

## RESEARCH ARTICLE

# Extended dissipative performance for a complex dynamical networks based on adaptive event-triggered filter method subject to hybrid attacks

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## Summary

The challenge of adaptive event-triggered based filter design for complex dynamical networks with time-varying coupling delays under hybrid attack is studied in this research. A new model of complex dynamical networks with input constraints, involving denial-of-service (DoS) attack, replay attack, and deception attack is designed. An adaptive event-triggered scheme is applied to filter design, which adaptively modifies the communication threshold level in order to reduce the effect of the transmission load. By using Lyapunov theory and inequality techniques, a novel necessary condition is proposed that guarantees mean-square asymptotically stability with extended dissipative performance. The proposed filter parameters are then given an explicit expression, which is solved using a set of linear matrix inequalities. Finally, two numerical results are presented to demonstrate the validity of the theoretical result suggested.

## KEYWORDS

adaptive event triggered scheme, complex dynamical networks, hybrid cyber attacks, Lyapunov-Krasvoskii functional

## 1 | INTRODUCTION

Due to the significant development in the size and complex of dynamical systems, more research has been directed on the modeling and control of complex dynamical networks (CDNs).<sup>1</sup> A complex network, in general, is a huge collection of interconnected nodes, each of which is a fundamental unit that can have different meanings in different situations. In the field of complex dynamical networks over the last few decades, controllers have mostly been applied to nodes to achieve desired dynamic characteristics such as stabilization, filtering, and so on.<sup>2-4</sup> Complex networks have received much attention in recent years due to their potential applications in a wide variety of fields, including biological systems, networked control systems, economic systems, power networks, information processing, financial networks, traffic networks, harmonic oscillation generation, food webs, secure communications, and biological networks.<sup>5-9</sup> This is the main reason for this paper, as many real world applications of complex dynamical networks are still unexplored.

In investigating CDNs, time delay is an essential consideration.<sup>10</sup> Delayed systems are frequent in our everyday lives, such as the stock exchange and communication.<sup>11</sup> When studying CDNs with filtering design, it is critical to consider the influence of coupling delays. Thus many experts investigate coupling delay issues.<sup>12-14</sup> Also a huge number of research

results around the time variable coupling delay with the filtering of delayed dynamical networks has become a focal point in investigating complex networks, which is an essential research content of complex dynamical networks.<sup>14-17</sup> Some recent articles about input constraints in Reference 18 show that input constraints can secure components and ensure good system performance to a considerable extent.

The study of CDNs brings certain difficulties in terms of theories and applications as an essential method to reduce the frequency of signal transfer between systems and controllers.<sup>19</sup> In regards to the event-triggered control (ETC), one goal is to ensure that the properties of systems under continuous channel can still be guaranteed by the ETC while communication resources are employed as efficiently as feasible and Zeno behavior is avoided. As a result, the ability to transmit data via a network is solely based on the system's limited resources. It has become an important and challenging problem to address how to minimize the signal transmission burden. Many experts have proposed an event-triggered scheme to solve the difficulties of the time-triggered scheme in order to reduce the strain of networked transmission more efficiently.<sup>20-23</sup> Event-triggered control (ETC) has been proposed and investigated in many fields to improve the performance of control systems and mitigate network communication usage.<sup>24-26</sup> In ETC, the control task is executed after the occurrence of an event that is triggered by some well-designed triggering condition.<sup>27,28</sup> The superiority of event-triggered control in decreasing information transmission and utilizing communication range has been demonstrated by researchers.<sup>29,30</sup> Relaxed resilient T-S fuzzy control of discrete-time systems via a higher order time-variant balanced matrix method discussed in Reference 31. Few research papers have been written about adaptive event-triggered strategies in complex dynamical networks, so this paper proposes a new adaptive event-triggered scheme.

With the advancement of network and modern technology, cyber security has subsequently become more crucial.<sup>32</sup> When it comes to system security, cyber-attacks are one of the most popular offenders, with the goal of weakening CDNs stability and reducing system performance.<sup>33-35</sup> However, hacker can easily target open and public networks, posing potential security vulnerabilities. The disruption of the stability and regular operation of networked facilities such as electricity network systems would have serious consequences for public safety and result in large economic losses. More importance is being given to the consequences of numerous cyber attacks, such as denial of service attacks,<sup>36</sup> replay attacks,<sup>37</sup> and deception attacks.<sup>38</sup> DoS attacks degrade data availability in control systems, causing in packet loss and a considerable delay in information interchange. In deception attacks, the adversary hijacks and interfere with the transmitted packets, forcing the next endpoints to receive misleading information.<sup>39</sup> As previously stated, there are still studies that investigate adaptive event-triggered control for the system when it is subjected to hybrid attacks, which randomly combines various kinds of attacks.<sup>40</sup> Simultaneously, the notion of hybrid cyber attacks, which combine several different cyber attack, was proposed.<sup>41-43</sup> Furthermore, many results have been presented to examine cyber-attacks when an event-triggered strategy is used to solve the stabilization problem of a networked control system.<sup>42-44</sup> So far as our author knows, no one has proposed an adaptive event-triggered filter method for complex dynamical networks that could be attacked by hybrid cyber attacks.

Inspired by the previous considerations, in this article, adaptive event-triggered filtering problem is investigated for a class of complex dynamical networks subject to the hybrid cyber-attacks. The following are the primary features of this article:

1. A novel adaptive-event-triggered filter error model for complex dynamical networks is first constructed based on the event-triggered scheme, taking into account hybrid cyber-attacks.
2. First time complex dynamical networks under hybrid attacks including deception attack, reply attack and DoS attack is proposed, which is different from existing works.<sup>32,33,38</sup>
3. In contrast to previous work,<sup>16,41</sup> the adaptive event-triggered filter design is proposed, which adaptively modifies the communication threshold level in order to reduce the effect of the transmission load.
4. For filtering error systems with a defined extended dissipative performance, efficient mean-square asymptotically stable criteria are achieved using the LMI technique.
5. Finally, the tunnel diode circuit application is employed to show the usefulness and efficiency of the obtained theoretical results.

**Notations:** Throughout the paper,  $R^l$  is the  $l$ -dimensional Euclidean space and  $\mathbb{R}^{l \times m}$  denotes the set of all  $l \times m$  real matrices.  $\otimes$  is kronecker product.  $I$  is an identity matrix with proper dimension.  $\|\cdot\|$  stands for the Euclidean vector norm. The symmetric elements of the symmetric matrix are represented by  $*$ . If  $\mathcal{A}$  is a positive definite matrix, then  $\mathcal{A} > 0$ . The expectation of  $\omega$  is indicated by  $\mathbf{E}\{\omega\}$ .

## 2 | PRELIMINARIES AND MODEL FORMULATIONS

Filtering-based complex dynamical networks under hybrid cyber attacks are examined in this paper. The proposed complex dynamical networks take the following form:

$$\begin{cases} \dot{\varphi}_i(\rho) = D\varphi_i(\rho) + Ew_i(\rho) + (1 - \eta(\rho))\sum_{j=1}^N \lambda_{ij} \Upsilon \varphi_j(\rho) + \eta(\rho)\sum_{j=1}^N \bar{\lambda}_{ij} \bar{\Upsilon} \varphi_j(\rho - a(\rho)), \\ \xi_i(\rho) = F\varphi_i(\rho), \\ z_i(\rho) = G\varphi_i(\rho). \end{cases} \quad (1)$$

where  $z_i(\rho)$  is actual output vector,  $\varphi_i(\rho) \in \mathbb{R}^n$  is the state vector,  $\xi_i(\rho)$  is the measurement of the sensor, and  $w(\rho)$  represents the external disturbance.  $G, F, E$  and  $D$  are constant matrices.  $\Upsilon$  and  $\bar{\Upsilon}$  are diagonal matrices representing non-delayed and delayed states of inner coupling matrices, respectively.  $\lambda_{ij}$  and  $\bar{\lambda}_{ij}$  are non-delayed and delayed states of outer coupling matrices, respectively. Let  $\Lambda = [\lambda_{ij}]_{N \times N}$ ,  $\bar{\Lambda} = [\bar{\lambda}_{ij}]_{N \times N}$ . The continuous function  $a(\rho)$  denotes the time-varying delay that satisfies the conditions:  $0 \leq a_1 \leq a(\rho) \leq a_2$  and  $\dot{a}(\rho) < \nu < 1$ . Where  $a_1, a_2, a(\rho)$  and  $\nu$  are known real constant matrices.

The filter is constructed as follows,

$$\begin{cases} \dot{\varphi}_f(\rho) = D_f \varphi_f(\rho) + E_f \tilde{\xi}(\rho), \\ z_f(\rho) = C_f \varphi_f(\rho) \end{cases} \quad (2)$$

where  $\tilde{\xi}(\rho)$  is the real input of the filter,  $z_f(\rho)$  is the estimation of  $z(\rho)$ , and  $\varphi_f(\rho)$  is the filter state. The filtering parameters  $E_f, D_f, C_f$  will be designed later.

A sensor, as we all know, monitors the condition information of a plant. However, data supplied by the sensor is frequently limited due to the influence of its own hardware and environment. The following restriction function is addressed in this article:

$$\bar{\xi}(\rho) = \begin{cases} \xi_{max}, & \text{if } \xi(\rho) > \xi_{max} \\ \xi(\rho), & \text{if } -\xi_{max} \leq \xi(\rho) \leq \xi_{max} \\ -\xi_{max}, & \text{if } \xi(\rho) < -\xi_{max}. \end{cases} \quad (3)$$

Inspired by Reference 44, the signal can be written as,

$$\bar{\xi}(\rho) = \xi(\rho) - \pi(\xi(\rho)), \quad (4)$$

$\phi(\xi(\rho))$  is a non-linear function and also satisfies the condition for  $\nu \in (0, 1)$ :

$$\nu \xi^T(\rho) \xi(\rho) \geq \phi^T(\xi(\rho)) \rho(\xi(\rho)). \quad (5)$$

An adaptive ET scheme is used to solve resource restrictions by taking into consideration the restricted network resources. The triggering conditions scheme is given by,

$$\rho_{k+1} h = \rho_k h + \inf_{p \geq 1} \{ p h | e^T(\rho_k h) \Theta e(\rho_k h) - \rho(\rho) \xi^T(\rho_k^p h) \Theta \xi(\rho_k^p h) \geq 0 \} \quad (6)$$

where  $h$  denotes sampling period,  $\rho_k h$  represents the triggering instants, and  $\Theta > 0$  is the weighting matrix to be designed.  $\rho_k^p h = \rho_k h + p h$  and  $e(\rho_k h) = \xi(\rho_k h) - \xi(\rho_k h + p h)$  represents the threshold error,  $p = 1, 2, \dots$ , and  $\rho(\rho)$  is given trigger threshold, then the adaptive law satisfies the following requirements:

$$\dot{\rho}(\rho) = \frac{\delta}{\rho(\rho)} \left[ \frac{1}{\rho(\rho)} - \pi \right] e^T(\rho) \Theta e(\rho) \quad (7)$$

with  $\delta > 0$  and  $0 < \rho(\rho) \leq 1$ .

*Remark 1.* Adaptive ETS is introduced, which is determined by a varying threshold  $\rho(\rho)$ . The adaptive law allows the release frequency to be adjusted by changing the threshold parameter. Furthermore, when the system is stable, the adaptive law (7) states that  $\rho(\rho)$  will remain constant, suggesting that the system will regress to the traditional event-triggered scheme. The adaptive event-triggered scheme switches to the time-triggered scheme if  $\rho(\rho) = 0$ .

Similar to the analysis in Reference 28, the interval  $[\rho_k h + v_{\rho_k}, \rho_{k+1} h + v_{\rho_{k+1}})$  can be divided into several subintervals. which can be expressed as  $[\rho_k h + v_{\rho_k}, \rho_{k+1} h + v_{\rho_{k+1}}) = \bigcup_{i=0}^{\theta} [\rho_k^i h + v_{\rho_k}, \rho_k^i h + h + v_{\rho_{k+1}})$ . The equivalent network-induced delay is  $v_{\rho_k}$ ,  $\theta = \rho_{k+1} - \rho_k - 1$ . Define  $b(\rho) = \rho - \rho_k^p h$ , and the range is simple to obtain if  $b(\rho)$  as  $0 \leq v_{\rho_k} \leq b(\rho) \leq h + b_{\rho_{k+q+1}} \triangleq b_N$ .

The signal sent to the network can then be described as

$$\hat{\xi}(\rho) = \bar{\xi}(\rho - b(\rho)) + e(\rho). \quad (8)$$

where  $e(\rho) = \text{col}_N\{e_i(\rho)\}$ .

When data is sent over a network, the first thing to consider is a deception attack. The deception attack signal  $\hat{h}(\cdot)$  which satisfies Assumption 1 can completely replace the normal transmission data in this work. Let  $\gamma(\rho)$  is a stochastic Bernoulli variable, which is represent the occurring of deception attacks and its expectation, variance is denoted as  $\mathbf{E}\{\gamma(\rho)\} = \bar{\gamma}$ ,  $\mathbf{E}\{(\gamma(\rho) - \bar{\gamma})^2\} = \tau_1^2$ , respectively.

The data sent as an outcome of the deception attack can then be written as

$$\xi_1(\rho) = (1 - \gamma(\rho))\hat{h}(\xi(\rho - c(\rho))) + \gamma(\rho)\hat{\xi}(\rho), \quad (9)$$

where,  $\hat{\xi}(\rho)$  denoted as the data transferred through the network.

**Assumption 1** (44). The attackers' modified nonlinear function  $\hat{h}(\xi(\rho))$  satisfies:

$$\|\hat{h}(\xi(\rho))\|_2 \leq \|H\xi(\rho)\|_2.$$

where  $H$  is a given matrix.

*Remark 2.* From (9), one knows that if  $\gamma(\rho) = 0$ , it means that deception attacks occur, with  $\hat{h}(\xi(\rho - c(\rho)))$  replacing the transmitted signal. If  $\gamma(\rho) = 1$ , it indicates that the network is secure of deception attacks, such that  $\xi_1(\rho) = \hat{\xi}(\rho)$ .

When a replay attack happens, the data transmitted is replaced with previously transmitted data. Bernoulli variable  $\vartheta(\rho)$  is introduced to represent the uncertainty of attack occurrences. Then, the signal sent through the network can be rewritten as

$$\xi_2(\rho) = (1 - \vartheta(\rho))\xi_o(\rho) + \vartheta(\rho)\xi_1(\rho). \quad (10)$$

where  $\xi_o(\rho) = \hat{\xi}(\rho - o(\rho))$  is the replay signal. The  $o(\rho)$  denotes the data that has been replayed from the previous  $o(\rho)$  seconds. Let  $\vartheta(\rho)$  is a stochastic Bernoulli variable, which is represent the occurring of replay attacks. Its expectation  $\mathbf{E}\{\vartheta(\rho)\} = \bar{\vartheta}$  and the variance  $\mathbf{E}\{(\vartheta(\rho) - \bar{\vartheta})^2\} = \tau_2^2$ . It should be noted that the data taken by attackers is usually limited in time, that is,  $o(\rho)$  has an upper bound  $o_N$ . Then we have  $0 < o(\rho) < o_N$ .

**Assumption 2** (34). The transmission data are supposed to be recorded from  $\rho_0$  to the present instant  $\rho$ , and then data from any random instant in the series is taken for replay.

Finally DoS attack is taken into consideration when the communication network transmits the measurement signals. The signal is generally transmitted to the filter when the DoS attack is sleeping. While the DoS attack is occurring, no signal can be transferred to the filter. The data that has been vulnerable to a DoS attack can be written as follows:

$$\xi_3(\rho) = \begin{cases} \xi_2(\rho), & \rho \in [x_n, d_n) \\ 0, & \rho \in [d_n, x_{n+1}), \end{cases} \quad (11)$$

where  $x_n$  denotes the start time of the DoS attack, when it enters sleeping for the  $n$ th time.  $t_n$  indicates the duration of the  $n$ th sleeping.  $d_n$  is the ending instant of the  $n$ th sleeping, that is,  $d_n = x_n + t_n$ , and the condition  $0 < x_0 < x_1 < d_1 < x_2 < d_2 < \dots < d_n$  is satisfied.

**Assumption 3** (37). Assume that there exist uniform bounds on the lengths of the DoS sleeping and active periods, respectively, as:

$$\begin{cases} t_{min} \leq \inf_{n \in \mathbb{N}} \{t_n\} \\ y_{max} \geq \sup_{n \in \mathbb{N}} \{x_n - d_{n-1}\}. \end{cases} \quad (12)$$

**Assumption 4** (37). Let  $d(\rho)$  denote the number of DoS attack sleep/active transitions occurring in the interval  $[0, \rho)$ . For given real number  $\varpi > h$ ,  $\mu_1 \geq 0$ , the DoS frequency  $d(\rho)$  satisfies the following condition:

$$d(\rho) \leq \mu_1 + \frac{\rho}{\varpi}. \quad (13)$$

The actual input filter can be obtained by combining (9)–(11), as

$$\tilde{\xi}(\rho) = \begin{cases} (1 - \vartheta(\rho))\xi_r(\rho) + \vartheta(\rho)(1 - \gamma(\rho))\hat{h}(\xi(\rho - c(\rho))) + \gamma(\rho)\vartheta(\rho)\hat{\xi}(\rho), & \rho \in [x_n, d_n), \\ 0, & \rho \in [d_n, x_{n+1}). \end{cases} \quad (14)$$

*Remark 3.* In this article, we use the attack detection methods to determine the attacks occur and its sequence, then adjusting the corresponding control strategies.

By combining (2), (4) and (15), we can obtain

$$\dot{\varphi}(\rho) = \begin{cases} D_f \varphi_f(\rho) + E_f[(1 - \vartheta(\rho))\xi_o(\rho) + (1 - \gamma(\rho))\beta(\rho)\hat{h}(\xi(\rho - c(\rho))) + \gamma(\rho)\vartheta(\rho)F\varphi(\rho - o(\rho)) \\ + \gamma(\rho)\vartheta(\rho)e(\rho) - \gamma(\rho)\vartheta(\rho)\rho(u(\rho - o(\rho)))], & \rho \in [x_n, d_n). \\ 0, & \rho \in [d_n, x_{n+1}). \end{cases} \quad (15)$$

Define  $\varpi(\rho) = \begin{bmatrix} \varphi(\rho) \\ \varphi_f(\rho) \end{bmatrix}$ ,  $\bar{z}(\rho) = z(\rho) - z_f(\rho)$ .

The filtering error based complex dynamical system can be written in the following compact form using the Kronecker product:

$$\dot{\varpi}(\rho) = \begin{cases} \bar{D}\varpi(\rho) + \bar{E}w(\rho) + (1 - \vartheta(\rho))\bar{B}L\varpi(\rho - o(\rho)) + (1 - \vartheta(\rho))\bar{B}e(\rho - o(\rho)) \\ + (1 - \gamma(\rho))\vartheta(\rho)\bar{B}\hat{h}(\xi(\rho - c(\rho))) + \gamma(\rho)\vartheta(\rho)\bar{F}(L(\varpi(\rho - o(\rho))) + e(\rho)) \\ - \gamma(\rho)\vartheta(\rho)\bar{B}\pi(\xi(\rho - d(\rho))) + \bar{D}\varpi(\rho - a(\rho)), & \rho \in [x_n, d_n), \\ \bar{D}\varpi(\rho) + \bar{E}w(\rho) & \rho \in [d_n, x_{n+1}) \end{cases} \quad (16)$$

$$\bar{z} = \bar{G}\varpi(\rho)$$

where  $\bar{D} = \begin{bmatrix} (I \otimes D) + (1 - \bar{\eta})(\Lambda \otimes Y) & 0 \\ * & (I \otimes D_f) \end{bmatrix}$ ,  $\bar{E} = \begin{bmatrix} (I \otimes E) \\ 0 \end{bmatrix}$ ,  $\bar{F} = \begin{bmatrix} 0 \\ (I \otimes E_f F) \end{bmatrix}$ ,  $\bar{L} = [I \ 0]$ ,  $\bar{B} = [0 \ (I \otimes E_f)]$ ,  $\bar{G} = [I \otimes G \quad -(I \otimes F_f)]$ ,  $\bar{K} = \begin{bmatrix} \eta(\bar{\Lambda} \otimes \bar{Y}) & 0 \\ * & 0 \end{bmatrix}$ .

**Definition 1** (24). For given real matrices  $\Psi_1 = \Psi_1^T \leq 0$ ,  $\Psi_2, \Psi_3 = \Psi_3^T > 0$ ,  $\Psi_4 = \Psi_4^T$ , and a known constant  $\alpha \in [0, 1]$ , the over all-closed loop system (16) achieves extended dissipativity with mean-square asymptotically stable, if for any  $\mathcal{X} > 0$  and non-zero  $w(\pi) \in \mathcal{L} \in [0, \infty)$  the following inequality satisfies:

$$\int_0^\infty \Psi(\Psi_1, \Psi_2, \Psi_3, \rho) d\rho \geq \alpha \sup_{0 \leq \rho \leq \mathcal{X}} \bar{z}^T(\rho) \Psi_4 \bar{z}(\rho) \quad (17)$$

where,  $\Psi(\Psi_1, \Psi_2, \Psi_3, \rho) \triangleq (1 - \alpha) \left[ \bar{z}^T(\rho) \Psi_1 \bar{z}(\rho) + 2\bar{z}^T(\rho) \Psi_2 w(\rho) \right] + w^T(\rho) \Psi_3 w(\rho)$ .

*Remark 4.* The condition (17) can be degradedly into a standard dissipative property index (or)  $l_2 - l_\infty$  performance index (or)  $H_\infty$  performance index (or) passive property index by tuning  $\alpha$ ,  $\Psi_1$ ,  $\Psi_2$ ,  $\Psi_3$  and  $\Psi_4$ . The

details are given as follows: (1) Let  $\alpha = 0$  and  $\Psi_1 = 0$  the condition (17) could turn into a passive property index, if  $\Psi_2 = \Psi_2^T \leq 0$  and  $\Psi_3^T = \Psi_3 = \tilde{\gamma}^2 I > 0$ . (2) Let  $\alpha = 0$  and  $\Psi_2 = 0$  the condition (17) could turn into a  $H_\infty$  performance index, if  $\Psi_1 = \Psi_1^T \leq 0$  and  $\Psi_3^T = \Psi_3 = \tilde{\gamma}^2 I > 0$ . (3) Let  $\alpha = 0$ , the condition (17) could turn into a  $l_2 - l_\infty$  performance index, if  $\Psi_3^T = \Psi_3 = \tilde{\gamma}^2 I > 0$ . (4) Let  $\alpha = 0$ , the condition (17) could turn into a standard dissipative property index, if  $\Psi_1 = \Psi_1^T \leq 0$  and  $\Psi_3^T = \Psi_3 = \tilde{\gamma}^2 I > 0$ .

**Lemma 1** (15). For any positive definite matrix  $O_1 \in \mathbb{R}^{n \times n}$ , scalar  $a_1 > 0$  and vector function  $\varpi(\rho) \in \mathbb{R}^n$ , the following integral inequality holds:

$$-a_1 \int_{\rho-a_1}^{\rho} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr \leq \begin{bmatrix} \varpi(\rho) \\ \varpi(\rho - a_1) \end{bmatrix}^T \begin{bmatrix} -(I \otimes O_{11}) & * \\ (I \otimes O_{11}) & -(I \otimes O_{11}) \end{bmatrix} \begin{bmatrix} \varpi(\rho) \\ \varpi(\rho - a_1) \end{bmatrix}. \quad (18)$$

**Lemma 2** (15). Suppose that  $\varpi(\rho) \in \mathbb{R}^n$ ,  $0 \leq a_1 \leq a(\rho) \leq a_2$  and  $0 \leq \beta \leq 1$ , then for any positive definite matrix  $O_1$  the following inequality holds:

$$(a_2 - a_1) \int_{\rho-a_2}^{\rho-a_1} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr \leq \begin{bmatrix} \varpi(\rho - a_1) \\ \varpi(\rho - a(\rho)) \\ \varpi(\rho - a_2) \end{bmatrix}^T \begin{bmatrix} \kappa_1 & \kappa_2 & 0 \\ * & \kappa_3 & \kappa_4 \\ * & * & \kappa_5 \end{bmatrix} \begin{bmatrix} \varpi(\rho - a_1) \\ \varpi(\rho - a(\rho)) \\ \varpi(\rho - a_2) \end{bmatrix}. \quad (19)$$

where,  $\kappa_1 = -(I \otimes O_1) - (1 - \beta)(I \otimes O_1)$ ,  $\kappa_2 = (I \otimes O_1) + (1 - \beta)(I \otimes O_1)$ ,  $\kappa_3 = -2(I \otimes O_1) - \beta(I \otimes O_1) - (1 - \beta)(I \otimes O_1)$ ,  $\kappa_4 = (I \otimes O_1) + \beta(I \otimes O_1)$ ,  $\kappa_5 = -(I \otimes O_1) - \beta(I \otimes O_1)$ .

**Lemma 3** (10). For a matrix  $J > 0$ ,  $0 \leq c_1 \leq c(\rho) \leq c_N$  and any known matrix  $F$  with proper dimension, which satisfy  $\begin{bmatrix} J & F \\ * & J \end{bmatrix} \geq 0$ , the following inequality holds.

$$-c_N \int_{\rho-c_N}^{\rho} \dot{\varpi}^T(r)(I \otimes J)\dot{\varpi}(r)dr \leq \begin{bmatrix} \varpi(\rho) - \varpi(\rho - c(\rho)) \\ \varpi(\rho - c(\rho)) - \varpi(\rho - c_N) \end{bmatrix}^T \begin{bmatrix} -(I \otimes J) & -(I \otimes F_1) \\ * & -(I \otimes J) \end{bmatrix} \begin{bmatrix} \varpi(\rho) - \varpi(\rho - c(\rho)) \\ \varpi(\rho - c(\rho)) - \varpi(\rho - c_N) \end{bmatrix}. \quad (20)$$

**Lemma 4.** Let  $\mathcal{M}$  and  $\mathcal{X}$  be a positive definite matrices and  $\epsilon$  be a scalar, the following inequality holds:

$$-\mathcal{M}\mathcal{X}^{-1}\mathcal{M} \leq -2\epsilon\mathcal{M} + \epsilon^2\mathcal{X}. \quad (21)$$

### 3 | MAIN RESULTS

**Theorem 1.** For given positive scalar  $\epsilon_j$ ,  $j = \{1, 2, 3, 4\}$  and  $\bar{\eta}(\rho), \bar{\vartheta}(\rho), \bar{\gamma}(\rho) \in [0, 1]$ . The filter parameters are  $D_f, C_f, F_f$ . The complex dynamical system (16) under the adaptive event triggered filter design subject to hybrid-cyber-attacks is asymptotically stable with extended dissipative performance, if the positive definite matrices  $P_z, V_{zx}, Q_{zx}, R_{zx}, W_{zz}, M_{zz}, O_{zz}$  ( $z = 1, 2$ ), ( $x = 1, 2, 3$ ) with the proper dimension and event-triggered weighting matrix  $\Theta$  exist, such that the following conditions hold.

$$\alpha \bar{G}^T \Psi_4 \bar{G} - P_1 < 0, \quad (22)$$

$$\Pi = \begin{bmatrix} \Gamma_{17 \times 17} & \mathfrak{R}_1 W_{11} & \mathfrak{R}_2 W_{12} & \mathfrak{R}_3 R_{11} & \mathfrak{R}_4 R_{12} & \mathfrak{R}_5 R_{13} \\ * & -W_{11} & 0 & 0 & 0 & 0 \\ * & * & -W_{12} & 0 & 0 & 0 \\ * & * & * & -R_{13} & 0 & 0 \\ * & * & * & * & -R_{12} & 0 \\ * & * & * & * & * & -R_{13} \end{bmatrix} < 0, \quad (23)$$

With

$$\begin{aligned}
\Gamma_{1,1} &= (I \otimes V_{11}) + (I \otimes V_{12}) + (I \otimes V_{13}) + (I \otimes W_{11}) + (I \otimes T_{11}) + (I \otimes M_{11}) - (I \otimes O_{11}) - (I \otimes R_{11}) + \\
&F_1^T H^T H F_1 + 2(I \otimes P_1) \bar{D} - (1 - \alpha) \bar{G}^T \bar{G}, \Gamma_{1,2} = (I \otimes O_{11}), \Gamma_{1,3}^1 = (I \otimes P_1) \bar{K}, \Gamma_{1,6}^1 = (I \otimes F_{11}), \Gamma_{1,5} = -(I \otimes F_1) + \\
&(I \otimes R_{11}), \Gamma_{1,7} = -(I \otimes F_2) + \bar{\gamma}(\rho) \bar{\vartheta}(\rho) (I \otimes P_1) \bar{F} L, \Gamma_{1,9} = -(I \otimes F_3) + (I \otimes P_1) \bar{B} L, \Gamma_{1,8} = (I \otimes F_2), \Gamma_{1,10} = \\
&(I \otimes F_3), \Gamma_{1,11} = \bar{\gamma}(\rho) \bar{\vartheta}(\rho) (I \otimes P_1) \bar{F}, \Gamma_{1,12}^1 = (1 - \bar{\vartheta}(\rho)) (I \otimes P_1) \bar{B}, \Gamma_{1,14} = (1 - \gamma(\rho)) \bar{\vartheta}(\rho) (I \otimes P_1) \bar{B}, \Gamma_{1,16} = -\bar{\gamma}(\rho) \\
&\bar{\vartheta}(\rho) (I \otimes F_2 P_1) \bar{B}, \Gamma_{1,17} = -(1 - \alpha) (I \otimes \bar{G}^T) \Psi_2, \Gamma_{2,2} = -(I \otimes V_{11}) - 2(I \otimes O_{11}) - (1 - \beta) (I \otimes O_{11}), \Gamma_{2,3} = (I \otimes O_{11}) + \\
&(1 - \beta) (I \otimes O_{11}), \Gamma_{3,3} = -(1 - v_1) (I \otimes V_{12}) - (I \otimes O_{11}) - \beta (I \otimes O_{11}), \Gamma_{3,4} = (I \otimes O_{11}) + \beta (I \otimes O_{11}), \Gamma_{4,4} = -(I \otimes \\
&V_{13}) - (I \otimes O_{11}) - \beta (I \otimes O_{11}), \Gamma_{5,6} = (I \otimes R_{11}) - (I \otimes F_1), \Gamma_{5,5} = -(1 - v_2) (I \otimes W_{11}) + (1 - v_2) (I \otimes W_{12}) - 2(I \otimes R_{11}) \\
&+ 2(I \otimes F_1), \Gamma_{6,6} = -(I \otimes W_{12}) - (I \otimes R_{11}), \Gamma_{7,7} = -(1 - v_3) (I \otimes T_{11}) + (1 - v_3) (I \otimes T_{12}) - 2(I \otimes R_{12}) + 2(I \otimes F_2) \\
&+ \zeta F_1^T F_1 + L^T \Theta L, \Gamma_{7,8} = (I \otimes R_{12}) - (I \otimes F_2), \Gamma_{8,8} = -(I \otimes T_{12}) - (I \otimes R_{12}), \Gamma_{12,12}^1 = -(I \otimes Q_{11}), \Gamma_{9,9} = -(1 - v_4) \\
&(I \otimes T_{11}) + (1 - v_4) (I \otimes T_{12}) - 2(I \otimes R_{13}) + 2(I \otimes F_3), \Gamma_{9,10} = (I \otimes R_{13}) - (I \otimes F_3), \Gamma_{10,10} = -(I \otimes M_{12}) - (I \otimes R_{13}), \\
\Gamma_{11,11} &= -(I \otimes \pi \Theta) + (I \otimes Q_1), \Gamma_{13,13} = -I + (I \otimes Q_{12}), \Gamma_{14,14} = -(I \otimes Q_{12}), \Gamma_{15,15} = -I + (I \otimes Q_{13}), \Gamma_{16,16} = \\
&(I \otimes Q_{13}), \Gamma_{17,17} = -\Psi_3, \mathfrak{R}_1 = a_1 \mathfrak{S}, \mathfrak{R}_2 = (a_2 - a_1) \mathfrak{S}, \mathfrak{R}_3 = c_N \mathfrak{S}, \mathfrak{R}_4 = b_N \mathfrak{S}, \mathfrak{R}_5 = o_N \mathfrak{S} \text{ and } \mathfrak{S} = [\bar{D} \ 0 \ \bar{K} \ 0 \ 0 \\
&0 \ \gamma(\rho) \bar{\vartheta}(\rho) \bar{F} L \ 0 \ (1 - \bar{\vartheta}(\rho)) \bar{B} L \ 0 \ \gamma(\rho) \bar{\vartheta}(\rho) \bar{F} \ (1 - \bar{\vartheta}(\rho)) \bar{B} \ 0 \ (1 - \gamma(\rho)) \bar{\vartheta}(\rho) \bar{B} \ 0 \ -\bar{\gamma}(\rho) \bar{\vartheta}(\rho) \bar{B} \ \bar{E}].
\end{aligned}$$

*Proof.* Choose the Lyapunov–Krasovskii functional candidate as follows:

$$\begin{aligned}
V_1^j(\rho) &= \varpi^T(\rho) (I \otimes P_j) \varpi(\rho), \\
V_2^j(\rho) &= \int_{\rho-a_1}^{\rho} \varpi^T(r) (I \otimes V_{j1}) \varpi(r) dr + \int_{\rho-a(\rho)}^{\rho} \varpi^T(r) (I \otimes V_{j2}) \varpi(r) dr + \int_{\rho-a_2}^{\rho} \varpi^T(r) (I \otimes V_{j3}) \varpi(r) dr, \\
V_3^j(\rho) &= \int_{\rho-c(\rho)}^{\rho} \varpi^T(r) (I \otimes W_{j1}) \varpi(r) dr + \int_{\rho-c_N}^{\rho-c(\rho)} \varpi^T(r) (I \otimes W_{j2}) \varpi(r) dr, \\
V_4^j(\rho) &= \int_{\rho-b(\rho)}^{\rho} \varpi^T(r) (I \otimes T_{j1}) \varpi(r) dr + \int_{\rho-b_N}^{\rho-b(\rho)} \varpi^T(r) (I \otimes T_{j2}) \varpi(r) dr, \\
V_5^j(\rho) &= \int_{\rho-o(\rho)}^{\rho} \varpi^T(r) (I \otimes M_{j1}) \varpi(r) dr + \int_{\rho-o_N}^{\rho-o(\rho)} \varpi^T(r) (I \otimes M_{j2}) \varpi(r) dr, \\
V_6^j(\rho) &= a_1 \int_{-a_1}^0 \int_{\rho+s}^{\rho} \dot{\varpi}^T(r) (I \otimes O_{j1}) \dot{\varpi}(r) dr ds + (a_2 - a_1) \int_{-a_2}^{-a_1} \int_{\rho+s}^{\rho} \dot{\varpi}^T(r) (I \otimes O_{j1}) \dot{\varpi}(r) dr ds, \\
V_7^j(\rho) &= c_N \int_{-c_N}^0 \int_{\rho+s}^{\rho} \dot{\varpi}^T(r) (I \otimes R_{j1}) \dot{\varpi}(r) dr ds + b_N \int_{-b_N}^0 \int_{\rho+s}^{\rho} \dot{\varpi}^T(r) (I \otimes R_{j2}) \dot{\varpi}(r) dr ds \\
&+ o_N \int_{-o_N}^0 \int_{\rho+s}^{\rho} \dot{\varpi}^T(r) (I \otimes R_{j3}) \dot{\varpi}(r) dr ds, \\
V_8^j(\rho) &= \int_{\rho-o(\rho)}^{\rho} e^T(r) (I \otimes Q_{j1}) e(r) dr + \int_{\rho-c(\rho)}^{\rho} \bar{h}^T(\xi(r)) (I \otimes Q_{j2}) \bar{h}(\xi(r)) dr \\
&+ \int_{\rho-b(\rho)}^{\rho} \phi^T(\xi(r)) (I \otimes Q_{j3}) \phi(\xi(r)) dr.
\end{aligned}$$

where,

$$j = \begin{cases} 1, & \rho \in [-h, 0] \cup (\cup_{n \in \mathbb{N}} [x_n, d_n]), \\ 2, & \rho \in \cup_{n \in \mathbb{N}} [d_n, x_{n+1}]. \end{cases}$$

When  $j = 1$ , calculating the time derivative of  $V(\rho)$  along the system's trajectory (16), we obtain to

$$\mathbf{E}(\dot{V}_1^1(\rho)) = \varpi^T(\rho) (I \otimes P_1) \dot{\varpi}(\rho) + \dot{\varpi}(\rho) (I \otimes P_1) \varpi^T(\rho), \quad (24)$$

$$\begin{aligned}
\mathbf{E}(\dot{V}_2^1(\rho)) &= \varpi^T(\rho) [(I \otimes V_{11}) + (I \otimes V_{12}) + (I \otimes V_{13})] \varpi(\rho) - \varpi^T(\rho - a_1) (I \otimes V_{11}) \varpi(\rho - a_1) \\
&- (1 - v_1) \varpi^T(\rho - a(\rho)) (I \otimes V_{12}) \varpi(\rho - a(\rho)) - \varpi^T(\rho - a_2) (I \otimes V_{13}) \varpi(\rho - a_2), \quad (25)
\end{aligned}$$

$$\begin{aligned}
\mathbf{E}(\dot{V}_3^1(\rho)) &= \varpi^T(\rho) (I \otimes W_{11}) \varpi(\rho) - (1 - v_2) \varpi^T(\rho - c(\rho)) (I \otimes W_{11}) \varpi(\rho - c(\rho)) \\
&+ (1 - v_2) \varpi^T(\rho - c(\rho)) (I \otimes W_{12}) \varpi(\rho - c(\rho)) - \varpi^T(\rho - c_N) (I \otimes W_{12}) \varpi(\rho - c_N), \quad (26)
\end{aligned}$$

$$\begin{aligned} \mathbf{E}(\dot{V}_4^1(\rho)) &= \varpi^T(\rho)(I \otimes T_{11})\varpi(\rho) - (1 - v_3)\varpi^T(\rho - b(\rho))(I \otimes T_{11})\varpi(\rho - b(\rho)) \\ &\quad + (1 - v_3)\varpi^T(\rho - b(\rho))(I \otimes T_{12})\varpi(\rho - b(\rho)) - \varpi^T(\rho - b_N)(I \otimes T_{12})\varpi(\rho - b_N), \end{aligned} \tag{27}$$

$$\begin{aligned} \mathbf{E}(\dot{V}_5^1(\rho)) &= \varpi^T(\rho)(I \otimes M_{11})\varpi(\rho) - (1 - v_4)\varpi^T(\rho - o(\rho))(I \otimes M_{11})\varpi(\rho - o(\rho)) \\ &\quad + (1 - v_4)\varpi^T(\rho - o(\rho))(I \otimes M_{12})\varpi(\rho - o(\rho)) - \varpi^T(\rho - o_N)(I \otimes M_{12})\varpi(\rho - o_N), \end{aligned} \tag{28}$$

$$\begin{aligned} \mathbf{E}(\dot{V}_6^1(\rho)) &= \dot{\varpi}^T(\rho)[a_1^2(I \otimes O_{11}) + (a_2 - a_1)^2(I \otimes O_{11})]\dot{\varpi}(\rho) - a_1 \int_{\rho-a_1}^{\rho} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr \\ &\quad - (a_2 - a_1) \int_{\rho-a_2}^{\rho-a_1} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr. \end{aligned} \tag{29}$$

By applying Lemmas 1 and 2, we get

$$-a_1 \int_{\rho-a_1}^{\rho} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr \leq \begin{bmatrix} \varpi(\rho) \\ \varpi(\rho - a_1) \end{bmatrix}^T \begin{bmatrix} -\kappa_{11} & * \\ \kappa_{11} & -\kappa_{11} \end{bmatrix} \begin{bmatrix} \varpi(\rho) \\ \varpi(\rho - a_1) \end{bmatrix} \tag{30}$$

$$(a_2 - a_1) \int_{\rho-a_2}^{\rho-a_1} \dot{\varpi}^T(r)(I \otimes O_{11})\dot{\varpi}(r)dr \leq \begin{bmatrix} \varpi(\rho - a_1) \\ \varpi(\rho - a(\rho)) \\ \varpi(\rho - a_2) \end{bmatrix}^T \begin{bmatrix} \kappa_1 & \kappa_2 & 0 \\ * & \kappa_3 & \kappa_4 \\ * & * & \kappa_5 \end{bmatrix} \begin{bmatrix} \varpi(\rho - a_1) \\ \varpi(\rho - a(\rho)) \\ \varpi(\rho - a_2) \end{bmatrix}. \tag{31}$$

where,  $\kappa_{11} = (I \otimes O_{11})$ ,  $\kappa_1 = -(I \otimes O_1) - (1 - \beta)(I \otimes O_1)$ ,  $\kappa_2 = (I \otimes O_1) + (1 - \beta)(I \otimes O_1)$ ,  $\kappa_3 = -2(I \otimes O_1) - \beta(I \otimes O_1) - (1 - \beta)(I \otimes O_1)$ ,  $\kappa_4 = (I \otimes O_1) + \beta(I \otimes O_1)$ ,  $\kappa_5 = -(I \otimes O_1) - \beta(I \otimes O_1)$ .

$$\begin{aligned} \mathbf{E}(\dot{V}_7^1(\rho)) &= c_N^2 \dot{\varpi}^T(\rho)(I \otimes R_{11})\dot{\varpi}(\rho) - c_N \int_{\rho-c_N}^{\rho} \dot{\varpi}^T(r)(I \otimes R_{11})\dot{\varpi}(r)dr + b_N^2 \dot{\varpi}^T(\rho)(I \otimes R_{12})\dot{\varpi}(\rho) \\ &\quad + o_N^2 \dot{\varpi}^T(\rho)(I \otimes R_{13})\dot{\varpi}(\rho) - d_N \int_{\rho-d_N}^{\rho} \dot{\varpi}^T(r)(I \otimes R_{12})\dot{\varpi}(r)dr \\ &\quad - o_N \int_{\rho-o_N}^{\rho} \dot{\varpi}^T(r)(I \otimes R_{13})\dot{\varpi}(r)dr. \end{aligned} \tag{32}$$

By applying Lemma 3, we get

$$\begin{aligned} &-c_N \int_{\rho-c_N}^{\rho} \dot{\varpi}^T(r)(I \otimes R_{11})\dot{\varpi}(r)dr \\ &\leq \begin{bmatrix} \varpi(\rho) - \varpi(\rho - c(\rho)) \\ \varpi(\rho - c(\rho)) - \varpi(\rho - c_N) \end{bmatrix}^T \begin{bmatrix} -(I \otimes R_{11}) & -(I \otimes F_1) \\ * & -(I \otimes R_{11}) \end{bmatrix} \begin{bmatrix} \varpi(\rho) - \varpi(\rho - c(\rho)) \\ \varpi(\rho - c(\rho)) - \varpi(\rho - c_N) \end{bmatrix}, \end{aligned} \tag{33}$$

$$\begin{aligned} &-b_N \int_{\rho-b_N}^{\rho} \dot{\varpi}^T(r)(I \otimes R_{12})\dot{\varpi}(r)dr \\ &\leq \begin{bmatrix} \varpi(\rho) - \varpi(\rho - b(\rho)) \\ \varpi(\rho - b(\rho)) - \varpi(\rho - b_N) \end{bmatrix}^T \begin{bmatrix} -(I \otimes R_{12}) & -(I \otimes F_1) \\ * & -(I \otimes R_{12}) \end{bmatrix} \begin{bmatrix} \varpi(\rho) - \varpi(\rho - b(\rho)) \\ \varpi(\rho - b(\rho)) - \varpi(\rho - b_N) \end{bmatrix}, \end{aligned} \tag{34}$$

$$\begin{aligned} &-o_N \int_{\rho-o_N}^{\rho} \dot{\varpi}^T(r)R_{13}\dot{\varpi}(r)dr \\ &\leq \begin{bmatrix} \varpi(\rho) - \varpi(\rho - o(\rho)) \\ \varpi(\rho - o(\rho)) - \varpi(\rho - o_N) \end{bmatrix}^T \begin{bmatrix} -(I \otimes R_{13}) & -(I \otimes F_3) \\ * & -(I \otimes R_{13}) \end{bmatrix} \begin{bmatrix} \varpi(\rho) - \varpi(\rho - o(\rho)) \\ \varpi(\rho - o(\rho)) - \varpi(\rho - o_N) \end{bmatrix}, \end{aligned} \tag{35}$$

$$\begin{aligned} \mathbf{E}(\dot{V}_8^1(\rho)) &= e^T(\rho)(I \otimes Q_{11})e(\rho) - e^T(\rho - o(\rho))(I \otimes Q_{11})e(\rho - o(\rho)) + \dot{h}^T(\xi(\rho))(I \otimes Q_{12})\dot{h}(\xi(\rho)) \\ &\quad - \dot{h}^T(\xi(\rho - c(\rho)))(I \otimes Q_{12})\dot{h}(\xi(\rho - c(\rho))) + \phi^T(\xi(\rho))(I \otimes Q_{13})\phi(\xi(\rho)) \\ &\quad - \phi^T(\xi(\rho - b(\rho)))(I \otimes Q_{13})\phi(\xi(\rho - b(\rho))). \end{aligned} \tag{36}$$



It is possible to obtain the input constraint from (5) as

$$\zeta \varpi^T(\rho - b(\rho))(I \otimes F_1^T F_1) \varpi(\rho - b(\rho)) - \phi^T(\xi(\rho)) \phi(\xi(\rho)) > 0. \quad (37)$$

From the Equations (6) and (7), adaptive event-triggered scheme we get

$$\varpi^T(\rho - b(\rho))(I \otimes L^T) \Theta (I \otimes L) \varpi(\rho - b(\rho)) - \pi e^T(t) \Theta e(t) > 0. \quad (38)$$

From the Assumption 1, we have

$$\varpi^T(\rho)(I \otimes F_1^T H^T)(I \otimes H F_1) \varpi(\rho) - \tilde{h}^T(\xi(\rho)) h(\xi(\rho)) \geq 0. \quad (39)$$

Denote  $\zeta^T(\rho) = \left[ \varpi^T(\rho) \ \varpi^T(\rho - a_1) \ \varpi^T(\rho - a(\rho)) \ \varpi^T(\rho - a_2) \ \varpi^T(\rho - c(\rho)) \ \varpi^T(\rho - c_N) \ \varpi^T(\rho - b(\rho)) \ \varpi^T(\rho - b_N) \ \varpi^T(\rho - o(\rho)) \ \varpi^T(\rho - o_N) \ e^T(\rho) \ e^T(\rho - o(\rho)) \ \tilde{h}^T(\xi(\rho)) \ \tilde{h}^T(\xi(\rho - c(\rho))) \ \phi^T(\xi(\rho)) \ \phi^T(\xi(\rho - b(\rho))) \ w^T(\rho) \right]$ . Furthermore combining (24)–(39), we have

$$\mathbf{E}\{\dot{V}^1(\rho) - \Psi(\Psi_1, \Psi_2, \Psi_3, \rho)\} \leq \zeta^T(\rho) \Pi \zeta(\rho). \quad (40)$$

According to (23), for any small scalar  $\varepsilon > 0$  under the initial condition, the following inequality holds

$$\mathbf{E}\{\dot{V}^1(\rho) - \Psi(\Psi_1, \Psi_2, \Psi_3, \rho)\} \leq -\varepsilon_1 \|\xi(\rho)\|^2, \quad (41)$$

In this similarly, when  $j = 2$  then we can obtain

$$\mathbf{E}\{\dot{V}^2(\rho) - \Psi(\Psi_1, \Psi_2, \Psi_3, \rho)\} \leq -\varepsilon_2 \|\xi(\rho)\|^2,$$

which means  $\lim_{\rho \rightarrow \infty} \|\xi(\rho)\| = 0$ . Therefore, based on Definition (2.4), the over all closed loop system (16) is mean-square asymptotically stable.

Furthermore, it follows from (41) that

$$\mathbf{E}\{\dot{V}^j(\rho) - (1 - \alpha)[\bar{z}^T(\rho) \Psi_1 \bar{z}^T(\rho) + 2\bar{z}^T(\rho) w(\rho)] - w^T(\rho) \Psi_3 w(\rho)\} \leq 0. \quad (42)$$

Let  $\zeta = 0$ , for any  $0 < \pi < S$  under zero initial conditions, we have

$$\begin{aligned} \mathbf{E}\{\dot{V}^j(\pi)\} &= \mathbf{E}\{\Psi(\Psi_1, \Psi_2, \Psi_3, \pi)\}, \\ 0 &\leq \mathbf{E}\{V^j(\pi)\} \leq \mathbf{E}\left\{\int_0^\theta \Psi(\Psi_1, \Psi_2, \Psi_3, \pi) d\pi\right\}. \end{aligned}$$

This means that

$$0 \leq \mathbf{E}\{\xi^T(\pi) \mathcal{O} \xi(\pi)\} \leq \mathbf{E}\left\{\int_0^X \Psi(\Psi_1, \Psi_2, \Psi_3, \pi) d\pi\right\},$$

which implies

$$0 \leq \mathbf{E}\left\{\int_0^X \Psi(\Psi_1, \Psi_2, \Psi_3, \pi) d\pi\right\}. \quad (43)$$

Let  $\alpha = 1$ , for any  $0 < \rho < X$ , we can get

$$\mathbf{E}\{V^j(\rho)\} \leq \mathbf{E}\left\{\int_0^\rho w^T(\pi) \Psi_3 w(\pi) d\pi\right\} \leq \mathbf{E}\left\{\int_0^X w^T(\pi) \Psi_3 w(\pi) d\pi\right\},$$

which means

$$\mathbf{E}\{\xi^T(\pi)\mathcal{P}\xi(\pi)\} \leq \mathbf{E}\left\{\int_0^X w^T(\pi)\Psi_3 w(\pi)d\rho\right\},$$

then, from (23), we get

$$\begin{aligned} \mathbf{E}\{\alpha G^T\Psi_4 G - P\} &\leq 0, \\ \mathbf{E}\{\alpha\xi^T(\rho)G^T\Psi_4 G\xi(\rho) - \xi^T(\rho)P\xi(\rho)\} &\leq 0. \end{aligned} \quad (44)$$

It means that

$$\mathbf{E}\{\alpha\bar{z}^T(\rho)\Psi_4\bar{z}(\rho)\} \leq \mathbf{E}\left\{\int_0^X w^T(\rho)\Psi_3 w(\rho)d\rho\right\}. \quad (45)$$

The entire closed-loop system (16) is mean-square asymptotically stable and satisfy an extended dissipative performance, according to Definition 2. The proof of Theorem 1 is therefore completed. ■

**Theorem 2.** For given positive scalar  $\epsilon_j$ ,  $j = \{1, 2, 3, 4\}$  and  $\bar{\eta}(\rho), \bar{\vartheta}(\rho), \bar{\gamma}(\rho) \in [0, 1]$ .  $D_f, C_f, F_f$  are filter parameters. The complex dynamical system (16) under the adaptive event triggered filter design subject to hybrid-cyber-attacks is asymptotically stable with extended dissipative performance, if positive definite matrices  $P_z, V_{zx}, Q_{zx}, R_{zx}, W_{zz}, M_{zz}, O_{zz}$  ( $z = 1, 2$ ), ( $x = 1, 2, 3$ ) with proper dimension and event-triggered weighting matrix  $\Theta$  exist, such that the following conditions hold.

$$W_i < \epsilon_i P_1, \quad R_\tau < \epsilon_{\tau+2} P_1, \quad (46)$$

where  $i = \{1, 2\}$  and  $\tau = \{1, 2, 3\}$

$$\hat{\Pi} = \begin{bmatrix} \hat{\Gamma}_{17 \times 17} & \hat{\mathfrak{R}}_1 & \hat{\mathfrak{R}}_2 & \hat{\mathfrak{R}}_3 & \hat{\mathfrak{R}}_4 & \hat{\mathfrak{R}}_5 \\ * & -W_{11} & 0 & 0 & 0 & 0 \\ * & * & -W_{12} & 0 & 0 & 0 \\ * & * & * & -R_{13} & 0 & 0 \\ * & * & * & * & -R_{12} & 0 \\ * & * & * & * & * & -R_{13} \end{bmatrix} < 0, \quad (47)$$

$$\begin{aligned} \text{With } \hat{\Gamma}_{1,1} &= \begin{bmatrix} \ell_1 & \ell_2 \\ * & \ell_3 \end{bmatrix}, \ell_1 = (I \otimes V_{11}) + (I \otimes V_{12}) + (I \otimes V_{13}) + (I \otimes W_{11}) + (I \otimes T_{11}) + (I \otimes M_{11}) \\ &\quad - (I \otimes O_{11}) - (I \otimes R_{11}) + F_1^T H^T H F_1 + 2P_1 D + 2P_1(1 - \bar{\eta})(\Lambda \otimes \Upsilon) - (1 - \alpha)G^T G, \ell_2 = -GX^{-1}M_3, \ell_3 = \\ &\quad (I \otimes V_{11}) + (I \otimes V_{12}) + (I \otimes V_{13}) + (I \otimes W_{11}) + (I \otimes T_{11}) + (I \otimes M_{11}) - (I \otimes O_{11}) - (I \otimes R_{11}) \\ &\quad + F_1^T H^T H F_1 + 2M_1 + (1 - \alpha)[\epsilon^2 I - 2\epsilon X^{-1}M_3], \hat{\Gamma}_{1,2} = \begin{bmatrix} (I \otimes O_1) & 0 \\ * & (I \otimes O_1) \end{bmatrix}, \hat{\Gamma}_{1,3} = \begin{bmatrix} (\bar{\Lambda} \otimes \bar{\Upsilon})\bar{\eta}P_1 & 0 \\ * & 0 \end{bmatrix}, \\ \hat{\Gamma}_{1,5} &= \begin{bmatrix} -F_1 + W_1 & 0 \\ * & -F_1 + W_1 \end{bmatrix}, \hat{\Gamma}_{1,6} = \begin{bmatrix} F_1 & 0 \\ * & F_1 \end{bmatrix}, \hat{\Gamma}_{1,7} = \begin{bmatrix} -F_2 & 0 \\ \bar{\gamma}(\rho)\bar{\vartheta}(\rho)M_2 F & -F_2 \end{bmatrix}, \hat{\Gamma}_{1,8} = \begin{bmatrix} F_2 & 0 \\ * & F_2 \end{bmatrix}, \\ \hat{\Gamma}_{1,9} &= \begin{bmatrix} -F_3 & 0 \\ (1 - \bar{\vartheta}(\rho))M_2 & -F_3 \end{bmatrix}, \hat{\Gamma}_{1,10} = \begin{bmatrix} F_3 & 0 \\ * & F_3 \end{bmatrix}, \hat{\Gamma}_{1,11} = \begin{bmatrix} 0 & 0 \\ \bar{\gamma}(\rho)\bar{\vartheta}(\rho)M_2 F & 0 \end{bmatrix}, \\ \hat{\Gamma}_{1,12} &= \begin{bmatrix} 0 & 0 \\ (1 - \bar{\vartheta}(\rho))M_2 & 0 \end{bmatrix}, \hat{\Gamma}_{1,14} = \begin{bmatrix} 0 & 0 \\ (1 - \gamma(\rho))\bar{\vartheta}(\rho)M_2 & 0 \end{bmatrix}, \hat{\Gamma}_{1,16} = \begin{bmatrix} 0 & 0 \\ -\gamma(\rho)\bar{\vartheta}(\rho)M_2 & 0 \end{bmatrix}, \\ \hat{\Gamma}_{1,17} &= \begin{bmatrix} -(1 - \alpha)L^T & (1 - \alpha)X^{-1}M_3 \\ 0 & 0 \end{bmatrix}, \hat{\Gamma}_{2,2} = \begin{bmatrix} \ell_4 & 0 \\ 0 & \ell_4 \end{bmatrix}, \hat{\Gamma}_{2,3} = \begin{bmatrix} \ell_5 & 0 \\ 0 & \ell_5 \end{bmatrix}, \hat{\Gamma}_{3,3} = \begin{bmatrix} \ell_6 & 0 \\ 0 & \ell_6 \end{bmatrix}, \\ \hat{\Gamma}_{3,4} &= \begin{bmatrix} O_1 + \beta O_1 & 0 \\ 0 & O_1 + \beta O_1 \end{bmatrix}, \hat{\Gamma}_{4,4} = \begin{bmatrix} \ell_7 & 0 \\ 0 & \ell_7 \end{bmatrix}, \hat{\Gamma}_{5,5} = \begin{bmatrix} \ell_8 & 0 \\ 0 & \ell_8 \end{bmatrix}, \hat{\Gamma}_{5,6} = \begin{bmatrix} \ell_9 & 0 \\ 0 & \ell_9 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}\hat{\Gamma}_{6,6} &= \begin{bmatrix} \ell_{10} & 0 \\ 0 & \ell_{10} \end{bmatrix}, \hat{\Gamma}_{7,7} = \begin{bmatrix} \ell_{11} & 0 \\ 0 & \ell_{11} \end{bmatrix}, \hat{\Gamma}_{7,8} = \begin{bmatrix} \ell_{12} & 0 \\ 0 & \ell_{12} \end{bmatrix}, \hat{\Gamma}_{8,8} = \begin{bmatrix} \ell_{13} & 0 \\ 0 & \ell_{13} \end{bmatrix}, \hat{\Gamma}_{9,9} = \begin{bmatrix} \ell_{14} & 0 \\ 0 & \ell_{14} \end{bmatrix}, \\ \hat{\Gamma}_{9,10} &= \begin{bmatrix} \ell_{15} & 0 \\ 0 & \ell_{15} \end{bmatrix}, \hat{\Gamma}_{10,10} = \begin{bmatrix} \ell_{16} & 0 \\ 0 & \ell_{16} \end{bmatrix}, \hat{\Gamma}_{11,11} = \begin{bmatrix} \ell_{17} & 0 \\ 0 & \ell_{17} \end{bmatrix}, \hat{\Gamma}_{13,13} = \begin{bmatrix} \ell_{18} & 0 \\ 0 & \ell_{18} \end{bmatrix}, \\ \hat{\Gamma}_{12,12} &= \begin{bmatrix} -(I \otimes Q_{11}) & 0 \\ 0 & -(I \otimes Q_{11}) \end{bmatrix}, \hat{\Gamma}_{14,14} = \begin{bmatrix} -(I \otimes Q_{12}) & 0 \\ 0 & -(I \otimes Q_{12}) \end{bmatrix}, \hat{\Gamma}_{15,15} = \begin{bmatrix} \ell_{19} & 0 \\ 0 & \ell_{19} \end{bmatrix}, \\ \hat{\Gamma}_{17,17} &= \begin{bmatrix} -\Psi_3 & 0 \\ 0 & -\Psi_3 \end{bmatrix}, \hat{\Gamma}_{16,16} = \begin{bmatrix} (I \otimes Q_{13}) & 0 \\ 0 & (I \otimes Q_{13}) \end{bmatrix},\end{aligned}$$

$\ell_4 = -(I \otimes V_{11}) - 2(I \otimes O_{11}) - (1 - \beta)O_1$ ,  $\ell_5 = (I \otimes O_{11}) + (1 - \beta)O_1$ ,  $\ell_6 = -(1 - v_1)(I \otimes V_{12}) - 2(I \otimes O_{11}) - \beta O_1$ ,  $\ell_7 = -(I \otimes V_{13}) - (I \otimes O_{11}) - \beta O_1$ ,  $\ell_8 = -(1 - v_2)(I \otimes W_{11}) + (1 - v_2)(I \otimes W_{12}) - 2(I \otimes R_{11}) + 2(I \otimes F_1)$ ,  $\ell_9 = (I \otimes R_{11}) - (I \otimes F_1)$ ,  $\ell_{10} = -(I \otimes W_{12}) - (I \otimes R_{11})$ ,  $\ell_{11} = -(1 - v_3)(I \otimes T_{11}) + (1 - v_3)(I \otimes T_{12}) - 2(I \otimes R_{12}) + 2(I \otimes F_2) + \zeta F_1^T F_1 + L^T \Theta L$ ,  $\ell_{12} = (I \otimes R_{12}) - (I \otimes F_1)$ ,  $\ell_{13} = -(I \otimes T_{12}) - (I \otimes R_{12})$ ,  $\ell_{14} = -(1 - v_4)(I \otimes T_{11}) + (1 - v_4)(I \otimes T_{12}) - 2(I \otimes R_{13}) + 2(I \otimes F_3)$ ,  $\ell_{15} = (I \otimes R_{13}) - (I \otimes F_3)$ ,  $\ell_{16} = -(I \otimes M_{12}) - (I \otimes R_{13})$ ,  $\ell_{17} = -(I \otimes \pi \Theta) + (I \otimes Q_{11})$ ,  $\ell_{18} = -I + (I \otimes Q_{12})$ ,  $\ell_{19} = -I + (I \otimes Q_{13})$ . Define  $\mathfrak{R}_i = \Delta_i \nabla_i \mathfrak{F}$  here  $i = \{1, 2, \dots, 5\}$ . Where  $\Delta_1 = a_1$ ,  $\Delta_2 = (a_2 - a_1)$ ,  $\Delta_3 = c_N$ ,  $\Delta_4 = b_N$ ,  $\Delta_5 = o_N$  and  $\nabla_1 = \epsilon_1 P_1$ ,  $\nabla_2 = \epsilon_2 P_1$ ,  $\nabla_3 = \epsilon_3 P_1$ ,  $\nabla_4 = \epsilon_4 P_1$ ,  $\nabla_5 = \epsilon_5 P_1$ .  $\mathfrak{F}$  is defined in Theorem 1.

The parameters of the designed filter can be obtained by,

$$D_f = M_1 P_1^{-1}, E_f = M_2 P_1^{-1}, F_f = M_2 \mathcal{X}^{-1}. \quad (48)$$

*Proof.* Define

$$\begin{cases} P_1 D_f = M_1 \\ P_2 E_f = M_2 \\ \mathcal{X} F_f = M_3. \end{cases} \quad (49)$$

Substituting (49) in Equation (23) and using Lemma 4.5, we get

$$\Gamma + \mathfrak{F}^T(\rho) [a_1^2 O_1 + (a_2 - a_1)^2 O_1 + c_N^2 W_1 + b_N^2 W_2 + d_N^2 W_1] \mathfrak{F}(\rho) < 0 \quad (50)$$

By using Schur complement, (50) is equals to (47). Hence the proof is accomplished. ■

## 4 | NUMERICAL EXAMPLES

The effectiveness of the proposed approach is illustrated using numerical examples.

**Example 1.** Consider a complex dynamical networks (16) based on adaptive event-triggered filter method subject to hybrid cyber-attacks with the following parameters:

$$\begin{aligned}D &= \begin{bmatrix} -3.2 & 0.4 \\ 0.6 & -2 \end{bmatrix}, E = \begin{bmatrix} -1.3 & 0.5 \\ 0.3 & -0.6 \end{bmatrix}, F = \begin{bmatrix} -1.3 & 0.4 \\ 0.4 & -1.3 \end{bmatrix}, G = \begin{bmatrix} -2.7 & 0.9 \\ 0.45 & -1.7 \end{bmatrix}, \\ H &= \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, X = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.5 \end{bmatrix},\end{aligned}$$

Choose the parameters  $\epsilon = 3.12$ ,  $\epsilon_1 = 5$ ,  $\epsilon_2 = 2$ ,  $\epsilon_3 = 2$ ,  $\epsilon_4 = 2$ ,  $a_1 = 3$ ,  $a_2 = 3$ ,  $\mu = 0.16$ ,  $\mu_1 = 0.2$ ,  $\mu_2 = 0.3$ ,  $\mu_3 = 0.5$ ,  $\mu_4 = 0.3$ ,  $F_1 = 0.25$ ,  $c_N = 0.08$ ,  $b_N = 0.05$ ,  $o_N = 0.06$ ,  $\pi = 0.3$ ,  $\gamma(\rho) = 0.2$ ,  $\bar{\vartheta}(\rho) = 0.3$ ,  $\bar{\eta}(\rho) = 0.5$

The inner coupling matrices are taken as  $\Upsilon = \text{diag}\{0.7, 0.7\}$ ,  $\bar{\Upsilon} = \text{diag}\{1, 1\}$ . The outer coupling matrices are

$$\Lambda = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \bar{\Lambda} = \begin{bmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix}.$$

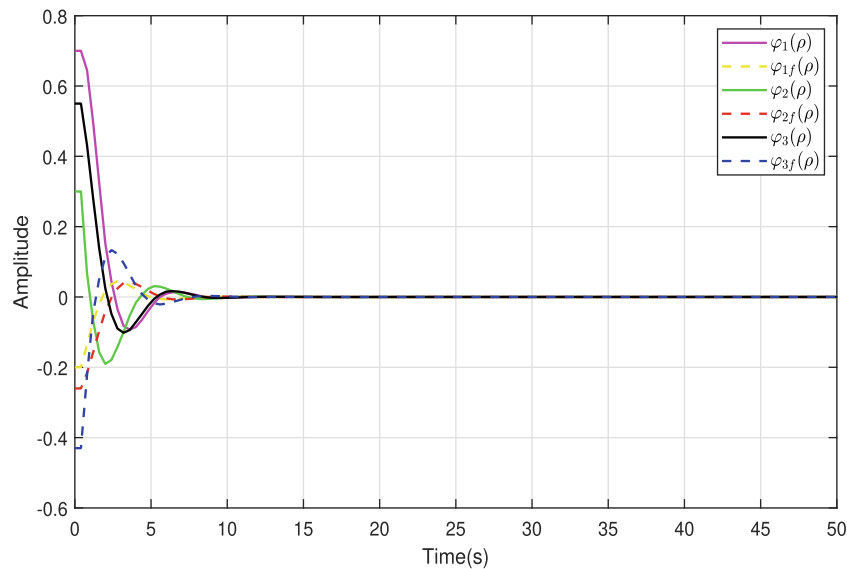
The deception attack signal chosen as  $h(\xi(\rho)) = \tanh(0.38\xi(\rho))$  and disturbance signal described as  $w(\rho) = \begin{cases} 1, & 5 \leq \rho \leq 8, \\ 0, & \text{else} \end{cases}$

**Case 1:** Let  $\Psi_1 = 0, \Psi_2 = 1, \Psi_3 = 1$  the condition (21) can be converted into a passive performance index. The filter parameters are determined by solving the criteria stated in Theorem 2 as follows:

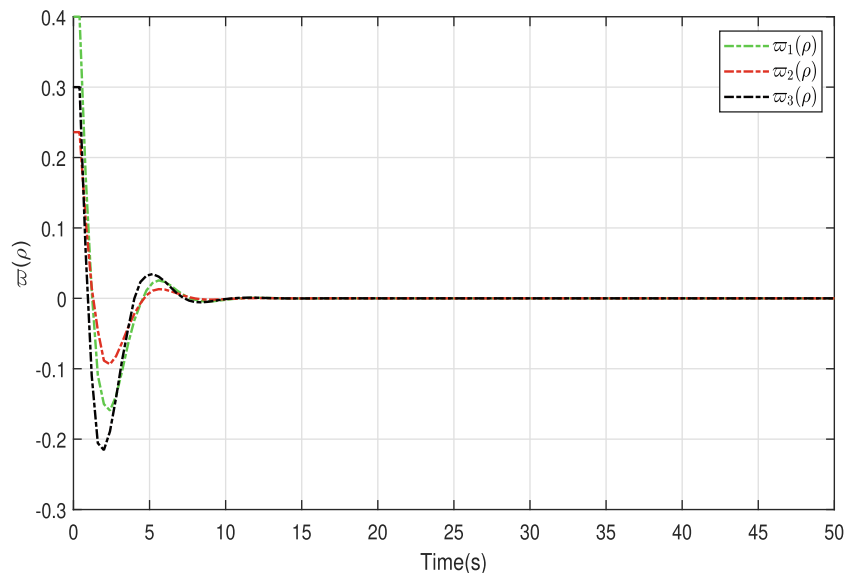
$$D_f = 10^{-7} \begin{bmatrix} -0.0788 & 0.2828 \\ 0.2828 & 0.7950 \end{bmatrix}, E_f = 10^{-4} \begin{bmatrix} 0.0386 & 0.1165 \\ 0.1165 & 0.3332 \end{bmatrix}, F_f = \begin{bmatrix} -0.6468 & 0.2214 \\ 0.2214 & -0.2867 \end{bmatrix},$$

and event-triggered weighting matrix  $\Lambda = 13.0094$ .

Choose the initial state conditions to be  $\varphi_1(0) = [0.7 \quad -0.7]^T, \varphi_{1f}(0) = [-0.2 \quad 0.3]^T, \varphi_2(0) = [0.3 \quad -0.2]^T, \varphi_{2f}(0) = [-0.26 \quad 0.5]^T, \varphi_3(0) = [0.67 \quad -0.3]^T, \varphi_{3f}(0) = [-0.43 \quad 0.2]^T$ . Using the filter design and the hybrid attacks into account, In Figures 1 and 2 illustrates the state trajectories  $\varphi_i(\rho)$  and its estimates  $\varphi_{if}(\rho)$ , respectively. The adaptive threshold parameter value  $\rho(\rho)$  is depicted in Figure 3. Adaptive ETS is introduced, which is determined by a



**FIGURE 1** The state trajectories of  $\varphi_i(\rho)$  and its estimate  $\varphi_{if}(\rho)$ , ( $i = 1, 2, 3$ ).



**FIGURE 2** The filtering error responses  $w_i(\rho)$ .

varying threshold  $\varrho(\rho)$ . The adaptive law allows the release frequency to be adjusted by changing the threshold parameter. The Bernoulli variable  $\gamma(\rho)$  represents the occurrence of the deception attack depicted in Figure 4. The Bernoulli variable  $\bar{\delta}(\rho)$  represents the occurrence of the reply attack depicted in Figure 5. In Figure 6, shows the occurrence of DoS attacks. Figure 7, shows the release instants and intervals of adaptive event-triggered schemes, which demonstrates the effectiveness of the proposed methodology.

**Case 2:** Let  $\Psi_1 = -1, \Psi_2 = 0, \Psi_3 = 1$  the condition (21) can be converted into a dissipative performance index. The filter parameters are determined by solving the criteria stated in Theorem 2 as follows:

$$D_f = 10^{-4} * \begin{bmatrix} -0.0276 & -0.0248 \\ -0.0248 & -0.1619 \end{bmatrix}, E_f = 10^{-3} \begin{bmatrix} -0.0880 & -0.1284 \\ -0.1284 & -0.3851 \end{bmatrix}, F_f = \begin{bmatrix} -2.7874 & -0.7496 \\ -0.7496 & -4.7355 \end{bmatrix}.$$

and event-triggered weighting matrix  $\Lambda = 4.8509$ .

Choose the initial state conditions to be  $\varphi_1(0) = [0.4 \quad -0.4]^T, \varphi_{1f}(0) = [-0.8 \quad 0.4]^T, \varphi_2(0) = [0.75 \quad -0.3]^T, \varphi_{2f}(0) = [-0.38 \quad 0.6]^T, \varphi_3(0) = [0.3 \quad -0.5]^T, \varphi_{3f}(0) = [-0.43 \quad 0.8]^T$ . Using the filter design and the hybrid attacks into account, In Figures 8 and 9 illustrates the state trajectories  $\varphi_i(\rho)$  and its estimates  $\varphi_{if}(\rho)$ , respectively.

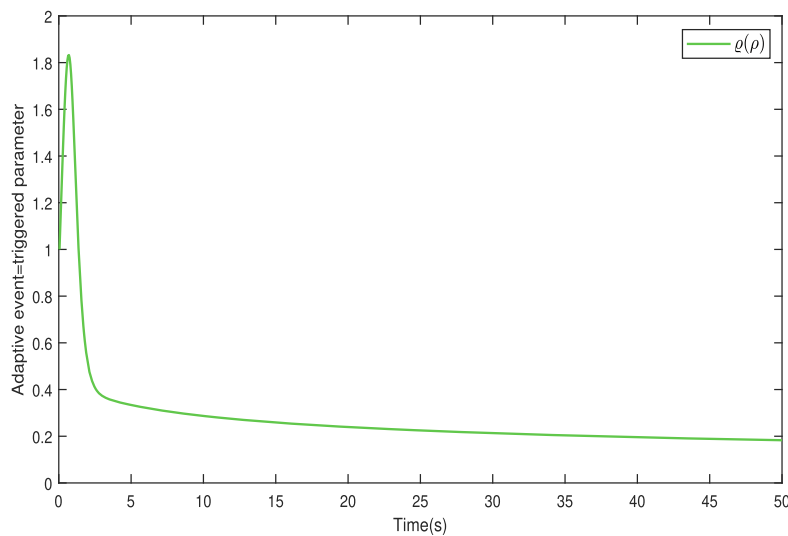


FIGURE 3 Values of the adaptive threshold parameter  $\varrho(\rho)$ .

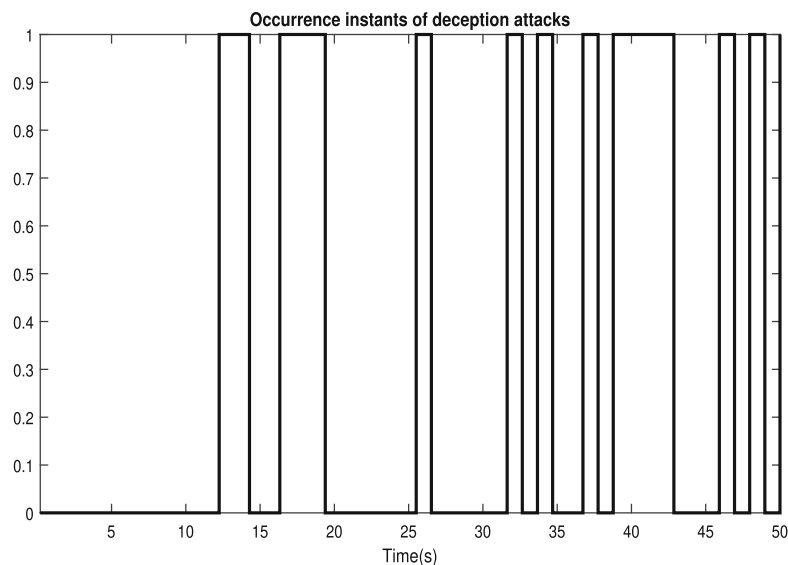


FIGURE 4 Occurrence of deception attacks.

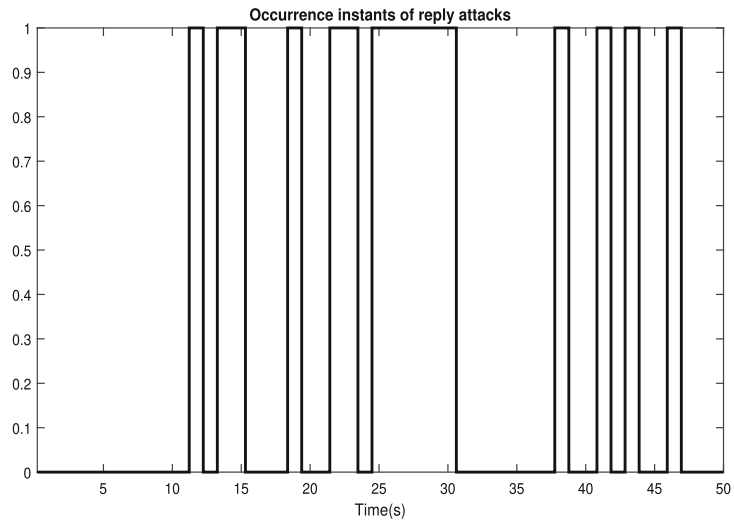


FIGURE 5 Occurrence of reply attacks.

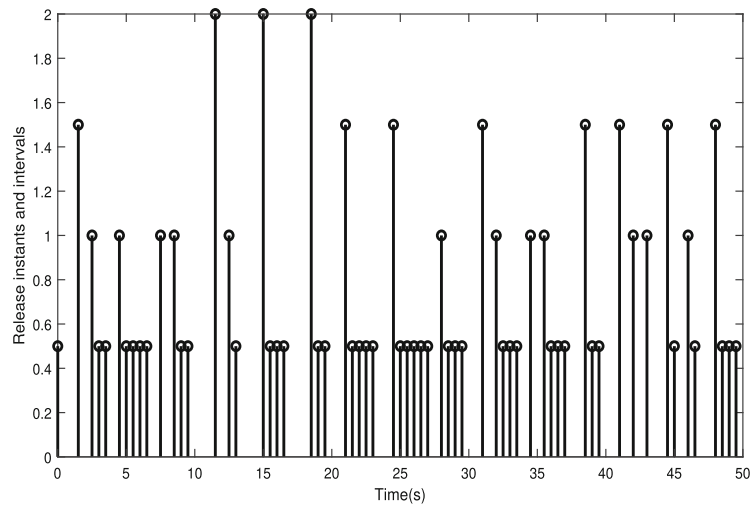


FIGURE 6 Adaptive event-triggered release instants and intervals.

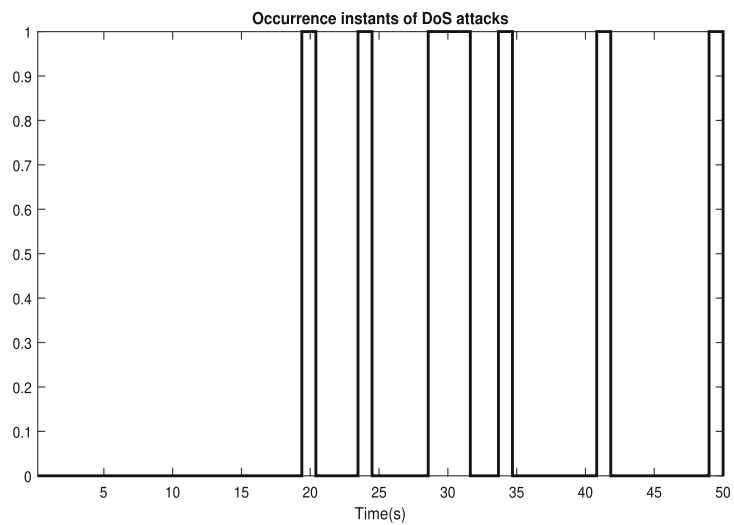


FIGURE 7 Occurrence of DoS attacks.

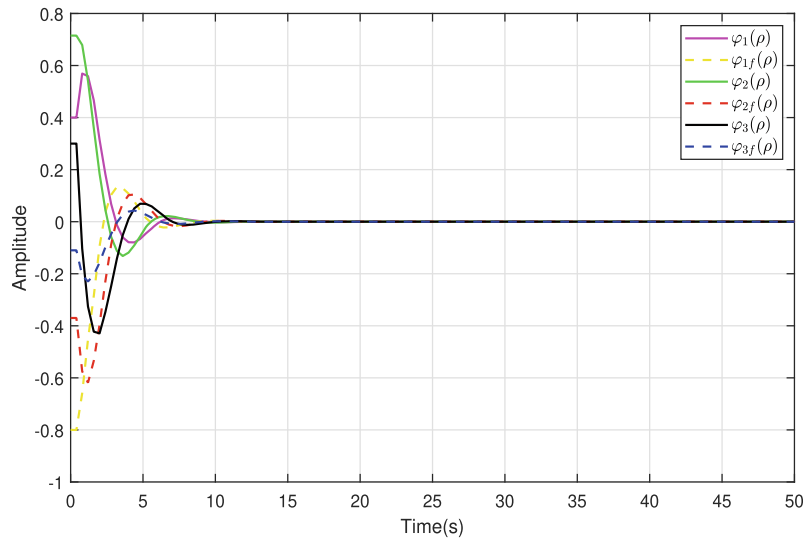


FIGURE 8 The state trajectories of  $\varphi_i(\rho)$  and its estimate  $\varphi_{if}(\rho)$ , ( $i = 1, 2, 3$ ).

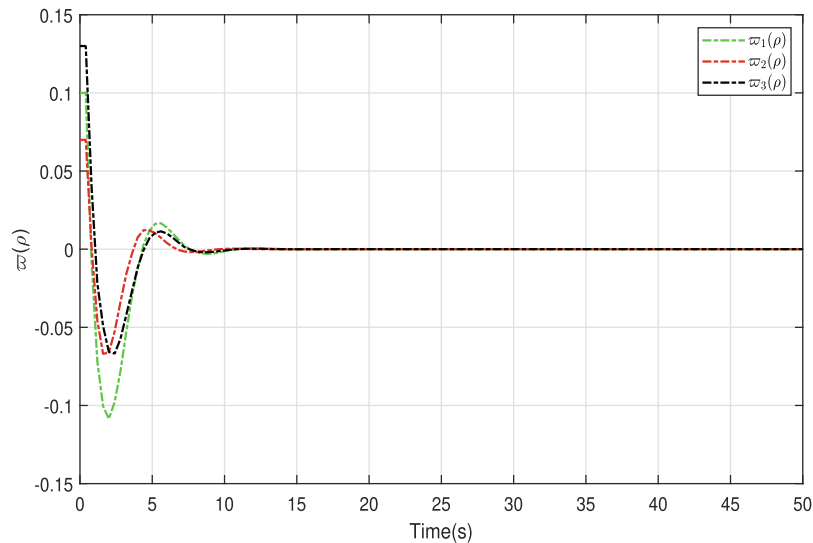


FIGURE 9 The filtering error responses  $w_i(\rho)$ .

The adaptive threshold parameter value  $\rho(\rho)$  is depicted in Figure 10. The Bernoulli variable  $\gamma(\rho)$  represents the occurrence of the deception attack depicted in Figure 11. The Bernoulli variable  $\bar{\gamma}(\rho)$  represents the occurrence of the reply attack depicted in Figure 12. In Figure 13, shows the occurrence of DoS attacks. In Figure 14, shows the release instants and intervals of adaptive event-triggered schemes, which demonstrates the effectiveness of the proposed methodology.

**Example 2.** As shown Figure 15, a tunnel diode circuit system is considered,<sup>18</sup> and expressed as follows.

$$\begin{cases} C\dot{V}_C(\rho) = -\frac{V_C(\rho)}{\mathcal{R}_D} + i_{\mathcal{L}}(\rho), \\ \mathcal{L}i_{\mathcal{L}}(\rho) = -V_C(\rho) - \mathcal{R}_E i_{\mathcal{L}}(\rho) + w(\rho). \end{cases} \quad (51)$$

Let  $\varphi(\rho) = [\varphi_1(\rho) \quad \varphi_2(\rho)]^T = [V_C(\rho) \quad i_{\mathcal{L}}(\rho)]^T$ , then the circuit system can be expressed as

$$\dot{\varphi}(\rho) = \begin{bmatrix} -\frac{1}{C\mathcal{R}_D} & -\frac{1}{C} \\ -\frac{1}{\mathcal{L}} & -\frac{\mathcal{R}_E}{\mathcal{L}} \end{bmatrix} \varphi(\rho) + \begin{bmatrix} 0 \\ \frac{1}{\mathcal{L}} \end{bmatrix} w(\rho)$$

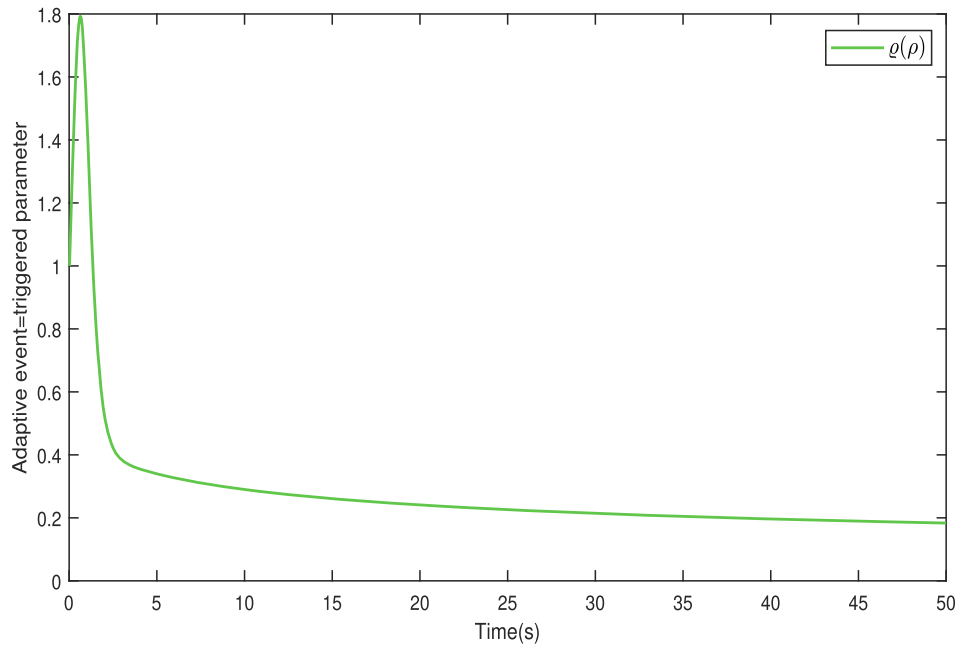


FIGURE 10 Values of the adaptive threshold parameter  $\rho(\rho)$ .

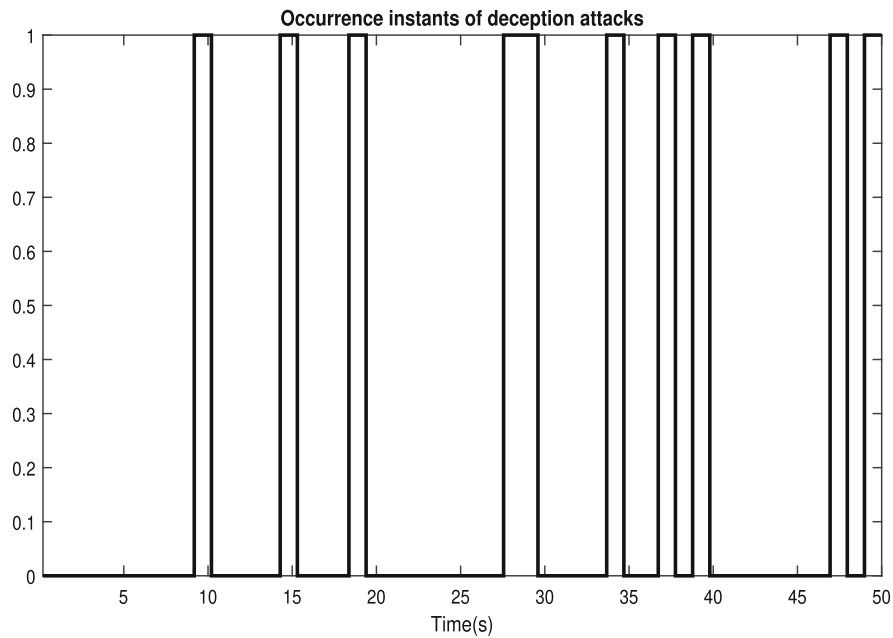


FIGURE 11 Occurrence of deception attacks.

Consider the circuit system in Figure 1 with  $C = 2F$ ,  $\mathcal{R}_\mathcal{E} = 2\Omega$ ,  $\mathcal{L} = 1000mH$  and  $\mathcal{R}_\mathcal{D} = 2\Omega$ . Choose the parameters of the system as  $\epsilon = 2.2$ ,  $\epsilon_1 = 6$ ,  $\epsilon_2 = 2.5$ ,  $\epsilon_3 = 2.5$ ,  $\epsilon_4 = 2.5$ ,  $a_1 = 2$ ,  $a_2 = 2$ ,  $\mu = 0.26$ ,  $\mu_1 = 0.25$ ,  $\mu_2 = 0.35$ ,  $\mu_3 = 0.55$ ,  $\mu_4 = 0.35$ ,  $F_1 = 0.35$ ,  $c_N = 0.18$ ,  $b_N = 0.15$ ,  $o_N = 0.16$ ,  $\pi = 0.35$ ,  $\gamma(\rho) = 0.25$ ,  $\bar{\vartheta}(\rho) = 0.35$ ,  $\bar{\eta}(\rho) = 0.55$

The inner coupling matrices are taken as  $\Upsilon = \text{diag}\{0.7, 0.7\}$ ,  $\bar{\Upsilon} = \text{diag}\{1, 1\}$ . The outer coupling matrices are

$$\Lambda = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \bar{\Lambda} = \begin{bmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & -3 \end{bmatrix}.$$



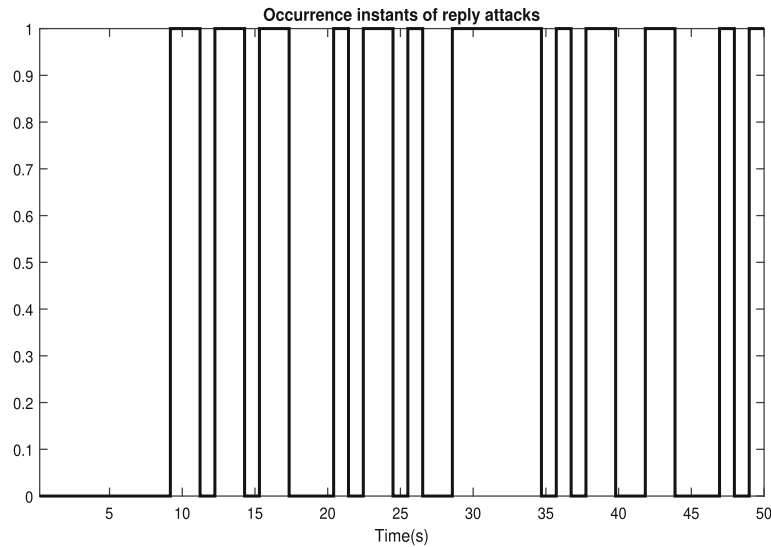


FIGURE 12 Occurrence of reply attacks.

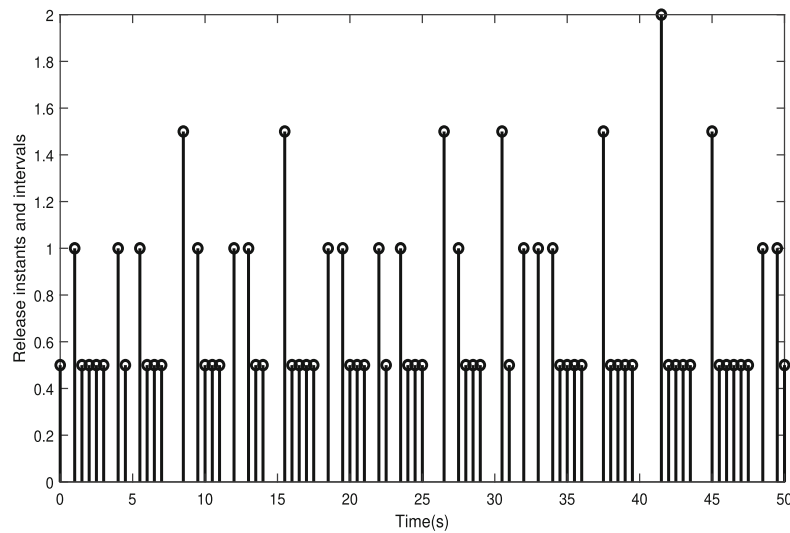


FIGURE 13 Adaptive event-triggered release instants and intervals.

The deception attack signal chosen as  $\hat{h}(\xi(\rho)) = \tanh(0.53\xi(\rho))$  and disturbance signal described as  $w(\rho) = \begin{cases} 1, & 5 \leq \rho \leq 8, \\ 0, & \text{else} \end{cases}$ . Let  $\Psi_1 = -1, \Psi_2 = 0, \Psi_3 = 1$  the condition (21) can be converted into a dissipative performance index. The filter parameters are determined by solving the criteria stated in Theorem 2 as follows:

$$D_f = 10^{-4} * \begin{bmatrix} -3.5843 & -0.0656 \\ -8.7892 & -5.7145 \end{bmatrix}, E_f = 10^{-3} \begin{bmatrix} -0.4307 \\ -0.1408 \end{bmatrix}, F_f = \begin{bmatrix} -6.9004 & -3.7090 \end{bmatrix}.$$

Choose the initial state conditions to be  $\varphi_1(0) = [0.28 \quad -0.3]^T, \varphi_{1f}(0) = [-0.25 \quad 0.3]^T, \varphi_2(0) = [0.39 \quad -0.5]^T, \varphi_{2f}(0) = [-0.5 \quad 0.2]^T, \varphi_3(0) = [0.7 \quad -0.2]^T, \varphi_{3f}(0) = [-0.2 \quad 0.6]^T$ . Using the filter design and the hybrid attacks into account, In Figures 16 and 17 illustrates the state trajectories  $\varphi_i(\rho)$  and its estimates  $\varphi_{if}(\rho)$ , respectively. The adaptive threshold parameter value  $\rho(\rho)$  is depicted in Figure 18. The Bernoulli variable  $\gamma(\rho)$  represents the occurrence of the deception attack depicted in Figure 19. The Bernoulli variable  $\bar{\vartheta}(\rho)$  represents the occurrence of the reply attack depicted in Figure 20. In Figure 21, shows the occurrence of DoS attacks. Figure 22, shows the release instants and intervals of adaptive event-triggered schemes, which demonstrates the effectiveness of the proposed methodology.

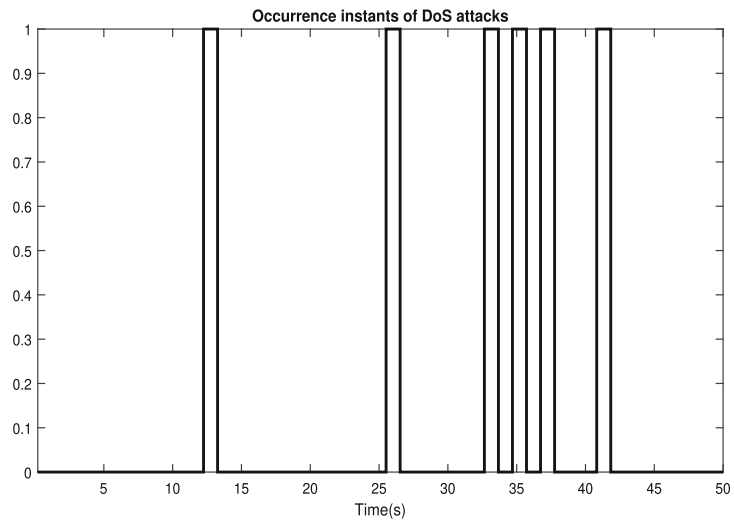


FIGURE 14 Occurrence of DoS attacks.

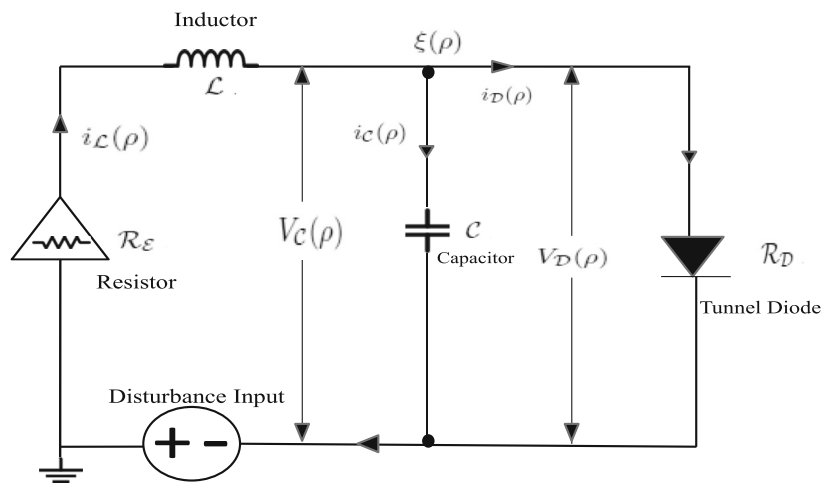


FIGURE 15 Structure of the tunnel diode circuit system.

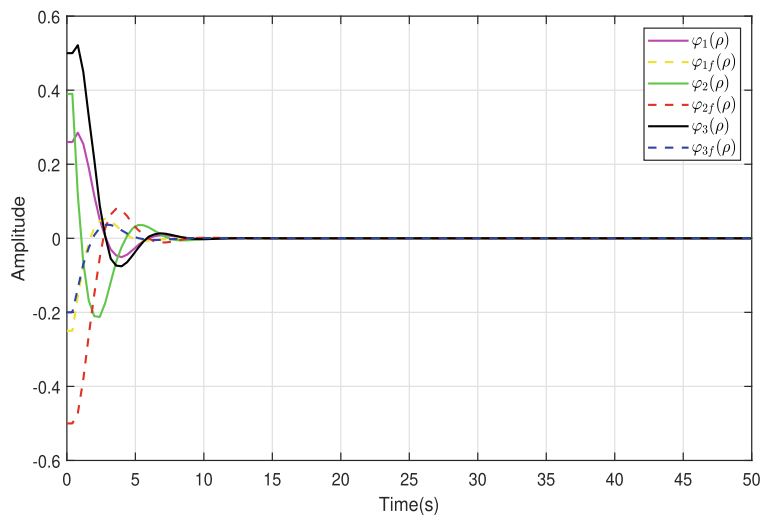


FIGURE 16 The state trajectories of  $\varphi_i(\rho)$  and its estimate  $\varphi_{if}(\rho)$ , ( $i = 1, 2, 3$ ).

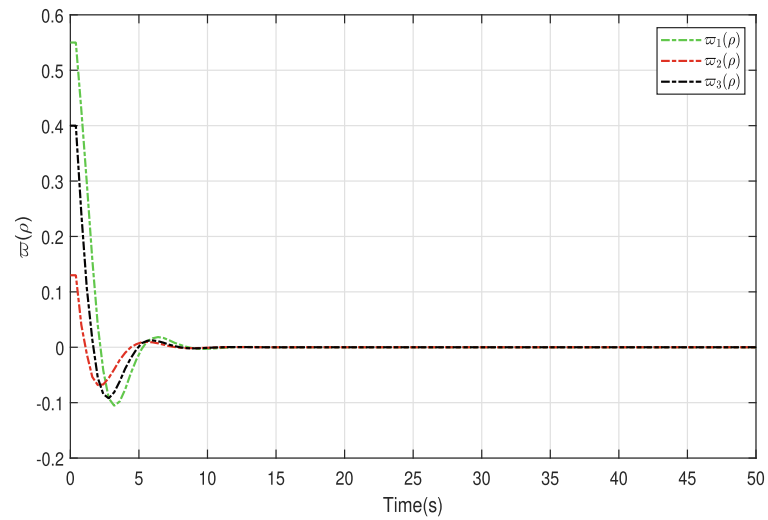


FIGURE 17 The filtering error responses  $\varpi_i(\rho)$ .

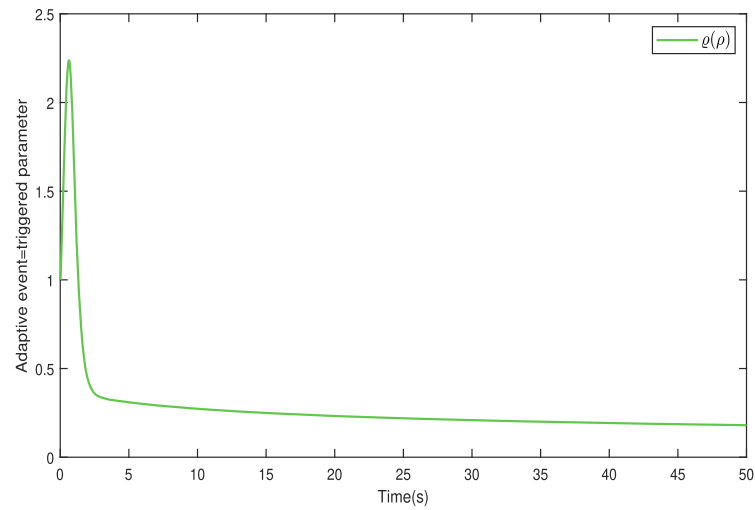


FIGURE 18 Values of the adaptive threshold parameter  $\rho(\rho)$ .

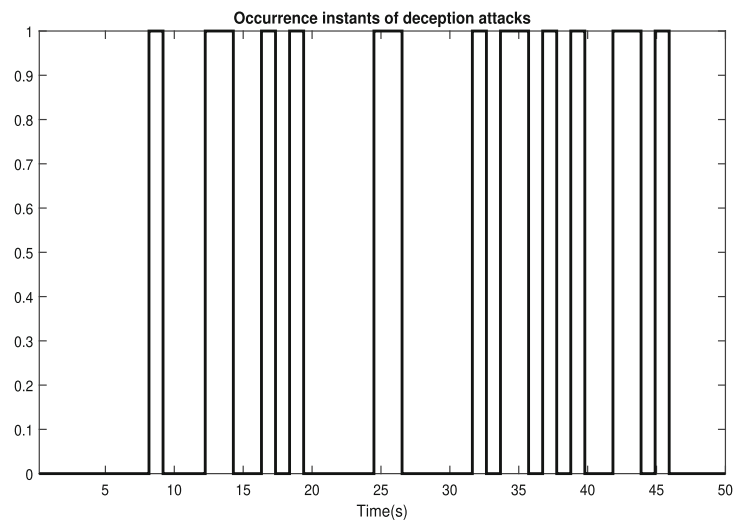


FIGURE 19 Occurrence of deception attacks.

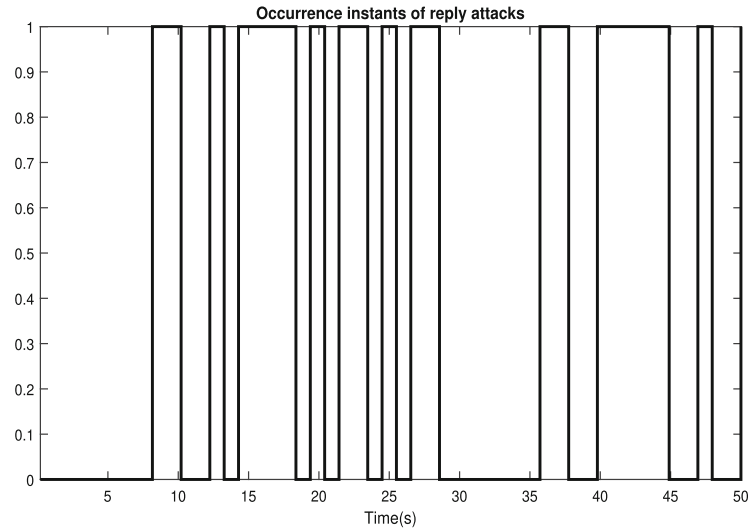


FIGURE 20 Occurrence of reply attacks.

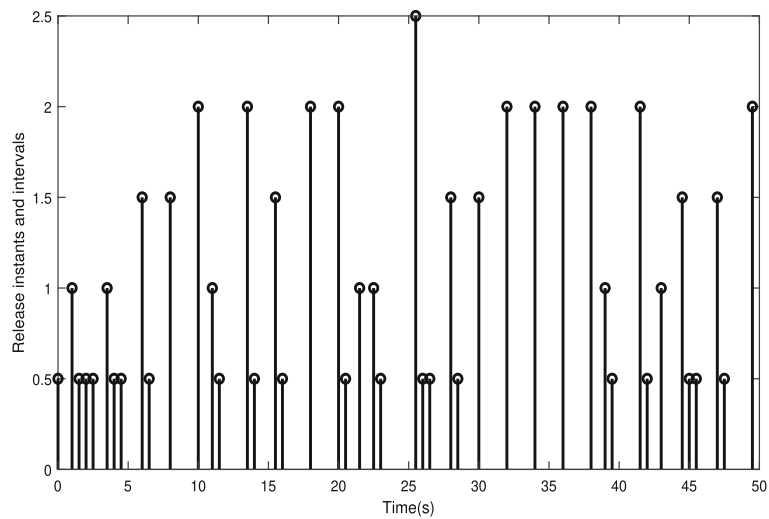


FIGURE 21 Adaptive event-triggered release instants and intervals.

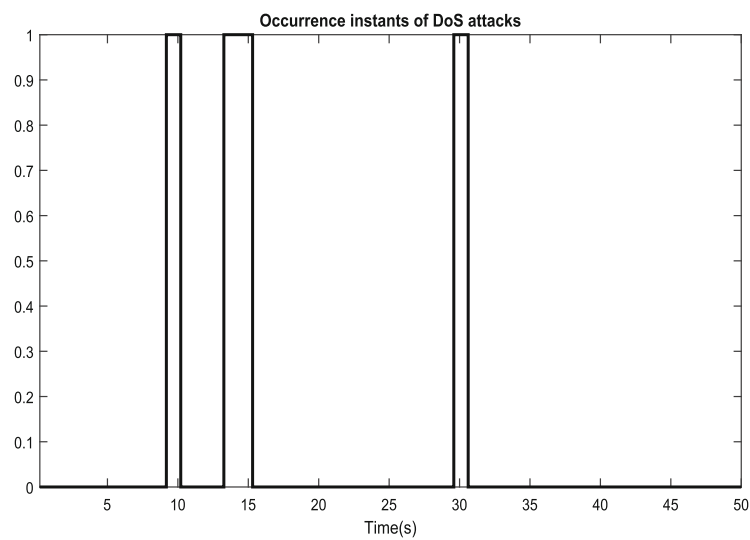


FIGURE 22 Occurrence of DoS attacks.

## 5 | CONCLUSION

This study investigated adaptive event-triggered-based filter design for complex dynamical networks with time-varying coupling delays under hybrid cyber-attacks. An adaptive event-triggered strategy reduces bandwidth use while keeping efficiency due to communications system instability and network resource constraints. An adaptive event-triggered scheme modifies the communication threshold level in the filter design to lower transmission load. A novel required condition is discovered using Lyapunov stability theory, which ensures mean-square asymptotically stable filter error systems with extended dissipative performance. By using linear matrix inequality methodology, the proposed filter parameters are determined. Finally, two examples were shown to demonstrate the benefits of the proposed strategy. In the future, sliding mode control for the filtering system is discussed.

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## CONFLICT OF INTEREST STATEMENT

No potential conflict of interest was reported by the author(s).

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