Alexandria Engineering Journal 79 (2023) 427–437

Contents lists available at ScienceDirect

Alexandria Engineering Journal

journal homepage: www.elsevier.com/locate/aej

ORIGINAL ARTICLE

Exploring the finite-time dissipativity of Markovian jump delayed neural networks

V.E. Sathishkumar [a], R. Vadivel [b], Jaehyuk Cho [a,*], Nallappan Gunasekaran [c]

[a] Department of Software Engineering, Jeonbuk National University, Jeonju, Republic of Korea
[b] Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Phuket- 83000, Thailand
[c] Eastern Michigan Joint College of Engineering, Beibu Gulf University, Qinzhou 535011, China

A R T I C L E I N F O

Keywords:
Linear matrix inequality
Lyapunov method
Finite-time dissipativity
Markovian jump neural networks
Time-varying delays

A B S T R A C T

In this paper, we study the finite-time dissipativity analysis of Markovian jump-delayed neural networks (MJDNNs). The goal is to establish less conservative results for extended dissipativity conditions for delayed MJDNNs. To achieve this, an appropriate Lyapunov-Krasovskii functional (LKF) with novel inequality like composite slack-matrix-based integral inequality (CSMBII). Next, the CSMBII and other sufficient conditions are employed to estimate the derivative of the constructed LKF. Using these techniques, a delay-dependent finite-time dissipativity condition is derived in terms of linear matrix inequalities (LMIs). These LMIs are used to formulate the finite dissipativity condition for the delayed MJNNs. The utility of the suggested approach is then confirmed by a number of interesting numerical examples, one of which has been confirmed by a real-world application of the benchmark problem that is associated with the designed MJDNNs. The illustrative simulation results conclusively demonstrate the superior performance and success of the developed CSMBII technique in this proposal, surpassing the limitations of existing techniques.

1. Introduction

Neural networks (NNs) are complicated network structures made up of a huge number of small processing units that are linked together. Because of their high application potential, neural networks (NNs) have received a lot of attention in recent years. It is critical to ensure that the NN model is globally asymptotically stable [1–4], while constructing NNs to handle problems such as linear programming and pattern recognition. However, information capture is common in NNS, implying that NNS may be limited to shifting from one mode to another at different times. As a result, in this case, the NNS can be thought of as a Markovian jump NNS [5–7]. Because information processing is restricted in speed, the existence of temporal delays in NNS frequently causes oscillations, divergence, or instability. In recent years, the stability problem of delayed NNS has received a great deal of theoretical and practical attention. This issue is gaining popularity in areas such as signal and image processing, artificial intelligence, and so forth. Delay-dependent stability analysis of delayed NNs is a significant challenge in dynamical systems since time delays occur in the actual world and might affect the stability of the system [8–15]. The point of this study was to make the stability criteria more conservative. Whereas the index, which shows how conservative the stability criteria are, is known to be the maximum bound of the delay, there has been a great deal of interest in creating and constructing an effective Lyapunov–Krasovskii function (LKF) to get a relatively high upper bound on the delay time so that the suggested system is nearly constant.

Dissipative theory for dynamical systems was first established in [16], and it has subsequently been broadened and intensively investigated for nonlinear systems in [17–19]. The idea of dissipativity gives a foundation for the analysis and design of process control that use input–output descriptions based on system energy concerns. Dissipativity theory is a fundamental concept that has been applied to a wide range of scientific and technical fields. This ties physics, system theory, and control theory together in a powerful way. Robotics, active vibration damping, mechanical systems, internal combustion, and circuit theory have all proven to be important control tools. Many important physical systems exhibit input parameters related to energy conservation, dissipation, and transport. As a result, both conceptually and empirically, dissipativity analysis in complex system dynamics has emerged as an important topic of research. In recent years, several solid studies on the

* Corresponding author.
E-mail addresses: srisathishkumarve@gmail.com (V.E. Sathishkumar), vadivelsr@yahoo.com (R. Vadivel), chojh@jbnu.ac.kr (J. Cho), gunasmaths@gmail.com (N. Gunasekaran).

https://doi.org/10.1016/j.aej.2023.07.073
Received 22 January 2023; Received in revised form 1 July 2023; Accepted 26 July 2023
Available online 17 August 2023

dissipativity of delayed and stochastic NNs have been suggested (see [20,21]) and references therein.

Markovian jump neural networks (MJNNs) are a type of neural network that incorporates Markov chain models to represent systems with variable dynamics. This approach is useful in modeling complex systems with varying behaviors, such as those that experience faults, failures, or environmental disturbances. The use of linear matrix inequalities (LMIs) in analyzing the dynamic behaviors of MJNNs has been an area of interest for researchers over the past 15 years. LMIs are a set of mathematical constraints that can be used to determine the stability and robustness of a system, and they have proven to be useful in analyzing the behavior of MJNNs [22–24]. Moreover, systems with Markovian jump parameters have Markov chain-governed transitions between models and take values from a finite set, acting as stochastic hybrid systems with two phases in the state. The first is a system of differential equations that represents the mode, while the second is a continuous-time finite-state Markovian process that represents the state. Several studies on Markovian jump systems (MJSs) have recently been published as a result of the extensive usage of such models in industrial systems, power systems, communication systems, and network-based control systems (see references [25–29]). Finite-settling-time behavior of systems with continuous dynamics, on the other hand, is examined in [30–35]. However, such systems have not been adequately investigated. The authors in [31] recently explored the subject of dissipative stabilization analysis of time-delayed NNs. In [38], developed some enhanced stability criteria for neural network stability analysis using convex inequality. The subject of asymptotic dissipativity stability analysis for static NNs with time-varying delays was explored in [39]. This is our primary incentive to conduct more study and apply it to a real-life electrical circuit model.

Motivated by the above discussion, finite-time dissipativity is a concept used in the analysis of systems that describes the ability of a system to dissipate energy over a finite period of time. In the context of Markovian jump-delayed neural networks, finite-time dissipativity is used as a criterion for analyzing the stability and execution of the system. A Markovian jump-delayed neural network is a type of neural network that incorporates the effects of random jumps in the system dynamics, as well as a time delay in the feedback loop. The use of finite-time dissipativity as a criterion for analysis allows for a more comprehensive understanding of the system's behavior, including its ability to handle disturbances and its overall robustness. In order to explore the finite-time dissipativity of Markovian jump-delayed NNs, researchers typically use a combination of mathematical analysis and simulation techniques. One key component of this analysis is the use of Lyapunov-Krasovskii functionals, which are mathematical constructs that can be used to describe the energy dissipation of a system over time. These functionals are used to derive conditions for the finite-time dissipativity of the system, which can then be used to analyze the stability and performance of the network. The subject of finite-time dissipativity (FTD) analysis and scheme for stochastic Markovian jump neural networks (MJNNs) with time-varying delay is studied in this research. This paper's main contributions are:

(1) This study examines finite-time dissipative stochastic MJNNs with time-varying delays.
(2) Lyapunov-Krasovskii functional and free-connection weighting matrices provide sufficient FTD requirements for stochastic MJNNs with time-varying delay.
(3) The dissipativity criteria, which depend on the upper bounds of the time-varying delay and its derivative, are linear matrix inequalities (LMI) that can be easily calculated using conventional numerical techniques.
(4) LMIs are arranged in a way that takes into consideration the connections between terms, and novel bound approaches are employed for integral terms.
(5) Finally, numerical examples are presented to illustrate the utility and reduced conservatism of the stability criteria are also established.

Notations: The notations throughout this paper are standard, which can be referred to [31]. Besides denote $\{\varrho_t, t \geq 0\}$ be a right-continuous Markov chain on a probability space $(\mathscr{X}, \mathscr{F}, \mathscr{P})$ taking values in a finite state space $\mathscr{N} = \{1, 2, ..., m\}$ with generator $\Pi = \{\pi_{pq}\}$ given by,

$$P\{\varrho_{t+\Delta} = q | \varrho_t = p\} = \begin{cases} \pi_{pq}\Delta + o(\Delta), & p \neq q \\ 1 + \pi_{pq}\Delta + o(\Delta), & p = q. \end{cases}$$

Here $\Delta > 0, \lim_{t\to+\infty} \frac{o(\Delta)}{\Delta} = 0, \pi_{pq} \geq 0$ is the transition rate from p to q if $q \neq p$ while $\pi_{pp} = -\sum_{q=1, q\neq p}^{m} \pi_{pq}$ for each mode p. Note that if $\pi_{pp} = 0$ for some $p \in \mathscr{N}$, then the pth mode is called "terminal mode" [22]. Furthermore, the transition probabilities matrix Π is denoted as follows:

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \dots & \pi_{1m} \\ \pi_{21} & \pi_{22} & \dots & \pi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{m1} & \pi_{m2} & \dots & \pi_{mm} \end{bmatrix}.$$

2. Problem statement

Consider the following Markovian jump-delayed neural networks:

$$\begin{cases} \dot{x}(t) = & -E_{\varrho_t} x(t) + A_{\varrho_t} f(x(t)) + B_{\varrho_t} f(x(t-\tau(t))) \\ & +G_{\varrho_t} \upsilon(t), \\ z(t) = & C_{\varrho_t} x(t) + H_{\varrho_t} \upsilon(t), \\ x(t) = & \phi(t), \ t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ indicates the state vector. The neuron activation function is given by the non-linear function $f(x(t)) \in \mathbb{R}^n$, and $\tau(t)$ means the time-varying delays. $\phi(t) \in \mathbb{R}^n$ is a vector-valued initial condition function; $\upsilon(t) \in \mathbb{R}^n$ signifies the input of the disturbance that belongs to $L_2[0, \infty)$; and $z(t) \in \mathbb{R}^n$ is recognized as the control output. Interconnection weight matrices with the appropriate dimensions for a modified mode ϱt are denoted by the notation $E_{\varrho t} = diag\{e_1, \cdots, e_n\} > 0$, $A_{\varrho t}$, $B_{\varrho t}$, $G_{\varrho t}$, $C_{\varrho t}$, $H_{\varrho t}$. Here $0 \leq \tau(t) \leq \tau$, $\dot{\tau} \leq \mu$, where τ and μ are positive constants. For ease of presentation, we designate the Markovian chain $\{\varrho_t, t \geq 0\}$ by i records, and the Markovian jump delayed network system (1) may be expressed as

$$\begin{aligned} \dot{x}(t) =& -E_i x(t) + A_i f(x(t)) + B_i f(x(t-\tau(t))) + G_i \upsilon(t), \\ z(t) =& C_i x(t) + H_i \upsilon(t). \end{aligned} \tag{2}$$

Throughout the study, some assumptions, definitions, and lemmas are used.

Assumption (H). [1] The activation functions $f_i(\cdot)$ in (1) are continuous and fulfill

$$F_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_i^+, \ i = 1, 2, \cdots, n$$

where $f_i(0) = 0, \alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2$ and F_i^- and F_i^+ denoted as scalars.

Assumption *A*. [26] The $\upsilon(t) \in \mathbb{R}$ fulfills

$$\int_0^T \upsilon^T(t)\upsilon(t)dt < d, \ d \geq 0. \tag{3}$$

Remark 2.1. Let c_1, c_2, T, d be given positive constants, matrix $\mathscr{R}_i > 0$. The $\upsilon(t)$ is always energy-bounded in practice, indicating that (A) is valid. Moreover, $(c_1, c_2, \mathscr{R}_i, T) = \Delta$ and $(c_1, c_2, \mathscr{R}_i, T, d) = \Delta_1$.

Definition 2.2. [26] For some given constants $c_2 > c_1 \geq 0$, and symmetric matrix $\mathscr{R}_i > 0$, such that the system (2) with $\upsilon(t) = 0$ for a given time constant T and is finite-time stable (FTS) with regard to Δ, then

$$\mathbb{E}\{\sup_{-\tau\le t\le 0}[x^T(t)\mathscr{R}_i x(t)]\}\le c_1$$
$$\Longrightarrow \mathbb{E}\{x^T(t)\mathscr{R}_i x(t)\}< c_2,\ \forall_t\in[0,T].$$

Definition 2.3. [17] For given time constant T and real function $\eta(\cdot),\eta(0)=0$, (2) is denoted as finite-time $(\bar{\bar{\mathscr{Z}}},\bar{\mathscr{S}},\bar{\mathscr{G}})$-dissipative (FTD) with regard to $(c_1,c_2,\mathscr{R}_i,T,d)$, then the following conditions are satisfied

(i). The (2) is finite-time bounded (FTB) in terms of Δ_1.

(ii). Under zero initial conditions $\psi(\theta)=0,\ \forall\ \theta\in[-\tau,0]$, system (2) satisfies

$$\mathbb{E}\left\{\int_0^T\begin{bmatrix}z(t)\\ v(t)\end{bmatrix}^T\begin{bmatrix}\bar{\bar{\mathscr{Z}}} & \bar{\mathscr{S}}\\ * & \bar{\mathscr{G}}\end{bmatrix}\begin{bmatrix}z(t))\\ v(t)\end{bmatrix}dt\right\}\ge-\eta(x_0),$$

when $v(t)$ satisfies (3).

Lemma 2.4. *[36] For every constant matrix* $\aleph>0$ *and scalars* $\beta>\alpha>0$ *with well-defined integration's,*

$$-(\alpha-\beta)\int_\beta^\alpha e^T(s)\aleph e(s)ds\le-\int_\beta^\alpha e^T(s)ds\ \aleph\int_\beta^\alpha e(s)ds.$$

Lemma 2.5. *[40] For every matrix* $R>0$*, any vector* ξ*, and any continuously differentiable function* $x:[-\tau,0]\to\mathbb{R}^n$*, and slack matrices* M,N*, it holds*

$$\begin{aligned}&-\tau\int_{t-\tau}^t\dot{x}^T(s)R\dot{x}(s)ds\\
&\le\xi^T(t)[\tau(t)M^T\hat{R}^{-1}M+h_\tau N^T\hat{R}^{-1}N]\\
&+(\frac{h_\tau}{\tau}+\frac{\tau(t)^2}{\tau^2})Sym[v^T(t-\tau(t),t)\Gamma_a^T M\\
&+v^T(t-\tau,t-\tau(t))\Gamma_a^T N]\xi\\
&-\{\frac{h_\tau}{\tau}v^T(t-\tau(t),t)\Gamma_a^T M\hat{R}\Gamma_a v(t-\tau(t),t)\\
&+\frac{\tau(t)^2}{\tau^2}v^T(t-\tau,t-\tau(t))\Gamma_a^T M\hat{R}\Gamma_a(t-\tau,t-\tau(t))\}.\end{aligned}$$

Lemma 2.6. *[37] If* $\mathscr{X}_{33}>0$*, any matrices* $\mathscr{X}_{11}$*,* $\mathscr{X}_{12}$*,* $\mathscr{X}_{13}$*,* $\mathscr{X}_{22}$ *and* $\mathscr{X}_{23}$ *such that* $[\mathscr{X}_{ij}]_{3\times3}\ge0$*, then*

$$-\int_{t-\tau}^t\dot{x}^T(s)\mathscr{X}_{33}\dot{x}(s)ds\le\int_{t-\tau}^t\varpi^T(t)\begin{bmatrix}\mathscr{X}_{11}&\mathscr{X}_{12}&\mathscr{X}_{13}\\ *&\mathscr{X}_{22}&\mathscr{X}_{23}\\ *&*&0\end{bmatrix}\varpi(t)ds,$$

where $\varpi(t)=[x^T(t)\quad x^T(t-\tau)\quad \dot{x}^T(s)]^T$.

3. Main results

In this part, we use the LMI technique to develop our primary results. The following definitions are provided for the convenience of notation:

$$F_1=diag\{F_1^-F_1^+,F_2^-F_2^+,...,F_n^-F_n^+\},\wp=\tau(t)$$
$$F_2=diag\left\{\frac{F_1^-+F_1^+}{2},\frac{F_2^-+F_2^+}{2},...,\frac{F_n^-+F_n^+}{2}\right\},$$
$$\Delta_i=c_1[\lambda_{max}(\bar{P}_i)+\lambda_{max}(F_2-F_1)+\lambda_{max}\bar{D}$$
$$+\lambda_{max}\bar{Q}+\tau\lambda_{max}\bar{R}+\frac{\tau^2}{2}\lambda_{max}\bar{X}+\frac{\tau^2}{2}\lambda_{max}\bar{Y}],$$
$$\mathscr{I}_l=[0_{n\times(l-1)n}\ I_n\ 0_{n\times(12-1)n}\ 0_{n\times m}]^T,l=1,2,...,12.$$

Theorem 3.1. *Under Assumption (H), for given scalars* τ*,* α*,* β*,* d *and* μ*, the system (2) is FTB in terms of* Δ_1*, if there exist matrices* $P_i>0$*,* $D_1>0$*,* $D_2>0$*,* $\mathscr{Q}=\begin{bmatrix}Q_1&Q_2\\ *&Q_3\end{bmatrix}>0$*,* $R>0$*,* $X>0$*,* $Y>0$*, matrices* $[R_{ij}]_{3\times3}\ge0$*, the diagonal matrices* $\Sigma_i>0,i=1,2,3$*, and any matrices* U_1*,* U_2*,* $\mathcal{Y}_1,\mathcal{Y}_2$*,* M *with sufficient dimensions, such that the following LMIs hold with* $\tau(t)=\{0,\tau\}$*:*

$$\begin{bmatrix}\bar{\Xi}(0,\dot{\wp})&\sqrt{\tau}\mathcal{Y}_2^T\\ *&-[X-R_{33}]\end{bmatrix}<0,\tag{4}$$

$$\begin{bmatrix}\bar{\Xi}(\tau,\dot{\wp})&\sqrt{\tau}\mathcal{Y}_1^T\\ *&-[X-R_{33}]\end{bmatrix}<0,\tag{5}$$

where

$$\Xi(\wp,\dot{\wp})=\bar{\Xi}_{11}(\wp,\dot{\wp})+\Xi_{02}(\wp),$$

$$\begin{aligned}\bar{\Xi}_{11}(\wp,\dot{\wp})=&\ \mathscr{I}_1^T[\tau^2Y+\sum_{j=1}^N\pi_{ij}P_j+Q_1+R\\
&-X+R_{33}-F_1\Sigma_1-F_1\Sigma_3+\tau^2R_{11}+\tau R_{13}^T\\
&+\tau R_{13}-2U_1E_i]\mathscr{I}_1+\mathscr{I}_1^T[2P_i-F_2D_1\\
&+F_1D_2-U_1-E_i^TU_2^T]\mathscr{I}_2+\mathscr{I}_1^T[X-R_{33}\\
&+\tau^2R_{12}-\tau R_{23}+\tau R_{23}^T]\mathscr{I}_4\\
&+\mathscr{I}_1^T[U_1A_i+Q_2+F_2\Sigma_1+F_2\Sigma_3]\mathscr{I}_5\\
&+\mathscr{I}_1^T[U_1B_i-F_2\Sigma_3]\mathscr{I}_6+\mathscr{I}_1^T[U_1G_i]\mathscr{I}_{12}\\
&+\mathscr{I}_2^T[U_2G_i]\mathscr{I}_{12}+\mathscr{I}_2^T[-2U_2+\tau^2X\ \mathscr{I}_2\\
&+\mathscr{I}_2^T[D_1^T-D_2^T+U_2A_i]\mathscr{I}_5+\mathscr{I}_4^T[-R-X\\
&+R_{33}+\tau^2R_{22}-\tau R_{23}+\tau R_{23}^T\mathscr{I}_4\\
&+\mathscr{I}_3^T[-(1-\mu)Q_1-F_1\Sigma_2-F_1\Sigma_3]\mathscr{I}_3\\
&+\mathscr{I}_4^T[-(1-\mu)Q_2+F_2\Sigma_2+F_2\Sigma_3]\mathscr{I}_6\\
&+\mathscr{I}_5[Q_3-\Sigma_1-\Sigma_3]\mathscr{I}_5\\
&+\mathscr{I}_6^T[-(1-\mu)Q_3-\Sigma_2-\Sigma_3]\mathscr{I}_6\\
&-\mathscr{I}_7^TY\mathscr{I}_7-\mathscr{I}_{12}^T\beta M\mathscr{I}_{12},\end{aligned}$$

$$\begin{aligned}\Xi_{02}(\wp)=&\ (\frac{\tau-\tau(t)}{\tau}+\frac{\tau(t)^2}{\tau^2})sym[\mathcal{Y}_1\chi_1+\mathcal{Y}_2\chi_2]\\
&-(\frac{\tau-\tau(t)}{\tau^2}\chi_1^T[X-R_{33}]\chi_1\\
&+\frac{\tau(t)}{\tau^2}\chi_2^T[X-R_{33}]\chi_2),\end{aligned}$$

$$\chi_1=col[\mathscr{I}_1-\mathscr{I}_3,\mathscr{I}_1+\mathscr{I}_3-2\mathscr{I}_8,\ \mathscr{I}_1-\mathscr{I}_3+6\mathscr{I}_8-12\mathscr{I}_9],$$

$$\chi_2=col[\mathscr{I}_3-\mathscr{I}_4,\mathscr{I}_3+\mathscr{I}_4-2\mathscr{I}_{10},\ \mathscr{I}_3-\mathscr{I}_4+6\mathscr{I}_{10}-12\mathscr{I}_{11}],$$

and

$$\frac{e^{\beta T}\left[\Delta_i+\beta d\lambda_{max}(M)\frac{1-e^{-\beta T}}{\beta}\right]}{\lambda_{min}(\bar{P}_i)}<c_2.\tag{6}$$

Proof. Consider the LKF candidate as follows

$$V(t)=\sum_{i=1}^4V_i(t),\tag{7}$$

where

$$V_1(t) = x^T(t)P_i x(t),$$

$$V_2(t) = 2\sum_{i=1}^{n} d_{1i}\int_0^{x_i(t)} (f_i(s) - F_i^- s)ds$$

$$+2\sum_{i=1}^{n} d_{2i}\int_0^{x_i(t)} (F_i^+ s - f_i(s))ds,$$

$$V_3(t) = \int_{t-\tau(t)}^{t} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^T \mathscr{Q} \begin{bmatrix} x(s)) \\ f(x(s)) \end{bmatrix} ds$$

$$+\int_{t-\tau}^{t} x^T(s)Rx(s)ds,$$

$$V_4(t) = \tau\int_{\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X\dot{x}(s)dsd\theta$$

$$+\tau\int_{\tau}^{0}\int_{t+\theta}^{t} x^T(s)Yx(s)dsd\theta.$$

For each mode i, it can be observed that

$$\mathcal{L}V_1(x(t),t,i) = 2\mathscr{J}_1^T(t)P_i\mathscr{J}_2 + \sum_{j=1}^{N}\pi_{ij}\mathscr{J}_1^T P_j\mathscr{J}_1, \tag{8}$$

$$\mathcal{L}V_2(x(t),t,i) = 2(\mathscr{J}_5 - F_2\mathscr{J}_1)^T D_1\mathscr{J}_2$$

$$+2(F_1\mathscr{J}_1 - \mathscr{J}_5)^T D_2\mathscr{J}_2, \tag{9}$$

$$\mathcal{L}V_3(x(t),t,i) \le \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \end{bmatrix}^T \mathscr{Q} \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \end{bmatrix}$$

$$-(1-\mu)\begin{bmatrix} \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix}^T \mathscr{Q} \begin{bmatrix} \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix}, \tag{10}$$

$$\mathcal{L}V_4(x(t),t,i) = \tau^2\mathscr{J}_1^T Y\mathscr{J}_1 + \tau^2\mathscr{J}_2^T X\mathscr{J}_2$$

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)[X - R_{33}]\dot{x}(s)ds$$

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)R_{33}\dot{x}(s)ds$$

$$-\tau\int_{t-\tau}^{t}x^T(s)Yx(s)ds. \tag{11}$$

Using Lemma 2.4 in (11), we have

$$-\tau\int_{t-\tau}^{t} x^T(s)\,Yx(s)ds \le -\mathscr{J}_7^T\ Y\mathscr{J}_7 ds. \tag{12}$$

Applying Lemma 2.6 and $-\tau\int_{t-\tau}^{t}\dot{x}^T(s)R_{33}\dot{x}(s)ds$ the following equation holds:

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)R_{33}\dot{x}(s)ds$$

$$\le \int_{t-\tau}^{t}\varpi^T(t)\begin{bmatrix} \tau R_{11} & \tau R_{12} & \tau R_{13} \\ * & \tau R_{22} & \tau R_{23} \\ * & * & 0 \end{bmatrix}\varpi(t)ds,$$

$$\le \mathscr{J}_1^T\tau^2 R_{11}\mathscr{J}_1 + 2\mathscr{J}_1^T\tau^2 R_{12}\mathscr{J}_4 + \mathscr{J}_4^T\tau R_{22}\mathscr{J}_4$$

$$+2\mathscr{J}_1^T R_{13}^T\mathscr{J}_7 + 2\mathscr{J}_4^T R_{23}^T\mathscr{J}_7,$$

$$= \mathscr{J}_1^T\left[\tau^2 R_{11} + \tau R_{13}^T + \tau R_{13}\right]\mathscr{J}_1$$

$$+2\mathscr{J}_1^T\left[\tau^2 R_{12} - \tau R_{13} + \tau R_{23}^T\right]\mathscr{J}_4$$

$$+\mathscr{J}_4^T\left[\tau^2 R_{22} - \tau R_{23} - \tau R_{23}^T\right]\mathscr{J}_4, \tag{13}$$

where $\varpi(t) = [x^T(t) \quad x^T(t-\tau) \quad \dot{x}^T(s)]^T$. The integral term in (11) can be rearranged according to Lemma 2.5

$$-\int_{t-\tau}^{t}\dot{x}^T(s)[X - R_{33}]\dot{x}(s)ds$$

$$\le \eta^T(t)[\tau(t)\mathcal{Y}_1^T[X - R_{33}]^{-1}\mathcal{Y}_1]$$

$$+(\tau - \tau(t))\mathcal{Y}_2^T[X - R_{33}]^{-1}\mathcal{Y}_2$$

$$+(\frac{\tau - \tau(t)}{\tau} + \frac{\tau(t)^2}{\tau^2})sym[\mathcal{Y}_1\chi_1 + \mathcal{Y}_1\chi_2]$$

$$-(\frac{\tau - \tau(t)}{\tau^2}\chi_1^T[X - R_{33}]\chi_1 + \frac{\tau(t)}{\tau^2}\chi_2^T[X - R_{33}]\chi_2),$$

$$= \eta^T(t)[\Xi_{01}(\wp) + \Xi_{02}(\wp)]\eta(t), \tag{14}$$

where, $\Xi_{01}(\wp) = \tau(t)\mathcal{Y}_1^T[X - R_{33}]^{-1}\mathcal{Y}_1 + (\tau - \tau(t))\mathcal{Y}_2^T[X - R_{33}]^{-1}\mathcal{Y}_2$.

$$0 \le \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \end{bmatrix}^T \Psi_1 \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \end{bmatrix},\ 0 \le \begin{bmatrix} \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix}^T \Psi_2 \begin{bmatrix} \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix},$$

$$0 \le \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \\ \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix}^T \Psi_3 \begin{bmatrix} \mathscr{J}_1 \\ \mathscr{J}_5 \\ \mathscr{J}_3 \\ \mathscr{J}_6 \end{bmatrix}, \tag{15}$$

where

$$\Psi_1 = \begin{bmatrix} -F_1\Sigma_1 & F_2\Sigma_1 \\ * & -\Sigma_1 \end{bmatrix},\ \Psi_2 = \begin{bmatrix} -F_1\Sigma_2 & F_2\Sigma_2 \\ * & -\Sigma_2 \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} -F_1\Sigma_3 & F_2\Sigma_3 & F_1\Sigma_3 & -F_2\Sigma_3 \\ * & -\Sigma_3 & -F_2\Sigma_3 & \Sigma_3 \\ * & * & -F_1\Sigma_3 & F_2\Sigma_3 \\ * & * & * & -\Sigma_3 \end{bmatrix}.$$

For any matrices U_1, U_2 with appropriate dimensions, we can get

$$0 = 2[\mathscr{J}_1^T U_1 + \mathscr{J}_2^T U_2][-\dot{x}(t) - E_i\mathscr{J}_1 + A_i\mathscr{J}_5$$

$$+B_i\mathscr{J}_6 + G_i\mathscr{J}_{12}]$$

$$= -2\mathscr{J}_1^T U_1\mathscr{J}_2 - 2\mathscr{J}_1^T U_1 E_i\mathscr{J}_1 + 2\mathscr{J}_1^T U_1 A_i\mathscr{J}_5$$

$$+2\mathscr{J}_1^T U_1 A_i B_i\mathscr{J}_6 + 2\mathscr{J}_1^T U_1 G_i\mathscr{J}_{12}$$

$$-2\mathscr{J}_2^T U_2\mathscr{J}_2 - 2\mathscr{J}_2^T U_2 E_i\mathscr{J}_2$$

$$+2\mathscr{J}_2^T U_2 A_i\mathscr{J}_5 + 2\mathscr{J}_1^T U_2 A_i B_i\mathscr{J}_6 + 2\mathscr{J}_2^T U_2 G_i\mathscr{J}_{12}. \tag{16}$$

By combining (8)–(16) it can be got that

$$\mathbb{E}\left\{\mathcal{L}V(x(t),t,i)\right\} \le \mathbb{E}\left\{\eta^T(t)\{\Xi(\wp,\dot{\wp})\right.$$

$$\left.+\Xi_{01}(\wp) + \Xi_{02}(\wp)\}\eta(t)\right\} < 0, \tag{17}$$

where

$$\eta^T(t) = \Big[x^T(t)\ \dot{x}^T(t)\ x^T(t-\tau(t))\ x^T(t-\tau)$$

$$f^T(x(t))\ f^T(t-\tau(t))\int_{t-\tau}^{t}x^T(s)ds$$

$$\frac{1}{\tau(t)}\int_{t-d}^{t}x^T(s)ds\ \frac{1}{\tau(t)^2}\int_{t-\tau(t)}^{t}\int_{u}^{t}x^T(s)dsdu$$

$$\frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)} x^T(s)ds$$

$$\frac{1}{(\tau-\tau(t))^2}\int_{t-\tau}^{t-\tau(t)}\int_{u}^{t-\tau(t)} x^T(s)dsdu\ \upsilon^T(t)\Big].$$

Using the Schur complement, we get the LMIs (4) and (5). Note that $\pi_{pq}<0$ holds for all $q=p$ and $\pi_{pq}\geq 0$ holds for all $q\neq p$. If $i\in\mathcal{N}$, the inequalities from (4)-(5) imply that

$$\begin{aligned}\mathcal{L}V(x(t),i) &< \beta x^T(t)P_i x(t)+\beta\upsilon^T(t)M\upsilon(t)\\ &< \beta V(x(t),i)+\beta\upsilon^T(t)M\upsilon(t).\end{aligned} \tag{18}$$

On the other hand, if $i\in\mathcal{N}$, inequalities (4)-(5) imply that (18) holds. Multiplying (18) by $e^{-\beta t}$, we get

$$\mathcal{L}(e^{-\beta t}V(x(t),i)) < \beta e^{-\beta t}\upsilon^T(s)M\upsilon(s). \tag{19}$$

Applying the Dynkins rule to (19), we get

$$\begin{aligned}&\mathbb{E}\{e^{-\beta t}V(x(t),i)\}-V(x_0,t_0)\\ &\quad<\beta\int_0^t e^{-\beta s}\upsilon^T(s)M\upsilon(s).\end{aligned} \tag{20}$$

Also,

$$\begin{aligned}E\{V(x(t),i)\} &< e^{\beta t}V(x_0,t_0)\\ &\quad+\beta e^{\beta t}\int_0^t e^{-\beta s}\upsilon^T(s)M\upsilon(s)ds\\ &< e^{-\beta t}\Big[V(x_0,t_0)\\ &\quad+\beta d\lambda_{max}(M)\frac{1-e^{-\beta t}}{\beta}\Big],\end{aligned} \tag{21}$$

and

$$\begin{aligned}V(x_0,t_0) =& x^T(0)P_i x(0)\\ &+2\sum_{i=1}^{n} d_{1i}\int_0^{x_i(0)}(f_i(s)-F_i^- s)ds\\ &+2\sum_{i=1}^{n} d_{2i}\int_0^{x_i(0)}(F_i^+ s-f_i(s))ds\\ &+\int_{t-\tau(t)}^{t}\begin{bmatrix}x(s)\\ f(x(s))ds\end{bmatrix}^T\mathscr{Q}\begin{bmatrix}x(s))\\ f(x(s))ds\end{bmatrix}\\ &+\int_{-\tau}^{0}x^T(s)Rx(s)ds\\ &+\tau\int_{\tau}^{0}\int_{\theta}^{0}\dot{x}^T(s)X\dot{x}(s)ds\\ &+\tau\int_{\tau}^{0}\int_{\theta}^{0}x^T(s)Yx(s)ds\\ &< e^{\beta t}\Big[\Delta_i+\beta d\lambda_{max}(M)\frac{1-e^{-\beta t}}{\beta}\Big],\end{aligned}$$

where
$\bar{P}_i=\mathscr{R}^{-\frac{1}{2}}P_i\mathscr{R}^{-\frac{1}{2}}$, $\bar{D}=\mathscr{R}^{-\frac{1}{2}}(D_1+D_2)\mathscr{R}^{-\frac{1}{2}}$, $\bar{Q}=\mathscr{R}^{-\frac{1}{2}}\otimes\mathscr{Q}\otimes\mathscr{R}^{-\frac{1}{2}}$, $\bar{X}=\mathscr{R}^{-\frac{1}{2}}X\mathscr{R}^{-\frac{1}{2}}$, $\bar{Y}=\mathscr{R}^{-\frac{1}{2}}Y\mathscr{R}^{-\frac{1}{2}}$ and $\Delta_i=c_1[\lambda_{max}(\bar{P}_i)+\lambda_{max}(F_2-F_1)+\lambda_{max}\bar{D}+\lambda_{max}\bar{Q}+\tau\lambda_{max}\bar{R}+\frac{\tau^2}{2}\lambda_{max}\bar{X}+\frac{\tau^2}{2}\lambda_{max}\bar{Y}]$.

Taking into account

$$\mathbb{E}\{V(x(t),i)\}\geq x^T(t)P_i x(t)\geq\lambda_{min}\bar{P}_i(x^T(t)\mathscr{R}_i x(t)), \tag{22}$$

gives rise to

$$\mathbb{E}\{x^T(t)\mathscr{R}_i x(t)\}\leq\frac{e^{\beta t}\Big[\Delta_i+\beta d\lambda_{max}(M)\frac{1-e^{-\beta t}}{\beta}\Big]}{\lambda_{min}(\bar{P}_i)}<c_2. \tag{23}$$

As a result, given the requirements of Theorem 3.1, for any $t\in[0,T]$, we get $\mathbb{E}\{\mathscr{J}_1^T\mathscr{R}_i\mathscr{J}_1\}<c_2$. Then, the system (2) is FTB with $(c_1,c_2,\mathscr{R}_i,T,d)$. The proof is completed. □

Remark 3.2. Consider $M=\frac{\mathscr{G}-\alpha I}{\beta}$ in Theorem 3.1 LMIs, then the conditions are FTB and satisfied with $(c_1,c_2,\mathscr{R}_i,T,d)$.

4. Finite-time dissipativity analysis

This section will offer an essential requirement to ensure that (2) is FTD.

Theorem 4.1. *Under Assumption (H), for given scalars τ, α, β, d and μ, the system (2) is FTB in terms of Δ_1, if there exist matrices $P_i>0$, $D_1>0$, $D_2>0$, $\mathscr{Q}=\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}>0$, $R>0$, $X>0$, $Y>0$, matrices $[R_{ij}]_{3\times 3}\geq 0$, the diagonal matrices $\Sigma_i>0, i=1,2,3$, and any matrices U_1, U_2 $\mathcal{Y}_1,\mathcal{Y}_2$, M with sufficient dimensions, such that the following LMIs hold with $\tau(t)=\{0,\tau\}$:*

$$\begin{bmatrix}\bar{\Psi}(0,\dot{\wp}) & \sqrt{\tau}\mathcal{Y}_2^T\\ * & -[X-R_{33}]\end{bmatrix}<0, \tag{24}$$

$$\begin{bmatrix}\bar{\Psi}(\tau,\dot{\wp}) & \sqrt{\tau}\mathcal{Y}_1^T\\ * & -[X-R_{33}]\end{bmatrix}<0, \tag{25}$$

where

$$\Psi(\wp,\dot{\wp})=\bar{\Psi}_{11}(\wp,\dot{\wp})+\Psi_{02}(\wp),$$

$$\begin{aligned}\bar{\Psi}_{11}(\wp,\dot{\wp})=&\mathscr{J}_1^T[\tau^2 Y+\sum_{j=1}^{N}\pi_{ij}P_j+Q_1+R+\beta P_i\\ &-X+R_{33}-F_1\Sigma_1-F_1\Sigma_3+\tau^2 R_{11}+\tau R_{13}^T\\ &+\tau R_{13}-2U_1E_i]\mathscr{J}_1+\mathscr{J}_1^T[2P_i-F_2D_1\\ &+F_1D_2-U_1-E_i^TU_2^T]\mathscr{J}_2+\mathscr{J}_1^T[X-R_{33}\\ &+\tau^2R_{12}-\tau R_{23}+\tau R_{23}^T]\mathscr{J}_4\\ &+\mathscr{J}_1^T[U_1A_i+Q_2+F_2\Sigma_1+F_2\Sigma_3]\mathscr{J}_5\\ &+\mathscr{J}_1^T[U_1B_i-F_2\Sigma_3]\mathscr{J}_6+\mathscr{J}_1^T[U_1G_i]\mathscr{J}_{12}\\ &+\mathscr{J}_2^T[U_2G_i]\mathscr{J}_{12}+\mathscr{J}_2^T[-2U_2+\tau^2X\mathscr{J}_2\\ &+\mathscr{J}_2^T[D_1^T-D_2^T+U_2A_i]\mathscr{J}_5+\mathscr{J}_4^T[-R-X\\ &+R_{33}+\tau^2R_{22}-\tau R_{23}+\tau R_{23}^T\mathscr{J}_4\\ &+\mathscr{J}_3^T[-(1-\mu)Q_1-F_1\Sigma_2-F_1\Sigma_3]\mathscr{J}_3\\ &+\mathscr{J}_4^T[-(1-\mu)Q_2+F_2\Sigma_2+F_2\Sigma_3]\mathscr{J}_6\\ &+\mathscr{J}_5[Q_3-\Sigma_1-\Sigma_3]\mathscr{J}_5\\ &+\mathscr{J}_6^T[-(1-\mu)Q_3-\Sigma_2-\Sigma_3]\mathscr{J}_6-\mathscr{J}_7^TY\mathscr{J}_7+\\ &[\mathscr{J}_1^TC_i^T+\mathscr{J}_{12}^TH_i^T]\bar{\mathscr{Z}}[C_i\mathscr{J}_1+H_i\mathscr{J}_{12}]\\ &+2\mathscr{J}_{12}^T\mathscr{S}[C_i\mathscr{J}_1+H_i\mathscr{J}_{12}]+\mathscr{J}_{12}^T(\mathscr{G}-\alpha I)\mathscr{J}_{12},\end{aligned}$$

$$\Psi_{02}(\wp)=(\frac{\tau-\tau(t)}{\tau}+\frac{\tau(t)^2}{\tau^2})sym[\mathcal{Y}_1\chi_1+\mathcal{Y}_2\chi_2]$$

$$-(\frac{\tau-\tau(t)}{\tau^2}\chi_1^T[X-R_{33}]\chi_1 + \frac{\tau(t)}{\tau^2}\chi_2^T[X-R_{33}]\chi_2),$$
$$\chi_1 = col[\mathscr{J}_1-\mathscr{J}_3, \mathscr{J}_1+\mathscr{J}_3-2\mathscr{J}_8, \mathscr{J}_1-\mathscr{J}_3+6\mathscr{J}_8-12\mathscr{J}_9],$$
$$\chi_2 = col[\mathscr{J}_3-\mathscr{J}_4, \mathscr{J}_3+\mathscr{J}_4-2\mathscr{J}_{10}, \mathscr{J}_3-\mathscr{J}_4+6\mathscr{J}_{10}-12\mathscr{J}_{11}].$$

Proof. Define the Lyapunov-Krasovskii functional candidate as follows:

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{26}$$

where

$$V_1(t) = x^T(t)P_i x(t),$$
$$V_2(t) = 2\sum_{i=1}^{n} d_{1i}\int_0^{x_i(t)} (f_i(s) - F_i^- s)ds + 2\sum_{i=1}^{n} d_{2i}\int_0^{x_i(t)} (F_i^+ s - f_i(s))ds,$$
$$V_3(t) = \int_{t-\tau(t)}^{t} \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix}^T \mathscr{Q} \begin{bmatrix} x(s)) \\ f(x(s)) \end{bmatrix} ds + \int_{t-\tau}^{t} x^T(s)Rx(s)ds,$$
$$V_4(t) = \tau\int_{\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X\dot{x}(s)dsd\theta + \tau\int_{\tau}^{0}\int_{t+\theta}^{t} x^T(s)Yx(s)dsd\theta.$$

The derivation is the same as that of Theorem 3.1. Therefore, it is not displayed here.

For each mode i, it can be defined that

$$\mathbb{E}\Big\{\mathcal{L}V(x(t),i) - \beta\mathscr{J}_1^T P_i\mathscr{J}_1 - z^T(t)\bar{\tilde{\mathscr{Z}}}\ z(t) - 2\mathscr{J}_{12}^T\bar{\mathscr{S}}z(t) - \mathscr{J}_{12}^T(\bar{\mathscr{G}}\ -\ \alpha\ I)\mathscr{J}_{12}\Big\} \le \mathbb{E}\Big\{\eta^T(t)\Psi(\wp,\dot{\wp})\eta(t)\Big\}. \tag{27}$$

According to the inequalities (4), (5), (6), (24), and (25), we have

$$\begin{aligned}\mathcal{L}V(x(t),i) \le& \beta\mathscr{J}_1^T P_i\mathscr{J}_1 + z^T(t)\bar{\tilde{\mathscr{Z}}}z(t) + 2\mathscr{J}_{12}^T\bar{\mathscr{S}}z(t) + \mathscr{J}_{12}^T(\bar{\mathscr{G}}-\alpha I)\mathscr{J}_{12}\\ \le& \beta V(x(t),i) + z^T(t)\bar{\tilde{\mathscr{Z}}}z(t) + 2\mathscr{J}_{12}^T\bar{\mathscr{S}}z(t) + \mathscr{J}_{12}^T(\bar{\mathscr{G}}-\alpha I)\mathscr{J}_{12},\\ =&\beta V(x(t),i) + [\mathscr{J}_1^T C_i^T + \mathscr{J}_{12}^T H_i^T] \times\bar{\tilde{\mathscr{Z}}}[C_i\mathscr{J}_1 + H_i\mathscr{J}_{12}] + 2\mathscr{J}_{12}^T\bar{\mathscr{S}}[C_i\mathscr{J}_1 + H_i\mathscr{J}_{12}] + \mathscr{J}_{12}^T(\bar{\mathscr{G}}-\alpha I)\mathscr{J}_{12},\end{aligned} \tag{28}$$

and

$$\mathcal{L}[e^{-at}V(x(t),i)] \le e^{-at}[z^T(t)\bar{\tilde{\mathscr{Z}}}z(t) + 2\mathscr{J}_{12}^T\bar{\mathscr{S}}z(t) + \mathscr{J}_{12}^T(\bar{\mathscr{G}}-\alpha I)\mathscr{J}_{12}]. \tag{29}$$

Using Dynkin's formula under zero initial conditions, we get

$$e^{-at}V(x(t),i) \le \int_0^T e^{-at}[z^T(t)\bar{\tilde{\mathscr{Z}}}z(t) + 2\mathscr{J}_{12}^T\bar{\mathscr{S}}z(t) + \mathscr{J}_{12}^T(\bar{\mathscr{G}}-\alpha I)\mathscr{J}_{12}]dt. \tag{30}$$

Moreover,

$$\mathbb{E}\Big\{\int_0^T \begin{bmatrix} z(t) \\ \mathscr{J}_{12}\end{bmatrix}^T \begin{bmatrix} \bar{\tilde{\mathscr{Z}}} & \bar{\mathscr{S}} \\ * & (\bar{\mathscr{G}}-\alpha I)\end{bmatrix}\begin{bmatrix} z(t)) \\ \mathscr{J}_{12}\end{bmatrix} dt\Big\} \ge -\eta(x_0). \tag{31}$$

According to Definition 2.3, the system (1) is FTD in terms of Δ_1. The proof is completed. □

Remark 4.2. It should be noted that the FTD with respect to Δ_1 in (31) represents a performance criterion of (1). To demonstrate this, consider the following significant cases.

(i). Setting $\bar{\tilde{\mathscr{Z}}} = -I$, $\bar{\mathscr{S}} = 0$, $(\bar{\mathscr{G}}-\alpha I) = \gamma^2 I$ in the system (1) yields FTB with disturbance attention γ.

(ii). Setting $\bar{\tilde{\mathscr{Z}}} = 0$, $\bar{\mathscr{S}} = I$, $(\bar{\mathscr{G}}-\alpha I) = 0$ in system (1) yields FTB with strictly positive real theorem;

(iii). Setting $\bar{\tilde{\mathscr{Z}}} = 0$, $\bar{\mathscr{S}} = I$, $(\bar{\mathscr{G}}-\alpha I) = \beta I$ in system (1) yields FTP.

Remark 4.3. When $G_{\varrho_t} = C_{\varrho_t} = D = 0$, and $\varrho_t = 1$, the system described by (1) reduces to delayed neural networks

$$\dot{x}(t) = -E_{\varrho_t}x(t) + A_{\varrho_t}f(x(t)) + B_{\varrho_t}f(x(t-\tau(t)))$$
$$x(t) = \phi(t),\ t\in[-\tau,0]. \tag{32}$$

Similarly, we may derive the following asymptotic stability requirement for delayed neural networks (32) from Theorem 3.1.

Remark 4.4. Our method for tackling the problem at hand is rooted in the utilization of the LKF method, in conjunction with the incorporation of slack variables and an optimized computation process. By comparing the number of variables required in Theorem 3.1 with the results found in previous research, we can see that our solution is less restrictive. Tables 1–3 are presented to provide a clear comparison of the results, and it can be inferred from these that our approach is less conservative.

Remark 4.5. It is noteworthy that in many industrial processes, the dynamical behaviors are generally complex and non-linear and their genuine mathematical models are always difficult to obtain. How to model the finite-time dissipativity of Markovian jump-delayed neural networks (MJDNNs) has become one of the main themes in our research work. More particularly, some pioneering works have been done in the finite time dissipativity of MJDNNs. In [30] and [31], the problem of Markovian jumping NNs has been studied, addressing finite-time stability and dissipative performance. Further results on dissipativity and stability analysis of Markov jump generalized neural networks with time-varying interval delays have been studied in [22]. Recently, finite-time stabilization was proposed in [32] positive Markovian jumping neural networks. The model considered in the present study is more practical than that proposed by [22,30–32], because they consider finite-time performance has been studied with SNNs based on stabilization conditions, but in this paper, we consider a new MJDNNs with the combination dissipative approach in the finite time interval with the practical application. Due to the many real-life application the combined study of finite time dissipativity effects on the system model is more important. In addition, the proposed dissipative analysis is the relation of applied energy to the system with energy started in the system, which is why we analyze Circuit realization of delayed neural networks with two neurons in this issue in our paper which may have many applications background, which is another advantage of our paper. Furthermore, it is mentioned that we utilize composite slack-matrix-based integral inequality techniques to estimate the derivative of a Lyapunov functional, such as defined in $\mathcal{L}V_4(x(t),t,i)$, which can induce tighter information on the delay of the considered system, which has been demonstrated in the numerical example section. Henceforth the investigation procedure and framework model proposed in this paper merit a lot of regard for fill such a demand all the more successfully.

5. Numerical example

To show how useful delayed neural networks can be, we will talk about a number of situations in which this method has been used and the improvements that have been seen as a result. These examples will range from easy to hard and show how useful this method can be in many different situations. The goal is to make it clear and easy to understand how this approach can be used and what can be gained from it.

Example 5.1. Take into account the subsequent delayed neural networks with $(i = 1,2)$:

$$\dot{x}(t) = -E_i x(t) + A_i f(x(t)) + B_i f(x(t-\tau(t))) + G_i v(t),$$
$$z(t) = C_i x(t) + D_i v(t), \tag{33}$$

Mode 1: $\pi_{11} = -0.8,\ \pi_{12} = 0.8$,

$$E_1 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.5 & -1.3 \\ 0.42 & 0.35 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.3 & 0.2 \\ 1.1 & 1.2 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 2 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ H_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Mode 2: $\pi_{21} = 0.4,\ \pi_{22} = -0.4$,

$$E_2 = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.1 & 0.3 \\ -0.22 & 0.25 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.3 & 1.2 \\ 1.1 & 1.2 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$F_1^- = F_2^- = -0.1,\ F_1^+ = F_2^+ = 0.9.$

In this example, we choose $\tau = 0.9$

$$\bar{\mathscr{Z}} = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.9 \end{bmatrix},\ \bar{\mathscr{T}} = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 1 \end{bmatrix},\ \bar{\mathscr{G}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let us consider $\mathscr{R} = 2I,\ c_1 = T = 1,\ \alpha = \beta = 0.1,\ d = 0.5$, and $\mu = 0.9$, we find the feasible solutions as follows by solving the LMIs in Theorem 4.1 using the Matlab LMI toolbox:

$$P_1 = \begin{bmatrix} 0.7551 & -0.0444 \\ -0.0444 & 0.5690 \end{bmatrix},\ P_2 = \begin{bmatrix} 1.2930 & -0.2823 \\ -0.2823 & 0.9281 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.9845 & -0.0932 \\ -0.0932 & 0.7130 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.9021 & -0.1305 \\ -0.1305 & 0.6981 \end{bmatrix},\ X = \begin{bmatrix} 0.1513 & -0.0336 \\ -0.0336 & 0.1086 \end{bmatrix},$$
$$Y = \begin{bmatrix} 7.4349 & 0.0020 \\ 0.0020 & 7.4337 \end{bmatrix}\ R = \begin{bmatrix} 7.4312 & -0.0000 \\ -0.0000 & 7.4312 \end{bmatrix},$$
$$\Sigma_1 = \begin{bmatrix} 7.4312 & 0 \\ 0 & 7.4312 \end{bmatrix},\ \Sigma_2 = \begin{bmatrix} 3.1234 & 0 \\ 0 & 3.1234 \end{bmatrix},$$
$$\Sigma_3 = \begin{bmatrix} 2.2821 & 0 \\ 0 & 2.2821 \end{bmatrix},\ U_1 = \begin{bmatrix} 2.5128 & -0.3178 \\ -0.3178 & 1.8038 \end{bmatrix},$$
$$U_2 = \begin{bmatrix} 0.2292 & -0.0354 \\ -0.0354 & 0.1678 \end{bmatrix},\ c_2 = 16.0036,$$

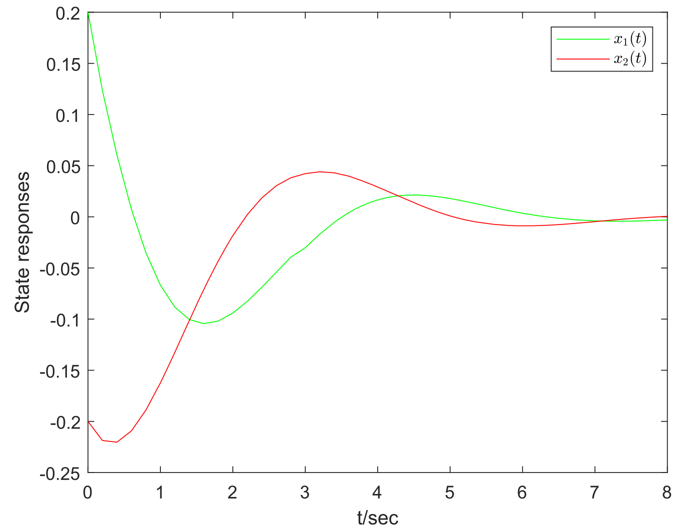

Fig. 1. Behavior of the state responses in Example 5.1.

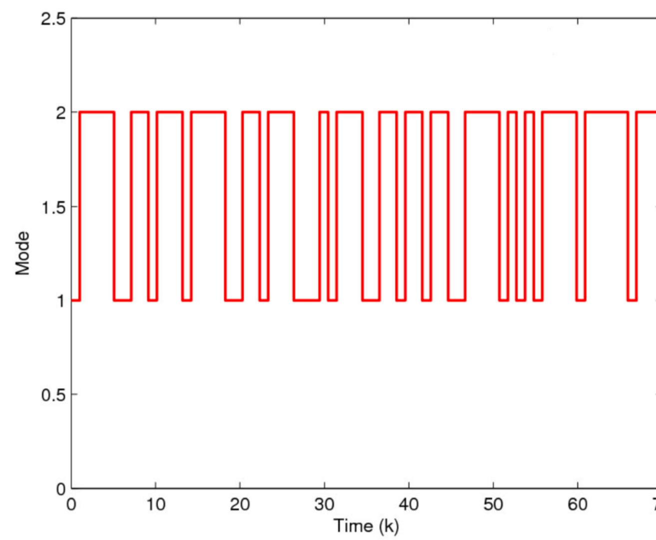

Fig. 2. Evolution of the modes in Example 5.1.

$$\bar{Q} = \begin{bmatrix} 28.0767 & -0.1228 & 4.1575 & 0.2788 \\ -0.1228 & 27.4979 & 0.2788 & 3.4528 \\ 4.1575 & 0.2788 & 6.4965 & 0.1395 \\ 0.2788 & 3.4528 & 0.1395 & 6.7023 \end{bmatrix}.$$

Furthermore, Fig. 1 depicts the state trajectories and Fig. 2 shows the mode evolution in Example 5.1. Fig. 3 also depicts the finite-time curve for the linked states. Using Theorem 4.1 and optimizing over value c_2, it is determined that delayed Markovian jumping neural networks (39) have FTB with regard to Δ_1 with minimal $c_2 = 16.0036$. This means that all the requirements of Theorem 4.1 are meet. Closed-loop stochastic MJNNs with time-varying delays are clearly finite-time dissipative.

6. Application of delayed neural networks

Example 6.1. This circuit is designed to perform a specific function by utilizing various electrical components. The activation function circuit is used to activate or deactivate certain elements in the circuit based on input signals. The time delay unit is responsible for introducing a delay in the circuit's response to input signals. The related unit is used to connect and integrate other components of the circuit, such as resistors, capacitors, inductance, operational amplifiers, and other commonly used electrical components. These components work together to

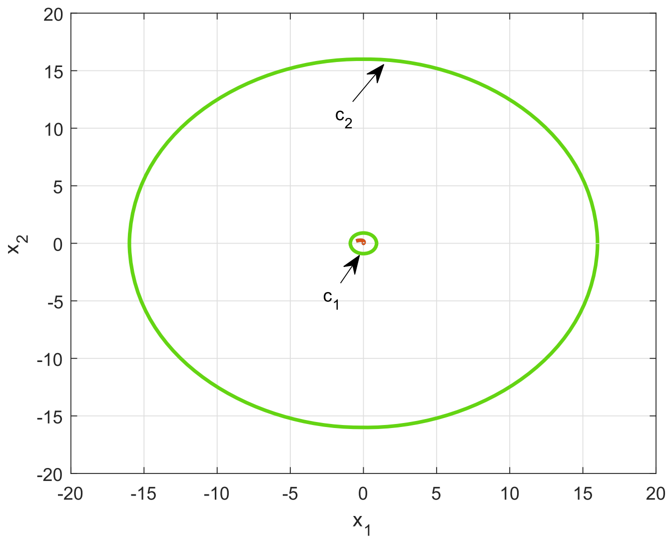

Fig. 3. Curves of $x^T(t)\mathscr{R}x(t)$ in Example 5.1.

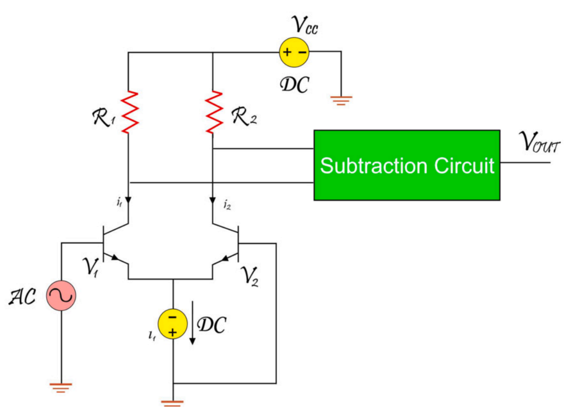

Fig. 4. Realization of the circuit of the hyperbolic tangent.

achieve the desired function of the circuit. Based on how complicated the models are, neural network models are turned into circuits.

To make it easier to understand how these circuits work, simple diagrams are used to show the different parts and how they are connected. As seen in Figs. 4 and 5, the suggested units can be used to construct a Hopfield-type neural network, and the system can be described as follows after analyzing the circuit node: The Hopfield-type neural network is a recurrent neural network with a unique design that can be used to do things like recognize patterns and find the best solution. The circuit has an activation function circuit, a time delay unit, and other parts such as resistors, capacitors, and inductances that work together. The activation function circuit is used to process the signals coming in and figure out what state the circuit is in when it is turned on. The time delay unit is used to slow down the circuit's response to input signals, which is needed for the network to be able to recognize and store patterns. The circuit node is evaluated to find out how well the system works, including how well it can recognize patterns and how quickly it can find the best solution. In general, using circuits to implement neural network models is a powerful method that makes it possible to process large amounts of data quickly and accurately.

The hyperbolic tangent (tanh) function is a common activation function used in neural networks. It can be realized in a circuit using a combination of operational amplifiers, resistors, and capacitors (see Fig. 4). Using a voltage-to-current converter and a differential amplifier is one way to put the tanh function into a circuit. The voltage-to-current converter converts the input voltage to a current, and the differential

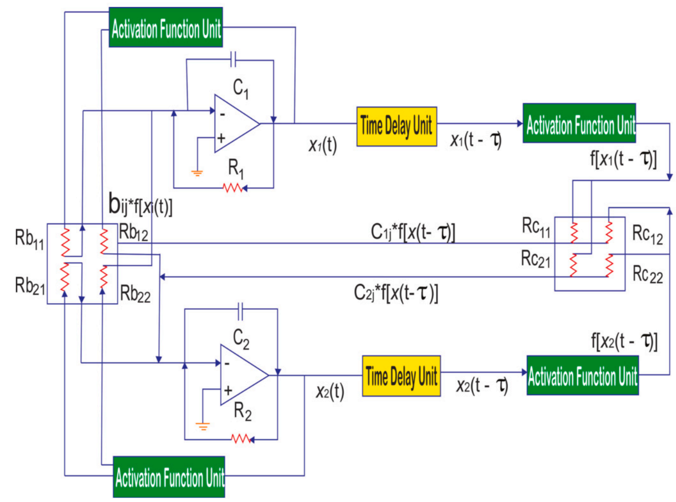

Fig. 5. Circuit realization of delayed neural networks with two neurons.

amplifier compares the current to a reference current and produces an output voltage. The voltage from the output then goes through a tanh function generator, which is usually made up of operational amplifiers and resistors. A precision rectifier circuit followed by a log-domain circuit is another way to use the tanh function in a circuit. The input voltage is turned into a current by the precision rectifier circuit. The current is then sent through a log-domain circuit, which turns the current into an output voltage. The voltage from this output is then sent to a tanh function generator, which can be made with operational amplifiers and resistors. In both cases, the implementation of the tanh function in a circuit requires precise design and the proper selection of the components to achieve the desired performance. The specific implementation depends on the requirements of the overall circuit and system.

In Fig. 5, a simple implementation of a delayed neural network with two neurons can be achieved by using two operational amplifiers, each representing a neuron. The first operational amplifier receives the input signal and produces an output signal. Before going to the second operational amplifier, this signal goes through a delay line, which causes a delay in the signal. The second operational amplifier receives the delayed signal and produces a final output signal. For the activation function to work, the output signal can be sent through a precision rectifier circuit before it goes to the second operational amplifier. This straightens out the signal and lets it go through the tanh function generator circuit, which is usually made up of operational amplifiers and resistors. For the second neuron to affect the first neuron, the circuit also needs a feedback loop. This can be done by connecting the output of the second operational amplifier to the input of the first operational amplifier with a gain. Stability analysis helps identify potential problems such as oscillations, diverging behavior, or other undesirable behaviors that may occur in the circuit. In particular, it's important to analyze the stability of the feedback loop in the circuit, as this is where the second neuron influences the first one. If the feedback loop is not set up correctly or the gain isn't set right, the circuit could become unstable and give results that are hard to predict. Stability analysis is also important for figuring out how stable the circuit's response is to signals from outside. The circuit should be able to respond to the input signals in a stable and predictable way. During the stability analysis, any problems with how the circuit responds should be found and fixed.

Stability analysis is important for the circuit realization of delayed neural networks with two neurons. This helps make sure that the circuit works as expected and does not become unstable over time. It also helps find problems, like oscillations or behavior that goes in different directions, and makes sure that the circuit's response to signals from outside is stable and predictable. Fig. 5 demonstrates how the proposed units, such as resistors, capacitors, operational amplifiers, and delay lines, can be assembled to create a neural network with two neurons. This network can be used for a variety of tasks, such as pattern recognition and optimization. By evaluating the circuit nodes, the system can be described mathematically as shown in equation (34). For our convenience to find stability analysis, we consider delayed neural networks with two neurons:

$$\frac{dx_p(t)}{dt} = -a_p x_p(t) + \sum_{q=1}^{n} b_{pq} f(x_q(t)) + \sum_{q=1}^{n} c_{pq} f(x_q(t-\tau)),\ p = 1, 2, \cdots, n, \tag{34}$$

where $a_p = \frac{1}{C_p R_p}, b_{pq} = \frac{1}{C_p R b_{pq}}$ and $c_{pq} = \frac{1}{C_p R c_{pq}}$. As illustrated in Figs. 4 and 5, the proposed units can be used to construct a neural network with two neurons. By analyzing the circuit nodes, the system can be described as follows.

$$\frac{dx_1(t)}{dt} = -\frac{1}{C_1 R_1} x_1(t) + \frac{1}{C_1 R b_{11}} f(x_1(t)) + \frac{1}{C_1 R b_{12}} f(x_2(t)) + \frac{1}{C_1 R c_{11}} f(x_1(t-\tau)) + \frac{1}{C_1 R c_{12}} f(x_2(t-\tau))$$

$$\frac{dx_2(t)}{dt} = -\frac{1}{C_2 R_2} x_1(t) + \frac{1}{C_2 R b_{21}} f(x_1(t)) + \frac{1}{C_2 R b_{22}} f(x_2(t)) + \frac{1}{C_2 R c_{21}} f(x_1(t-\tau)) + \frac{1}{C_2 R c_{22}} f(x_2(t-\tau))$$

As demonstrated in Figs. 4 and 5, neural networks with two neurons can be constructed using the proposed units. By analyzing the circuit nodes, it can be seen that the system can be described mathematically as shown in equation (34). Furthermore, this same approach can be extended to systems with more neurons by simply increasing the number of neurons in the circuit, the system with n neurons can be represented by the same equation (34). This demonstrates the scalability of this circuit-based approach to neural networks.

$$\begin{cases} \frac{dx_1(t)}{dt} &= -a_1 x_1(t) + \sum_{p=1}^{n} b_{1q} f(x_q(t)) + \sum_{q=1}^{n} c_{1q} f(x_q(t-\tau)) \\ \frac{dx_2(t)}{dt} &= -a_2 x_2(t) + \sum_{p=1}^{n} b_{2q} f(x_q(t)) + \sum_{q=1}^{n} c_{2q} f(x_q(t-\tau)) \\ \vdots &= \vdots \qquad \vdots \qquad \vdots \\ \frac{dx_n(t)}{dt} &= -a_n x_n(t) + \sum_{p=1}^{n} b_{nq} f(x_q(t)) + \sum_{q=1}^{n} c_{nq} f(x_q(t-\tau)) \end{cases} \tag{35}$$

In this electronic circuit, some parameters are set to a fixed value. Specifically, b_{22} and c_{22} vary while maintaining $b_{22} = c_{22} = 0.5$. This study focuses on neural networks represented by Equation (35) with two neurons. The system can be simplified by considering the fixed values of b_{22} and c_{22} and expressing it as a function of these parameters.

$$\dot{x}(t) = -\mathcal{A}x(t) + \mathcal{B}f(x(t)) + \mathcal{C}f(x(t-\tau)), \tag{36}$$

where

$$\mathcal{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathcal{B} = \begin{bmatrix} 2 & -0.1 \\ -5 & b_{22} \end{bmatrix},\ \mathcal{C} = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & c_{22} \end{bmatrix},$$

b_{22}, c_{22} are allowed to be changeable values, the activation function is $f(x(t)) = \tanh(x(t))$, and by using the Matlab LMI toolbox and solving the LMIs in (36), a group of feasible solutions can be acquired. Along these lines, we conclude that Remark 4.3 infers that the system (36) is finite-time stable. The state response of system (36) is shown in the Fig. 6 with the initial condition $x(t) = [-3, 2]^T$.

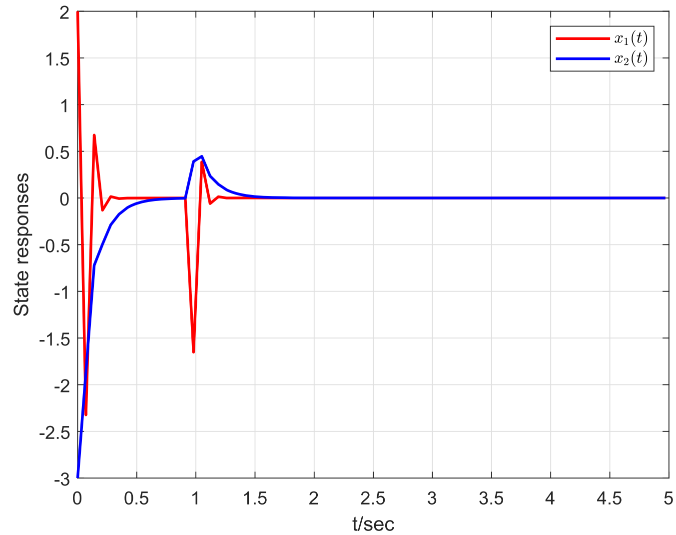

Fig. 6. Behavior of the state responses in Example 6.1.

Table 1
The maximum allowable delay bounds (MADBs) of τ for various μ in Example 7.1.

Methods	$\mu = 0.8$	$\mu = 0.9$
[8]	2.9105	2.1854
[9]	4.8167	3.4245
Theorem 3.1	4.9196	3.8262

7. Comparison examples

In this study, we consider the delayed neural networks represented by Equation (32), which includes specific parameters that are used in circuit design. These parameters are likely to affect the behavior and performance of the network and therefore need to be carefully chosen and optimized to achieve the desired results.

Example 7.1. Consider the delayed neural networks (32) with the following parameters:

$$E_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix},$$

$$F_1^- = F_2^- = 0,\ F_1^+ = 0.4,\ F_2^+ = 0.8.$$

It is possible to demonstrate that the LMIs (4)-(5) is practical. Table 1 shows the acceptable upper boundaries of τ for various values of μ. It is evident that the delay-dependent stability solution given in Theorem 3.1 yields less conservative findings than [8,9].

Example 7.2. Consider the delayed neural networks (32) with the following parameters:

$$E_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.7 \end{bmatrix},\ A_1 = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},\ F_1^- = F_2^- = 0,$$

$$F_1^+ = 0.3,\ F_2^+ = 0.8.$$

Table 2
MADBs of τ for various μ in Example 7.2.

Methods	$\mu = 0.4$	$\mu = 0.45$	$\mu = 0.5$	$\mu = 0.55$
[6]	5.4036	4.6017	4.3121	4.1582
[10](Corollary. 1)	5.6504	4.7596	4.4276	4.2450
[10](Corollary. 2)	7.5919	6.6339	6.2829	6.0999
[10](Corollary. 3)	7.4203	6.6190	6.3428	6.2095
[10](Corollary. 4)	7.5049	6.6401	6.3715	6.0237
[10](Corollary. 5)	5.4065	4.6401	4.3715	4.2224
[10](Theorem. 1)	7.6697	6.7287	6.4126	6.2569
Theorem 3.1	7.9999	7.0012	6.8695	6.6666

Table 3
MADBs of τ for various μ in Example 7.3.

Methods	$\mu = 0.1$	$\mu = 0.5$	$\mu = 0.9$
[7]	3.4984	2.7243	2.2029
[10](Corollary. 1)	3.9055	3.0997	2.6944
[10](Corollary. 2)	4.2729	3.0666	2.7687
[10](Corollary. 3)	4.1838	3.1510	2.8347
[10](Corollary. 4)	4.2732	3.0666	2.7648
[10](Corollary. 5)	3.8554	3.0278	2.6526
[10](Theorem. 1)	4.2993	3.1577	2.8371
Theorem 3.1	4.8051	3.6270	3.0006

We compared our results with the current results in [6,10], and we gave various values of μ. Table 2 shows the comparing findings. According to Table 2, the results in this study are less conservative than those in [6,10].

Example 7.3. Consider the delayed neural networks (32) with the following parameters:

$$E_1 = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.578 \\ -0.1311 & 0.3253 & -0.9534 & -0.501 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix},$$

$$F_1^- = F_2^- = F_3^- = F_4^- = 0, \ F_1^+ = 0.1137,$$

$$F_2^+ = 0.1279, \ F_3^+ = 0.7994, \ F_4^+ = 0.2368.$$

The findings in [7,10] are provided. Table 3 shows the comparing findings. According to Table 3, the results in this study are less conservative than those in [7,10].

8. Conclusion

In this study, we present a unique method for analyzing the finite-time dissipativity of stochastic Markovian jump-delayed neural networks. LMIs are developed by constructing an appropriate LKF and using Newton-Leibniz methods to enumerate delay-dependent dissipativity requirements. This new strategy yields dissipativity criteria that are less conservative. The simulation results back up the theoretical reasons made for the suggested theorems. Numerical results are provided to demonstrate the utility of the suggested technique. Outlining illustrations have been used to compare the benefits of the suggested methodology to existing works in the literature. This paper focuses on finite-time dissipativity analysis for Markovian jump-delayed neural networks and the analytical method developed may be extended to additional control issues such as sampled-data control [15] and event-triggered control [21], also focus on the retarded time delay system [41,42] to reduce the conservatism issue, all of which will be investigated in future research.

CRediT authorship contribution statement

All authors contributed equally and significantly to the writing of this article and typed, read, and approved the final manuscript.

Declaration of competing interest

I confirm that neither I with which I am associated have any personal interest in or potential for personal gain from any of the organizations.

Availability of data and materials

Data sharing is not applicable to this article as data sets were not generated or analyzed during the current study.

Acknowledgement

This work was supported by the Institute of Information and Communications Technology Planning and Evaluation funded by the Korea Government, Ministry of Science and ICT (Building a Digital Open Lab as open innovation platform) under Grant 2021-0-00546.

References

[1] Y. Liu, Z. Wang, X. Liu, Global exponential stability of generalized recurrent neural networks with discrete and distributed delays, Neural Netw. 19 (2006) 667–675.
[2] J. Sun, G.P. Liu, J. Chen, D. Rees, Improved stability criteria for neural networks with time-varying delay, Phys. Lett. A 373 (2009) 342–348.
[3] Y. Zhou, Z. Zeng, Event-triggered finite-time stabilization of fuzzy neural networks with infinite time delays and discontinuous activations, IEEE Trans. Fuzzy Syst. (2023), https://doi.org/10.1109/TFUZZ.2023.3287202.
[4] Y. Tong, Z. Ren, D. Tong, Z. Fan, X. Feng, Combined finite-time state feedback design for discrete-time neural networks with time-varying delays and disturbances, Neural Process. Lett. (2023), https://doi.org/10.1007/s11063-023-11289-y.
[5] L. Yu, D. Zhang, Exponential state estimation for Markovian jumping neural networks with time-varying discrete and distributed delays, Neural Netw. 35 (2012) 103–111.
[6] W. Chen, Q. Ma, G. Miao, Y. Zhang, Stability analysis of stochastic neural networks with Markovian jump parameters using delay-partitioning approach, Neurocomputing 103 (2013) 22–28.
[7] H. Zhang, F. Yang, X. Liu, Q. Zhang, Stability analysis for neural networks with time-varying delay based on quadratic convex combination, IEEE Trans. Neural Netw. Learn. Syst. 24 (2013) 513–521.
[8] X.B. Zhou, J.K. Tian, H.J. Ma, S.M. Zhong, Improved delay-dependent stability criteria for recurrent neural networks with time-varying delays, Neurocomputing 129 (2014) 401–408.
[9] H-B. Zeng, Y. He, M. Wu, S-P. Xiao, Stability analysis of generalized neural networks with time-varying delays via a new integral inequality, Neurocomputing 161 (2015) 148–154.
[10] C-K. Zhang, Y. He, L. Jiang, M. Wu, Stability analysis for delayed neural networks considering both conservativeness and complexity, IEEE Trans. Neural Netw. Learn. Syst. 27 (2016) 1486–1501.
[11] H.B. Zeng, J.H. Park, C.-F. Zhang, W. Wang, Stability and dissipativity analysis of static neural networks with interval time-varying delay, J. Franklin Inst. 352 (2015) 1284–1295.
[12] T. Radhika, G. Nagamani, Q. Zhu, S. Ramasamy, R. Saravanakumar, Further results on dissipativity analysis for Markovian jump neural networks with randomly occurring uncertainties and leakage delays, Neural Comput. Appl. 30 (2018) 3565–3579.
[13] C.K. Ahn, P. Shi, R.K. Agarwal, J. Xu, L_∞ performance of single and interconnected neural networks with time-varying delay, Inf. Sci. 346 (2016) 412–423.
[14] C-D. Zheng, Y. Gu, W. Liang, Z. Wang, Novel delay-dependent stability criteria for switched Hopfield neural networks of neutral type, Neurocomputing 158 (2015) 117–126.
[15] M. Syed Ali, N. Gunasekaran, Q. Zhu, State estimation of T-S fuzzy delayed neural networks with Markovian jumping parameters using sampled-data control, Fuzzy Sets Syst. 306 (2017) 87–104.

[16] G. Nagamani, Y.H. Joo, T. Radhika, Delay-dependent dissipativity criteria for Markovian jump neural networks with random delays and incomplete transition probabilities, Nonlinear Dyn. 348 (2018) 2503–2522.
[17] X. Gao, L. Lian, W. Qi, Finite-time dissipativity analysis and design for stochastic Markovian jump systems with generally uncertain transition rates and time-varying delay, Trans. Inst. Meas. Control 39 (2017) 807–819.
[18] L. Wu, W.X. Zheng, H. Gao, Dissipativity-based sliding mode control of switched stochastic systems, IEEE Trans. Autom. Control 58 (2013) 785–793.
[19] H.B. Zeng, J.H. Park, J.W. Xia, Further results on dissipativity analysis of neural networks with time-varying delay and randomly occurring uncertainties, Nonlinear Dyn. 79 (2015) 83–91.
[20] G. Nagamani, T. Radhika, P. Gopalakrishnan, Dissipativity and passivity analysis of Markovian jump impulsive neural networks with time delays, Int. J. Comput. Math. 194 (2017) 1479–1500.
[21] R. Vadivel, P. Hammachukiattikul, N. Gunasekaran, R. Saravanakumar, H. Dutta, Strict dissipativity synchronization for delayed static neural networks: an event-triggered scheme, Chaos Solitons Fractals 150 (2021) 111212.
[22] J. Shiyu, H. Shen, Y. Wei, X. Huang, Z. Wang, Further results on dissipativity and stability analysis of Markov jump generalized neural networks with time-varying interval delays, Appl. Math. Comput. 336 (2018) 338–350.
[23] Y. Kao, L. Shi, J. Xie, H.R. Karimi, Global exponential stability of delayed Markovian jump fuzzy cellular neural networks with generally incomplete transition probability, Neural Netw. 63 (2015) 18–30.
[24] H.R. Karimi, Passivity-based output feedback control of Markovian jump systems with discrete and distributed time-varying delays, Int. J. Syst. Sci. 44 (2013) 1290–1300.
[25] Y.G. Kao, C.H. Wang, J. Xie, H.R. Karimi, W. Li, H_∞ sliding mode control for uncertain neutral-type stochastic systems with Markovian jumping parameters, Inf. Sci. 314 (2015) 200–211.
[26] W.H. Qi, X.W. Gao, Finite-time H_∞ control for stochastic time-delayed Markovian switching systems with partly known transition rates and nonlinearity, Int. J. Syst. Sci. 14 (2016) 637–646.
[27] Y. Kao, C. Wang, J. Xie, H.R. Karimi, New delay-dependent stability of Markovian jump neutral stochastic systems with general unknown transition rates, Int. J. Syst. Sci. 47 (2016) 2499–2509.
[28] H. Wang, Y. Ni, J. Wang, J. Tian, C. Ge, Sampled-data control for synchronization of Markovian jumping neural networks with packet dropout, Appl. Intell. (2015) 1–12, https://doi.org/10.1007/s10489-022-03379-6.
[29] P. Balasubramaniam, G. Nagamani, Passivity analysis of neural networks with Markovian jumping parameters and interval time-varying delays, Nonlinear Anal. Hybrid Syst. 4 (2010) 853–864.
[30] S. Li, Y. Ma, Finite-time dissipative control for singular Markovian jump systems via quantizing approach, Nonlinear Anal. Hybrid Syst. 27 (2018) 323–340.
[31] S. Saravanan, M. Syed Ali, R. Saravanakumar, Finite-time non-fragile dissipative stabilization of delayed neural networks, Neural Process. Lett. 49 (2019) 573–591.
[32] C. Ren, S. He, Finite-time stabilization for positive Markovian jumping neural networks, Appl. Math. Comput. 365 (2020) 124631.
[33] Y. Zhang, P. Shi, S.K. Nguang, J. Zhang, H.R. Karimi, Finite-time boundedness for uncertain discrete neural networks with time-delays and Markovian jumps, Neurocomputing 140 (2014) 1–7.
[34] C. Zheng, J. Cao, M. Hu, X. Fan, Finite-time stabilization for discrete-time T-S fuzzy model system with channel fading and two types of parametric uncertainty, Int. J. Syst. Sci. 48 (2017) 34–42.
[35] Y. Wu, J. Cao, A. Alofi, A. AL-Mazrooei, A. Elaiw, Finite-time boundedness and stabilization of uncertain switched neural networks with time-varying delay, Neural Netw. 69 (2015) 135–143.
[36] K. Gu, An integral inequality in the stability problem of time-delay systems, in: Proc. 39th IEEE Conf. Decision and Control, Sydney, Australia, 2000, pp. 2805–2810.
[37] P. Liu, Robust exponential stability for uncertain time-varying delay systems with delay dependence, J. Franklin Inst. 346 (2009) 958–968.
[38] H.C. Lin, H.B. Zeng, X.M. Zhang, W. Wang, Stability analysis for delayed neural networks via a generalized reciprocally convex inequality, IEEE Trans. Neural Netw. Learn. Syst. (2022), https://doi.org/10.1109/TNNLS.2022.3144032.
[39] D.T. Hong, N.H. Sau, M.V. Thuan, New criteria for dissipativity analysis of fractional-order static neural networks, Circuits Syst. Signal Process. 41 (4) (2022) 2221–2243.
[40] Y. Tian, Z. Wang, Composite slack-matrix-based integral inequality and its application to stability analysis of time-delay systems, Appl. Math. Lett. 120 (2021) 107252.
[41] G. Zhuang, S.-F. Su, J. Xia, W. Sun, HMM-based asynchronous H_∞ filtering for fuzzy singular Markovian switching systems with retarded time-varying delays, IEEE Trans. Cybern. 120 (2021) 1189–1203.
[42] G. Zhuang, J. Xia, J.-e. Feng, W. Sun, B. Zhang, Admissibilization for implicit jump systems with mixed retarded delays based on reciprocally convex integral inequality and Barbalat's lemma, IEEE Trans. Syst. Man Cybern. Syst. 51 (2021) 6808–6818.