Received: 26 February 2022 | Revised: 22 August 2022 | Accepted: 24 September 2022

DOI: 10.1002/asjc.2965

RESEARCH ARTICLE

WILEY

Event-triggered synchronization for stochastic delayed neural networks: Passivity and passification case

R. Vadivel[1] | P. Hammachukiattikul[1] | Quanxin Zhu[2,3] | Nallappan Gunasekaran[4]

[1]Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Phuket, Thailand

[2]School of Mathematics and Statistics, Hunan Normal University, Changsha, China

[3]The Key Laboratory of Control and Optimization of Complex Systems, College of Hunan Province, Hunan Normal University, Changsha, China

[4]Computational Intelligence Laboratory, Toyota Technological Institute, Aichi, Nagoya, Japan

Correspondence
Quanxin Zhu, School of Mathematics and Statistics, Hunan Normal University, Changsha 410081, China.
Email: zqx22@126.com

Funding information
This work was jointly supported by the National Natural Science Foundation of China (62173139) and the Science and Technology Innovation Program of Hunan Province (2021RC4030).

Abstract

This article investigates the event-triggered synchronization problem of stochastic neural networks under passivity and passification cases. For saving communication resources, an event-triggered approach is engaged in the design of synchronization for the delayed stochastic neural networks. To decrease network trouble, an event-triggered scheme is suggested between the sampler and communication network. A nonfragile event-triggered controller is intended to guarantee the finite-time stability of the subsequent closed-loop system. By applying the Lyapunov–Krasovkii functional (LKF) and the novel integral inequalities, a stability criteria for an interval-time varying delay error system ensure the designed controller can fulfill the necessities of passivity and passification performance. The desired control gain and event-triggered parameters are then found based on the linear matrix inequalities (LMIs). Finally, illustrative examples are given to show the benefits and validity of the desired control law.

KEYWORDS

event-triggered control, Lyapunov–Krasovskii functional, passivity, stochastic neural networks, synchronization

1 | INTRODUCTION

Neural networks (NNs) are generally considered as one of the simplified models of neural processing in human brains. Besides that, stochastic neural networks (SNNs) model has been broadly researched and effectively implemented in numerous areas such as information sciences, associative memory, and optimization problems [1–3]. Moreover, in the electronic circuit usage of the NNs, time-varying delays brought about by the limited exchanging pace of amplifiers and communication time, the above affect factors often give rise to oscillation, poor performance, and even instability [4, 5]. In addition, the vast majority of the previously discussed NNs model are unavoidable, and they are reasonable to the situation when there is no perturbation is delivered by the stochastic circumstance. Stochastic perturbations frequently occur in real-world systems because of the presence of environmental noise and human disturbances. Therefore, delayed SNNs state likely better deliberate the absoluteness and gotten significant considerations, a huge number of research outcomes have been detailed in the literature [4–6].

Typically, from the purpose of engineering, the finite-time stability study of the considered framework is the important one in further utilization. The stability of neural networks has additionally been paid a lot of care in advanced research and is one of the hottest topics. Certainly, most of the research studies was about whether the networks belong to stability in infinite time.

For example, asymptotic stability [7] and exponential stability [8], all these stability issues make the system stable in the infinite time interval. Although, for practical application purposes, all the networks deal with the finite time interval and reached the stability condition [9, 10]. Since finite-time stability (FTS) demands that the system state does not overcome a specific threshold throughout the limit of time interval, it typically exhibits quicker convergence rates, better accuracies, and stronger disturbance rejection properties, leading to the analysis of finite-time stability for SNNs have recently attracted a lot of attention from researchers [11–13]. To name a few, in Lv and Li [14], finite-time stability of coupled impulsive NNs have been considered with time-varying delays and saturating actuators. In Wang and Shen [15], FTS and instability of memristive NNs with the nonlinear discontinuous controller have been discussed. Subsequently, for the superiority of finite-time convergence, Zhang and Cao [16] introduced the concept of synchronization guideline for inertial NNs with time delays and a new inequality method. On the other hand, the passivity theory has played an important tool to study the stability analysis of dynamical systems [17], the passification, similar to the problem of stabilization for a dynamical system, is to design a controller to ensure the passivity of the system [18, 19]. The previously mentioned literature maximum studied the passivity-based control techniques and obtained the stability in the infinite time interval. In this manner, recently, researchers choose finite-time with some control techniques that likely meet the practical requirements [13–15, 18].

Due to the pioneering work of synchronization between master and slave NNs (MSNNs), the issue of stochastic synchronization process has been sparked by science and technology because of its wide range of applications, such as cryptography [20], secure communication [21], and circuit process [22]. It is notable that the synchronization of SNNs is essential to recognize how random noise influences the system dynamic behavior responses. It is thus a remarkable and essential one to SNNs with master and slave effects. Over the past 10 years, some fundamental results have been discussed to the SNNs relating to synchronization. For instance, a class of synchronization of uncertain stochastic reaction-diffusion NNs have been studied in Ding et al. [23] and an intermittent nonfragile control scheme was proposed in terms of linear matrix inequalities (LMIs). Pinning impulsive synchronization of stochastic delayed NNs were examined in Pan et al. [24] via uniformly stable function. In Wang et al. [25], stochastic synchronization for Markovian coupled NNs has been discussed with the time-varying mode-delays technique. Although exponential synchronization allows two complex networks to achieve perfect synchronization at the fastest speed, it is complicated to apply to complex practical systems. As a result, many researchers have shifted their focus to nearly certainly asymptotic synchronization with the finite-time case. The above related cited synchronization can only be achieved when the time interval closes to infinite. Instead of the above literature, in this work, it is fascinating and beneficial to achieve synchronization in a finite time case. As is acknowledged, it is important and deserving of examining the finite-time synchronization issues of SNNs in the passivity case. Handling these concerns is one of the first motivations.

However, to get the desired system performance of SNNs like stability, synchronization, passivity, and so forth, a very few good control techniques have been implemented, including sampled-data control [26], intermittent control [27], and state feedback control [28]. It is noticed that all the previously mentioned control methods require continuous feedback signals to SNNs. In any case, in execution, the communication resources of SNNs are constantly limited. For practical purposes, some effective control scheme has been needed to save the communication resources of SNNs. Control systems are applied in real situations using digitally sampled data. The most typical method is to send sampled measurements on a regular basis. Although it is preferable for modeling and evaluating systems in a periodic way, the communication burden is frequently overlooked. As a result, for systems with limited network resources and large communication loads, an efficient technique to reducing wasteful communication resource use while preserving control performance would be required. Recently, an event-triggered technique has been presented as an alternate way to conserving restricted network resources, in which the sample signal is released only if an event-triggering condition connected to system state is violated [29, 30]. If compared to traditional time-triggered communication, an important feature of the event triggering approach is its potential to reduce redundant transmissions while maintaining assured system performance. In recent years, event-triggered strategies for stochastic models have received increased attention, with numerous notable results reported in the literature [30–33]. It should be noted, however, that many proven conclusions pertaining to event-triggering techniques are in the framework of continuous-time and discrete-time systems [34]. Using the event-triggered approach, beneficial results on the investigation of SNNs for several stability issues, such as finite-time synchronization [30, 32, 35, 36], have been established. Meanwhile, due to the implementation, limitations of the system equipment, and impact of environments, some fluctuations certainly happen in the control process of SNNs. However, these fluctuations may cause the instability of the synchronization system [37, 38]. Hence, it is necessary to model nonfragile terms in the ETC because these can effectively save the communication resources. Nevertheless, finite-time event-triggered synchronization for SNNs via passivity and passification approach has not been well addressed yet. It is the primary interest of this current study.

Inspired by the above analysis, this article is devoted to pursuing finite-time event-triggered synchronization for SNNs along with the passivity and passification conditions. The suggested method not only achieves the finite-time synchronization apart from that modification of the MSS to save precious communication resources. The major contributions of our currently proposed research are as follows:

(I) Nonfragile ETC is designed for SNNs. Compared to different available control schemes, the designed one is more applicable and can adequately save the limited network bandwidth.

(II) Finite-time boundedness and finite-time passivity are presented for the resulting synchronization error system with respect to stochastic analysis technique, integral inequality approaches, and novel LKF methods

(III) Nonfragile event-triggered control is proposed to ensure the SNNs using linear matrix inequalities (LMIs).

(IV) Finally, numerical examples are provided to demonstrate the effectiveness of the designed ETC law, which are able to be solved easily by the familiar computing software MATLAB.

Notation: For a matrix S, S^{-1} indicates inverse and S^T represents the transpose; $\mathbb{R}^m$ and $\mathbb{R}^{n\times m}$ denotes the n-dimensional Euclidean space and set of $n\times m$ real matrix, respectively. $\mathcal{Y}$ is a positive (negative) definite matrix, such that $\mathcal{Y} > 0, (\mathcal{Y} < 0)$, and I_n denotes the identity matrix of dimension n; mathematical expectation $\mathbb{E}\{\cdot\}$ denotes the stochastic process; $\star$ in a symmetric matrix indicates the entry means symmetry; we define $\lambda_{\min}(\mathscr{P})/\lambda_{\max}(\mathscr{P})$ as the minimal/maximal eigenvalue of $\mathscr{P}$.

2 | PRELIMINARIES

In this paper, we consider the following SNNs with mixed time-varying delays

$$\begin{aligned} d\hat{x}(t) &= [-W_1\hat{x}(t) + W_2 g(\hat{x}(t)) + W_3 g(\hat{x}(t-\tau(t)))]dt \\ &\quad + (B\hat{x}(t) + C\hat{x}(t-\tau(t)))dw(t), \\ z(t) &= Ag(\hat{x}(t)), \\ \hat{x}(t) &= \varphi(t),\ t \in [-\tau_2, 0], \end{aligned} \tag{1}$$

where $\hat{x}(t) = (\hat{x}_1(t), \hat{x}_2(t), \dots, \hat{x}_n(t)) \in \mathbb{R}^n$ denotes the state vector of the neuron; $W_1 = diag(w_{11}, w_{12}, \dots, w_{1n})$ is a positive diagonal matrix, $W_2 = (w_{2_{ij}})_{n\times n}$, and $W_3 = (w_{3_{ij}})_{n\times n}$ are known constant matrices; $A \in \mathbb{R}^n$, $B \in \mathbb{R}^n$, and $C \in \mathbb{R}^n$ are the known constant matrices; $\varphi(t)$ is the initial condition; $w(t) = (w_1(t), w_2(t), \dots, w_n(t))^T$ is an n-dimensional Brownian motion, and it is defined as a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathscr{P})$ and satisfying $\mathbb{E}[dw(t)] = 0$ and $\mathbb{E}[dw^2(t)] = dt$; $f(\hat{x}(t)) = (f_1(\hat{x}_1(t)), f_2(\hat{x}_2(t)), \dots, f_n(\hat{x}_n(t)))^T$ represents a neuron activation function at time t; $z(t) \in \mathbb{R}^p$ is the output; $\tau(t) > 0$ is the time-varying delay and is supposed to achieve $\tau_1 \leq \tau(t) \leq \tau_2$, $\dot{\tau}(t) \leq \tau_3$ where τ_1, τ_2, and τ_3 are constants.

For the master system (1), the slave system is considered as follows:

$$\begin{aligned} d\check{x}(t) &= [-W_1\check{x}(t) + W_2 g(\check{x}(t)) + W_3 g(\check{x}(t-\tau(t))) \\ &\quad + W_4 u(t) + W_5 \wp(t)]dt + (B\check{x}(t) + C\check{x}(t-\tau(t)))dw(t), \\ \check{z}(t) &= Ag(\check{x}(t)), \\ \check{x}(t) &= \psi(t), \end{aligned} \tag{2}$$

where $\check{x}(t) = (\check{x}_1(t), \check{x}_2(t), \dots, \check{x}_n(t)) \in \mathbb{R}^n$ are the state responses of the controlled system. $u(t) \in \mathbb{R}^m$ is the control input. $\wp(t) \in \mathbb{R}^l$ is the external disturbance that belongs to $\mathbb{L}_2[0, \infty)$. W_4 and W_5 are known real constant matrices with appropriate dimensions. The error is $x(t) = \check{x}(t) - \hat{x}(t), \tilde{z}(t) = \check{z}(t) - z(t)$, and $f(x(t)) = g(\check{x}(t)) - g(\hat{x}(t))$.

Then, one can obtain the error system as follows:

$$\begin{aligned} dx(t) &= [-W_1 x(t) + W_2 f(x(t)) + W_3 f(x(t-\tau(t))) \\ &\quad + W_4 u(t) + W_5 \wp(t)]dt + (Bx(t) + Cx(t-\tau(t))))dw(t), \\ \tilde{z}(t) &= Af(x(t)) \\ x(t) &= \psi(t) - \varphi(t) = \phi(t). \end{aligned} \tag{3}$$

The following common assumption is needed for activation function:

(H1) For any $j = [1, 2, \dots, n], g_j(0) = 0$ and satisfying constants $0 \leq l_j^- < l_j^+$ such that

$$l_j^- \leq \frac{g_j(\beta_1) - g_j(\beta_2)}{\beta_1 - \beta_2} \leq l_j^+, \beta_1 \neq \beta_2. \tag{4}$$

From Assumption (H1), it is easy to see that in error system (3), it holds $l_j^- \leq \frac{f_j(\hat{\beta})}{\hat{\beta}} \leq l_j^+, \forall \hat{\beta} \neq 0$.

(H2) For a given parameter $d > 0$, such that $\wp(t)$ satisfies $\int_0^T \wp^T(t)\wp(t)dt \leq d$.

In the network communication, an event-generator constructed on a periodically sampled state $x(\jmath\hat{h})$, $\hat{h}$ is a constant sampling period. For every triggered scheme, sampled data can be planned to decide whether or not the current sampled data have to be sent to the controller to

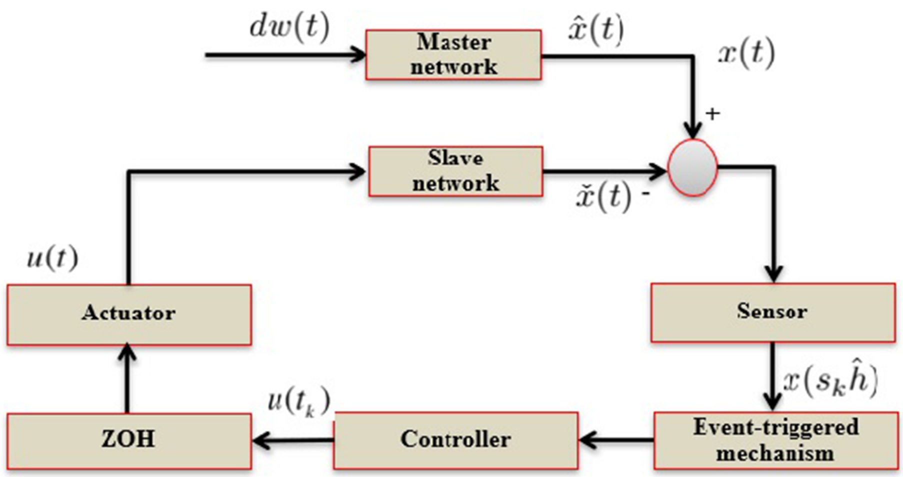

FIGURE 1 The diagrammatic of master slave neural networks [Color figure can be viewed at wileyonlinelibrary.com]

mitigate unnecessary waste of network resources. Next, sampled stated $x(s_k\hat{h}+\jmath\hat{h})$ is delivered by the event generator at the current sampling instant $x(s_k\hat{h}+\jmath\hat{h})$ and the recent transmitted one $x(s_k\hat{h})$ fulfill the accompanying condition:

$$\begin{aligned}\mathbb{E}\{[x(s_k\hat{h}+\jmath\hat{h})-x(s_k\hat{h})]^T\tilde{\Theta}_1[x(s_k\hat{h}+\jmath\hat{h})-x(s_k\hat{h})]\}\\ \leq \kappa\mathbb{E}\{x^T(s_k\hat{h}+\jmath\hat{h})\tilde{\Theta}_1 x(s_k\hat{h}+\jmath\hat{h})\},\end{aligned} \tag{5}$$

where $\kappa \in [0,1)$, $\tilde{\Theta}_1 > 0$ is a matrix, which is symmetry and to be determined. With the network induced delay, the real input of the actuator can be modeled as

$$u(t)=\hat{K}x(s_k\hat{h}), t\in[s_k\hat{h}+\eta_{s_k}, s_{k+1}\hat{h}+\eta_{s_{k+1}}), \tag{6}$$

where $\eta_{s_k}\in[0,\eta]$, $k\in\mathbb{N}$ is the network-induced delay. In this study, an actual nonfragile controller is constructed as follows:

$$u(t)=(\hat{K}+\Delta\hat{K}(t))x(s_k\hat{h}), t\in[s_k\hat{h}+\eta_{s_k}, s_{k+1}\hat{h}+\eta_{s_{k+1}}), \tag{7}$$

where $\mathbb{k}=\hat{K}+\Delta\hat{K}(t)$, $\hat{K}$ is control gain matrix, which is to be evaluated, and gain variation $\Delta\hat{K}(t)$ is characterized by the norm-bounded uncertainty form as in (8)

$$\Delta\hat{K}(t)=H\mathcal{M}_4(t)E, \tag{8}$$

where H and E are known real matrices of proper dimensions, and $\mathcal{M}_4(t)$ is an unknown time-varying matrix satisfying $\mathcal{M}_4^T(t)\mathcal{M}_4(t)\leq I$. Now, we defined the time interval as $[s_k\hat{h}+\eta_{t_k}, s_{k+1}\hat{h}+\eta_{t_{k+1}})$ into $t_{k+1}-t_k$ subintervals

$$\begin{aligned}[s_k\hat{h}+\eta_{t_k}, s_{k+1}\hat{h}+\eta_{t_{k+1}})=\bigcup_{j=0}^{s_{k+1}-s_k-1}[s_k\hat{h}+j\hat{h}+\eta_{t_k+j},\\ s_k\hat{h}+(j+1)\hat{h}+\eta_{t_k+j+1}.\end{aligned} \tag{9}$$

Define $\eta(t)=s-s_k\hat{h}-j\hat{h}$ and $\hat{e}(t)=x(s_k\hat{h})-x(s_k\hat{h}+j\hat{h})$ for $t\in[s_k\hat{h}+j\hat{h}+\eta_{t_k}, s_k\hat{h}+(j+1)\hat{h}+\eta_{t_{k+1}}$, $j=0,1,\ldots,s_{k+1}-s_k-1$. Moreover, the time-varying delay is a piecewise-linear function satisfying $\dot{\eta}(t)=1$ and $0\leq\eta_1\leq\eta(t)\leq\hat{h}+\eta\leq\eta_2$. A simple schematic diagram is shown in Figure 1, which describes the influences of stochastic noises and event-triggered control for synchronization NNs. Based on the above discussion, the system (3) can be represented as

$$\begin{aligned}dx(t)&=[-W_1(t)x(t)+W_2f(x(t))+W_3f(x(t-\tau(t)))\\&\quad+W_4\mathbb{k}[x(t-\eta(t))+e(t)]+W_5\wp(t)]dt\\&\quad+(Bx(t)+Cx(t-\tau(t))))dw(t),\\ \hat{z}(t)&=Af(x(t))\\ x(t)&=\psi(t), t\in[-\tau_{12},0].\end{aligned} \tag{10}$$

where $\tau_{12}=max[\tau_2,\eta_2]$. In addition, the following definitions and lemmas, which are crucial to derive the main results, are presented.

Definition 2.1 (Qi et al. [19]). For given constants $c_1 > 0, c_2 > 0, T > 0$, and symmetric matrix $\mathcal{W} > 0$, the error system (10) is finite-time bounded (FTB) with respect to $(c_1,c_2,d,T,\mathcal{W})$, if there exist constants $c_2 > c_1$ and $\wp(t)$ satisfying (10), such that $\mathbb{E}\{\sup_{-\tau_{12}\leq t\leq 0}[x^T(t)\mathcal{W}x(t), \nu^T(t)\mathcal{W}\nu(t), \delta^T(t)\mathcal{W}\delta(t)]\}\leq c_1\Rightarrow\mathbb{E}\{x^T(t)\mathcal{W}x(t)\}\leq c_2$, for $t\in[0,T]$

Definition 2.2 (Qi et al. [19]). For given constants $c_1>0, c_2>0$ and $T>0$ and the symmetric matrix $\mathcal{W}>0$, the system (10) with nonfragile event-triggered controller (7) is finite-time passive (FTP) in terms of $(c_1,c_2,d,T,\mathcal{W})$, constants $c_2>c_1$, in such a manner error system (10) is FTB and there exists constant $\beta>0$, which satisfies

$$2\int_0^T\hat{z}^T(t)\wp(t)dt\geq\beta\int_0^T\wp^T(t)\wp(t)dt. \tag{11}$$

Lemma 2.3 (Ren et al. [17]). *For matrix $\mathbb{M}>0$, the following inequalities hold for all differentiable $x\in$*

$[\ell_1, \ell_2] \to \mathbb{R}^n$:

$$\int_{\ell_1}^{\ell_2} x^T(\theta)\mathbb{M}x(\theta)d\theta \geq \frac{1}{\ell_2 - \ell_1}[\aleph_1^T \mathbb{M}\aleph_1 + 3\aleph_2^T \mathbb{M}\aleph_2 + 5\aleph_3^T \mathbb{M}\aleph_3]$$

where

$$\begin{aligned}
\aleph_1 &= \int_{\ell_1}^{\ell_2} x(\theta)d\theta, \\
\aleph_2 &= \int_{\ell_1}^{\ell_2} x(\theta)d\theta + \frac{2}{\ell_2 - \ell_1}\int_{\ell_1}^{\ell_2}\int_{\alpha}^{\ell_2} x(\theta)d\theta d\alpha, \\
\aleph_3 &= \int_{\ell_1}^{\ell_2} x(\theta)d\theta - \frac{6}{\ell_2 - \ell_1}\int_{\ell_1}^{\ell_2}\int_{\alpha}^{\ell_2} x(\theta)d\theta d\alpha \\
&\quad + \frac{12}{\ell_2 - \ell_1}\int_{\ell_1}^{\ell_2}\int_{\alpha}^{\ell_2}\int_{s}^{\ell_2} x(\theta)d\theta ds d\alpha.
\end{aligned}$$

Lemma 2.4 (Vadivel et al. [30]). *Let $\mathcal{Y}$, $\mathcal{Q}$, and $\Delta(t)$ be real matrices with adjustable dimensions. Furthermore, $\Delta(t)$ satisfies $\Delta^T(t)\Delta(t) \leq I$. Then, for any constant $\epsilon > 0$, the subsequent inequality holds:*

$$\mathcal{Y}\Delta(t)\mathcal{Q} + \mathcal{Q}^T\Delta^T(t)\mathcal{Y}^T \leq \epsilon\mathcal{Y}\mathcal{Y}^T + \epsilon^{-1}\mathcal{Q}^T\mathcal{Q}.$$

Remark 2.5. According to triggered schemes (5)–(9), for $t \in [s_k\hat{h} + \eta_{s_k}, s_{k+1}\hat{h} + \eta_{s_{k+1}})$, we can derive that

$$\mathbb{E}\{e^T(t)\tilde{\Theta}_1 e(t)\} \leq \kappa\mathbb{E}\{x^T(t-\eta(t))\tilde{\Theta}_1 x(t-\eta(t))\}.$$

Remark 2.6. With the analysis of (5), suppose $s_k\hat{h}(k = 0, 1, 2, \ldots)$ denotes the release times, where $s_0 = 0$ is the initial time. Thus, $t_k\hat{h} = s_{k+1}\hat{h} - s_k\hat{h}$ will be noted as the release period that corresponds to the sampling period given by the event mechanism (5). Let time-varying induced delay be $\eta_{s_k} \in [0, \eta], \eta > 0$. Hence, the triggered data $x(s_0\hat{h}), x(s_1\hat{h}), x(s_2\hat{h}), \ldots$ can reach the controller at instants $s_0\hat{h} + \eta_{s_0}, s_1\hat{h} + \eta_{s_1}, s_2\hat{h} + \eta_{s_2}, \ldots$, respectively.

3 | MAIN RESULTS

In this section, we provided passivity criteria of error system (10) with interval time-varying delay. The control gains are designed by solving linear matrix inequalities (LMIs). Various matrices are established for the following main results.

$$G_1 = diag\left\{l_1^- l_1^+, l_2^- l_2^+, \ldots, l_n^- l_n^+\right\}, z(s) = \begin{bmatrix} x(s) \\ f(x(s)) \end{bmatrix},$$

$$G_2 = diag\left\{\frac{l_1^- + l_1^+}{2}, \frac{l_2^- + l_2^+}{2}, \ldots, \frac{l_n^- + l_n^+}{2}\right\},$$

$$\mho_1 = \begin{bmatrix} Q_1 & Q_2 \\ \star & Q_3 \end{bmatrix}, \mho_2 = \begin{bmatrix} M_1 & M_2 \\ \star & M_3 \end{bmatrix}, \mho_3 = \begin{bmatrix} O_1 & O_2 \\ \star & O_3 \end{bmatrix},$$

$$\begin{aligned}
\xi^T(t) = \Big[&x^T(t)x^T(t-\tau_1)x^T(t-\tau(t))x^T(t-\tau_2)f^T(x(t)) \\
&\times f^T(x(t-\tau(t)))f^T(x(t-\tau_1)) \\
&\times f^T(x(t-\tau_2))v^T(t)x^T(t-\eta_1) \\
&\times x^T(t-\eta(t))x^T(t-\eta_2)e^T(t)\int_{t-\eta(t)}^{t-\eta_1} x^T(s)ds \\
&\times \int_{t-\eta_2}^{t-\eta(t)} x^T(s)ds \\
&\times \int_{t-\eta_1}^{t} x^T(s)ds\frac{1}{\eta(t)-\eta_1}\int_{-\eta(t)}^{-\eta_1}\int_{t+\theta}^{t-\eta_1} x^T(s)dsd\theta \\
&\times \frac{1}{\eta_2-\eta(t)}\int_{-\eta_2}^{-\eta(t)}\int_{t+\theta}^{t-\eta(t)} x^T(s)dsd\theta \\
&\times \frac{1}{\eta_1}\int_{t-\eta_1}^{t}\int_{t+\theta}^{t} x^T(s)dsd\theta \\
&\times \frac{1}{(\eta(t)-\eta_1)^2}\int_{-\eta(t)}^{-\eta_1}\int_{\alpha}^{-\eta_1}\int_{t+\theta}^{t-\eta_1} x^T(s)dsd\theta d\alpha \\
&\times \frac{1}{(\eta_2-\eta(t))^2}\int_{-\eta_2}^{-\eta(t)}\int_{\alpha}^{-\eta(t)}\int_{t+\theta}^{t-\eta(t)} x^T(s)dsd\theta d\alpha \\
&\times \frac{1}{\eta_1^2}\int_{t-\eta_1}^{t}\int_{\alpha}^{t}\int_{t+\theta}^{t} x^T(s)dsd\theta d\alpha\wp^T(t)\Big], \hat{\tau}_m = \tau_2 - \tau_1, \\
&\times e_l = [0_{n\times(l-1)n} I_n 0_{n\times(23-l)n}], l = 1, 2, \ldots 23.
\end{aligned}$$

Theorem 3.1. *For given $\hat{K}$, positive scalars $c_1, \tau_1, \tau_2, \tau_3, \eta_1, \eta_2$, and positive constants $T, \tilde{\alpha}, \varepsilon_1, \varepsilon_2$, the considered error system described by (10) is FTB with respect to $(c_1, c_2, d, T, \mathcal{W})$ and satisfying Assumption (H_1), if there exist symmetric positive definite matrices $\mathbb{P} \in \mathbb{R}^{n\times n}$, $\mho_l \in \mathbb{R}^{2n\times 2n}$, $Z_j, Q_4, Q_5, S_1, S_2 \in \mathbb{R}^{n\times n}$, $l = 1, 2, 3$, $j = 1, 2, 3, 4, 5$, appropriate dimension matrix $G, \tilde{\Theta}_1, \hat{Y}, \mathcal{K}, \bar{\mathcal{L}}, \mathcal{M}$, diagonal matrices $\vartheta = diag(v_1, v_2, \ldots, v_n) \geq 0$, $\Phi = diag(\phi_1, \phi_2, \ldots, \phi_n) \geq 0$, and $\mathscr{H}_l \geq 0 (l = 1, 2)$, such that the subsequent LMI holds:*

$$\Psi = \begin{bmatrix} \sum_{i=1}^{8}\Phi_i & \varepsilon_1\hat{Y}W_5 & \tau_1\mathcal{K} & \hat{\tau}_m\bar{\mathcal{L}} & \varpi_{15} \\ \star & -\tilde{\alpha}G & 0 & 0 & 0 \\ \star & \star & -\tau_1 Z_5 & 0 & 0 \\ \star & \star & \star & -\hat{\tau}_m Z_4 & 0 \\ \star & \star & \star & \star & \varpi_{55} \end{bmatrix} < 0, \quad (12)$$

$$\begin{aligned}
&c_1\big(\lambda_{\max}(\bar{\mathbb{P}})+\lambda_{\max}(\bar{\vartheta})[\max(\mathbb{L}^2-G_1)]+\lambda_{\max}(\bar{\phi})[\max(G_2\\
&\quad-\mathbb{L}^2)]+\tau_2\lambda_{\max}(\bar{\mho}_1)+\hat{\tau}_m\lambda_{\max}(\bar{\mho}_2)+\tau_1\lambda_{\max}(\bar{\mho}_3)\\
&\quad+\frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_4+\frac{\tau_1^2}{2}\lambda_{\max}\bar{Z}_5+\frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_2\\
&\quad+\frac{\tau_1^2}{2}\lambda_{\max}\bar{Z}_3+\frac{(-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_4+\frac{(\eta_2-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_5\\
&\quad+\eta_1\lambda_{\max}(\bar{S}_1)+(\eta_2-\eta_1)\lambda_{\max}(\bar{S}_2)+\eta_2\lambda_{\max}(\bar{Z}_1)\big)\\
&\quad+d\lambda_{\max}(G)(1-e^{\tilde{\alpha}T})<c_2\lambda_{\min}(\bar{\mathbb{P}})e^{-\tilde{\alpha}T},\\
&\Phi_1=e_1\mathbb{P}e_9^T+2(e_5-G_1e_1)\vartheta e_9^T+2(G_1e_1-e_5)\phi e_9^T,\\
&\Phi_2=\begin{bmatrix}e_1\\e_5\end{bmatrix}\mho_1\begin{bmatrix}e_1\\e_5\end{bmatrix}^T-(1-\mu)\begin{bmatrix}e_3\\e_6\end{bmatrix}\mho_1\begin{bmatrix}e_3\\e_6\end{bmatrix}^T\\
&\quad+\begin{bmatrix}e_2\\e_7\end{bmatrix}\mho_2\begin{bmatrix}e_2\\e_7\end{bmatrix}^T-\begin{bmatrix}e_4\\e_8\end{bmatrix}\mho_2\begin{bmatrix}e_4\\e_8\end{bmatrix}^T\\
&\quad+\begin{bmatrix}e_1\\e_2\end{bmatrix}\mho_3\begin{bmatrix}e_1\\e_2\end{bmatrix}^T-\begin{bmatrix}e_2\\e_7\end{bmatrix}\mho_3\begin{bmatrix}e_2\,e_7\end{bmatrix}^T,\\
&\Phi_3=e_9(\hat{\tau}_mZ_4+\tau_1Z_5)e_9^T,\ \Phi_5=e_{10}S_1e_{10}^T-e_{12}S_2e_{12}^T,\\
&\Phi_4=e_1(\eta_1Q_4+(\eta_2-\eta_1)Q_5)e_1^T\\
&\quad-\frac{1}{\eta(t)-\eta_1}\mathcal{A}diag(Q_5,3Q_5,5Q_5)\mathcal{A}^T\\
&\quad-\frac{1}{\eta_2-\eta(t)}\mathcal{B}diag(Q_5,3Q_5,5Q_5)\mathcal{B}^T\\
&\quad-\frac{1}{\eta_1}C\,diag(Q_4,3Q_4,5Q_4)C^T,\\
&\Phi_6=2[\varepsilon_1e_1+\varepsilon_2e_9]\hat{Y}[-W_1e_1^T+W_2e_5^T\\
&\quad+W_3e_6^T+W_4\hat{K}e_{11}^T+W_4\hat{K}e_{13}^T]+2\varepsilon_2e_9\hat{Y}W_5e_{23}^T,\\
&\Phi_7=-e_1\mathscr{H}_1G_1e_1^T+e_1\mathscr{H}_1G_2e_5^T-e_5\mathscr{H}_1e_5^T,\\
&\quad-e_3\mathscr{H}_2G_1e_3^T+e_3\mathscr{H}_2G_2e_6^T-e_6\mathscr{H}_2e_6^T\\
&\Phi_8=e_{11}\tilde{\Theta}_1e_{11}^T-\kappa e_{13}\tilde{\Theta}_1e_{13}^T,\ \varpi_{15}=[\hat{\sigma}_1,\hat{\sigma}_2,\hat{\sigma}_3,\hat{\sigma}_4,\hat{X}],\\
&\hat{\sigma}_1^T=[\hat{\tau}_m\mathcal{M}^T\underbrace{000}_{14\,times}]^T,\ \hat{\sigma}_2^T=[\mathcal{K}^T\underbrace{000}_{14\,times}]^T,\\
&\hat{\sigma}_3^T=[\bar{\mathcal{L}}^T\underbrace{000}_{14\,times}]^T,\hat{\sigma}_4^T=[\bar{\mathcal{L}}^T\underbrace{000}_{14\,times}]^T\\
&\hat{X}^T=[\hat{P}B0\hat{P}C\underbrace{000}_{20\,times}],\ \mathbb{L}=\max(|l_j^-,l_j^+|),\\
&\hat{P}=\mathbb{P}+2(\vartheta+\phi)(G_2-G_1)+\hat{\tau}_mZ_2+\tau_1Z_3,\\
&\varpi_{55}=diag\{-\hat{\tau}_mZ_4,-Z_2,-Z_3,-Z_3,-\hat{P}\},\\
&\kappa^T=[\kappa_\iota^T],\ \kappa=\mathcal{K},\bar{\mathcal{L}},\mathcal{M}\,and\,\iota=1,2,\ \dots,9.
\end{aligned}\tag{13}$$

Proof. For simplicity presentation, we denote

$$\begin{aligned}
v(t)&=-W_1x(t)+W_2f(x(t))+W_3f(x(t-\tau(t)))\\
&\quad+W_4u(t)+W_5\wp(t),
\end{aligned}\tag{14}$$

$$\delta(t)=Bx(t)+Cx(t-\tau(t));$$

then, system (10) can be written as

$$dx(t)=v(t)dt+\delta(t)dw(t).\tag{15}$$

Choose a Lyapunov–Krasovskii functional candidate as

$$V(x(t),t)=\sum_{i=1}^{6}V_i(x(t),t),\tag{16}$$

where

$$\begin{aligned}
V_1(x(t),t)&=x^T(t)\mathbb{P}x(t)+2\sum_{l=1}^{n}\vartheta_l\int_0^{x_l}(f_l(\alpha)-l_l^-(\alpha))d\alpha\\
&\quad+2\sum_{l=1}^{n}\phi_l\int_0^{x_l}(l_l^+(\alpha)-f_l(\alpha))d\alpha,\\
V_2(x(t),t)&=\int_{t-\tau(t)}^{t}z^T(s)\mho_1z(s)ds+\int_{t-\tau_2}^{t-\tau_1}z^T(s)\mho_2z(s)ds\\
&\quad+\int_{t-\tau_1}^{t}z^T(s)\mho_3z(s)ds,\\
V_3(x(t),t)&=\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}v^T(s)Z_4v(s)dsd\theta\\
&\quad+\int_{-\tau_1}^{0}\int_{t+\theta}^{t}v^T(s)Z_5v(s)dsd\theta,\\
V_4(x(t),t)&=\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\delta^T(s)Z_2\delta(s)dsd\theta\\
&\quad+\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\delta^T(s)Z_3\delta(s)dsd\theta,\\
V_5(x(t),t)&=\int_{-\eta_1}^{0}\int_{t+\theta}^{t}x^T(s)Q_4x(s)dsd\theta\\
&\quad+\int_{-\eta_2}^{-\eta_1}\int_{t+\theta}^{t}x^T(s)Q_5x(s)dsd\theta,\\
V_6(x(t),t)&=\int_{t-\eta_1}^{t}x^T(s)S_1x(s)ds+\int_{t-\eta_2}^{t-\eta_1}x^T(s)S_2x(s)ds\\
&\quad+(\eta_2-\eta(t))x^T(t-\eta(t))Z_1x(t-\eta(t)).
\end{aligned}\tag{17}$$

In the following, $\mathcal{L}$ is the weak infinitesimal operator of the stochastic process $\epsilon_t, t\geq 0$. By the rule of Itô's differential, the stochastic derivative of $V(x(t),t)$ along the trajectory of model (10) can be obtained as

$$dV(x(t),t)=\mathcal{L}V(x(t),t)dt+2x^T(t)\mathbb{P}\delta(t)dw(t),\tag{18}$$

where $\mathcal{L}V(x(t),t) = \mathcal{L}\sum_{i=1}^{6} V(x(t),t)$. Therefore, we have

$$\begin{aligned}\mathcal{L}_1V(x(t),t) \leq\ & 2x^T(t)\mathbb{P}\nu(t) + 2\delta^T(t)\mathbb{P}\delta(t)\\ & + 2(f(x(t)) - G_1x(t))^T\vartheta\nu(t)\\ & + 2\delta^T(t)\vartheta(G_2 - G_1)\delta(t)\\ & + 2(G_2x(t) - f(x(t)))^T\phi\nu(t)\\ & + 2\delta^T(t)\phi(G_2 - G_1)\delta(t)\\ =\ & \xi^T(t)\Phi_1\xi(t),\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{L}_2V(x(t),t) \leq\ & z^T(t)\mho_1z(t) - (1-\tau_3)z^T(t-\tau(t))\mho_1\\ & \times z(t-\tau(t)) + z^T(t-\tau_1)\mho_2z(t-\tau_1)\\ & - z^T(t-\tau_2)\mho_2z(t-\tau_2) + z^T(t)\mho_3z(t)\\ & - z^T(t-\tau_1)\mho_3z(t-\tau_1)\\ =\ & \xi^T(t)\Phi_2\xi(t),\end{aligned} \tag{20}$$

$$\begin{aligned}\mathcal{L}_3V(x(t),t) =\ & \hat{\tau}_m\nu^T(t)Z_4\nu(t) - \int_{t-\tau_2}^{t-\tau_1}\nu^T(s)Z_4\nu(s)ds\\ & + \tau_1\nu^T(t)Z_5\nu(t) - \int_{t-\tau_1}^{t}\nu^T(s)Z_5\nu(s)ds,\end{aligned} \tag{21}$$

$$\begin{aligned}\mathcal{L}_4V(x(t),t) =\ & \hat{\tau}_m\delta^T(t)Z_2\delta(t) + \tau_1\delta^T(t)Z_3\delta(t)\\ & - \int_{t-\tau_2}^{t-\tau_1}\delta^T(s)Z_2\delta(s)ds - \int_{t-\tau_1}^{t}\delta^T(s)Z_3\delta(s)ds.\end{aligned} \tag{22}$$

By (21), we see that for any matrices $\mathcal{K}, \bar{\mathcal{L}}, \mathcal{M}$, the subsequent equations hold:

$$\begin{aligned}0 =\ & 2\xi^T(t)\mathcal{K}\left[x(t) - x(t-\tau_1) - \int_{t-\tau_1}^{t}\nu(s)ds\right.\\ & \left. - \int_{t-\tau_1}^{t}\delta(s)dw(s)\right],\\ 0 =\ & 2\xi^T(t)\bar{\mathcal{L}}\left[x(t-\tau_1) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1}\nu(s)ds\right.\\ & \left. - \int_{t-\tau(t)}^{t-\tau_1}\delta(s)dw(s)\right],\\ 0 =\ & 2\xi^T(t)\mathcal{M}\left[x(t-\tau(t)) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)}\nu(s)ds\right.\\ & \left. - \int_{t-\tau_2}^{t-\tau(t)}\delta(s)dw(s)\right].\end{aligned} \tag{23}$$

From Lemma 2.9 [30], we get

$$\begin{aligned}-2\xi^T(t)\mathcal{K}\int_{t-\tau_1}^{t}\delta(s)dw(s) \leq\ & \hat{\xi}^T(t)\mathcal{K}Z_3^{-1}\mathcal{K}^T\hat{\xi}(t)\\ & + \left(\int_{t-\tau_1}^{t}\delta(s)dw(s)\right)^T Z_3\left(\int_{t-\tau_1}^{t}\delta(s)dw(s)\right),\\ -2\xi^T(t)\bar{\mathcal{L}}\int_{t-\tau(t)}^{t-\tau_1}\delta(s)dw(s) \leq\ & \hat{\xi}^T(t)\bar{L}Z_2^{-1}\bar{L}^T\hat{\xi}(t)\\ & + \left(\int_{t-\tau(t)}^{t-\tau_1}\delta(s)dw(s)\right)^T Z_2\left(\int_{t-\tau(t)}^{t-\tau_1}\delta(s)dw(s)\right),\\ -2\xi^T(t)\mathcal{M}\int_{t-\tau_2}^{t-\tau(t)}\delta(s)dw(s) \leq\ & \hat{\xi}^T(t)\mathcal{M}Z_2^{-1}\mathcal{M}^T\hat{\xi}(t)\\ & + \left(\int_{t-\tau_2}^{t-\tau(t)}\delta(s)dw(s)\right)^T Z_2\left(\int_{t-\tau_2}^{t-\tau(t)}\delta(s)dw(s)\right).\end{aligned} \tag{24}$$

Therefore, combining (21)–(24), we get

$$\begin{aligned}\mathcal{L}_iV(x(t),t) \leq\ & \xi^T(t)\{\Phi_3 + \tau_1\mathcal{K}Z_5^{-1}\mathcal{K}^T + \hat{\tau}_m\bar{\mathcal{L}}Z_4^{-1}\bar{\mathcal{L}}^T\\ & + \hat{\tau}_m\mathcal{M}Z_4^{-1}\mathcal{M}^T + \mathcal{K}Z_3^{-1}\mathcal{K}^T + \bar{\mathcal{L}}Z_2^{-1}\bar{\mathcal{L}}^T\\ & + \mathcal{M}Z_2^{-1}\mathcal{M}^T\}\xi(t) - \int_{t-\tau_1}^{t}[\xi^T(t)\mathcal{K} + \nu^T(s)Z_5]\\ & \times Z_5^{-1}[\mathcal{K}^T\xi(t) + Z_5\nu(s)]ds - \int_{t-\tau(t)}^{t-\tau_1}[\xi^T(t)\bar{\mathcal{L}}\\ & + \nu^T(s)Z_4] \times Z_4^{-1}[\bar{\mathcal{L}}^T\xi(t) + Z_4\nu(s)]ds\\ & - \int_{t-\tau_2}^{t-\tau(t)}[\xi^T(t)\mathcal{M} + \nu^T(s)Z_4]Z_4^{-1}\\ & \times[\mathcal{M}^T\xi(t) + Z_4\nu(s)]ds, i = 3, 4,\end{aligned} \tag{25}$$

$$\begin{aligned}\mathcal{L}_5V(x(t),t) =\ & x^T(t)\left(\eta_1Q_4 + (\eta_2 - \eta_1)Q_5\right)x(t)\\ & - \int_{t-\eta_1}^{t}x^T(s)Q_4x(s)ds\\ & - \int_{t-\eta_2}^{t-\eta_1}x^T(s)Q_5x(s)ds.\end{aligned} \tag{26}$$

Since $[t-\eta_2, t-\eta_1] = [t-\eta_2, t-\eta(t) \cup (t-\eta(t), t-\eta_1])$, the following inequalities about the integration terms of $\mathcal{L}_5$ are obtained using Lemma 2.3:

$$
\begin{aligned}
&-\int_{t-\eta(t)}^{t-\eta_1} x^T(s)Q_5x(s)ds \\
&\qquad \leq \frac{-1}{\eta(t)-\eta_1}\mathcal{A}^T diag\{Q_5,\ 3Q_5,\ 5Q_5\}\mathcal{A}, \\
&-\int_{t-\eta_2}^{t-\eta(t)} x^T(s)Q_5x(s)ds \\
&\qquad \leq \frac{-1}{\eta_2-\eta(t)}\mathcal{B}^T diag\{Q_5,\ 3Q_5,\ 5Q_5\}\mathcal{B}, \\
&-\int_{t-\eta_1}^{t} x^T(s)Q_4x(s)ds \\
&\qquad \leq \frac{-1}{\eta_1}\mathcal{C}^T diag\{Q_4,\ 3Q_4,\ 5Q_4\}\mathcal{C},
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
\mathcal{A} = \Bigg[&\int_{t-\eta(t)}^{t-\eta_1} x^T(s)ds, \int_{t-\eta(t)}^{t-\eta_1} x^T(s)ds - \frac{2}{\eta(t)-\eta_1}\int_{-\eta(t)}^{-\eta_1} \\
&\times \int_{t+\theta}^{t-\eta_1} x^T(s)dsd\theta, \int_{t-\eta(t)}^{t-\eta_1} x^T(s)ds - \frac{6}{\eta(t)-\eta_1}\int_{-\eta(t)}^{-\eta_1} \\
&\times \int_{t+\theta}^{t-\eta_1} x^T(s)dsd\theta + \frac{12}{(\eta(t)-\eta_1)^2}\int_{-\eta(t)}^{-\eta_1}\int_{\alpha}^{-\eta_1}\int_{t+\theta}^{t-\eta_1} \\
&x^T(s)dsd\theta d\alpha\Bigg], \\
\mathcal{B} = \Bigg[&\int_{t-\eta_2}^{t-\eta(t)} x^T(s)ds, \int_{t-\eta_2}^{t-\eta(t)} x^T(s)ds - \frac{2}{\eta_2-\eta(t)}\int_{-\eta_2}^{-\eta(t)} \\
&\times \int_{t+\theta}^{t-\eta(t)} x^T(s)dsd\theta, \int_{t-\eta_2}^{t-\eta(t)} x^T(s)ds - \frac{6}{\eta_2-\eta(t)}\int_{-\eta_2}^{-\eta(t)} \\
&\times \int_{t+\theta}^{t-\eta_1} x^T(s)dsd\theta + \frac{12}{(\eta_2-\eta(t))^2}\int_{-\eta_2}^{-\eta(t)}\int_{\alpha}^{-\eta(t)}\int_{t+\theta}^{t-\eta(t)} \\
&x^T(s)dsd\theta d\alpha\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C} = \Bigg[&\int_{t-\eta_1}^{t} x^T(s)ds, \int_{t-\eta_1}^{t} x^T(s)ds - \frac{2}{\eta_1}\int_{t-\eta_1}^{t} \\
&\times \int_{t+\theta}^{t} x^T(s)dsd\theta, \int_{t-\eta_1}^{t} x^T(s)ds - \frac{6}{\eta_1}\int_{t-\eta_1}^{t} \\
&\times \int_{\theta}^{t} x^T(s)dsd\theta + \frac{12}{(\eta_1)^2}\int_{t-\eta_1}^{t}\int_{\alpha}^{t}\int_{\theta}^{t} x^T(s)dsd\theta d\alpha\Bigg].
\end{aligned}
$$

Therefore, from (26) and (27), we get

$$
\begin{aligned}
\mathcal{L}_5V(x(t),t) \leq\ & x^T(t)\left(\eta_1Q_4 + (\eta_2-\eta_1)Q_5\right)x(t) \\
&\frac{-1}{\eta(t)-\eta_1}\mathcal{A}^T diag\{Q_5,\ 3Q_5,\ 5Q_5\}\mathcal{A} \\
&\frac{-1}{\eta_2-\eta(t)}\mathcal{B}^T diag\{Q_5,\ 3Q_5,\ 5Q_5\}\mathcal{B} \\
&\frac{-1}{\eta_1}\mathcal{C}^T diag\{Q_4,\ 3Q_4,\ 5Q_4\}\mathcal{C} \\
=\ & \xi^T(t)\Phi_4\xi(t),
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
\mathcal{L}_6V(x(t),t) \leq\ & x^T(t)S_1x(t) - x^T(t)(t-\eta_1)S_1(t-\eta_1)x(t) \\
&+ x^T(t-\eta_1)S_1x(t-\eta_1) - x^T(t-\eta_2) \\
&\times S_2x(t-\eta_2) - x^T(t-\eta(t))Z_1x(t-\eta(t)) \\
=\ & \xi^T(t)\Phi_5\xi(t).
\end{aligned} \tag{29}
$$

Moreover, for any matrix $\hat{Y}$ with suitable dimension and given scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, we have

$$
\begin{aligned}
0 =\ & 2[\varepsilon_1x^T(t) + \varepsilon_2v^T(t)]\hat{Y}[-W_1x(t) + W_2f(x(t)) \\
&+ W_3f(x(t-\tau(t))) + W_4\hat{K}(x(t-\eta(t)) + e(t)) \\
&+ W_5\wp(t) - v(t)], \\
=\ & \xi^T(t)\Phi_6\xi(t).
\end{aligned} \tag{30}
$$

In addition to Assumption (H1), for any positive diagonal matrices $\mathscr{H}_1$ and $\mathscr{H}_2$, the following can be obtained:

$$
\begin{aligned}
0 \leq\ & z^T(t)\mathscr{H}_1\begin{bmatrix} -G_1 & G_2 \\ 0 & -I \end{bmatrix}z(t) \\
&+ z^T(t-\tau(t))\mathscr{H}_2\begin{bmatrix} -G_1 & G_2 \\ 0 & -I \end{bmatrix}z(t-\tau(t)), \\
=\ & \xi^T(t)\Phi_7\xi(t).
\end{aligned} \tag{31}
$$

Therefore, for a given $\tilde{\alpha} > 0$ and combining (19)–(31) with event-triggered condition, we get

$$
\mathcal{L}V(x(t),t) - \tilde{\alpha}V(x(t),t) - \tilde{\alpha}\wp^T(t)G\wp(t) \leq \xi^T(t)\Psi\xi(t), \tag{32}
$$

where Ψ is outlined in Theorem 3.1. Then by (32) and the conditions in Theorem 3.1, we obtain

$$
\begin{aligned}
&\mathcal{L}V(x(t),t) - \tilde{\alpha}V(x(t),t) - \tilde{\alpha}\wp^T(t)G\wp(t) \leq 0,\ i.e.\ \Psi < 0, \\
&\mathcal{L}V(x(t),t) \leq \tilde{\alpha}V(x(t),t) + \tilde{\alpha}\wp^T(t)G\wp(t).
\end{aligned} \tag{33}
$$

From (33), multiplying $e^{-\tilde{\alpha}t}$ leads to

$$
\mathcal{L}(e^{-\tilde{\alpha}t}V(x(t),t)) < \tilde{\alpha}e^{-\tilde{\alpha}t}\wp^T(t)G\wp(t). \tag{34}
$$

Utilizing Dynkin's formula to inequality (34), we get

$$
e^{-\tilde{\alpha}t}V(x(t),t) - V(x(0),0) < \tilde{\alpha}\int_0^T e^{-\tilde{\alpha}s}\wp^T(s)G\wp(s)ds, \tag{35}
$$

which means

$$
\begin{aligned}
V(x(t),t) &< e^{\tilde{\alpha}t}\left(V(x(0),0) + \tilde{\alpha}\int_0^T e^{-\tilde{\alpha}s}\wp^T(s)G\wp(s)ds\right) \\
&< e^{\tilde{\alpha}t}\left(V(x(0),0) + \tilde{\alpha}d\lambda_{\max}(G)\frac{(1-e^{-\tilde{\alpha}T})}{\tilde{\alpha}}\right).
\end{aligned} \tag{36}
$$

Let $\bar{\mathbb{P}} = \mathcal{W}^{-1/2}\mathbb{P}\mathcal{W}^{-1/2}, \bar{\vartheta} = \mathcal{W}^{-1/2}\vartheta\mathcal{W}^{-1/2}, \bar{\phi} = \mathcal{W}^{-1/2}\phi\mathcal{W}^{-1/2}, \bar{\mho}_i = \mathcal{W}^{-1/2}\mho_i\mathcal{W}^{-1/2}, i = 1,2,3, \bar{Z}_j = \mathcal{W}^{-1/2}Z_j\mathcal{W}^{-1/2}, j = 1,2,3,4,5, \bar{Q}_j = \mathcal{W}^{-1/2}Q_j\mathcal{W}^{-1/2}, \bar{S}_l = \mathcal{W}^{-1/2}S_l\mathcal{W}^{-1/2}, l = 1,2$, on the other hand

$$V(x(t),t) < e^{\tilde{\alpha}T}[c_1\tilde{\wedge} + d\lambda_{\max}(G)(1-e^{-\tilde{\alpha}T})], \tag{37}$$

where $\tilde{\wedge} = \{\lambda_{\max}(\bar{\mathbb{P}}) + \lambda_{\max}(\bar{\vartheta})[\max(|l_j^-, l_j^+|^2 - G_1)] + \lambda_{\max}(\bar{\phi})[\max(G_2 - |l_j^-, l_j^+|^2)] + \tau_2\lambda_{\max}(\bar{\mho}_1) + (\tau_2 - \tau_1)\lambda_{\max}(\bar{\mho}_2) + \tau_1\lambda_{\max}(\bar{\mho}_3) + \frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_4 + \frac{(\tau_1)^2}{2}\lambda_{\max}\bar{Z}_5 + \frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_2 + \frac{(\tau_1)^2}{2}\lambda_{\max}\bar{Z}_3 + \frac{(-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_4 + \frac{(\eta_2-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_5 + \eta_1\lambda_{\max}(\bar{S}_1) + (\eta_2 - \eta_1)\lambda_{\max}(\bar{S}_2) + \eta_2\lambda_{\max}(\bar{Z}_1)\} \times \sup_{-\tau_{12}\le\theta\le 0}\{x^T(\theta)\mathcal{W}x(\theta), v^T(\theta)\mathcal{W}v(\theta)\}$, considering that

$$\mathbb{E}\{V(x(t),t)\} \ge x^T(t)\bar{\mathbb{P}}x(t) \ge \lambda_{\min}(\bar{\mathbb{P}})x^T(t)\mathcal{W}x(t), \tag{38}$$

we get

$$\mathbb{E}\{x^T(t)\mathcal{W}x(t)\} \le e^{\tilde{\alpha}T}\left[\frac{c_1\tilde{\wedge} + (1-e^{\tilde{\alpha}T})}{\lambda_{\min}(\bar{\mathbb{P}})}\right] < c_2. \tag{39}$$

By Definition 2.1, system (10) is FTB subject to $(c_1, c_2, T, d, \tilde{\alpha}, \mathcal{W})$.

Remark 3.2. Suppose there is no $\wp(t) = 0$, the definition of FTB will decrease to one of FTS. Also, most extraordinary distinction between Lyapunov stability and FTS. In the FTS NNs, once the time interval is fixed, the system states never exceed some particular limits over this period of time interval. Also, an FTS NNs cannot be Lyapunov stable and a Lyapunov stable system cannot be FTS only if the transient state exceeds recommended limits.

4 | FTP-ANALYSIS

In this section, we introduce the sufficient condition to assurance FTP algorithms for the synchronization of SNNs.

Theorem 4.1. *Consider the known gain matrix $\hat{K}$, positive constants $c_1, \tau_1, \tau_2, \tau_3, \eta_1, \eta_2$, and scalars $T, \tilde{\alpha}, \beta, \epsilon_1, \epsilon_2$, the considered error system described by (10) is FTP with respect to $(c_1, c_2, T, d, \mathcal{W})$ and satisfying Assumption (H_1), if there exist symmetric positive definite matrices $\mathbb{P} \in \mathbb{R}^{n\times n}, \mho_l \in \mathbb{R}^{2n\times 2n}, Z_j, Q_4, Q_5, S_1, S_2 \in \mathbb{R}^{n\times n}, l = 1,2,3, j = 1,2,3,4,5$, appropriate matrix $, \tilde{\Theta}_1, \hat{Y}, \mathcal{K}, \bar{\mathcal{L}}, \mathcal{M}$, diagonal matrices $\vartheta = diag(v_1, v_2, \dots, v_n) \ge 0, \Phi = diag(\phi_1, \phi_2, \dots, \phi_n) \ge 0$ and $\mathscr{H}_l \ge 0 (l = 1,2)$ such that the following matrix inequalities hold:*

$$\Xi = \begin{bmatrix} \sum_{i=1}^{8}\Phi_i & \Gamma_1 & \tau_1\mathcal{K} & \hat{\tau}_m\bar{\mathcal{L}} & \varpi_{15} \\ \star & -\beta I & 0 & 0 & 0 \\ \star & \star & -\tau_1 Z_5 & 0 & 0 \\ \star & \star & \star & -\hat{\tau}_m Z_4 & 0 \\ \star & \star & \star & \star & \varpi_{55} \end{bmatrix} < 0, \tag{40}$$

$$\begin{aligned} c_1 \Big(&\lambda_{\max}(\bar{\mathbb{P}}) + \lambda_{\max}(\bar{\vartheta})[\max(\mathbb{L}^2 - G_1)] \\ &+ \lambda_{\max}(\bar{\phi})[\max(G_2 - \mathbb{L}^2)] \\ &+ \tau_2\lambda_{\max}(\bar{\mho}_1) + \hat{\tau}_m\lambda_{\max}(\bar{\mho}_2) + \tau_1\lambda_{\max}(\bar{\mho}_3) \\ &+ \frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_5 + \frac{(\tau_1)^2}{2}\lambda_{\max}\bar{Z}_4 + \frac{\hat{\tau}_m^2}{2}\lambda_{\max}\bar{Z}_2 \\ &+ \frac{(\tau_1)^2}{2}\lambda_{\max}\bar{Z}_3 + \frac{(-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_4 + \frac{(\eta_2-\eta_1)^2}{2}\lambda_{\max}\bar{Q}_5 \\ &+\eta_1\lambda_{\max}(\bar{S}_1) + (\eta_2 - \eta_1)\lambda_{\max}(\bar{S}_2) + \eta_2\lambda_{\max}(\bar{Z}_1)\Big) \\ &+ d\frac{\beta}{\tilde{\alpha}}(1 - e^{\tilde{\alpha}T}) < c_2\lambda_{\min}(\bar{\mathbb{P}})e^{-\tilde{\alpha}T}, \end{aligned}$$

where $\Gamma_1^T = [\epsilon_1\hat{Y}^T W_5 000 - A^T \underbrace{000}_{17\,times}]$ and the other terms are listed in (12).

Proof. Following the similar proof and LKF of Theorem 3.1, the subsequent condition holds

$$\begin{aligned}\mathcal{L}V(x(t),t) - \tilde{\alpha}V(x(t),t) - 2\hat{z}^T(t)\wp(t) - \beta\wp^T(t)\wp(t) \\ \le \xi^T(t)\Xi\xi(t),\end{aligned} \tag{41}$$

where Ξ is listed in (40). According to the process of (40) and Theorem 4.1, we obtain

$$\mathcal{L}V(x(t),t) \le \tilde{\alpha}V(x(t),t) + 2\hat{z}^T(t)\wp(t) + \beta\wp^T(t)\wp(t). \tag{42}$$

Multiplying the above inequality (42) by $e^{-\tilde{\alpha}t}$, yield

$$\mathcal{L}(e^{-\tilde{\alpha}t}V(x(t),t)) < e^{-\tilde{\alpha}t}[2\hat{z}^T(t)\wp(t) + \beta\wp^T(t)\wp(t)]. \tag{43}$$

Integrating (43) from 0 and T and then taking expectation yield

$$\begin{aligned}\mathbb{E}(e^{-\tilde{\alpha}t}V(x(t),t)) < \mathbb{E}\Big[2\int_0^T e^{-\tilde{\alpha}t}\hat{z}^T(t)\wp(t)dt \\ + \beta\int_0^T e^{-\tilde{\alpha}t}\wp^T(t)\wp(t)dt\Big].\end{aligned} \tag{44}$$

Therefore, we have

$$0 < \mathbb{E}(V(x(t),t)) < 2e^{\tilde{\alpha}t}\mathbb{E}\left[\int_0^T \hat{z}^T(t)\wp(t)dt + \beta\int_0^T e^{-\tilde{\alpha}t}\wp^T(t)\wp(t)dt\right].$$

Thus, we obtain

$$2\mathbb{E}\left[\int_0^T \hat{z}^T(t)\wp(t)dt\right] > -\mathbb{E}\left[\beta\int_0^T \wp^T(t)\wp(t)dt\right].$$

Hence, it can be concluded from the error system (10) can be FTP with respect to $(c_1, c_2, T, d, \mathcal{W})$ and Definition 2.2, Thus the proof of Theorem 4.1 is completed.

5 | NONFRAGILE EVENT-TRIGGERED CONTROLLER VIA FINITE-TIME PASSIFICATION APPROACH

In this part, we considered the passification problem; that is, a non-fragile ETC is to be constructed to make the closed-loop system is passive.

Theorem 5.1. *For given positive constants $c_1, \tau_1, \tau_2, \tau_3, \eta_1, \eta_2$, and positive scalars $\epsilon_1, \epsilon_2, T, \tilde{\alpha}, \beta, \epsilon$, the considered SNNs described by (10) is FTP with respect to $(c_1, c_2, T, d, \mathcal{W})$ and satisfying Assumption (H_1), if there exist symmetric positive definite matrices $\bar{\bar{\mathbb{P}}} \in \mathbb{R}^{n\times n}, \mho_l \in \mathbb{R}^{2n\times 2n}, \hat{Z}_j, \hat{Q}_4, \hat{Q}_5, \hat{S}_1, \hat{S}_2 \in \mathbb{R}^{n\times n}, l = 1,2,3, j = 1,2,3,4,5, L$, appropriate dimensional matrix $\bar{Y}, \tilde{\bar{\Theta}}_1, \bar{\mathcal{K}}, \bar{\tilde{\mathcal{L}}}, \bar{\mathcal{M}}$, diagonal matrices $\vartheta = diag(v_1, v_2, \dots, v_n) \geq 0$, $\Phi = diag(\phi_1, \phi_2, \dots, \phi_n) \geq 0$ and $v_l = diag(v_{l1}, v_{l2}, \dots, v_{ln})$ and $\mathscr{H}_l \geq 0 (l = 1, 2)$ such that the following LMIs hold:*

$$\begin{bmatrix} \sum_{i=1}^{8}\Phi_i & \Gamma_1 & \tau_1\mathcal{K} & \hat{\tau}_m\bar{\mathcal{L}} & \varpi_{15} & \varpi_{16} \\ \star & -\beta I & 0 & 0 & 0 & 0 \\ \star & \star & -\tau_1 Z_5 & 0 & 0 & 0 \\ \star & \star & \star & -\hat{\tau}_m Z_4 & 0 & 0 \\ \star & \star & \star & \star & \varpi_{55} & 0 \\ \star & \star & \star & \star & \star & \varpi_{66} \end{bmatrix} < 0, \tag{45a}$$

$$\begin{bmatrix} \bar{\bar{\Lambda}} & \sqrt{c_1} \\ \star & -\lambda_1 \end{bmatrix} < 0, \tag{45b}$$

where

$$\varpi_{16} = [\bar{\bar{\Lambda}}_p \,\bar{\bar{\Lambda}}_q],\ \varpi_{66} = diag\{-\epsilon I - \epsilon I\},$$

$$\bar{\bar{\Lambda}}_p = [\epsilon_1 W_4^T \underbrace{000}_{7\,times} \epsilon_2 W_4^T \underbrace{000}_{22\,times}]^T H,$$

$$\bar{\bar{\Lambda}}_q = [\underbrace{000}_{10\,times} E\bar{Y}0E\bar{Y} \underbrace{000}_{18\,times}],$$

$$\Phi_6 = 2[\epsilon_1 e_1 + \epsilon_2 e_9]\hat{Y}[-W_1 e_1^T + W_2 e_5^T + W_3 e_6^T] + 2[\epsilon_1 e_1 + \epsilon_2 e_9][Le_{11}^T + Le_{13}^T] + 2\epsilon_2 e_9 W_5 e_{23}^T,$$

$$\Gamma_1^T = [\epsilon_1 W_5 \bar{Y} 000 - \bar{Y}A^T \underbrace{000}_{17\,times}],$$

$$\bar{\bar{\Lambda}} = c_1\lambda_2 + c_1\lambda_3 + \tau_2 c_1\lambda_{\bar{\zeta}} + \tau_m c_1\lambda_{\bar{j}} + \tau_1 c_1\lambda_{\bar{k}} + \frac{\hat{\tau}_m^2}{2}c_1\lambda_{15} + \frac{(\tau_1)^2}{2}c_1\lambda_{16} + \frac{\hat{\tau}_m^2}{2}c_1\lambda_{17} + \frac{(\tau_1)^2}{2}c_1\lambda_{18} + \frac{(\eta_1)^2}{2}c_1\lambda_7 + \frac{(\eta_2-\eta_1)^2}{2}c_1\lambda_8 + \eta_1 c_1\lambda_{20} + \eta_2 c_1\lambda_{19} + (\eta_2-\eta_1)c_1\lambda_{21} - c_2 e^{-\tilde{\alpha}T},$$

and the other elements are defining in Theorem 3.1, then for $\hat{K} = \hat{Y}^{-1}L$, the slave system with event-triggered controller (7) and the master system (1) are synchronized in the finite time case.

Proof. The proof of Theorem 5.1 immediately follows that of Theorem 4.1 and replacing $\hat{K}$ by $\hat{K} + \Delta\hat{K}$ in (39), we have

$$\Xi + \tilde{\Lambda}_p\mathcal{M}_4(t)\tilde{\Lambda}_q + \tilde{\Lambda}_q\mathcal{M}_4^T(t)\tilde{\Lambda}_p^T, \tag{46}$$

where

$$\tilde{\Lambda}_p = [\epsilon_1(\hat{Y}W_4)^T \underbrace{000}_{7\,times} \epsilon_2(\hat{Y}W_4)^T \underbrace{000}_{22\,times}]H$$

$$\tilde{\Lambda}_q = [\underbrace{000}_{10\,times} E0E \underbrace{000}_{18\,times}].$$

By using Lemma 2.4, there exists a scalar $\epsilon > 0$, such that

$$\Xi + \epsilon^{-1}\tilde{\Lambda}_p\tilde{\Lambda}_p^T + \epsilon\tilde{\Lambda}_q^T\tilde{\Lambda}_q < 0, \tag{47}$$

Utilizing the Schur complement lemma, we get

$$\begin{bmatrix} \Xi & \Xi_1 \\ \star & \Xi_2 \end{bmatrix} < 0 \tag{48}$$

$$\Xi_1 = [\tilde{\Lambda}_p \epsilon\tilde{\Lambda}_q],\ \Xi_2 = diag\{-\epsilon I - \epsilon I\}.$$

Defining $\bar{Y} = \hat{Y}^{-1}, L = K\hat{Y}, \hat{Q}_{\bar{\zeta}} = \bar{Y}Q_{\bar{\zeta}}\bar{Y}, \bar{\zeta} = 1,2,3,4,5, \hat{M}_l = \bar{Y}M_l\bar{Y}, l = 1,2,3, \hat{O}_l = \bar{Y}O_l\bar{Y}, \hat{Z}_{\bar{\zeta}} = \bar{Y}Z_{\bar{\zeta}}\bar{Y}, \hat{S}_1 = \bar{Y}S_1\bar{Y}, \hat{S}_2 = \bar{Y}S_2\bar{Y}, \tilde{\bar{\Theta}}_1 = \bar{Y}\tilde{\Theta}_1\bar{Y}, \bar{\mathcal{K}} = \bar{Y}\mathcal{K}\bar{Y}, \bar{\tilde{\mathcal{L}}} = \bar{Y}\bar{\mathcal{L}}\bar{Y}, \bar{\mathcal{M}} = \bar{Y}\mathcal{M}\bar{Y}$. Then premultiplying and postmultiplying (48) by $diag\{\underbrace{\bar{Y}\ \dots\ \bar{Y}I}_{22\,times}\underbrace{\bar{Y}\ \dots\ \bar{Y}II}_{8\,times}\}$ yields (45a). Moreover, $\bar{\bar{\mathbb{P}}} = \mathcal{W}^{-1/2}\mathbb{P}\mathcal{W}^{-1/2}$, $\bar{\bar{\vartheta}} = \mathcal{W}^{-1/2}\bar{\vartheta}\mathcal{W}^{-1/2}$, $\bar{\bar{\phi}} = \mathcal{W}^{-1/2}\bar{\phi}\mathcal{W}^{-1/2}$, $\bar{\bar{\mho}}_i = \mathcal{W}^{-1/2}\bar{\mho}_i\mathcal{W}^{-1/2}, i = 1,2,3$, $\bar{\bar{Z}}_j = \mathcal{W}^{-1/2}\bar{Z}_j\mathcal{W}^{-1/2}, j = 2,3,4,5$, $\bar{\bar{Q}}_l = \mathcal{W}^{-1/2}\bar{Q}_l\mathcal{W}^{-1/2}$, $\bar{\bar{S}}_l = \mathcal{W}^{-1/2}\bar{S}_l\mathcal{W}^{-1/2}, l = 1,2$ and taking $\lambda_{\max}(\bar{\bar{\mathbb{P}}}) = 1/\lambda_{\min}(\bar{\bar{\mathbb{P}}})$ into consideration, we have

$$\lambda_1 \le \lambda_{\min}(\bar{\bar{\mathbb{P}}}),\ \lambda_{\max}(\bar{\bar{\mathbb{P}}}) < 1,\ \lambda_{\max}(\bar{\bar{\vartheta}}) \le \lambda_2,\ \lambda_{\max}(\bar{\bar{\theta}}) \le \lambda_3,$$
$$\lambda_{\max}(\bar{\bar{Q}}_i) \le \lambda_j, i = 1,2,3,4,5,\ j = 4,5,6,7,8,\ \lambda_{\max}(\bar{\bar{M}}_s)$$
$$\le \lambda_f, s = 1,2,3\ f = 9{,}10{,}11,\ \lambda_{\max}(\bar{\bar{O}}_s) \le \lambda_l,\ f = 12,$$
$$13{,}14,\ \lambda_{\max}(\bar{\bar{Z}}_v) \le \lambda_z, v = 1,2,3,4,5,\ z = 15{,}16{,}17{,}18{,}19,$$
$$\lambda_{\max}(\bar{\bar{S}}_p) \le \lambda_v, p = 1,2, v = 20{,}21.$$

Therefore,

$$\begin{aligned}&\frac{c_1}{\lambda_1} + c_1\lambda_2 + c_1\lambda_3 + \tau_2 c_1\lambda_{\tilde{\zeta}} + \tau_m c_1\lambda_{\tilde{j}} + \tau_1 c_1\lambda_{\tilde{k}} + \frac{\hat{\tau}_m^2}{2}c_1\lambda_{15}\\ &+ \frac{(\tau_1)^2}{2}c_1\lambda_{16} + \frac{\hat{\tau}_m^2}{2}c_1\lambda_{17} + \frac{(\tau_1)^2}{2}c_1\lambda_{18}\\ &+ \frac{(\eta_1)^2}{2}c_1\lambda_7 + \frac{(\eta_2-\eta_1)^2}{2}c_1\lambda_8 + \eta_1 c_1\lambda_{20} + \eta_2 c_1\lambda_{19}\\ &+ (\eta_2-\eta_1)c_1\lambda_{21} + d\frac{\beta}{\tilde{\alpha}}(1-e^{-\tilde{\alpha}T}) < c_2 e^{-\tilde{\alpha}T},\\ &\tilde{\zeta} = 4,5,6,\ \tilde{j} = 9{,}10{,}11,\ \tilde{k} = 12{,}13{,}14.\end{aligned} \tag{49}$$

It is clear that the inequality (48) is equivalent to (44b).

Corollary 5.2. *The master and slave system without finite time case considered are as follows:*

$$\begin{cases} d\hat{x}(t) &= [-W_1\hat{x}(t) + W_2 g(\hat{x}(t)) + W_3 g(\hat{x}(t-\tau(t)))]dt\\ &\quad +(B\hat{x}(t) + C\hat{x}(t-\tau(t))))dw(t),\\ 0z(t) &= Ag(\hat{x}(t)), \text{ and}\end{cases} \tag{50}$$

$$\begin{cases} d\check{x}(t) &= [-W_1\check{x}(t) + W_2 g(\check{x}(t)) + W_3 g(\check{x}(t-\tau(t)))\\ &\quad +u(t)]dt + (B\check{x}(t) + C\check{x}(t-\tau(t))))dw(t),\\ \check{z}(t) &= Ag(\check{x}(t)).\end{cases} \tag{51}$$

Under Assumption (H_1) *and for given scalars* $\tau_1, \tau_2, \tau_3, \eta_1,$ *and* η_2, *the considered SNNs, the master–slave systems (50)–(51) can be synchronized when the event-triggered controller is given by*

$$u(t) = \hat{K}x(s_k\hat{h}),$$

where $\hat{K} = \hat{Y}^{-1}L$ *and the symmetric positive definite matrices* $\hat{P} \in \mathbb{R}^{n\times n}, \mho_l \in \mathbb{R}^{2n\times n}, Z_j, Q_4, Q_5, S_1, S_2 \in \mathbb{R}^{n\times n}, l = 1,2,3, j = 1,2,3,4,5,$ *appropriate dimensional matrix* $\bar{Y}, \tilde{\Theta}_1, \mathcal{K}, \bar{\mathcal{L}}, \mathcal{M}$, *diagonal matrices* $\vartheta = diag(\nu_1, \nu_2, \dots, \nu_n) \ge 0$, $\Phi = diag(\phi_1, \phi_2, \dots, \phi_n) \ge 0$, *and* $\mathscr{H}_l \ge 0 (l = 1,2)$ *satisfying the following LMI:*

$$\begin{bmatrix} \sum_{i=1}^{8}\Phi_i & \tau_1\mathcal{K} & \hat{\tau}_m\bar{\mathcal{L}} & \varpi_{15}\\ \star & -\tau_1 Z_4 & 0 & 0\\ \star & 0 & -\hat{\tau}_m Z_5 & 0\\ \star & \star & 0 & \varpi_{55}\end{bmatrix} < 0, \tag{52}$$

with the other elements are same as in Theorem 5.1.

Proof. Similar to the proof of Theorem 5.1, we can easily derive the result. Its proof is straightforward and hence omitted.

Remark 5.3. It is noteworthy that in many industrial processes, the dynamical behaviors are generally complex and nonlinear and their genuine mathematical models are always difficult to obtain. How to model the event-triggered synchronization of SNNs with respect to passivity performance has become one if the main theme in our research work. More particularly, some pioneering works have been done in finite time event-triggered control for SNNs. In Cao et al. [31] and Vadivel et al. [30], the problem of synchronization of Markovian jumping SNNs have been studied with event-triggered control. Exponential synchronization of SNNs with time-varying delays and Lévy noises via event-triggered control has been studied in [33]. Recently, finite-time stabilization was proposed in Liu et al. [32] for semi-Markov jump neural networks based on the event-triggered control. The model considered in the present study is more practical than that proposed by previous works [30–33], because they consider usual event-triggered control has been studied with SNNs based on finite time stability, but in this paper, we consider a new nonfragile event-triggered control with the combination of passivity and passifi-

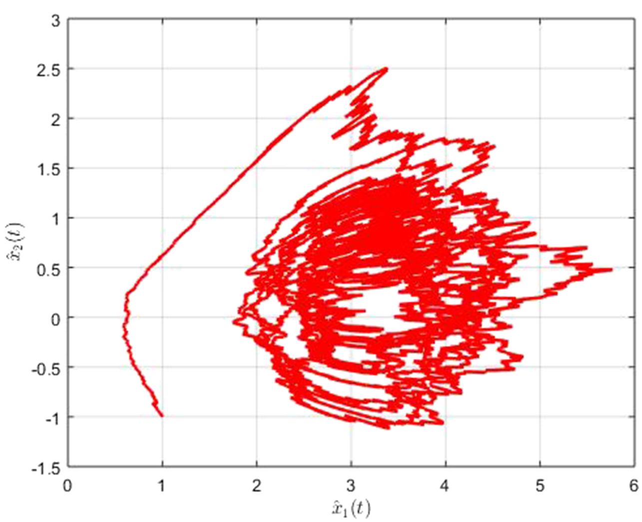

FIGURE 2 Chaotic behavior of master system (1) in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

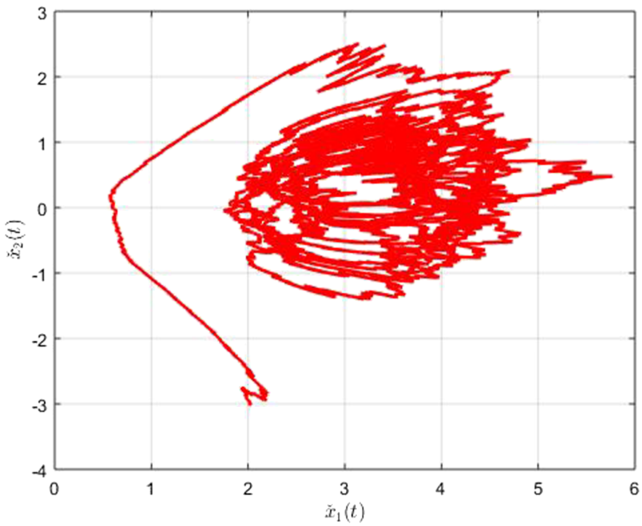

FIGURE 3 Chaotic behavior of slave system (2) in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

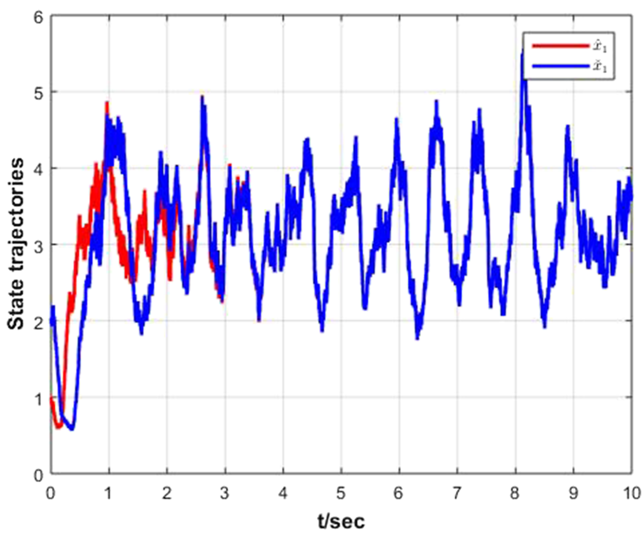

FIGURE 4 A trajectory of $\hat{x}_1(t), \check{x}_1(t)$ in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

cation approach in the finite time interval. Due to the many real-life application, the combined study of finite time passivity and passification effects to the system model is more important. The purpose of this study is to establish synchronization conditions of nonfragile SNNs by applying the Lyapunov functional theory and event-triggered communication scheme. In event-triggered control, the measured error acts a key role during the event-triggered controller design. An event will be triggered to update the event-triggered controller when its magnitude reaches the prescribed value. Additionally, it is mentioned that we utilize orthogonal polynomials based on two general inequality techniques to estimate the derivative of a Lyapunov functional, such as defined in $V_5(x(t), t)$, which can induce tighter information on the delay of the considered system. Henceforth, the investigation procedure and framework model proposed in this paper merit a lot of regard to fill such a demand all the more successfully.

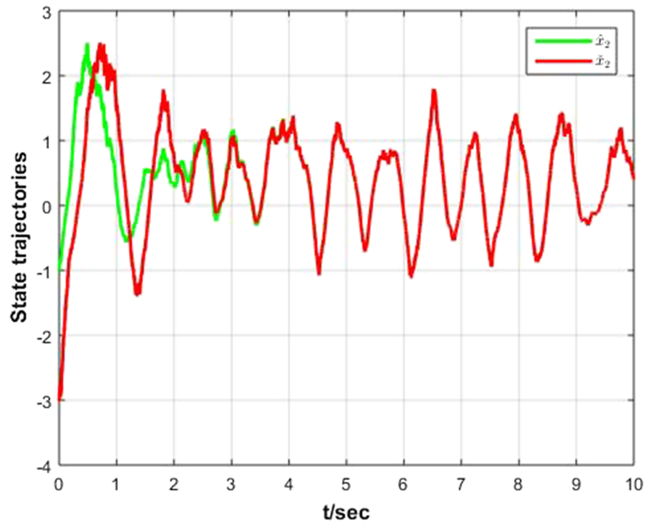

FIGURE 5 A trajectory of $\hat{x}_2(t), \check{x}_2(t)$ in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

Remark 5.4. Computational burden will become a major concern as the size of the LMIs and the number of decision variables increase. Theorems 3.1,4.1, and 5.1 employ the maximum number of decision variables in our LMIs. Furthermore, larger LMIs produce greater results. As a result of the proposed conditions being used by multiple integral inequalities in this work, considerable computational load may arise in this technique. Finsler's Lemma will be used in future work to reduce the number of decision variables in order to reduce computing complexity load and time processing. Furthermore, in a future study, we will concentrate on reducing the computational complexity of stability concerns while preserving the appropriate system performance.

6 | SIMULATION RESULTS

In this section, the efficiency of our results, which have been derived in the preceding section, will be verified by the following examples.

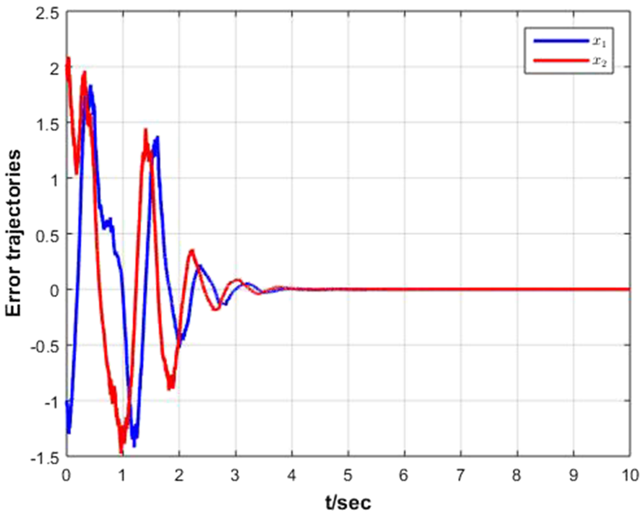

FIGURE 6 Synchronization of error responses $x_1(t)$, $x_2(t)$ in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

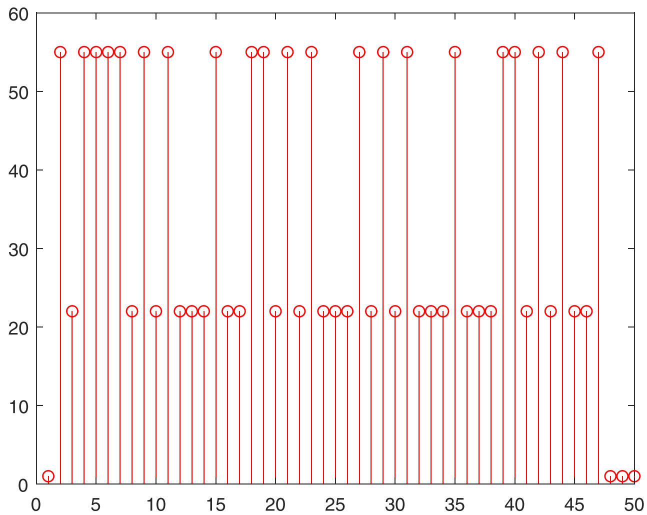

FIGURE 7 Release time intervals in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

Example 6.1. Consider the two-neuron SNNs (10), in which the parameters are given by

$$W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W_2 = \begin{bmatrix} \sqrt{2} * \frac{\pi}{2} & 2 \\ 10 & \sqrt{2} * \frac{\pi}{2} \end{bmatrix},$$

$$W_3 = \begin{bmatrix} \sqrt{2} * \frac{\pi}{4} & 10 \\ 10 & -1.3 * \sqrt{2} * \frac{\pi}{2} \end{bmatrix}, W_4 = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, A = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

The parameters $\tau(t)$ of time-varying delays satisfy $0.5 \le \tau(t) \le 1$, $0.3 \le \eta(t) \le 0.5$, and select the activation function as $f_1(x) = \frac{1}{20}(|x+1| + |x-1|)$, $f_2(x) = \frac{1}{10}(|x+1| + |x-1|)$, such that $G_1 = diag\{-0.01 - 0.04\}$, $G_2 = 0$. For given scalars, $\tau_3 = 0.5$, $c_1 = 0.2$, $d = 0.1$, $T = 10$, $\tilde{\alpha} = 0.2$, and matrix $\mathcal{W} = I$. Meanwhile, utilizing the parameter mentioned above and MATLAB LMI toolbox to evaluate the sufficient conditions (44a)–(44b) in Theorem 5.1, we obtained the minimum value of $c_2 = 9.5$ and the controller gain matrix, and the transmission scheme weighting matrix is given by

$$\hat{K} = \begin{bmatrix} 0.3550 & 0.0031 \\ 0.0007 & 0.1330 \end{bmatrix}, \tilde{\Theta}_1 = \begin{bmatrix} 2.7335 & -0.0349 \\ -0.0349 & 3.0273 \end{bmatrix}.$$

With the above estimated gain values by Theorem 5.1, Figures 2 and 3 depicted the chaotic behavior and chose the initial conditions $\hat{x}(t) = [1-1]^T$, $\check{x}(t) = [2-3]^T$; the synchronization responses are given in Figures 4 and 5. Consequently, one can easily see that state responses of the subsequent ETC (6) are pictured in Figures 4 and 5. With the impact of event-triggered controller (6), we can obtain the state responses curve $(\hat{x}_1(t), \check{x}_1(t))$ and $(\hat{x}_1(t), \check{x}_1(t))$ of SNNs (10), and it can be displayed in Figures 4 and 5. It shows that the trajectories of SNNs (10) can achieve the corresponding states and designed controller in this paper is still effective. Under the initial conditions $x(0) = [-1 2]^T$, synchronization error was depicted in Figure 6, which explains the error trajectories converge (synchronize) to the corresponding states. Clearly, from the above simulations, the error system (10) is stable, with the effect of control input. The release instants and release intervals are depicted in Figure 7. Also, the corresponding time history figures are provided in Figure 8a,b. Here, it is easily seen from Figure 8 that the curve of $\mathbb{E}\{x^T(t)\mathcal{W}x(t)\}$ does not exceed the prescribed bound value c_2.

Moreover, the optimum bound value of c_2 is determined for various values of τ_2, and it is provided in Table 1, in which the delay bound is increased, the optimum bound value c_2 is also increased. From the above simulation results, the designed controller obtained by Theorem 5.1 with passivity performance can effectively stabilize the unstable system; that is, the considered SNNs (10) reaches the desired synchronization during the particular finite-time interval. So that the master system and the slave system are FTP and synchronous.

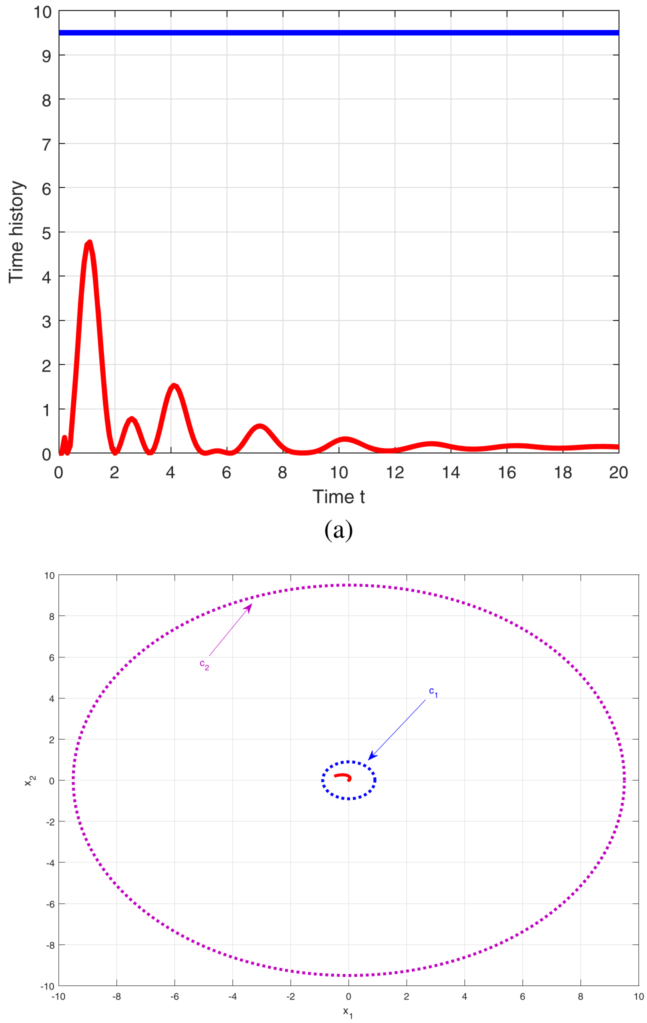

FIGURE 8 Evolution of time history $E\{x^T(t)Wx(t)\}$ in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

Example 6.2. Consider a master chaotic neural network as follows:

$$\begin{cases} d\hat{x}(t) = \left(-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix} g(\hat{x}(t)) \right. \\ \quad \left. + \begin{bmatrix} -1.3\sqrt{2}\frac{\pi}{4} & 0.1 \\ 0.1 & -1.3\sqrt{2}\frac{\pi}{4} \end{bmatrix} g(\hat{x}(t-\tau(t))) \right) dt \\ \quad + \left(\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \right. \\ \quad \left. \times \hat{x}(t-\tau(t)) \right) dw(t), \\ \hat{z}(t) = \begin{bmatrix} 0.1 & 1 \\ 1 & 0.2 \end{bmatrix} g(\hat{x}(t)), \end{cases}$$

TABLE 1 Calculated parameters of c_2 for different parameters of τ_2

τ_2	0.2	0.3	0.4	0.5	0.6
c_2	2.5631	3.4012	4.0682	4.5578	6.2563

TABLE 2 Upper bound of η_2 for different values of τ_3 and $\eta_1 = 0.2$

τ_3	0.10	0.15	0.20	0.25	0.30
Theorem 5.1	0.0371	0.0365	0.0331	0.0320	0.0315
Corollary 5.2	0.4362	0.4353	0.3920	0.3915	0.3880

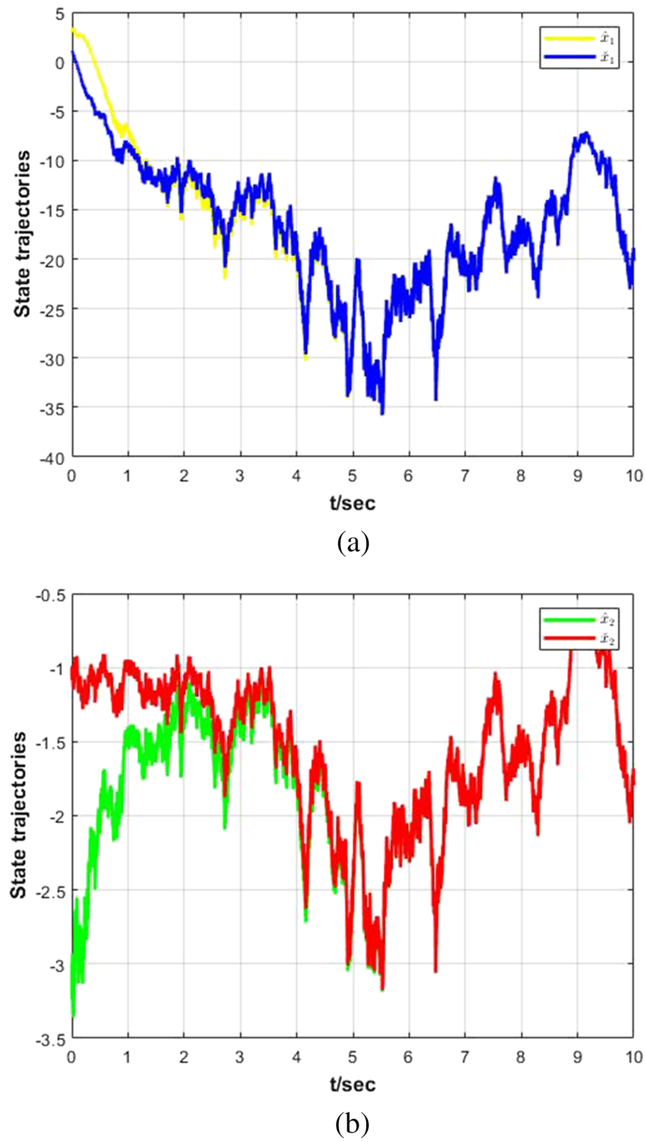

FIGURE 9 Simulations of (a) $\hat{x}_1, \hat{x}_2$, (b) $\check{x}_1, \check{x}_2$ in Example 6.2 [Color figure can be viewed at wileyonlinelibrary.com]

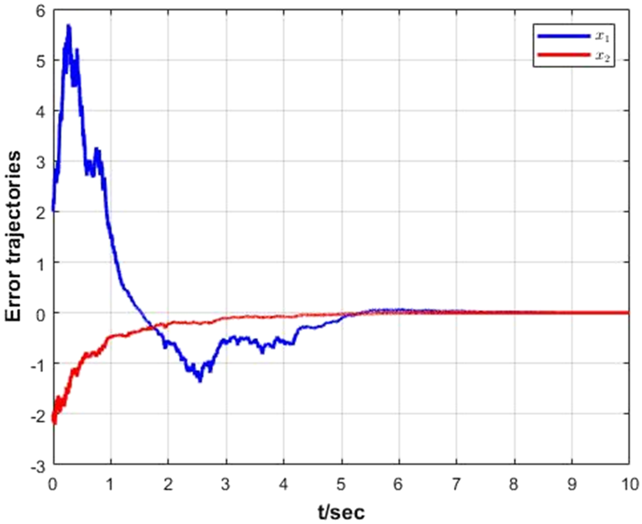

FIGURE 10 The error responses in Example 6.2 [Color figure can be viewed at wileyonlinelibrary.com]

The corresponding response system can be

$$\begin{cases} d\check{x}(t) = \left(-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\check{x}(t) + \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix} g(\check{x}(t)) \right. \\ \quad + \begin{bmatrix} -1.3\sqrt{2}\frac{\pi}{4} & 0.1 \\ 0.1 & -1.3\sqrt{2}\frac{\pi}{4} \end{bmatrix} g(\check{x}(t-\tau(t))) \\ \quad \left. + \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & 1 \end{bmatrix} \mathbb{k}\left[\check{x}(t-\eta(t)) + e(t)\right]\right) dt \\ \quad + \left(\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}\check{x}(t) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \right. \\ \quad \left. \times \check{x}(t-\tau(t))\right) dw(t), \\ \check{z}(t) = \begin{bmatrix} 0.1 & 1 \\ 1 & 0.2 \end{bmatrix} g(\check{x}(t)), \end{cases} \tag{53}$$

A. Numerical results

Because the restriction of the time-varying delay is merely on the lower and upper bounds, the time-varying delay $\tau(t)$ is generated randomly in simulation. Supposing that time-varying delay $\tau(t) = \frac{e^t}{e^t+1}$, thus we can get $\tau_1 = 0.1, \tau_2 = 0.7$, and $\dot{\tau}(t) = \frac{e^t}{(e^t+1)^2} \le 0.25$, which implies $\tau_3 = 0.6$. By selecting $f_i(x) = tanh(x), i = 1, 2.$ and utilizing the LMI in Corollary 5.2, the corresponding control gain can be worked out and given by

$$\hat{K} = \begin{bmatrix} 0.4641 & 0.1212 \\ 0.0852 & 0.2626 \end{bmatrix}.$$

Moreover, the obtained upper bound η_2 of period $\eta(t)$ for different values of τ_3 and $\eta_1 = 0.2$ based on Corollary 5.2 is listed in Table 2. From Table 2, we can find that the upper bound η_2 of period $\eta(t)$ by Corollary 5.2 is much larger than that by Theorem 5.1. Therefore, Corollary 5.2 is less conservative than Theorem 5.1.

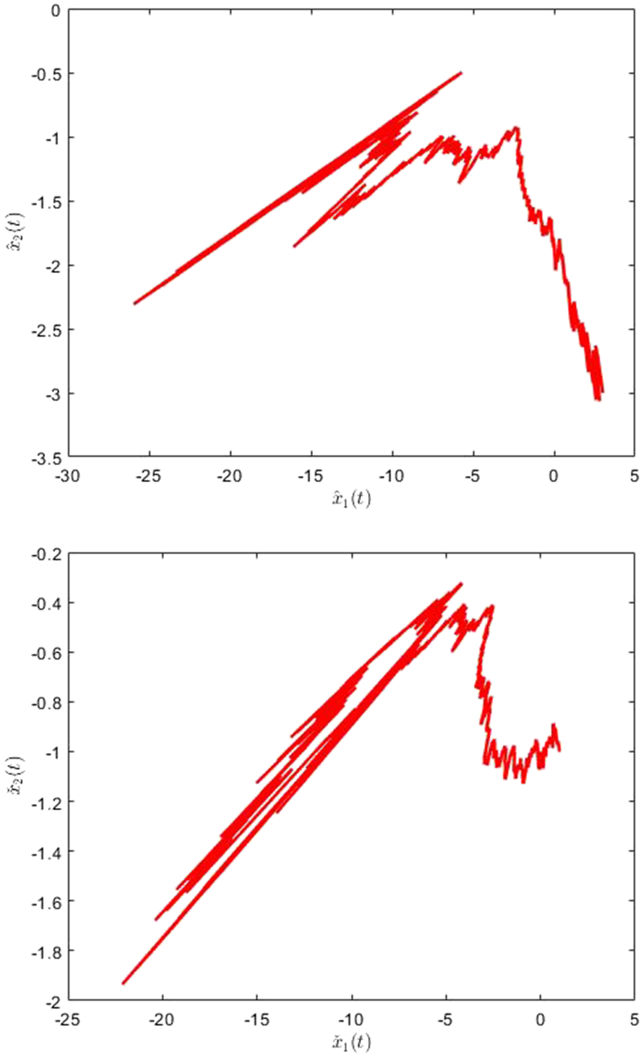

FIGURE 11 Chaotic behavior of master and slave system described in Example 6.1 [Color figure can be viewed at wileyonlinelibrary.com]

B. Simulation results

Furthermore, we assume that the initial state of the master system and the slave system is $\hat{x}(t) = [1 - 1]^T$ and $\check{x}(t) = [2 - 3]^T$, which has been shown in Figure 9a,b. Based on the obtained ETC by Corollary 5.2, Figure 10 shows the error responses, Figure 11 displayed chaotic behavior, and the release instants are depicted in Figure 12. Therefore, the ETC obtained by the Corollary 5.2 can effectively stabilize the unstable system so that the master system and the slave system are asymptotically stable and synchronous.

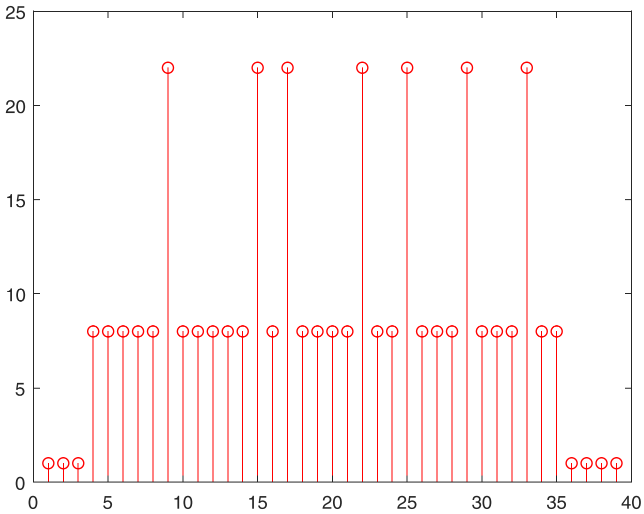

FIGURE 12 Triggered instants in Example 6.2 [Color figure can be viewed at wileyonlinelibrary.com]

7 | CONCLUSION AND FUTURE DIRECTIONS

In this paper, the finite-time event-triggered control synchronization for delayed SNNs is studied with passivity and passification approach. A nonfragile ETC has been proposed to diminish the communication load during the networked transmission. In view of the finite-time analysis techniques, impact of the transmission delay on the system, and adequate conditions have been acquired to guarantee the SNNs are FTP and FTB within the interval, subject to synchronization criteria. Then, the desired controller gains and triggered parameters have been derived, which are expressed in the form of LMIs. Finally, simulation examples are given to verify the effectiveness of the proposed controller. The presented results and approaches in this article can be extended to many complex dynamic systems, such as stochastic switched delayed NNs with tracking error constraints, semi-Markovian jump-delayed SNNs. Moreover, the model proposed in this work can be also extended event triggered mechanism to the fuzzy SNNs with imperfect communication, such as packet dropouts and quantization, which makes the model more practical, which will be investigated in our future work.

CONFLICT OF INTEREST

The authors declare that they have no competing interests.

AUTHOR CONTRIBUTIONS

R. Vadivel: conceptualization, methodology, software. **P. Hammachukiattikul:** conceptualization, investigation, methodology, software, visualization. **Quanxin Zhu:** conceptualization, methodology, supervision, visualization. **Nallappan Gunasekaran:** conceptualization, investigation, visualization.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

ORCID

Quanxin Zhu https://orcid.org/0000-0003-3130-4923
Nallappan Gunasekaran https://orcid.org/0000-0003-0375-7917

REFERENCES

1. K. Shi, H. Zhu, S. Zhong, Y. Zeng, and Y. Zhang, *Less conservative stability criteria for neural networks with discrete and distributed delays using a delay-partitioning approach*, Neurocomputing **140** (2014), 273–282.
2. H. Huang and G. Feng, *Delay-dependent stability for uncertain stochastic neural networks with time-varying delay*, Phys. A: Stat. Mech. Appl. **381** (2007), 93–103.
3. G. Nagamani, S. Ramasamy, and P. Balasubramaniam, *Robust dissipativity and passivity analysis for discrete-time stochastic neural networks with time-varying delay*, Complexity **21** (2016), no. 3, 47–58.
4. M. S. Ali and P. Balasubramaniam, *Exponential stability of uncertain stochastic fuzzy BAM neural networks with time-varying delays*, Neurocomputing **72** (2009), no. 4-6, 1347–1354.
5. O. M. Kwon, S. M. Lee, and J. H. Park, *Improved delay-dependent exponential stability for uncertain stochastic neural networks with time-varying delays*, Phys. Lett. A **374** (2010), no. 10, 1232–1241.
6. R. Vadivel, M. S. Ali, and F. Alzahrani, *Robust H_∞ synchronization of Markov jump stochastic uncertain neural networks with decentralized event-triggered mechanism*, Chinese J. Phys. **60** (2019), 68–87.
7. Y. Xu, J. Yu, W. Li, and J. Feng, *Global asymptotic stability of fractional-order competitive neural networks with multiple time-varying-delay links*, Appl. Math. Comput. **389** (2021), 125498.
8. G. Zhang and Z. Zeng, *Exponential stability for a class of memristive neural networks with mixed time-varying delays*, Appl. Math. Comput. **321** (2018), 544–554.
9. D. Ouyang, J. Shao, H. Jiang, S. Wen, and S. K. Nguang, *Finite-time stability of coupled impulsive neural networks with time-varying delays and saturating actuators*, Neurocomputing **453** (2020), 590–598.
10. C. Long, G. Zhang, Z. Zeng, and J. Hu, *Finite-time lag synchronization of inertial neural networks with mixed infinite time-varying delays and state-dependent switching*, Neurocomputing **433** (2021), 50–58.
11. P. Wan, D. Sun, and M. Zhao, *Finite-time and fixed-time anti-synchronization of Markovian neural networks with stochastic disturbances via switching control*, Neural Netw. **123** (2020), 1–11.
12. L. Wang, Z. Wang, G. Wei, and F. E. Alsaadi, *Finite-time state estimation for recurrent delayed neural networks with component-based event-triggering protocol*, IEEE Trans. Neural Netw. Learn. Syst. **29** (2017), no. 4, 1046–1057.

13. Y. Zhou, X. Wan, C. Huang, and X. Yang, *Finite-time stochastic synchronization of dynamic networks with nonlinear coupling strength via quantized intermittent control*, Appl. Math. Comput. **376** (2020), 125157.
14. X. Lv and X. Li, *Finite time stability and controller design for nonlinear impulsive sampled-data systems with applications*, ISA Trans. **70** (2017), 30–36.
15. L. Wang and Y. Shen, *Finite-time stabilizability and instabilizability of delayed memristive neural networks with nonlinear discontinuous controller*, IEEE Trans. Neural Netw. Learn. Syst. **26** (2015), no. 11, 2914–2924.
16. Z. Zhang and J. Cao, *Novel finite-time synchronization criteria for inertial neural networks with time delays via integral inequality method*, IEEE Trans. Neural Netw. Learn. Syst. **30** (2018), no. 5, 1476–1485.
17. J. Ren, X. Liu, H. Zhu, and S. Zhong, *Passivity-based non-fragile control for Markovian jump delayed systems via stochastic sampling*, Int. J. Control **92** (2019), no. 4, 755–777.
18. R. Vadivel and Y. H. Joo, *Finite-time sampled-data fuzzy control for a non-linear system using passivity and passification approaches and its application*, IET Control Theory Appl. **14** (2020), no. 8, 1033–1045.
19. W. Qi, X. Gao, and J. Wang, *Finite-time passivity and passification for stochastic time-delayed Markovian switching systems with partly known transition rates*, Circ. Syst. Signal Process. **35** (2016), no. 11, 3913–3934.
20. P. Muthukumar, P. Balasubramaniam, and K. Ratnavelu, *A novel cascade encryption algorithm for digital images based on anti-synchronized fractional order dynamical systems*, Multimed. Tools Appl. **76** (2017), no. 22, 23517–23538.
21. A. M. Alimi, C. Aouiti, and E. A. Assali, *Finite-time and fixed-time synchronization of a class of inertial neural networks with multi-proportional delays and its application to secure communication*, Neurocomputing **332** (2019), 29–43.
22. N. Gunasekaran, G. Zhai, and Q. Yu, *Sampled-data synchronization of delayed multi-agent networks and its application to coupled circuit*, Neurocomputing **413** (2020), 499–511.
23. K. Ding, Q. Zhu, and L. Liu, *Extended dissipativity stabilization and synchronization of uncertain stochastic reaction-diffusion neural networks via intermittent non-fragile control*, J. Franklin Inst. **356** (2019), no. 18, 11690–11715.
24. L. Pan, Q. Song, J. Cao, and M. Ragulskis, *Pinning impulsive synchronization of stochastic delayed neural networks via uniformly stable function*, IEEE Trans. Neural Netw. Learn. Syst. **33** (2021), 4491–4501.
25. J. Wang, H. Zhang, Z. Wang, and H. Liang, *Stochastic synchronization for Markovian coupled neural networks with partial information on transition probabilities*, Neurocomputing **149** (2015), 983–992.
26. K. Shi, J. Wang, S. Zhong, Y. Tang, and J. Cheng, *Non-fragile memory filtering of T-S fuzzy delayed neural networks based on switched fuzzy sampled-data control*, Fuzzy Sets Syst. **394** (2020), 40–64.
27. Y. Feng, X. Yang, Q. Song, and J. Cao, *Synchronization of memristive neural networks with mixed delays via quantized intermittent control*, Appl. Math. Comput. **339** (2018), 874–887.
28. T. Zhao and S. Dian, *State feedback control for interval type-2 fuzzy systems with time-varying delay and unreliable communication links*, IEEE Trans. Fuzzy Syst. **26** (2017), no. 2, 951–966.
29. C. Peng and F. Li, *A survey on recent advances in event-triggered communication and control*, Inform. Sci. **457** (2018), 113–125.
30. R. Vadivel, M. S. Ali, and F. Alzahrani, *Robust H_∞ synchronization of Markov jump stochastic uncertain neural networks with decentralized event-triggered mechanism*, Chinese J. Phys. **60** (2019), 68–87.
31. D. Cao, Y. Jin, and W. Qi, *Synchronization for stochastic semi-Markov jump neural networks with dynamic event-triggered scheme*, J. Franklin Inst. (2021).
32. M. Liu, H. Wu, and W. Zhao, *Event-triggered stochastic synchronization in finite time for delayed semi-Markovian jump neural networks with discontinuous activations*, Comput. Appl. Math. **39** (2020), no. 2, 1–47.
33. D. Lu, D. Tong, Q. Chen, W. Zhou, J. Zhou, and S. Shen, *Exponential synchronization of stochastic neural networks with time-varying delays and Lévy noises via event-triggered control*, Neural Processing Lett. **53** (2021), no. 3, 2175–2196.
34. X. Xie, Q. Zhou, D. Yue, and H. Li, *Relaxed control design of discrete-time Takagi–Sugeno fuzzy systems: An event-triggered real-time scheduling approach*, IEEE Trans. Syst. Man Cybern.: Syst. **48** (2017), no. 12, 2251–2262.
35. Y. Luo, Y. Yao, Z. Cheng, X. Xiao, and H. Liu, *Event-triggered control for coupled reaction–diffusion complex network systems with finite-time synchronization*, Phys. A: Stat. Mech. Appl. **562** (2021), 125219.
36. W. Cui, W. Jin, and Z. Wang, *Novel finite-time synchronization criteria for coupled network systems with time-varying delays via event-triggered control*, Adv. Differ. Equa. **2020** (2020), no. 1, 1–13.
37. J. Hu, Y. Yang, H. Liu, D. Chen, and J. Du, *Non-fragile set-membership estimation for sensor-saturated memristive neural networks via weighted try-once-discard protocol*, IET Control Theory Appl. **14** (2020), no. 13, 1671–1680.
38. Y. Liu, A. Arumugam, S. Rathinasamy, and F. E. Alsaadi, *Event-triggered non-fragile finite-time guaranteed cost control for uncertain switched nonlinear networked systems*, Nonlin. Anal.: Hybrid Syst. **36** (2020), 100884.

AUTHOR BIOGRAPHIES

R. Vadivel received the B.Sc., M.Sc., and M.Phil. degrees in Mathematics from Sri Ramakrishna Mission Vidyalaya College of Arts and Science affiliated to Bharathiar University, Coimbatore, Tamil Nadu, India, in 2007, 2010, and 2012, respectively. He was awarded the Ph.D. degree in 2018 from Department of Mathematics, Thiruvalluvar University, Vellore, Tamil Nadu, India. He was a Post-Doctoral Research Fellow in the Research Center for Wind Energy Systems, Kunsan National University, Gunsan, South Korea, from 2018 to 2019. Currently, he is working as a lecturer at Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Thailand.

P. Hammachukiattikul received the Ph.D. degree in applied mathematics from the King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, Thailand. She is currently working with the Department of Mathematics, Phuket Rajabhat University (PKRU), Phuket, Thailand. Her research interests include mathematic model, climate change, atmospheric model, Lyapunov theory and neural networks, stability analysis of dynamical systems, synchronization, and chaos theory.

Quanxin Zhu received the M.S. degree in probability and statistics from Hunan Normal University, Changsha, in 2002 and the Ph.D. degree in probability and statistics from Sun Yat-sen (Zhongshan) University, Guangzhou, China, in 2005. He is currently a professor with Hunan Normal University, distinguished professor of Hunan Province and Leading talent of scientific and technological innovation in Hunan Province, deputy director of the Key Laboratory of computing and stochastic mathematics of the Ministry of education, and director control and optimization of complex systems of Hunan Key Laboratory of colleges and Universities. He won the Alexander von Humboldt Foundation of Germany and the Highly Cited Researcher Award by Clarivate Analytics in 2018–2021. He won First Prize of Hunan Natural Science Award and the list of Top 100000 scientists in the world in 2020–2022. He is an associate editor of some international journals, the lead guest editor of several journals, and a reviewer of more than 50 other journals. He is the author or the coauthor of more than 300 journal articles. His research interests include stochastic control, stochastic systems, stochastic stability, stochastic delayed systems, Markovian jump systems, and stochastic complex networks.

Nallappan Gunasekaran received the BSC from Mahendra Arts and Science College, Periyar University, Salem, Tamil Nadu, India, in 2009. He received his post-graduation from Jamal Mohamed College affiliated to Bharathidasan University, Trichy, Tamil Nadu, India, in 2012. He was awarded Master of Philosophy in 2013 from Bharathidasan University, Trichy, India, and the Ph.D. degree from Thiruvalluvar University, Vellore, India, in 2018, in mathematics. He was a Post-Doctoral Research Fellow in the Research Center for Wind Energy Systems, Kunsan National University, Gunsan, South Korea, from 2017 to 2018. Currently, he is working as a Post-Doctoral Research Fellow in Department of Advanced Science and Technology, Computational Intelligence Laboratory, Toyota Technological Institute, Nagoya,468-8511, Japan. His research interests are complex-valued NNs, complex dynamical networks, control theory, stability analysis, sampled-data control, multi-agent systems, T-S fuzzy theory, and cryptography. Dr. Gunasekaran serves as a reviewer for various SCI journals. He has authored and co-authored of more than 30 research articles in various SCI journals.

How to cite this article: R. Vadivel, P. Hammachukiattikul, Q. Zhu, and N. Gunasekaran, *Event-triggered synchronization for stochastic delayed neural networks: Passivity and passification case*, Asian J. Control, (2022), 1–18. https://doi.org/10.1002/asjc.2965