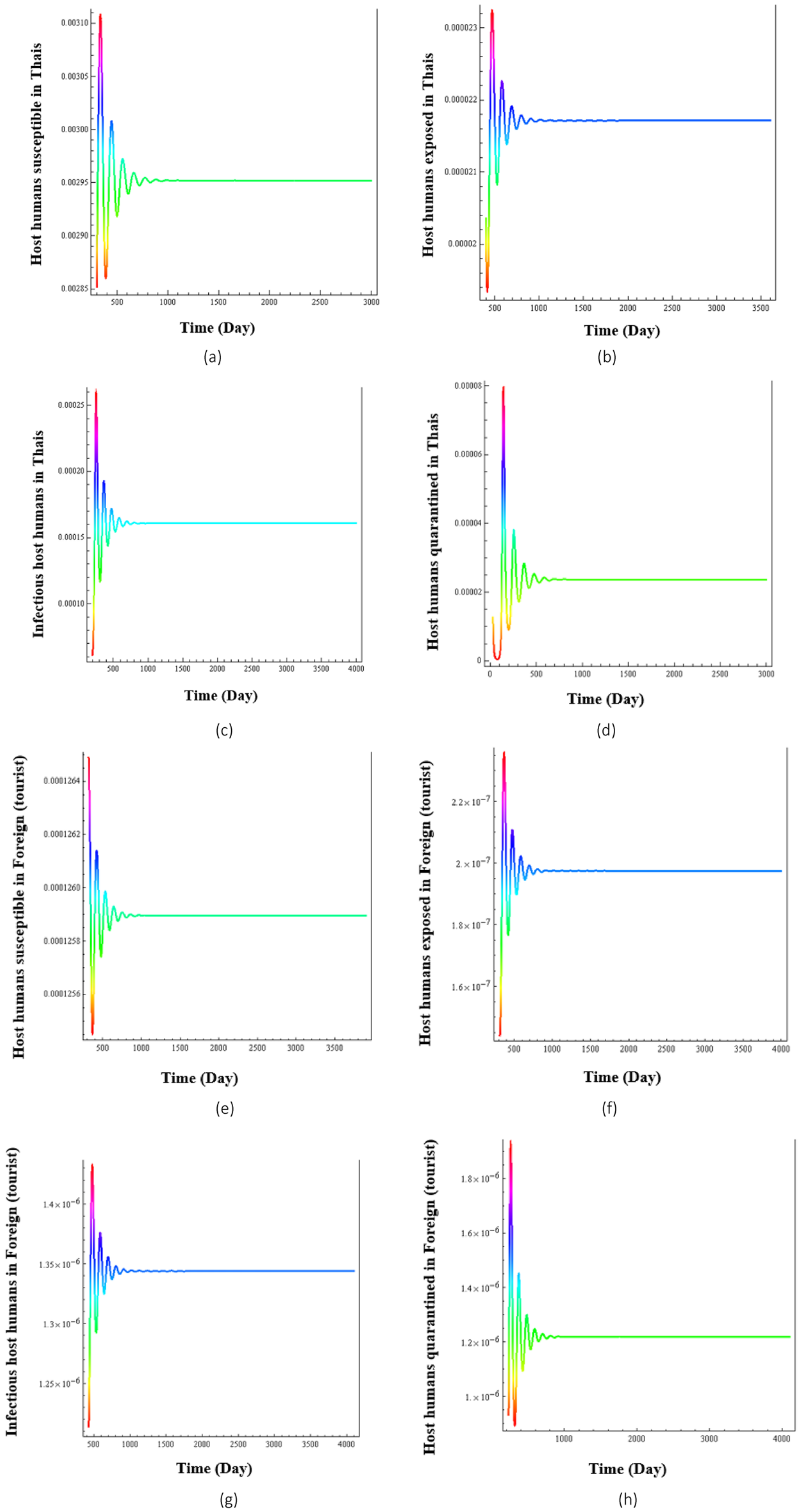

Fig. 6. Numerical simulations of each population for the disease state.

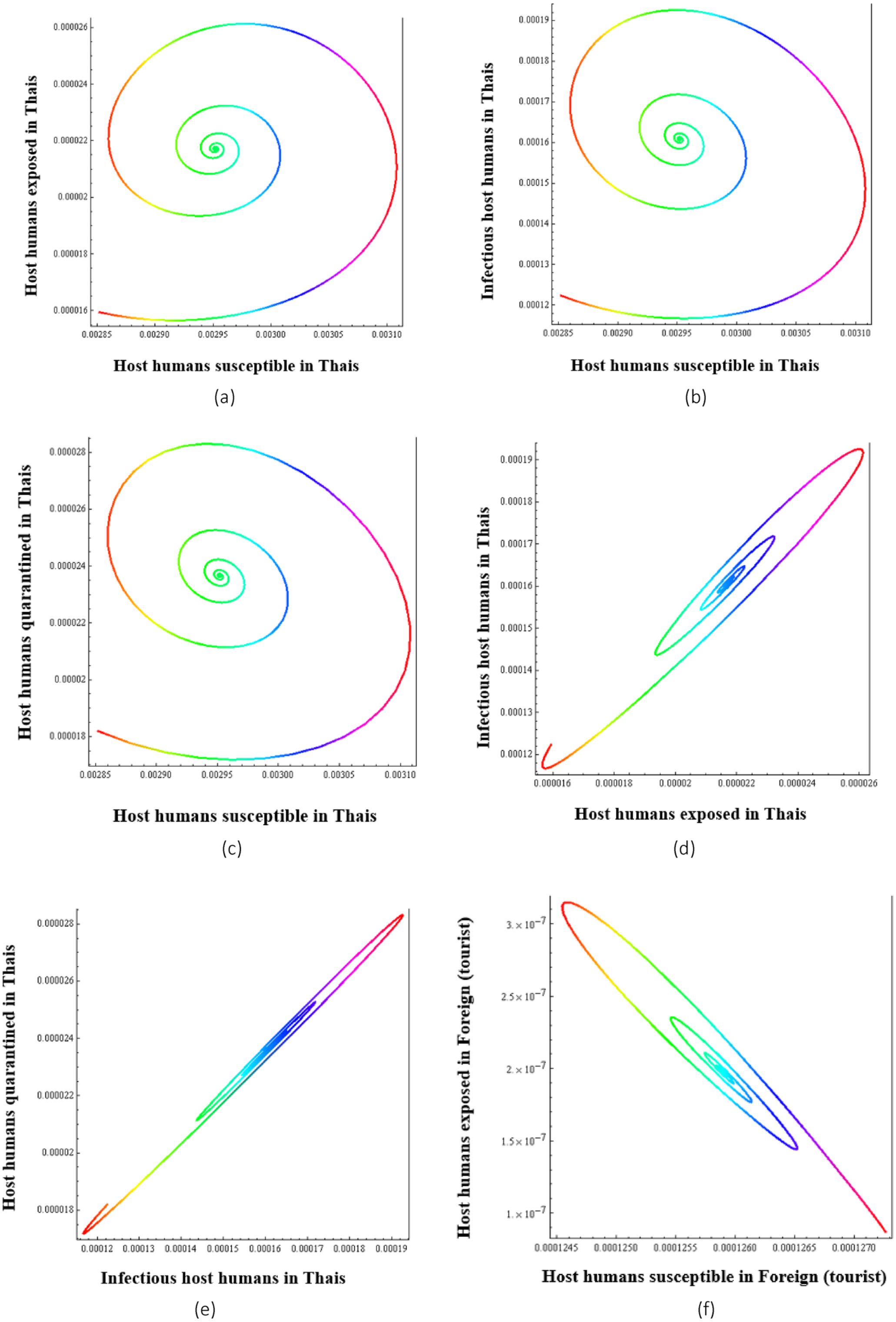

Fig. 7. The trajectories of the numerical projected onto the 2D (**a**)(S_1, E_1), (**b**) (S_1, I_1), (**c**) (S_1, Q_1), (**d**) (E_1, I_1), (**e**) (I_1, Q_1), (**f**) (S_2, E_2), (**g**) (S_2, Q_2), (**h**) (I_2, Q_2), (**i**) (E_1, E_2), (**j**) (I_1, I_2) and (**k**) (Q_1, Q_2). planes when there was no vertical transmission and equilibrium state the endemic state.

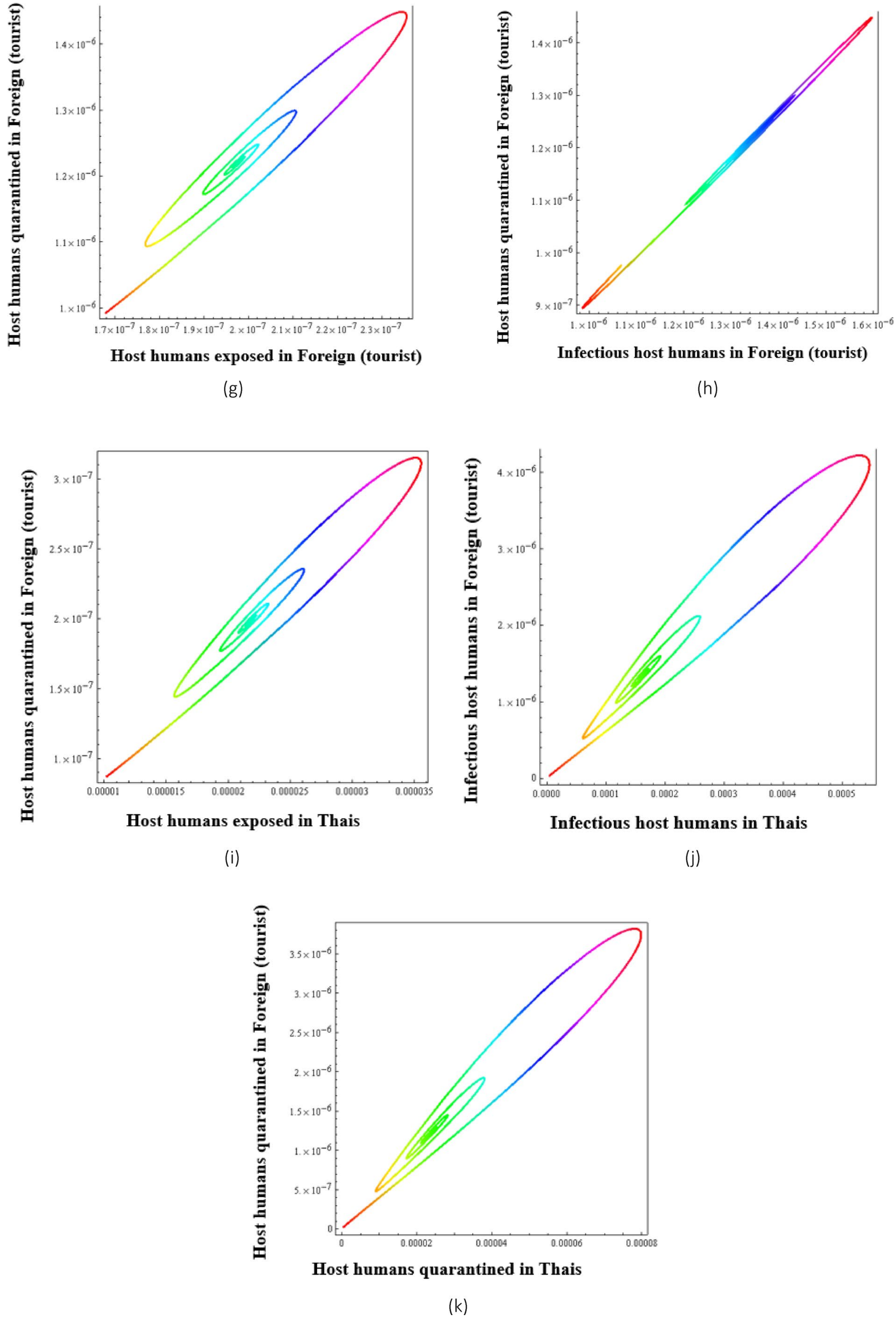

Fig. 7. (continued)

, and

$$\begin{aligned}
\kappa_9 =& W_1^4(W_8^4W_9 - W_{10}^3(2W_5W_6 + 3W_4W_7 + W_3W_8 + 4W_2W_9) - W_{10}W_8^2(W_7W_8 + 4W_6W_9) + W_{10}^2(3W_6W_7W_8 \\
&+ 2W_6^2W_9 + W_8(W_5W_8 + 4W_4W_9))) + W_1^3(W_9(W_8(-W_6^3W_7 + W_6(W_5W_6 + 3W_4W_7)W_8 - (2W_4W_5 + W_3W_6 + 3W_2W_7W_8^2) \\
&+ (W_6^4 - 4W_4W_6^2W_8 + 2(W_4^2 + 2W_2W_6)W_8^2 - 4W_8^3)W_9) + W_{10}(W_7(W_6^3W_7 - W_6(W_5W_6 + 3W_4W_7)W_8 + (2W_4W_5 + W_3W_6 + 3W_2W_7)W_8^2) \\
&+ (W_6^2 - (-2W_5W_6 + W_4W_7) + (4W_4W_5W_6 - 5W_4^2W_7 + W_6(3W_3W_6 + 2W_2W_7))W_8 \\
&+ (W_3W_4 + 2W_2W_5 + 7W_7)W_8^2)W_9 + 4w(W_6(W_4^2 - W_2W_6) + 2(-W_2W_4 + W_6)W_8)W_9^2) + W_{10}^3(W_3^2W_6 + W_5(2W_2W_5 + 5W_7) + W_3(3W_4W_5 + 4W_2W_7 + 5W_9)) - W_{10}^2(3W_7^3(-W_4^2 \\
&+ W_2W_6 + W_8) - W_5^2(W_6^2 - 2W_4W_8) + (-5W_2W_4 + 7W_6)W_7W_9 + 2(-3W_2^2 + 2W_4)W_9^2 + W_5(-W_4W_6W_7 + W_3W_6W_8 + 5W_2W_7W_8 + 3W_4^2W_9 - 2W_2W_6W_9 + 6W_8W_9) + W_3(2W_6^2W_7 \\
&+ W_4W_7W_8 + 5W_4W_6W_9 + W_2W_8W_9))) + W_1(W_{10}^3W_3^3(W_2W_3 + 5W_5) + W_{10}(W_7(W_7(2W_4^2W_5W_7 - W_4W_6(2W_5^2 + W_3W_7) + 3W_6(W_3W_5W_6 + W_7^2)) - (2W_4W_5^2 \\
&+ 3W_3W_5^2W_6 + 4W_3W_4W_5W_7 + W_3^2W_6W_7 + 4W_5W_7^2)W_8 + 4W_3^2W_5W_8^2) \\
&+ (-6W_4^2W_5^2W_7 - 3W_5W_6(2W_3W_5W_6 + 3W_7^2) + (11W_3^2W_5W_6 + 3W_5^2W_7 + 13W_3W_7^2)W_8 \\
&+ W_3^2W_8^2 + W_4(4W_5^3W_6 + 5W_3W_5W_6W_7 - 2W_7^3 - W_3(2W_5^2 + 5W_3W_7)W_8))W_9 + (W_3^2W_4W_6 + 4W_5(2W_5W_6 + W_4W_7) + W_3(6W_4^2W_5 + W_6W_7 - 2W_5W_8))W_9^2 \\
&+ (-7W_3W_4 + 5W_7)W_9^3 + W_2^3(W_7^4 - 4W_5W_7^2W_9 + 2(W_5^2 + 2W_3W_7)W_9^2) + W_2^2(-W_7(W_5^2W_6W_7 + 2W_3W_5^2W_7 + 4W_3W_5W_6W_7 + W_3W_4W_7^2 + W_7^3 + W_3(W_5^2 \\
&- 5W_3W_7)W_8)W_9 - (5W_3W_4W_5 + 3W_3^2W_6 + 3W_5W_7)W_9^2 + 7W_3W_9^3) + W_2(W_7(W_7(W_3^2W_6^2 + W_7(W_5W_6 - 3W_4W_7) + W_3W_4(-W_5W_6 + W_4W_7)) - (W_3W_5(-W_4W_5 + W_3W_6) \\
&+ (2W_3^2W_4 + W_5^2)W_7 - 5W_3W_7^2)W_8 + W_3^3W_8^2) + (W_5W_7(-2W_5W_6 + 11W_4W_7) + W_3(2W_4W_5^2W_6 - 3W_4^2W_5W_7 - 5W_4W_7) \\
&+ W_3^2(3W_4^3 - 8W_8))W_9^2 - 6W_5W_9^3)) + W_9(W_8(-2W_4^3W_5W_7^3 + W_3^2W_8(W_6W_7 - 4W_5W_8) + 3W_3W_6W_5(-W_6W_7 + W_5W_8) \\
&+ W_7^2(-3W_6W_7 + 4W_5W_8) + W_4(2W_5^2W_6W_7 + W_3W_6W_7^2 - 2W_5^3W_8 + 4W_3W_5W_7W_8)) + (W_6(2W_4^2W_5W_7 - W_4W_6(2W_5^2 \\
&+ W_3W_7) + 3W_6(W_3W_5W_6 + W_7^2)) + (4W_4^2W_5^2 + W_6(-W_3^2W_6 + 2W_5W_7) + 2W_4(-4W_3W_5W_6 + W_7^2))W_8)))).
\end{aligned}$$

Numerical results

Numerical simulations of the impact of the strategies to control the spread of coronavirus disease 19 (COVID-19) in the Thais population when there are Foreign (tourist) also present. Numerical values of various parameters and data points needed for the numerical calculations in Table 2. The Data collected were from the official website of the Ministry of Public Health and World Health Organization (WHO)[1–6] and[24–30]. Using the numerical values in Table 2, we obtained the time evolutions of a susceptible Thais individual, an exposed Thais, an infectious Thais, a quarantined Thais, a recovered Thais, a susceptible Foreigner (tourist), an exposed Foreigner (tourist), an infectious Foreigner (tourist), a quarantined Foreigner (tourist), and a recovered Foreigner (tourist). The values of the parameters were first chosen to lead to R_0 to be less than one so the equilibrium state will be disease free State (0.80893). The time evolutions of the ten states were plotted in Figs. 5. Next, we change the values of the parameters so that the value of R_0 will be greater than one, meaning that the equilibrium state will be the endemic state (9.4175). In Fig. 6, we see the evolution of the ten categories of individuals (susceptible Thais, exposed Thais, infectious Thais, quarantined Thais, recovered Thais, susceptible Foreign (tourist), exposed Foreign (tourist), infectious Foreign (tourist), quarantined Foreign (tourist), recovered Foreign (tourist)) converge to their epidemic equilibrium values (0.002951, 0.0000217, 0.0001608, 0.0000236, 0.000125, 0.0000001974, 0.00000134, 0.000001218).

The behavior's of the endemic, we has plotted the 2-D trajectories of the following thirteen pairs (Thais susceptible-Thais exposed), (Thais susceptible-Thais infectious), (Thais susceptible-Thais quarantined), (Thais exposed-Thais infectious), (Thais exposed-Thais quarantined), (Thais infectious-Thais quarantined), (susceptible Foreign (tourist) -exposed Foreigner), (susceptible Foreign (tourist) -infectious Foreign (tourist)), (exposed Foreign (tourist) -quarantined Foreign (tourist)), (infectious Foreign (tourist) -quarantined Foreign (tourist)), (infectious Thais-exposed Foreign (tourist)), (infectious Thais-infectious Foreign (tourist)) and (quarantined Thais-quarantined Foreign (tourist). These 2D trajectories are shown in Fig. 7. We can see that all the trajectories converge to a central point (the equilibrium pot).

Global Stability of disease free equilibrium for model

The solutions to Eq. (2) were asymptotically stable locally in section "Analysis of the model". We have now proved that the two equilibrium points are asymptotically stable globally through the following theorem.

Theorem 6: *If* $R_0 \leq 1$, *then the disease—free equilibrium* E^* *is globally asymptotically stable, by*

$$\varphi_1 = \frac{\delta_1 + \rho_1}{S_1^*} \quad \text{and} \quad \varphi_2 = \frac{\delta_2 + \rho_2 + \vartheta}{S_2^*} \qquad (*)$$

Proof of Theorem 6 The Lyapunov function may be constructed for the model (1) through the use of the function

$$P(t) = (S_1 - S_1^* \ln S_1) + E_1 + I_1 + Q_1 + R_1 + (S_2 - S_2^* \ln S_2) + E_2 + I_2 + Q_2 + R_2 \tag{17}$$

Differentiating with respect to time yields.

$$\dot{P}(t) = \left(S_1 - \frac{S_1^*}{S_1}\right) + +\dot{E}_1 + \dot{I}_1 + \dot{Q}_1 + \dot{R}_1 + \left(S_2 - \frac{S_2^*}{S_2}\right) + \dot{E}_2 + \dot{I}_2 + \dot{Q}_2 + \dot{R}_2$$

$$=\left((\mu N_h-\varphi_1 S_1(E_1+I_1)-\delta_1 S_1+\alpha_1 R_1-\varphi_{12}S_1(E_2+I_2))\left(S_1-\frac{S_1^*}{S_1}\right)\right)+(\varphi_1 S_1(E_1+I_1)+\varphi_{12}S_1(E_2+I_2)-\delta_1 E_1$$
$$-\frac{1}{IIP_1}E_1)+(\frac{1}{IIP_1}E_1-q_{2T}I_1-(\delta_1 I_1+\rho_1 I_1))+(q_{2T}I_1-\gamma_1 Q_1-\delta_1 Q_1+g_1(E_1+I_1))+(\gamma_1 Q_1-(\delta_1+\alpha_1)R_1)+((CN_T$$
$$-\varphi_2 S_2(E_2+I_2)-\varphi_{21}S_2(E_1+I_1)-\delta_2 S_2-\vartheta S_2+\alpha_2 R_2)\left(S_2-\frac{S_2^*}{S_2}\right))+(\varphi_2 S_2(E_2+I_2)+\varphi_{21}S_2(E_1+I_1)-\delta_2 E_2-\vartheta E_2-\frac{1}{IIP_2}E_2)$$
$$+(\frac{1}{IIP_2}E_2-q_{3T}I_2-(\delta_2+\rho_2+\vartheta)I_2)+(q_{3T}I_2-\gamma_2 Q_2-(\delta_2+\rho_2+\vartheta)Q_2+g_2(E_2+I_2))+(\gamma_2 Q_2-(\delta_2+\vartheta+\alpha_2)R_2)$$

$$=\mu N_h\left(1-\frac{S_1^*}{S_1}\right)+CN_T\left(1-\frac{S_2^*}{S_2}\right)-\delta_1 S_1^*\left(1-\frac{S_1}{S_1^*}\right)-\delta_2 S_2^*\left(1-\frac{S_2^*}{S_2}\right)-\vartheta S_2^*$$
$$\left(1-\frac{S_2}{S_2^*}\right)+\varphi_1 I_1 S_1^*+\varphi_1 S_1^* Q_1-\alpha_1 R_1\left(\frac{S_1^*}{S_1}\right)-\delta_1 R_1-\delta_1 E_1-(\delta_1+\rho_1)I_1-\delta_1 Q_1$$
$$+\varphi_2 I_2 S_2^*+\varphi_2 Q_2 S_2^*-\alpha_2 R_2\left(\frac{S_2^*}{S_2}\right)-(\delta_2+\vartheta)R_2-(\delta_2+\vartheta)E_2-(\delta_2+\rho_2+\vartheta)I_2-(\delta_2+\vartheta)Q_2$$

$$=\mu N_h\left(1-\frac{S_1^*}{S_1}\right)+CN_T\left(1-\frac{S_2^*}{S_2}\right)-\delta_1 S_1^*\left(1-\frac{S_1}{S_1^*}\right)-\delta_2 S_2^*\left(1-\frac{S_2^*}{S_2}\right)-\vartheta S_2^*\left(1-\frac{S_2}{S_2^*}\right)$$
$$-\delta_1 E_1-\left(\delta_1+\rho_1-\varphi_1 S_1^*\right)I_1-\left(\delta_1-\varphi_1 S_1^*)Q_1-R_1(\alpha_1 R_1\left(\frac{S_1^*}{S_1}\right)+\delta_1\right) \tag{17a}$$
$$-(\delta_2+\vartheta)E_2-(\delta_2+\rho_2+\vartheta-\varphi_2 S_2^*)I_2-(\delta_2+\vartheta-\varphi_2 S_2^*)Q_2-(\alpha_2\left(\frac{S_2^*}{S_2}\right)+(\delta_2+\vartheta))R_2$$

Using the condition (17), Eq. (17a) may be rewrite as,

$$\dot{P}(t)=\mu N_h\left(1-\frac{S_1^*}{S_1}\right)+CN_T\left(1-\frac{S_2^*}{S_2}\right)-\delta_1 S_1^*\left(1-\frac{S_1}{S_1^*}\right)-\delta_2 S_2^*\left(1-\frac{S_2^*}{S_2}\right)$$
$$-\vartheta S_2^*\left(1-\frac{S_2}{S_2^*}\right)-\delta_1 E_1-(\delta_1+\rho_1-\varphi_1 S_1^*)I_1-\left(\delta_1-\varphi_1 S_1^*)Q_1-R_1(\alpha_1 R_1\left(\frac{S_1^*}{S_1}\right)+\delta_1\right)-(\delta_2+\vartheta)E_2$$
$$-(\delta_2+\rho_2+\vartheta-\varphi_2 S_2^*)I_2-(\delta_2+\vartheta-\varphi_2 S_2^*)Q_2-(\alpha_2\left(\frac{S_2^*}{S_2}\right)+(\delta_2+\vartheta))R_2$$

Substitute with Eq. (17) we obtain

$$\dot{P}(t)=\mu N_h\left(1-\frac{S_1^*}{S_1}\right)+\delta_1 S_1^*\left(1-\frac{S_1}{S_1^*}\right)+CN_T\left(1-\frac{S_2^*}{S_2}\right)$$
$$+\delta_2 S_2^*\left(1-\frac{S_2^*}{S_2}\right)+\vartheta S_2^*\left(1-\frac{S_2}{S_2^*}\right)-\delta_1 E_1-(\delta_2+\vartheta)E_2-\delta_1 R_1-(\delta_2+\vartheta)R_2$$

Note that on Ω, we have $S_1^*=\frac{\mu N_h}{\delta_1}$ and $S_2^*=\frac{CN_T}{\delta_2+\vartheta}$ with this in mind, Eq. (17) becomes

$$\dot{P}(t)=\mu N_h\left(1-\frac{S_1^*}{S_1}\right)+\delta_1\left(\frac{\mu N_h}{\delta_1}\right)\left(1-\frac{S_1}{S_1^*}\right)+CN_T\left(1-\frac{S_2^*}{S_2}\right)+(\delta_2+\vartheta)\left(\frac{CN_T}{\delta_2+\vartheta}\right)\left(1-\frac{S_2^*}{S_2}\right)$$
$$-\delta_1 E_1-(\delta_2+\vartheta)E_2-\delta_1 R_1-(\delta_2+\vartheta)R_2$$

$$=\mu N_h\left(2-\frac{S_1^*}{S_1}-\frac{S_1}{S_1^*}\right)+CN_T\left(2-\frac{S_2^*}{S_2}-\frac{S_2}{S_2^*}\right)-\delta_1 E_1-(\delta_2+\vartheta)E_2-\delta_1 R_1-(\delta_2+\vartheta)R_2$$

$$\dot{P}(t)=-\mu N_h\left(\frac{(S_1^*-S_1)^2}{S_1^* S_1}\right)-CN_T\left(\frac{(S_2^*-S_2)^2}{S_2^* S_2}\right)-\delta_1 E_1-(\delta_2+\vartheta)E_2-\delta_1 R_1-(\delta_2+\vartheta)R_2\le 0$$

Hence, $\dot{P}(t)\le 0$. By using LaSalle' s (1976)[31–37] extension to Lyapunov method, the limit of each solution is contained in the largest invariant set for which $S_1=S_1^*, E_1=0, I_1=0, Q_1=0, R_1=0, S_2=S_2^*, E_2=0, I_2=0, Q_2=0$ and $R_2=0$ which is the singleton $\{E_0\}$. This means that the disease—free equilibrium $E^*=\left\{S_1^*, \overset{*}{E}_1, \overset{*}{I}_1, \overset{*}{Q}_1, \overset{*}{R}_1, \overset{*}{S}_2, \overset{*}{E}_2, \overset{*}{I}_2, \overset{*}{Q}_2, \overset{*}{R}_2\right\}$ is globally asymptotically stable on Ω. This achieves the proof of the theorem.

Theorem 7: *If $R_0>1$, then the positive endemic equilibrium state of system (1) exists and is globally asymptotically stable on Ω, by assuming that $\varphi_1=\frac{\delta_1+\rho_1}{S_1^*}, \rho_1=\gamma_1, \varphi_2=\frac{\delta_2+\rho_2+\vartheta}{S_2^*}$,*

$$\rho_2 = \gamma_2, \mu N_h = S_1^{**}(\delta_1) \text{ and } CN_T = S_2^{**}(\delta_2 + \vartheta). \tag{18}$$

Proof of Theorem 7 We construct the Lypunov function from the model as follows

$$\dot{\omega}(t) = (S_1 - S_1^{**} \ln S_1) + E_1 + I_1 + Q_1 + R_1 + (S_2 - S_2^{**} \ln S_2) + E_2 + I_2 + Q_2 + R_2,$$

$$\dot{\omega}(t) = \dot{S}_1\left(1 - \frac{S_1^{**}}{S_1}\right) + \dot{E}_1 + \dot{I}_1 + \dot{Q}_1 + \dot{R}_1 + \dot{S}_2\left(1 - \frac{S_2^{**}}{S_2}\right) + \dot{E}_2 + \dot{I}_2 + \dot{Q}_2 + \dot{R}_2,$$

$$\begin{aligned}
=&\big(\mu N_h - \varphi_1 S_1(E_1 + I_1) - \delta_1 S_1 - \varphi_{12} S_1(E_2 + I_2) + \alpha_1(S_1 + E_1 + I_1 + Q_1)\big)\left(1 - \frac{S_1^{**}}{S_1}\right)\\
&+ (\varphi_1 S_1(E_1 + I_1) + \varphi_{12} S_1(E_2 + I_2) - \delta_1 E_1 - \frac{1}{IIP_1} E_1)\\
&+ (\frac{1}{IIP_1} E_1 - q_{2T} I_1 - (\delta_1 I_1 + \rho_1 I_1) + q_{2T} I_1) + (q_{2T} I_1 - (\gamma_1 + \delta_1) Q_1 + g_1(E_1 + I_1))+)\\
&+ ((CN_T - \varphi_2 S_2(E_2 + I_2) - \varphi_{21} S_2(E_1 + I_1) - (\delta_2 + \vartheta) S_2 + \alpha_2((S_2 + E_2 + I_2 + Q_2))\left(1 - \frac{S_2^{**}}{S_2}\right)\\
&+ (\varphi_2 S_2(E_2 + I_2) + \varphi_{21} S_2(E_1 + I_1) - (\delta_2 + \vartheta) E_2 - \frac{1}{IIP_2} E_2)\\
&+ (\frac{1}{IIP_2} E_2 + q_{3T} I_2 - (\delta_2 + \rho_2 + \vartheta) I_2) + (q_{3T} I_2 - (\delta_2 + \rho_2 + \vartheta + \gamma_2) Q_2 + g_2(E_2 + I_2)),
\end{aligned}$$

$$\begin{aligned}
=&\mu N_h\left(1 - \frac{S_1^{**}}{S_1}\right) + \varphi_1(E_1 + I_1) S_1^{**} + \varphi_{12}(E_2 + I_2) S_1^{**} - \delta_1 S_1 + \delta_1 S_1^{**} + \alpha_1 - \alpha_1\left(\frac{S_1^{**}}{S_1}\right) - \alpha_1 S_1 + \alpha_1 S_1^{**}\\
&- \alpha_1 E_1 + \alpha_1 E_1\left(\frac{S_1^{**}}{S_1}\right) - \alpha_1 I_1 + \alpha_1 I_1\left(\frac{S_1^{**}}{S_1}\right) - \alpha_1 Q_1 + \alpha_1 Q_1\left(\frac{S_1^{**}}{S_1}\right)\\
&- \left(\delta_1 + \frac{1}{IIP_1}\right) E_1 - \delta_1 I_1 - \rho_1 I_1 - \gamma_1 Q_1 - \delta_1 Q_1 - q_{2T} I_1\\
&+ CN_T\left(1 - \frac{S_2^{**}}{S_2}\right) + \varphi_2(E_2 + I_2) S_2^{**} + \varphi_{21}(E_1 + I_1) S_2^{**} - \delta_2 S_2 + \delta_2 S_2^{**} - \vartheta S_2 + \vartheta S_2^{**} + \alpha_2 - \alpha_2\left(\frac{S_2^{**}}{S_2}\right)\\
&- \alpha_2 S_2 + \alpha_2 S_2^{**} - \alpha_2 E_2 + \alpha_2 E_2\left(\frac{S_2^{**}}{S_2}\right) - \alpha_2 I_2 + \alpha_2 I_2\left(\frac{S_2^{**}}{S_2}\right) - \alpha_2 Q_2 + \alpha_2 Q_2\left(\frac{S_2^{**}}{S_2}\right)\\
&- \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right) E_2 - (\delta_2 + \rho_2 + \vartheta) I_2 + q_{3T} I_2 - (\delta_2 + \rho_2 + \vartheta + \gamma_2) Q_2,
\end{aligned}$$

Parameters	Sensitivity indices	Parameters	Sensitivity indices
δ_1	Negative	δ_2	Negative
α_1	Positive	α_2	Positive
IIP_1	Negative	IIP_2	Positive
g_1	Positive	g_2	Positive
q_{2T}	Negative	q_{3T}	Positive
γ_1	Negative	γ_2	Negative
ρ_1	Negative	ρ_2	Negative
μ	Positive	C	Positive
φ_1	Positive	φ_{12}	Positive
φ_2	Positive	φ_{21}	Positive
ϑ	Negative		

Table 3. The sensitivity index (S.I).

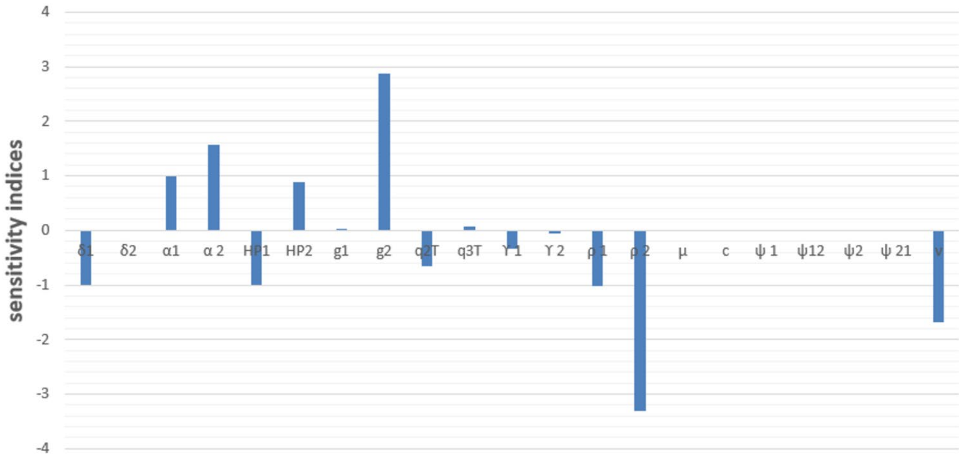

Fig. 8. A bar chart showing the measurement of the sensitivity indices with various parameters of model (2) and the reference values as indicated in Table 3.

$$
\begin{aligned}
=&\mu N_h\left(1-\frac{S_1^{**}}{S_1}\right)+\delta_1 S_1^{**}\left(1-\frac{S_1}{S_1^{**}}\right)+\alpha_1(1-S_1-E_1-I_1-Q_1)\left(1-\frac{S_1^{**}}{S_1}\right)-\delta_1 E_1\\
&-\frac{1}{IIP_1}E_1-(\delta_1+\rho_1-\varphi_1 S_1^{**})I_1-(\gamma_1+\delta_1-\varphi_1 S_1^{**})Q_1\\
&+CN_T\left(1-\frac{S_2^{**}}{S_2}\right)+(\delta_2+\vartheta)S_2^{**}\left(1-\frac{S_2}{S_2^{**}}\right)\\
&+\alpha_2(1-S_2-E_2-I_2-Q_2)\left(1-\frac{S_2^{**}}{S_2}\right)\\
&-\left(\delta_2+\vartheta+\frac{1}{IIP_2}\right)E_2-(q_{3T}+\delta_2+\rho_2+\vartheta)I_2+q_{3T}-\varphi_2 S_2^{**})I_2\\
&-(\delta_2+\rho_2+\vartheta+\gamma_2-\varphi_2 S_2^{**})Q_2
\end{aligned}
\tag{18a}
$$

Since

$$
S_1^{**}=\frac{R_1^*\alpha_1+\mu}{\varphi_1(E_1^*+I_1^*)+\delta_1+\varphi_{12}(E_2^*+I_2^*)}\quad\text{and}\quad S_2^{**}=\frac{C+R_2^*\alpha_2}{\varphi_2(E_2^*+I_2^*)-\varphi_{21}(E_1^*+I_1^*)+(\delta_2+\vartheta)}
$$

$$
\begin{aligned}
\dot{\omega}(t)=&\mu N_h\left(1-\frac{S_1^{**}}{S_1}\right)+\delta_1\left(\frac{R_1^*\alpha_1+\mu}{\varphi_1(E_1^*+I_1^*)+\delta_1+\varphi_{12}(E_2^*+I_2^*)}\right)\left(1-\frac{S_1}{S_1^{**}}\right)+\alpha_1(1-S_1-E_1-I_1-Q_1)\left(1-\frac{S_1^{**}}{S_1}\right)\\
&-\delta_1 E_1-\frac{1}{IIP_1}E_1-(\delta_1+\rho_1-\varphi_1 S_1^{**})I_1-(\gamma_1+\delta_1-\varphi_1 S_1^{**})Q_1\\
&+CN_T\left(1-\frac{S_2^{**}}{S_2}\right)+(\delta_2+\vartheta)(\frac{C+R_2^*\alpha_2}{\varphi_2(E_2^*+I_2^*)+\varphi_{21}(E_1^*+I_1^*)+(\delta_2+\vartheta)})\left(1-\frac{S_2}{S_2^{**}}\right)\\
&+\alpha_2(1-S_2-E_2-I_2-Q_2)\left(1-\frac{S_2^{**}}{S_2}\right)-\left(\delta_2+\vartheta+\frac{1}{IIP_2}\right)E_2\\
&-(q_{3T}+\delta_2+\rho_2+\vartheta)I_2+q_{3T}-\varphi_2 S_2^{**})I_2-(\delta_2+\rho_2+\vartheta+\gamma_2-\varphi_2 S_2^{**})Q_2
\end{aligned}
$$

Substituting the relations in Eqs. (18), we have

$$
\mu N_h=\delta_1(\frac{R_1^*\alpha_1+\mu}{\varphi_1(E_1^*+I_1^*)+\delta_1+\varphi_{12}(E_2^*+I_2^*)}),
$$

$$CN_T = (\delta_2 + \vartheta)(\frac{C + R_2^*\alpha_2}{\varphi_2(E_2^* + I_2^*) - \varphi_{21}(E_1^* + I_1^*) + (\delta_2 + \vartheta)}),$$

$$\begin{aligned}\dot{\omega}(t) =&\mu N_h\left(1 - \frac{S_1^{**}}{S_1}\right) + \mu N_h\left(1 - \frac{S_1}{S_1^{**}}\right)\delta_1 + \alpha_1(1 - S_1 - E_1 - I_1 - Q_1)\left(1 - \frac{S_1^{**}}{S_1}\right)\\ &+ \alpha_1(1 - S_1 - E_1 - I_1 - Q_1)\left(1 - \frac{S_1}{S_1^{**}}\right) - \delta_1 E_1 - \frac{1}{IIP_1}E_1 - (\gamma_1 + \delta_1 - \varphi_1 S_1^{**})Q_1 + CN_T\left(1 - \frac{S_2^{**}}{S_2}\right)\\ &+ CN_T\left(1 - \frac{S_2}{S_2^{**}}\right) + \alpha_2(1 - S_2 - E_2 - I_2 - Q_2)\left(1 - \frac{S_2^{**}}{S_2}\right)\\ &+ \alpha_2(1 - S_2 - E_2 - I_2 - Q_2)\left(1 - \frac{S_2}{S_2^{**}}\right) - \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right)E_2 - -(\delta_2 + \rho_2 + \vartheta + \gamma_2 - \varphi_2 S_2^{**})Q_2,\end{aligned}$$

$$\begin{aligned}\dot{\omega}(t) = &-\mu N_h\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}} - \alpha_1(1 - S_1 - E_1 - I_1 - Q_1)\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}}\\ &- \delta_1 E_1 - \frac{1}{IIP_1}E_1 - CN_T\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}}\\ &- \alpha_2(1 - S_2 - E_2 - I_2 - Q_2)\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}} - \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right)E_2.\end{aligned}$$

Substituting the relations in Eq. (18a), we have

$$\begin{aligned}\dot{\omega}(t) = &- \mu N_h\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}} - \alpha_1(1 - S_1 - E_1 - I_1 - Q_1)\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}}\\ &- \delta_1 E_1 - \frac{1}{IIP_1}E_1 - CN_T\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}}\\ &- \alpha_2(1 - S_2 - E_2 - I_2 - Q_2)\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}} - \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right)E_2,\end{aligned}$$

$$\begin{aligned}\dot{\omega}(t) = &-\Bigg[\mu N_h\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}} + \alpha_1(1 - S_1 - E_1 - I_1 - Q_1)\frac{\left(S_1^{**} - S_1\right)^2}{S_1 S_1^{**}}\\ &+\delta_1 E_1 + \frac{1}{IIP_1}E_1 + CN_T\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}}\alpha_2\\ &(1 - S_2 - E_2 - I_2 - Q_2)\frac{\left(S_2^{**} - S_2\right)^2}{S_2 S_2^{**}} + \left(\delta_2 + \vartheta + \frac{1}{IIP_2}\right)E_2\Bigg] \leq 0.\end{aligned}$$

Hence, the condition (16) show that $\dot{\omega}(t) \leq 0$ of all terms. Then the equilibrium steady state $B_1 = \left(S_1^*, \underset{1}{\overset{*}{E}}, \underset{1}{\overset{*}{I}}, \underset{1}{\overset{*}{Q}}, \underset{1}{\overset{*}{R}}, \underset{2}{\overset{*}{S}}, \underset{2}{\overset{*}{E}}, \underset{2}{\overset{*}{I}}, \underset{2}{\overset{*}{Q}}, \underset{2}{\overset{*}{R}}\right)$ is the globally asymptotically stable in the Ω.

Sensitivity analysis

The model of the parameters will affect the spread and spread of COVID-19, the results of insertion into model (2) will be subjected to a sensitivity analysis. We begin by first introducing the following definitions of[30–36].

Definition 1: The normalized forward sensitivity index of the variable (R_0), depending on the parameter difference, is given as: $E_\zeta^\emptyset = \frac{\partial\emptyset}{\partial\zeta} \times \frac{\zeta}{\emptyset}$. A new expression for R_0 is introduced as:

$$R_0 = \frac{\alpha_3 C(1 + \alpha_2 IIP_2)\varphi_2\left(\gamma_1 + \delta_1 + q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\delta_1 + \rho_1))\varphi_1}{\alpha_1\left(\alpha_2\alpha_3 + g_2 q_{3T}\right)IIP_2(\delta_2 + \vartheta)\delta_1(1 + IIP_1\delta_1)\left(g_1 q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)}.$$

Then the sensitivity indices of the basic reproduction number (R_0), with respect to the system model depends on the nineteenth parameter are computed as below.

$$\chi_{\alpha_1}^{R_0} = \left(\frac{\partial R_0}{\partial\alpha_1}\right)\left(\frac{\alpha_1}{R_0}\right) = 0,\ \chi_{\alpha_2}^{R_0} = \left(\frac{\partial R_0}{\partial\alpha_2}\right)\left(\frac{\alpha_2}{R_0}\right) = 0,\ \chi_C^{R_0} = \left(\frac{\partial R_0}{\partial C}\right)\left(\frac{C}{R_0}\right) = 0,$$

$$\chi_\mu^{R_0} = \left(\frac{\partial R_0}{\partial\mu}\right)\left(\frac{\mu}{R_0}\right) = \frac{IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2}{1 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2},$$

$$\chi_{\delta_1}^{R_0} = (IIP_1\left(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)(\delta_1 + (1 + IIP_1\delta_1)\mu\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right)\left(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)(1 + IIP_1)$$

$$\left(\frac{\partial R_0}{\partial \delta_1}\right)\left(\frac{\delta_1}{R_0}\right) = \left(q_{2T} + \gamma_1 + 2\delta_1 + \rho_1\right)(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2 - \frac{1}{IIP_1}\delta_1(1 + IIP_1\delta_1)\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right)\left(q_{2T} + \gamma_1 + 2\delta_1 + \rho_1\right)$$

$$(1 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1(\delta_1 + \rho_1))(1 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1(\delta_1 + \rho_1))$$

$$(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2 - \frac{1}{IIP_1}\delta_1(1 + IIP_1\delta_1)\left(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)$$

$$(1 + IIP_1)\left(q_{2T} + \gamma_1 + \rho_1\right)\mu\left(1 + IIP_1\,(\delta_1 + \rho_1)(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2) - \delta_1\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right)\left(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)(1 + IIP_1)\left(\left(q_{2T} + \gamma_1 + \delta_1\right)\mu\, 1 + IIP_1(\delta_1 + \rho_1)\right)(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)\right)$$

$$- \frac{1}{IIP_1}(1 + IIP_1\delta_{\,1}\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right) - \left(g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)(1 + IIP_1)\left(q_{2T} + \gamma_1 + \rho_1\right)\mu(1 + IIP_1)\,(\delta_1 + \rho_1)$$

$$\left(\left(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1)\right)(1 + IIP_1)\left(q_{2T} + \gamma_1 + \rho_1\right)\mu(1 + IIP_1)\,(\delta_1 + \rho_1)\,(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)\right)/$$

$$\left(1 + IIP_1\delta_1)\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right)(-g_1\, q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1))^2\,(1 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\,(\delta_1 + \rho_1)\,(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2))\right.$$

$$\chi_{\delta_2}^{R_0} = \left(\frac{\partial R_0}{\partial \delta_2}\right)\left(\frac{\delta_2}{R_0}\right) = \left(\delta_2\left(-g_2\, q_{3T} + q_{3T} + \gamma_2 + \delta_2\right)(\delta_2 + \vartheta + \rho_2)\right)\left(-q_{3T} + \gamma_2 + \delta_2\right)^2$$

$$\left(2\delta_2 + \vartheta + \rho_2 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\,(\delta_1 + \rho_1)\right)$$

$$(\gamma_2 + \vartheta + \delta_2)^2\varphi_1\varphi_2 + g_2\, q_{3T}\left(q_{3T} + \gamma_2 - \vartheta - IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)(\delta_1 + \rho_1)\right)$$

$$\left((\vartheta + \delta_2)^2 - \left(q_{3T} + \gamma_2\right)\rho_2 + \vartheta + \rho_2)\varphi_1\varphi_2\right)\right)/\left(\left(q_{3T} + \gamma_2 + \delta_2\right)\right.$$

$$(\vartheta + \delta_2(g_2\, q_{3T} - \left(q_{3T} + \gamma_2 + \delta_2\right)(\vartheta + \gamma_2 + \delta_2))^2$$

$$(1 + IIP_1\left(\left(\left(q_{2T} + \gamma_1 + \delta_1\right)\mu(1 + IIP_1)\,(\delta_1 + \rho_1)\right)(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)\right)$$

$$\chi_{IIP_1}^{R_0} = \left(\frac{\partial R_0}{\partial IIP_1}\right)\left(\frac{IIP_1}{R_0}\right) = (\frac{IIP_2\left(\frac{1}{IIP_2} + \delta_1 + \vartheta\right)\left(-1 - 2IIP_1\delta_1 + IIP_1{}^2\left(g_{2T} + \gamma_1 + \delta_1\right)\mu\rho_1((\delta_2 + \upsilon + \rho_2)\varphi_1\varphi_2)\right)}{((1 + IIP_1\delta_1(1 + IIP_2(\delta_1 + \vartheta))(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + HP_1(\delta_1 + \rho_1))((\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)),}$$

$$\chi_{IIP_2}^{R_0} = \left(\frac{\partial R_0}{\partial HP_2}\right)\left(\frac{HP_2}{R_0}\right) = \frac{1}{1 + IIP_2(\delta_1 + \vartheta)},$$

$$\chi_{\rho_1}^{R_0} = \left(\frac{\partial R_0}{\partial \rho_1}\right)\left(\frac{\rho_1}{R_0}\right) = -(IIP_2(q_{2T} + \gamma_1 + \delta_1)(\frac{1}{IIP_2} + \delta_1 + \vartheta)\rho_1(-g_1 q_{2T} + (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1))$$

$$(1 + IIP_1(q_{2T} + g_1 q_{2T} + \gamma_1 + \delta_1)\mu(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)/((1 + IIP_2(\delta_1 + \rho_1))(g_1 q_{2T} - (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1))^2$$

$$(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)),$$

$$\chi_{\rho_2}^{R_0} = \left(\frac{\partial R_0}{\partial \rho_2}\right)\left(\frac{\rho_2}{R_0}\right) = -(IIP_2(\frac{1}{HP_2} + \delta_1 + \vartheta)\rho_2(-g_2 q_{3T} + (q_{3T} + \gamma_2 + \delta_2)(\delta_2 + \upsilon + \rho_2))$$

$$(q_{3T} + \gamma_2 + \delta_2 + g_2 q_{3T} IIP_1(q_{2T} + \gamma_1 + \delta_1)$$

$$\mu(1 + IIP_1(\delta_1 + \rho_1))\varphi_1\varphi_2))/((1 + IIP_2(\delta_1 + \vartheta))(g_2 q_{3T} - (q_{3T} + \gamma_2 + \delta_2)(\delta_2 + \vartheta + \rho_2))^2$$

$$(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)),$$

$$\chi_{\gamma_1}^{R_0} = \left(\frac{\partial R_0}{\partial \gamma_1}\right)\left(\frac{\gamma_1}{R_0}\right) = \frac{\gamma_1(q_{2T} + \delta_1 + \rho)}{(q_{2T} + \gamma_1 + \delta_1)((\gamma_1 + \delta_1) + (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho))},$$

$$\chi_{\varphi_1}^{R_0} = \left(\frac{\partial R_0}{\partial \varphi_1}\right)\left(\frac{\varphi_1}{R_0}\right)\frac{IIP_1((q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))\varphi_1\varphi_2))}{(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2))},$$

$$\chi_{\varphi_{12}}^{R_0} = \left(\frac{\partial R_0}{\partial \varphi_{12}}\right)\left(\frac{\varphi_{12}}{R_0}\right) = 0,$$

$$\chi_{\varphi_{21}}^{R_0} = \left(\frac{\partial R_0}{\partial \varphi_{21}}\right)\left(\frac{\varphi_{21}}{R_0}\right) = 0,$$

$$\chi_{g_1}^{R_0}\left(\frac{\partial R_0}{\partial g_1}\right)\left(\frac{g_1}{R_0}\right) = -\frac{g_1 q_{2T}}{g_1 q_{2T} - (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1)},$$

$$\chi_{\varphi_2}^{R_0} = \left(\frac{\partial R_0}{\partial \varphi_2}\right)\left(\frac{\varphi_2}{R_0}\right) = \frac{IIP_1((q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2}{(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2},$$

$$\chi_{g_2}^{R_0} = \left(\frac{\partial R_0}{\partial g_2}\right)\left(\frac{g_2}{R_0}\right) = -\frac{g_2 q_{3T}}{g_2 q_{3T} - (q_{3T} + \gamma_3 + \delta_3)(\delta_2 + \rho_2 + \vartheta)},$$

$$\begin{aligned}\chi_{\gamma_1}^{R_0} =& \left(\frac{\partial R_0}{\partial \gamma_1}\right)\left(\frac{\gamma_1}{R_0}\right) = -(IIP_2\gamma_1(\frac{1}{HP_2} + \delta_1 + \vartheta)(-g_1 q_{2T} + (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1))(\delta_1 + \rho_1 + g_1 q_{2T} IIP_1\mu(1 + IIP_1 \\ &(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2))/((1 + IIP_1(\delta_1 + \rho_1))(-g_1 q_{2T} + (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1))^2 \\ &(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)),\end{aligned}$$

$$\chi_{\gamma_2}^{R_0} = \left(\frac{\partial R_0}{\partial \gamma_2}\right)\left(\frac{\gamma_2}{R_0}\right) = -\frac{g_2 q_{3T}\gamma_2}{(q_{3T} + \gamma_2 + \delta_2)(-g_2 q_{3T} + (q_{3T} + \gamma_2 + \delta_2)(\delta_2 + \vartheta + \rho_2))},$$

$$\begin{aligned}\left(\frac{\partial R_0}{\partial q_{2T}}\right)\left(\frac{q_{2T}}{R_0}\right) =& (q_{2T} IIP_2(\frac{1}{HP_2} + \delta_1 + \vartheta)(-g_1 q_{2T} + (q_{2T} + \gamma_1 + \delta_1)(\delta_1 + \rho_1)) \\ &(g_1 - \delta_1 - \rho_1 + g_1 IIP_1(\gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2))/((1 + IIP_1(\delta_1 + \rho_1)) \\ &(-g_1 q_{2T} + \left(q_{2T} + \gamma_1 + \delta_1\right)(\delta_1 + \rho_1))^2\left(1 + IIP_1\left(q_{2T} + \gamma_1 + \delta_1\right)\right. \\ &\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)),\end{aligned}$$

$$\chi_{q_{3T}}^{R_0}\left(\frac{\partial R_0}{\partial q_{3T}}\right)\left(\frac{q_{3T}}{R_0}\right) = \frac{g_2 q_{3T}(\gamma_2 + \delta_2)}{(q_{3T} + \gamma_2 + \delta_2)(-g_2 q_{3T} + (q_{3T} + \gamma_2 + \delta_2)(\delta_2 + \vartheta + \rho_2))},$$

$$\begin{aligned}\chi_{\vartheta}^{R_0} =& \left(\frac{\partial R_0}{\partial \vartheta}\right)\left(\frac{\vartheta}{R_0}\right) = (\vartheta(-g_2 q_{3T} + (q_{3T} + \gamma_2 + \delta_2)(\delta_2 + \vartheta + \rho_2))(-(q_{3T} + \gamma_2 + \delta_2) \\ &(2\delta_2 + 2IIP_2\delta_1\delta_2 + IIP_2{\delta_2}^2 + 2\vartheta + 2IIP_2\delta_1\vartheta + 4IIP_2\delta_2\vartheta + 3IIP_2\upsilon^2 + \rho_2 \\ &+ IIP_2\delta_1\rho_2 + IIP_2\delta_2\rho_2 + 2IIP_2\vartheta\rho_2 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_2(\delta_1 + \delta_2 + 2\vartheta)) \\ &(1 + IIP_1(\gamma_1 + \delta_1))(\delta_2 + \vartheta + \rho_2)^2\varphi_1\varphi_2) + g_2 q_{3T}(1 + IIP_2(\delta_1 + \delta_2 + 2\vartheta) + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IiP_1(\gamma_1 + \delta_1)) \\ &(\rho_2 + IiP_2((\delta_2 + \vartheta)^2 + (\delta_1 + \delta_2 + 2\vartheta)\rho_2))\varphi_1\varphi_2)))/(IiP_2(\frac{1}{IIP_2} + \delta_2 + \vartheta)(g_2 q_{3T} - (q_{3T} \\ &+ \gamma_2 + \delta_2)(\delta_2 + \vartheta + \rho_2))^2(1 + IIP_1(q_{2T} + \gamma_1 + \delta_1)\mu(1 + IIP_1(\delta_1 + \rho_1))(\delta_2 + \vartheta + \rho_2)\varphi_1\varphi_2)).\end{aligned}$$

We can estimate the sensitivity indices (S.I) of the basic reproduction number (R_0), taking into account the parameter of the model (2). The signs of sensitivity indices (S.I) are shown in the Table 3 and bar chart Fig. 8.

The effects of changing parameter values on the functional value of the reproduction number R_0 are obtainable in this section. The necessary parameters must be found, which may be important criteria in disease management. The desirable changes of the occurred when their changes produce a positive effect, i.e., when their sensitivity indices have positive sign, i.e.$\alpha_1, \alpha_2, IIP_2, g_1, g_2, q_{3T}, \mu, C, \varphi_1, \varphi_{12}, \varphi_2$ and φ_{21} have a positive effect on R_0. The determine that the increase in the number of two exposed population (E_1, E_2) and two infectious host population (I_1, I_2) with the value IIP_1, g_1, g_2, IIP_2 may lead to an outbreak. On the other hand, the negative sign of the sensitivity indices (S.I) in the R_0 i.e. $\delta_1, \delta_2, IIP_1, q_{2T}, \rho_1, \gamma_1, \vartheta, \gamma_2$ and ρ_2 has a negative effect on the spread of disease according to the system (2). Thus, the sensitivity indices (S.I) of the Covid-19 (2) shows that there will been appreciable change at the beginning of the transmission the disease. This would help the public health official to plan on how best to develop a reasonable interference strategy to prevent and manage the spread of the disease.

Conclusions and discussion

Tourism has become an important source of foreign currency for many countries. This is especially true for Thailand. It is the second major source of currency. This means that tourists are coming to Thailand every year. When combined with the need for temporary, seasonal farmer workers to support the main source of income in Thailand, that is the agricultural industry, foreigners (tourists), and these foreign workers diseases can be brought into Thailand. Thailand must always be aware of the arrival of new infectious diseases. Most recently, the novel coronavirus COVID-19 appeared in China. From a few hundred infections in Wuhan, China, it is quickly evolved into a pandemic, which spread to five continents public health authorities in Thailand have initiated public health measures to control the spread and stop the spread of this virus in the United States. More than one million people have died from the disease. In this article a standard SEIQR model has been introduced for the transmission dynamics of COVID-19 infection in Thailand and for Foreign (tourists) entering the Thai population. Affecting the change of COVID-19 among Thais people, they took the SEIQR model for each population and linked them together, allowing members of each population to cross-infection with each other. The impact of factors causing changes in the spread of COVID-19 is examined. After that, we performed a basic reproductive number analysis and saw how homeostasis changes. Taking cross-infection (mixed) into account, we find that our model achieves an infection-free equilibrium. When the basic reproductive number is less than one. This model achieves local equilibrium at multiple points when the number is greater than one. Our analysis shows that the rate of recovery rate of both Thais and tourists would be affected by decreases in the recruitment rates and death rates. This result shows that the recovery rate for both Thais and foreigners has increased. This is because changes in recruitment and death rates will result in a decrease in the basic reproductive number. However, changes IIP_1 (capita rate of progression of Thais population from the exposed state to the infectious state), IIP_2 (capita rate of progression of foreign human from the exposed state to the infectious state), q_{2T} (the number of infected Thais that leave the quarantine period with the virus intact) and q_{3T} (the number of infected Foreign that leave the quarantine period with the virus intact), g_1 (the rate at which the exposed Thais are put into quarantine from the exposed and infected Thais) and g_2 (the rate at which the exposed Foreign (tourists) are put into quarantine from the exposed and infected Foreign (tourists)), φ_1(transmission rate of virus between population in Thais population), φ_2 (transmission rate of virus between population in Foreign (tourists) population), μ (recruitment term of the susceptible population in Thais) and C(recruitment term of the susceptible population in Foreign (tourists)) would cause the basic reproductive number to increase meaning increases in the severity of the pandemic, more people being infected by the COVID-19 coronavirus. Therefore, we controlled the number of new confirmed cases or new infections significantly by introducing a positive change in the parameter memory value in the sensitivity analysis.

In summary, from the study of the spread of the COVID-19 virus, which is an infectious disease. This disease is a health problem, leading to a rapid decline in the impact on the economy. Although governments and the World Health Organization have implemented international control measures and prevented interference. By creating a mathematical model that uses data from disease outbreaks between Thais population and Foreign (tourist) entering Thailand. To determine some parameters affecting the outbreak under proper control of the disease. By relying on the strategies of the government and the World Health Organization, including controlling the spread of infection, incubation, treatment and prevention of fever. It was found that controlling the disease transmission will be a guideline for reducing the spread and reducing the number of cases between Thai people and foreigners.

Data availability

The data in the analysis is taken from Bureau of Epidemiology Ministry of Public Health, Thailand (https://ddc.moph.go.th/viralpneumonia/eng/index.php).

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