Passivity and Anti-Synchronization Analysis of Delayed Inertial Neural Networks via Memory Based Coupled Sampled Data Control

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***Abstract*—In this article deals Memory based sampled-data control with coupling terms (MSDC) to analyze the passivity and anti-synchronization of inertial neural networks (INNs). In addition, Lyapunov-Krasovskii functional (LKF), enhanced inequality techniques, designed feedback controllers, and linear matrix inequality (LMI) are used to provide the conditions that guarantee passivity and anti-synchronization between uncontrolled and regulated systems. To demonstrate the viability and validity of the suggested methods and standards, an example is provided at the end of the study.**

Keywords—synchronization, sampled-data control, inertial neural networks, linear matrix inequality

I. INTRODUCTION

Neural networks have garnered more and more attention from academics in the past several decades. In particular, a lot of research has focused on inertial neural networks (INNs), which are thought of as engineering systems. For instance, the concept of inertia contributes to the understanding of how neurons interact with their quasi-active membranes and squid axons. Particularly referring to the passivity and anti-synchronization of these networks, the dynamic behaviors of INNs are essential for improving coordination and efficiency in information interchange and task cooperation [1],[2],[3]. A number of important results on the synchronization and passivity of INNs have been achieved recently.

Recently, synchronization and passivity analysis have been studied in connection with a range of control strategies [4]. Coupling terms with memory sampled-data control (MSDC) have attracted a lot of attention as digital technologies have developed and control networks with coupling components have enhanced their communication systems [5],[6],[7]. MSDC has the benefit of requiring less maintenance and transmission bandwidth because it can only update the controller at predefined sample intervals [8]. The authors have examined several remarkable outcomes involving the synchronization of MSDC, and more study is necessary to address the issue of anti-synchronization analysis using MSDC research methodologies.

Based on the author's information, much research hasn't been done on the combined MSDC, passivity and anti-synchronization analysis of INNs. To sum up, the following are the main contributions made by this paper: Using memory sampled data control (SDC) with coupling terms, it (i) examines INNs and incorporates a Bernoulli distributed sequence into the controller design; (ii) provides LMI-based criteria to achieve passivity synchronization of INNs by employing various inequality techniques, an appropriate LKF, and sufficient conditions.

Notations: Refer to [4].

II. PROBLEM FORMULATION

This article will examine the INNs model, which is defined as follows:

$$\begin{cases} \ddot{y}(t) &= -\aleph\dot{y}(t) - \mathcal{A}y(t) + \mathscr{B}g(y(t)) + \mathcal{C}g(y(t-\eta(t))), \\ z(t) &= g(y(t)) \end{cases} \tag{1}$$

where the state variable is denoted by $y(t) \in \mathbb{R}^n$; the system's inertia term is denoted by (1); the output is represented by *z(t);* the diagonal matrices are $\mathcal{A} > 0$and $\aleph > 0$; $\mathscr{B}$ and $\mathcal{C}$ the connection weight matrices; $g(y(t)), g(y(t-\eta(t))) \in \mathbb{R}^n$ indicated as activation functions; $\eta(t)$ denotes the time-varying delays with $0 \leq \eta(t) \leq \eta, \dot{\eta}(t) \leq \tilde{\mu} < 1$, where $\eta > 0$ and $\tilde{\mu} > 0$ are constants. $y(t) = \phi(s), \frac{dy(s)}{dt} = \tau(s)$ indicated the initial conditions of (1). Where $s \in [-\eta, 0]$ and $\phi(s), \tau(s)$ are continuous.

$\mathbb{H}_1$: The following Lipschitz condition is satisfied by the neuron activation function g_j: scalar $\pi_j > 0$, such that for $j = 1,2,3,\cdots,n$,

$||g_j(y) - g_j(\hat{y})|| \leq \pi_j ||y - \hat{y}|| \forall y, \hat{y} \in \mathbb{R}$. Define $h(t) = \frac{dy(t)}{dt} + y(t)$, then (1) can be outlined as master system

$$\begin{cases} \frac{dy(t)}{dt} = -y(t) + h(t), z(t) = g(y(t)), \\ \frac{dh(t)}{dt} = \Omega y(t) - \Theta h(t) + \mathscr{B}g(y(t)) + \mathcal{C}g(y(t-\eta(t))), \end{cases} \tag{2}$$

where $\Omega = \mathcal{A} - \aleph + I,\ \Theta = \aleph - I$. The corresponding slave system is expressed as

$$\begin{cases}\frac{d\hat{y}(t)}{dt} &= -\hat{y}(t)+\hat{h}(t)+\mathcal{U}_1(t), z(t)=g(\hat{y}(t)) \\ \frac{d\hat{h}(t)}{dt} &= \Omega\hat{y}(t)-\Theta\hat{h}(t)+\mathcal{B}g(\hat{y}(t))+\mathcal{C}g(\hat{y}(t-\eta(t))) \\ & +\mathcal{U}_2(t)+H_1w(t),\end{cases} \quad (3)$$

where $\mathcal{U}_1(t)$ and $\mathcal{U}_2(t)$ are the controllers to be designed. Let $y(t)=col\{y_1(t),y_2(t),\cdots,y_n(t)\}$, $g(y(t))=col\{g_1(y_1(t)),g_2(y_2(t)),\cdots,g_n(y_n(t))\}$. Let $\varpi(t)=\hat{y}(t)+y(t), \wp(t)=\hat{h}(t)+h(t)$, the anti-synchronization error system from (2) and (3) is given as follows

$$\begin{cases}\frac{d\varpi(t)}{dt} &= -\varpi(t)+\wp(t)+\mathcal{U}_1(t),\ \bar{z}(t)=\mathscr{F}((t)) \\ \frac{d\wp(t)}{dt} &= \Omega\varpi(t)-\Theta\wp(t)+\mathcal{B}\mathscr{F}(\varpi(t))+\mathcal{C}\mathscr{F}(\varpi(t-\eta(t))) \\ & +\mathcal{U}_2(t)+H_1w(t),\end{cases} \quad (4)$$

where $\mathscr{F}(\varpi(t))=g(\hat{y}(t))+g(y(t))$ and $\mathscr{F}(\varpi(t-\eta(t)))=g(\hat{y}(t-\eta(t)))+g(y(t-\eta(t)))$, Then, (4) can be written as

$$\frac{d\varrho(t)}{dt}=\hat{\mathcal{A}}\varrho(t)+\hat{\mathcal{B}}\hat{\mathscr{F}}(\varrho(t))+\hat{\mathcal{C}}\hat{\mathscr{F}}(\varrho(t-\eta(t)))+\mathcal{U}(t)+\hat{H}_1w(t),$$

$$\bar{z}(t)=\mathscr{F}(t) \quad (5)$$

Where $\varrho(t)=\begin{bmatrix}\varpi(t)\\ \wp(t)\end{bmatrix}$, $\hat{\mathscr{F}}(\varrho(t))=\begin{bmatrix}\mathscr{F}(\varpi(t))\\ 0\end{bmatrix}$, $\hat{\mathscr{F}}(\varrho(t-\eta(t)))=\begin{bmatrix}\mathscr{F}(\varpi(t-\eta(t)))\\ 0\end{bmatrix}$, $\hat{\mathcal{A}}=\begin{bmatrix}I & -I\\ \Omega & \Theta\end{bmatrix}$, $\hat{\mathcal{C}}=\begin{bmatrix}0 & 0\\ \mathcal{C} & 0\end{bmatrix}$, $\hat{\mathcal{B}}=\begin{bmatrix}0 & 0\\ \mathcal{B} & 0\end{bmatrix}$, $\mathcal{U}(t)=\begin{bmatrix}\mathcal{U}_1(t)\\ \mathcal{U}_2(t)\end{bmatrix}$, $\hat{H}_1=\begin{bmatrix}0 & 0\\ H_1 & 0\end{bmatrix}$, $w(t)=\begin{bmatrix}w(t)\\ 0\end{bmatrix}$

III Design of Coupling memory sampled-data control (CMSDC)

The control signal is assumed to emerge through ZOH in order to construct the SDC. The hold times are supplied as follows: $0<t_0<t_1\ \cdots<t_k<\cdots<\lim_{n\to\infty}=+\infty$ relevant input $\mathcal{U}(t)$ CMSDC is developed in the following manner based on this

$$\begin{cases}\mathcal{U}(t)=\alpha(t)\mathcal{K}_1\varrho(t_k-\sigma)+(1-\alpha(t))\mathcal{K}_2\varrho(t_k), \\ \qquad t\in[t_k,t_{k+1})\end{cases} \quad (6)$$

where $\varrho(t_k)$ denotes the state $\varrho(t)$ as observed at the sampling time (ST) t_k,$\kappa$is a constant delay, and it is assumed that $0<t_{k+1}-t_k=\tau_k\le\sigma,\sigma>0,\forall k\ge 0$. Then, the ST $t_k$may written as $t_k=t-\sigma(t)$, where $0\le\sigma(t)\le\sigma$, by defining $0\le\sigma(t)\le\sigma$for $t\ne t_k$. Additionally, $\alpha(t)$ noted as the variable-related stochastic approach that links MSDC and sampled-data proportional control with

$$\alpha(t)=\begin{cases}1 & \text{Signal was sent and received without any issues}\\ 0 & \text{else}\end{cases}$$

where $\alpha(t)$ is a white sequence following a Bernoulli distribution with $Pr\{\alpha(t)=1\}=\xi\{\alpha(t)\}=\alpha$ and $Pr\{\alpha(t)=0\}=1-\xi\{\alpha(t)\}=1-\alpha$. Then

$$\mathcal{U}(t)=\alpha(t)\mathcal{K}_1\varrho(t-\sigma(t)-\kappa)+(1-\alpha(t))\mathcal{K}_2\varrho(t-\sigma(t)) \quad (7)$$

Combining (5) and (7), the INNs become

$$\frac{d\varrho(t)}{dt}=\hat{\mathcal{A}}\varrho(t)+\hat{\mathcal{B}}\hat{\mathscr{F}}(\varrho(t))+\hat{\mathcal{C}}\hat{\mathscr{F}}(\varrho(t-\eta(t)))+\alpha(t)\mathcal{K}_1\varrho(t-\sigma(t)-\kappa)+(1-\alpha(t))\mathcal{K}_2\varrho(t-\sigma(t))+H_1w(t),$$

$$z(t)=\mathscr{F}(t). \quad (8)$$

Definition: 1[3]

System (8) is concluded passive, if there exists a scalar $\gamma>0$that satisfies $2\int_0^{t_s}z^T(s)w(s)ds\ge -\gamma\int_0^{t_s}w^T(s)w(s)ds\forall t_s>0$ satisfied with the initial condition is zero.

Lemma: 1[9]

For all matrix R>0, vector ξ a function $x:[-\tau,0]\to\mathbb{R}^n$ that is continuously differentiable, and slack matrices $\mathcal{M},\mathcal{N}$. Please refer [9] for the remaining terms in the below inequality

$$-\int_{t-v}^{t}\dot{x}^T(s)H\dot{x}(s)ds\le\xi^T\left[\eta(t)\mathcal{M}^T\tilde{H}^{-1}\mathcal{M}\right]+(\eta-\eta(t))\mathcal{N}^T\tilde{H}^{-1}\mathcal{N}]\xi+(\tfrac{\eta-\eta(t)}{\eta}+\tfrac{\eta(t)}{\eta^2})\mathrm{He}(\partial^T(t-\eta(t),t))\vartheta^T(s)\mathcal{M}+\partial^T(t-\eta,t-\eta(t)))\vartheta^T(s)\mathcal{N}]\text{-}[\tfrac{\eta-\eta(t)}{\eta^2}\partial^T(t-\eta(t),t)\times\vartheta^T(s)\tilde{H}\vartheta(s)\partial(t-\eta(t),t)+\frac{\eta(t)}{\eta^2}\partial^T(t-\eta,t-\eta(t))\times\vartheta^T(s)\tilde{H}\vartheta(s)\partial(t-\eta,t-\eta(t))]$$

$\tilde{H}=diagH,3H,\dots,(2s+1)H$

$\vartheta(s)=col[\tilde{\pi}_s(0),\tilde{\pi}_s(1),\dots,\tilde{\pi}_s(S)]$

The aforementioned lemma and definition form the basis of the primary findings and demonstrate that the suggested CMSDC for the system under consideration (8) is feasible.

Remark: 1

The purpose of this work is to define the anti-synchronization and passivity criteria for the error system (8) with CMSDC by means of the LKF approach. The following goals are met for the system (8) in order to examine the stability conditions:(i) A generalized CMSDC technique that incorporates the time delay effect is the Bernoulli distributed sequence, which uses LMIs, derived in the form of delay-dependent adequate criteria, and satisfies Definition 1 (ii) By solving LMIs, the CMSDC gain matrices $\mathcal{U}_1$ and $\mathcal{U}_2$ are determined.

IV. Main results

Here, we examine the anti-synchronization and passivity criteria using the CMSDC methodology. An appropriate LKF will be selected, and adequate circumstances will be configured. We will start by addressing a few important indications.

$\mathcal{J}_s=[0_{n\times(l-1)n}\ I_n\ 0_{n\times(16-1)n}\ 0_{n\times m}]^T$, $s=1,2,\dots,16$, $\tilde{\eta}=\eta(t)$, $\tilde{\tilde{\eta}}=\eta-\eta(t)$, $\theta^T(t)=[\varrho^T(t)\ \varrho^T(t_k)]$, $\xi_1^T=[\frac{1}{\tilde{\eta}}\int_{t-\tilde{\eta}}^{t}\varrho^T(s)ds,\frac{1}{\eta-\tilde{\eta}}\int_{t-\eta}^{t-\tilde{\eta}}\varrho^T(s)ds]$, $\chi^T(s)=\varrho(s)-\varrho(t_\kappa)$, $\xi_2^T=[\frac{1}{\tilde{\eta}^2}\int_{t-\tilde{\eta}}^{t}\int_{\theta}^{t}\varrho^T(s)dsd\theta,\frac{1}{(\eta-\tilde{\eta})^2}\int_{t-\eta}^{t-\tilde{\eta}}\int_{\theta}^{t-\tilde{\eta}}\varrho^T(s)dsd\theta]$, $\Lambda_1=col[\mathcal{J}_1-\mathcal{J}_2,\mathcal{J}_1+\mathcal{J}_2-2\mathcal{J}_4,\mathcal{J}_1-\mathcal{J}_2+6\mathcal{J}_4-12\mathcal{J}_6]$, $\Lambda_2=col[\mathcal{J}_2-\mathcal{J}_3,\mathcal{J}_2+\mathcal{J}_3-2\mathcal{J}_5,\mathcal{J}_2-\mathcal{J}_3+6\mathcal{J}_5-12\mathcal{J}_7]$, $\tilde{Q}=\begin{bmatrix}Q_1+Q_1^T & Q_1+Q_2\\ * & -Q_2-Q_2^T\end{bmatrix}$, $t_\kappa=t-\sigma(t)-\kappa$, $\mathcal{J}_{01}=\mathcal{J}_1-\mathcal{J}_{10}$, $\mathcal{J}_s=\mathcal{J}_{10}-\mathcal{J}_{11}$

Theorem: 1

Given scalars $\eta,\tilde{\mu},\sigma,\gamma>0$, constant delay $\kappa>0$, if there exists positive symmetric matrices $P_i,Q_1,Q_2,T_1,\ T_2$, S, any matrices $\mathcal{G}_i,(i=1,2,3),\mathcal{Y}_1,\mathcal{Y}_2,T_3$, diagonal matrices $\mathcal{N}_1,\mathcal{N}_2$, the subsequent inequality satisfied $\forall\ \eta(t)\in\{0,\eta\}$

$$\begin{bmatrix}T_2 & T_3\\ * & T_2\end{bmatrix}\ge 0,\begin{bmatrix}\Xi_{\{0\}} & \sqrt{\eta}\mathcal{Y}_2^T & \Upsilon_1\\ * & -\tilde{\mathcal{S}} & 0\\ * & * & -\gamma I\end{bmatrix}<0,$$

$$\begin{bmatrix} \Xi_{\{\eta\}} & \sqrt{\eta}\mathcal{Y}_1^T & \Upsilon_1 \\ * & -\tilde{\mathcal{S}} & 0 \\ * & * & -\gamma I \end{bmatrix} < 0, \quad (9)$$

where

$$\Xi = \Xi_{11} + \Xi_{12} + \Xi_{13}, \Xi_{11} = 2\mathscr{J}_1^T P_1 \mathscr{J}_9 + \mathscr{J}_1^T (P_2 + P_3)\mathscr{J}_1$$
$$-\mu\mathscr{J}_2^T P_3 \mathscr{J}_2 - \mathscr{J}_3^T P_2 \mathscr{J}_3 + \hat{\psi}_{11}(\tilde{\eta}) + \hat{\psi}_{12}(\tilde{\eta}),$$
$$\Xi_{12} = 2\sigma[\mathscr{J}_1^T \ \mathscr{J}_9^T]\tilde{Q}[\mathscr{J}_8 \ 0] - [\mathscr{J}_1^T \ \mathscr{J}_9^T]\tilde{Q}[\mathscr{J}_1 \ \mathscr{J}_9] + \mathscr{J}_1^T T_1 \mathscr{J}_1 - \mathscr{J}_{11}^T T_1 \mathscr{J}_{11} - \mathscr{J}_{01}^T T_2 \mathscr{J}_{01} - \mathscr{J}_{01}^T T_3 \mathscr{J}_s - \mathscr{J}_s^T T_2 \mathscr{J}_s - \mathscr{J}_{14}^T I \mathscr{J}_{16} + \sigma^2 \mathscr{J}_8^T (T_1 + V)\mathscr{J}_8 - \pi^2/4(\mathscr{J}_{13} - \mathscr{J}_{12})^T V (\mathscr{J}_{13} - \mathscr{J}_{12}), \ \Xi_{13} = 2[\mathscr{J}_1^T + \epsilon_1 \mathscr{J}_{12}^T + \epsilon_2 \mathscr{J}_9^T][-G\mathscr{J}_9 + \hat{\mathcal{A}} G \mathscr{J}_1 + \hat{\mathscr{B}} G \mathscr{J}_{14} + \hat{\mathcal{C}} G \mathscr{J}_{15} + \alpha \hat{X} \mathscr{J}_{12} + (1-\alpha)\hat{Y}\mathscr{J}_{10} + G H_1 \mathscr{J}_{16}] + 2\mathscr{J}_1^T \mathscr{L} \mathcal{N}_1 \mathscr{J}_{14} - \mathscr{J}_{14}^T \mathcal{N}_1 \mathscr{J}_{14} + 2\mathscr{J}_1^T \mathscr{L} \mathcal{N}_2 \mathscr{J}_{15} - \mathscr{J}_{15}^T \mathcal{N}_2 \mathscr{J}_{15},$$
$$\tilde{\mathcal{S}} = diag\{s, 3s, 5s\} \qquad \Upsilon_1 = col[\mathcal{G}H_1, \overbrace{0,0,0}^{6\ times}, \epsilon_2 \mathcal{G} H_1, 0,0,0, \epsilon_1 \mathcal{G} H_1, 0,0,0].$$

then the master-slave systems (2) and (3) are anti-synchronized and passive and the appropriate gain matrices are provided by $\mathcal{K}_1 = \mathcal{G}^{-1}\hat{X}, \mathcal{K}_2 = \mathcal{G}^{-1}\hat{Y}$.

Proof:

Subsequent LKF is given as:

$$\mathbb{V}(t) = \mathbb{V}_1(t) + \mathbb{V}_2(t) + \mathbb{V}_3(t) + \mathbb{V}_4(t), \quad (10)$$

where

$$\mathbb{V}_1(t) = \varrho^T(t) P_1 \varrho(t) + \int_{t-\tilde{\eta}}^{t} \varrho^T(s) P_3 \varrho(s) ds,$$
$$\mathbb{V}_2(t) = \int_{t-\eta}^{t} \varrho^T(s) P_2 \varrho(s) ds + \int_{t-\eta}^{t}\int_{t+\theta}^{t} \dot{\varrho}^T(s) S \dot{\varrho}(s) ds d\theta,$$
$$\mathbb{V}_3(t) = (t_{k+1} - t)\theta^T(t)\tilde{Q}\theta(t) + \int_{t-\sigma}^{t} \varrho^T(t) T_1 \varrho(t) + \sigma \int_{-\sigma}^{0}\int_{t+\theta}^{t} \dot{\varrho}^T(s) T_2 \dot{\varrho}(s) ds d\theta,$$
$$\mathbb{V}_4(t) = \sigma^2 \int_{t_\kappa}^{t} \dot{\varrho}^T(s) V \dot{\varrho}(s) ds - \frac{\pi^2}{4}\int_{t_\kappa}^{t-\kappa} \chi^T(s) V \chi(s) ds.$$

The derivative of (10) can be computed as

$$\dot{\mathbb{V}}_1(t) = 2\varrho^T(t) P_1 \dot{\varrho}(t) + \varrho^T(t) P_3 \varrho(t) - \tilde{\mu}\varrho^T(t-\tilde{\eta}) P_3 \varrho(t-\tilde{\eta}), \quad (11)$$
$$\dot{\mathbb{V}}_2(t) = \varrho^T(t) P_2 \varrho(t) - \varrho^T(t-\eta) P_2 \varrho(t-\eta) + \eta \dot{\varrho}^T(t) S \dot{\varrho}(t) - \int_{t-\eta(t)}^{t} \dot{\varrho}^T(s) S \dot{\varrho}(s) ds. \quad (12)$$

By Lemma 1, yields

$$-\int_{t-\eta(t)}^{t} \dot{\varrho}^T(s) S \dot{\varrho}(s) ds \le \xi^T(t)[\eta(t)\mathcal{Y}_1^T \tilde{\mathcal{S}}^{-1}\mathcal{Y}_1 + (\eta - \eta(t))\mathcal{Y}_2^T \tilde{\mathcal{S}}^{-1}\mathcal{Y}_2 + (\frac{(\eta-\eta(t))}{\eta} + \frac{\eta(t)^2}{\eta^2}) He[\mathcal{Y}_1 \Lambda_1 + \mathcal{Y}_1 \Lambda_2] - (\frac{(\eta-\eta(t))}{\eta^2}\Lambda_1^T \tilde{\mathcal{S}} \Lambda_1 + \frac{\eta(t)^2}{\eta^2}\Lambda_2^T \tilde{\mathcal{S}}\Lambda_2))]\xi(t),$$
$$= \xi^T(t)[\hat{\psi}_{11}(\eta(t)) + \hat{\psi}_{12}(\eta(t))]\xi(t), \quad (13)$$

where

$$\hat{\psi}_{11}(\tilde{\eta}) = \tilde{\eta}\mathcal{Y}_1^T \tilde{\mathcal{S}}^{-1}\mathcal{Y}_1 + \tilde{\tilde{\eta}}\mathcal{Y}_2^T \tilde{\mathcal{S}}^{-1}\mathcal{Y}_2, \hat{\psi}_{12}(\tilde{\eta}) = (\frac{\tilde{\tilde{\eta}}}{\eta} + \frac{\tilde{\eta}^2}{\eta^2}) He[\mathcal{Y}_1 \Lambda_1 + \mathcal{Y}_1 \Lambda_2] - (\frac{\tilde{\tilde{\eta}}}{\eta^2}\Lambda_1^T \tilde{\mathcal{S}}\Lambda_1 + \frac{\tilde{\eta}^2}{\eta^2}\Lambda_2^T \tilde{\mathcal{S}}\Lambda_2).$$

Utilizing the Reciprocal convex lemma [10], we get

$$\dot{\mathbb{V}}_3(t) = 2(t_{k+1} - t)\theta^T(t)\tilde{Q}[\dot{\varrho}^T(t) \ 0^T]^T - \theta^T(t)\tilde{Q}\theta(t) + \varrho^T T_1 \varrho(t) - \varrho^T(t-\sigma) T_1 \varrho(t-\sigma) + \sigma^2 \dot{\varrho}^T(t) T_2 \dot{\varrho}(t) - \mathscr{J}_{01}^T T_2 \mathscr{J}_{01} - \mathscr{J}_{01}^T T_3 \mathscr{J}_s - \mathscr{J}_s^T T_2 \mathscr{J}_s, \quad (14)$$
$$\dot{\mathbb{V}}_4(t) = \sigma^2 \dot{\varrho}^T(t) V \dot{\varrho}(t) - \frac{\pi^2}{4}(\varrho^T(t-\kappa) - \varrho^T(t-\sigma(t)-\kappa)) V (\varrho(t-\kappa) - \varrho(t-\sigma(t)-\kappa)). \quad (15)$$

From $(\mathbb{H}_1)$, the subsequent inequalities satisfy with $\mathcal{N}_1 > 0, \mathcal{N}_2 > 0$ and $\mathscr{L} = \{l_1, l_2, \cdots l_n\}$

$$2[\varrho^T(t)\mathscr{L}\mathcal{N}_1 \mathscr{F}(\varrho(t)) - \mathscr{F}^T(\varrho(t))\mathcal{N}_1 \mathscr{F}(\varrho(t))] \ge 0,$$
$$2[\varrho^T(t-\eta(t))\mathscr{L}\mathcal{N}_2 \mathscr{F}(\varrho(t-\eta(t))) - \mathscr{F}^T(\varrho(t-\eta(t))) \mathcal{N}_2 \mathscr{F}(\varrho(t-\eta(t)))] \ge 0. \quad (16)$$

Additionally, on the basis of system (8), the requirements are true for every suitably dimensioned matrix $\mathcal{G}_i (i = 1,2,3)$.

$$0 = 2[\varrho^T(t)\mathcal{G}_1 + \varrho^T(t-\sigma(t)-\kappa)\mathcal{G}_2 + \dot{\varrho}^T(t)\mathcal{G}_3][-\dot{\varrho}(t) + \hat{\mathcal{A}}\varrho(t) + \hat{\mathscr{B}}\mathscr{F}(\varrho(t)) + \hat{\mathcal{C}}\mathscr{F}(\varrho(t-\eta(t))) + \alpha(t)\mathcal{K}_1 \varrho(t-\sigma(t)-\kappa) + (1-\alpha(t))\mathcal{K}_2 \varrho(t-\sigma(t)) + H_1 w(t)]. \quad (17)$$

Finally, from (11)-(17), and $\mathcal{G}_1 = \mathcal{G}, \ \mathcal{G}_2 = \epsilon_1 \mathcal{G}, \ \mathcal{G}_3 = \epsilon_2 \mathcal{G}, \hat{X} = \mathcal{G}_1 \mathcal{K}_1, \ \hat{Y} = \mathcal{G}_1 \mathcal{K}_2$, we have

$$\dot{\mathbb{V}}(t) - \gamma w^T(t) w(t) - 2z^T(t) w(t) < \xi^T(t)(\hat{\psi}_{11}(\tilde{\eta}) + \hat{\psi}_{12}(\tilde{\eta}) + \hat{\psi}_{13})\xi(t) < 0, \quad (18)$$

where $\xi^T(t) = [\varrho^T(t), \varrho^T(t-\eta(t)), \varrho^T(t-\eta), \xi_1^T, \xi_2^T, \dot{\varrho}^T(t), \varrho^T(t_k), \varrho^T(t-\sigma(t)), \varrho^T(t-\sigma), \varrho^T(t-\sigma(t)-\kappa), \varrho^T(t-\kappa), \mathscr{F}^T(\varrho(t), \mathscr{F}^T(\varrho(t-\eta(t))), w^T(t)]$ and moreover $\hat{\psi}_{11}(\tilde{\eta}), \hat{\psi}_{12}(\tilde{\eta}), \hat{\psi}_{13} = \Xi$ is defined in Theorem 1 and employing schur-complement Lemma, we get LMI (9) (i.e.) $\dot{\mathbb{V}}(t) < 0$. According to LMI (9), we get

$$\dot{\mathbb{V}}(t) - \gamma w^T(t) w(t) - 2z^T(t) w(t) \le 0. \quad (19)$$

By integrating (19) over the time interval from 0 to t_s, we derive that $2\int_0^{t_s} z^T(t) w(t) dt \ge \mathbb{V}(T) - \mathbb{V}(0) - \gamma \int_0^{t_s} w^T(t) w(t) dt. \quad (20)$

Since $\mathbb{V}(0) = 0$and $\mathbb{V}(T) \ge 0$, then Definition 1 satisfies, which implies that system (8) is passive.

V. SIMULATION EXAMPLE

In this part, simulation example of INNs (8) under CMSDC are offered to manifest the benefits of the technique that is discussed in this study.

Example:1

Take into consideration the system (8) with the following parameter values

$$\aleph = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \mathcal{A} = \begin{bmatrix} 2.2 & 0 \\ 0 & 2.2 \end{bmatrix}, \mathscr{B} = \begin{bmatrix} -0.5 & -1.3 \\ 0.4 & 0.3 \end{bmatrix},$$
$$\mathcal{C} = \begin{bmatrix} 0.3 & 0.2 \\ 0.9 & 1.1 \end{bmatrix}, H_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.2 \end{bmatrix},$$

using the above values, we get $\{\hat{\mathcal{A}}, \hat{\mathscr{B}}, \hat{\mathcal{C}}, \hat{H}_1\}_{4\times 4}$ matrix values and choose the activation functions as $g_i(y_i(t)) = tanh(y_i(t)), (i = 1,2), \mathscr{L} = diag\{1, \ 1,1,1\}, \eta(t) = 0.1sin|(t)|$, other known scalar values, and sampling period $\sigma = 0.1, \kappa = 0.02$. Engaging MATLAB, the gain values are accumulated from Theorem 1 LMIs

$$\mathcal{K}_1 = \begin{bmatrix} -1.4215 & -0.1523 & 0.0475 & 0.0042 \\ 0.1541 & -1.5214 & -0.0002 & -0.0578 \\ -0.0254 & -0.0088 & -1.5422 & -0.2509 \\ 0.0014 & -0.1023 & -0.2558 & -1.2001 \end{bmatrix},$$

$$\mathcal{K}_2 = \begin{bmatrix} -3.6524 & -0.1522 & 0.4001 & 0.24645 \\ 0.3244 & -3.9534 & -0.0245 & 1.3243 \\ -1.0654 & 1.0127 & -3.9787 & -0.0012 \\ 0.0056 & -0.8632 & -0.2558 & -4.2012 \end{bmatrix}.$$

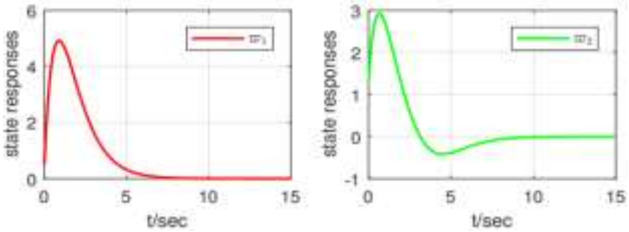

Fig. 1. Error actions of the system in Example 1.

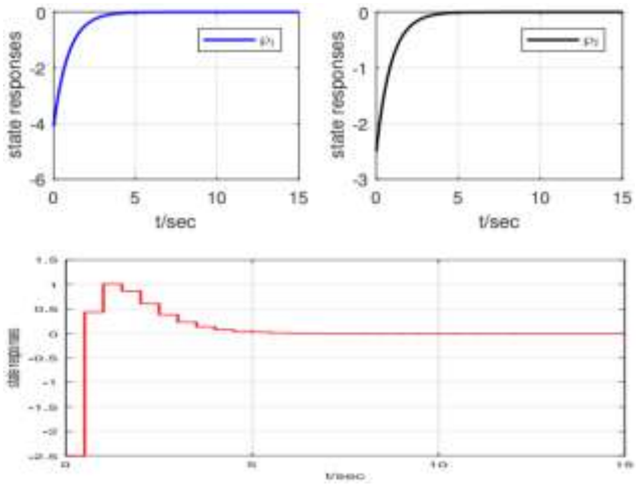

Fig. 2. Actions of the control in Example 1.

Furthermore, Fig. 1 demonstrates that the error state trajectories converge to zero with the suggested CMSDC when the randomized initial condition is used, which demonstrates the stability of the error model. Fig. 2 displays the trajectories of the control inputs. Owing to the page restriction, some figure replies and workable solutions have been left out. This implies that the performance of the CMSDC.

VI. Conclusion

This paper examined the passivity and anti-synchronization of delayed INNs using a CMSDC method, concentrating on the first and second-order derivatives. Sufficient LKF and additional prerequisites, depending on enhanced integral inequality techniques that ensure the passivity synchronization criteria of the recommended INNS, based on the LMI methodology. Numerical simulations corroborate the efficacy of the proposed methods.

References

[1] X. Wei, Z. Zhang, C. Lin, and J. Chen, "Synchronization and anti-synchronization for complex-valued inertial neural networks with time-varying delays," Applied Mathematics and Computation, vol. 403, pp. 126194, August 2021.

[2] J. Shi and Z. Zeng, "Anti-synchronization of delayed state-based switched inertial neural networks," IEEE Transactions on Cybernetics, vol. 51, pp. 2540-2549, September 2019.

[3] M. V. Thuan, H. Trinh, L. V. Hien, "New inequality-based approach to passivity analysis of neural networks with interval time-varying delay," Neurocomputing, vol. 194, pp. 301-307, June 2016.

[4] J. Wang, L. Su, H. Shen, Z. G. Wu, and J. H. Park, "Mixed H_∞ passive sampled-data synchronization control of complex dynamical networks with distributed coupling delay," Journal of the Franklin Institute, vol. 354, pp. 1302-1320, February 2017.

[5] R. Vadivel, Y. H. Joo, "Finite-time sampled-data fuzzy control for a non-linear system using passivity and passification approaches and its application," IET Control Theory & Applications, vol. 14, pp. 1033-1045, May 2020.

[6] R. Vadivel, S. Sabarathinam, Y. Wu, K. Chaisena, and N. Gunasekaran, "New results on T-S fuzzy sampled-data stabilization for switched chaotic systems with its applications," Chaos, Solitons & Fractals, vol. 164, pp. 112741, November 2022.

[7] J. Cheng, D. Zhang, W. Qi, J. Cao, and K. Shi, "Finite-time stabilization of T-S fuzzy semi-Markov switching systems: A coupling memory sampled-data control approach," Journal of the Franklin Institute, vol. 357, pp. 11265-11280, November 2020.

[8] K. Sanjay, R. V. Aravind, and P. Balasubramaniam, ``Stabilization analysis for nonlinear interconnected system with memory based coupled sampled data control via Quantum-inspired genetic algorithm," Information Sciences, pp.120857, June 2024.

[9] C. K. Zhang, Y. He, L. Jiang, M. Wu, and H. B. Zeng, "Stability analysis of systems with time-varying delay via relaxed integral inequalities," Systems & Control Letters, vol. 92, pp. 52-61, June 2016.

[10] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time varying delays," Automatica, vol. 47, pp. 235–238, January 2011.