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Novel adaptive strategies for synchronization control mechanism in nonlinear dynamic fuzzy modeling of fractional-order genetic regulatory networks

G. Narayanan [a], M. Syed Ali [b], Rajagopal Karthikeyan [a], Grienggrai Rajchakit [c,*], Anuwat Jirawattanapanit [d,*]

[a] Center for Nonlinear Systems, Chennai Institute of Technology, Chennai 600069, India
[b] Complex Systems and Networked Science Laboratory, Department of Mathematics, Thiruvalluvar University, Vellore 632 115, Tamilnadu, India
[c] Department of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand
[d] Department of Mathematics, Faculty of Science, Phuket Rajabhat University (PKRU), 6 Thepkasattree Road, Raddasa, Phuket 83000, Thailand

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ABSTRACT

The synchronization problem for a nonlinear dynamic fuzzy modeling of fractional-order genetic regulatory networks with adaptive feedback controllers is addressed in this article. To address such a problem, a fuzzy feedback MAX template and a fuzzy feedback MIN template of fuzzy modeling are introduced. We developed adaptive based criteria to ensure the asymptotic and finite-time stability of the error system by constructing a suitable Lyapunov functional with linear matrix inequality (LMI) approach, which ensures the drive system synchronizes with the response system. Meanwhile, two distinct controllers, linear feedback and adaptive feedback control are being developed. It should be noted that the presented results are expressed in terms of LMI, which can be efficiently solved using Matlab LMI toolbox. Numerical simulations are carried out to illustrate the theoretical results.

1. Introduction

Genetic networks are gaining popularity in biomedical sciences, biology, engineering, and other fields of study. A key purpose of control of gene regulatory networks (GRNs) is to derive intervention strategies to avoid undesirable states, such as those associated with the disease. One of the fundamental issues for biologists is to reveal the regulatory mechanisms and functions of GRNs, thereby providing clues for finding new therapeutics for some fatal diseases such as cancer [1]. As we all know, stability of dynamical systems is one of the most important topics. GRNs can be naturally modeled as dynamical systems, which are powerful tools for gaining biological insights based on their complex dynamical behaviors [2,3]. In the past decades, many researchers have made intensive efforts to understand the dynamical properties of genetic regulation inside cells, including system modeling, stability analysis, filtering and so on (see [4–6] and the references therein). Furthermore, the GRNs, due to the slow processes of the transcription and translation, the mRNA and protein concentrations not only depend on the current state, but also rely on the delayed states. In other words, the time delay exists in the GRNs, which affects stability performance. The authors [7,8] investigated the various types of delayed GRNs using Lyapunov stability theory and LMI methods. The authors of [9,10] used the M-matrix technique to develop stable conditions for GRNs to reduce computational complexity. So, it is of great significance to carry out research on the dynamical properties of genetic regulation mechanisms, which has inspired a great deal of interest for the biological scopes of delayed GRNs.

Recently, fractional-order models are an excellent tool for describing memory and hereditary properties of various materials and processes in comparison to classical integer-order models [11–16]. The fractional differential equations, which are based on biochemical reactions between genes, show the concentration of genes in response to the actions of themselves and other genes. Fractional-order gene regulatory networks (FOGRNs) are a highly nonlinear, high-dimensional systems with a wide range of dynamic behaviors, such as chaos [17], bifurcation [18], bursting oscillation [19,20], Mittag-Leffler stability [21,22] and synchronization [23,24]. However, there have been relatively few studies on FOGRNs. In addition the LMI conditions, the global stability of GRNs can be analyzed with ease and intuitive tool for analyzing

* Corresponding authors.
E-mail addresses: narayanang@citchennai.net (G. Narayanan), syedgru@gmail.com (M. Syed Ali), rkarthiekeyan@gmail.com (R. Karthikeyan), kreangkri@mju.ac.th (G. Rajchakit), anuwat.j@pkru.ac.th (A. Jirawattanapanit).

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system stability, especially for integer-order GRNs. Even though the aforementioned references have studied LMI stability, so far no result LMI based on FOGRNs has been reported. Thus, our work fills this, which also inspires our research.

The fuzzy logic theory is required to solve several challenging problems in bioinformatics which are beyond the capabilities of other existing approaches. Fuzzy logic systems which were applied in a wide range of areas [25,26] and applications [27] can be employed to simulate nonlinear systems. It is an excellent way to represent a complex nonlinear system and cope with its stability [28–30]. For nonlinear fractional-order systems, fuzzy logic (fuzzy AND and fuzzy OR) systems have become popular [31,32]. As a result, incorporating fuzzy logic (fuzzy AND and fuzzy OR) into GRNs appeared to be a promising option. Because ambiguity or uncertainty is unavoidable in the implementation of GRNs, fuzzy genetic regulatory networks (FGRNs) were introduced in [33–35]; they combine fuzzy logic with the fast nonlinear learning ability of GRNs. Ratnavelu et al. [33] investigated the global asymptotic stability of the FGRNs with time delay in their recent work. The global exponential stability of weighted pseudo-almost periodic solutions of FGRNs with various delays were studied by Ayachi [35]. According to [33,35], fuzzy logic can be used in integer-order FGRNs as:

$$\begin{cases} \dot{\phi}_r^{\varsigma}(t) = & -a_r\phi_r^{\varsigma} + \sum_{s=1}^{q} b_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t))) + \bigwedge_{s=1}^{q} \mu_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t))) \\ & + \bigvee_{s=1}^{q} \varphi_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t))) + \mathcal{J}_r, \\ \dot{\psi}_r^{\varsigma}(t) = & -c_r\psi_r^{\varsigma}(t) + d_r\phi_r^{\varsigma}(t-\sigma(t)) + \bigwedge_{s=1}^{q} \gamma_{rs}\phi_s^{\varsigma}(t-\sigma(t)) \\ & + \bigvee_{s=1}^{q} \rho_{rs}\phi_s^{\varsigma}(t-\sigma(t)) + \hat{\mathcal{J}}_r, \end{cases} \tag{1}$$

more detailed in [33,35]. However, its applications in the area of FGRNs have not been fully explored. These works inspired us to the stability analysis of FGRNs with using fuzzy AND and fuzzy OR, where more biological information can be incorporated. Surprisingly, the FGRNs (1) has not been extended to the fractional-order FGRNs (FOFGRNs) despite its application potential, and also it is very important to take some control strategies to FOFGRNs, this vacancy still needs to be filled.

In recent years, the importance of synchronization problems in biological science and theoretical research has been recognized [36,37]. Many results on synchronization of GRNs with various features, such as passive synchronization of Markov jump GRNs [38], finite-time synchronization of GRNs with stochastic noise [39] and exponential synchronization of switched GRNs with delays [40], have been obtained. Due to the slow rate of genetic regulation, time delay is common in genetic activities. The delay can destroy the steady or synchronized states of GRNs, resulting in bifurcation or oscillation [41]. As a result, some recent efforts have been made to investigate the effects of time-varying delays on GRNs [42,43]. On the other hand, the control gains of feedback controllers are usually much larger than those needed in practical applications because of the conservativeness of theoretical analysis, while adaptive feedback controllers can avoid the high control gains effectively. Based on adaptive feedback time-delayed control, Li and Huang [44], and Chen et al. [45] have formulated sufficient criteria for the synchronization of master–slave chaotic delayed neural networks. In [46], the problem of synchronization for neural networks with the fractional differential order with uncertain parameters is resolved with an adaptive controller scheme. Some few works have concerned the adaptive control of GRNs [47,48], but there are no results on the adaptive synchronization control problem of FOFGRNs in the literature. Therefore, it is of great significance to study the influence of fuzzy logic (fuzzy AND and fuzzy OR) based on the development and application of FOFGRNs. Even so, the adaptive synchronization control problem of FOFGRNs is still in its primitive stage.

The above discussion inspires us with a new topic: adaptive synchronization problem in nonlinear dynamic fuzzy modeling for FOGRNs with time-varying delays based on LMI approaches. The main contributions are:

Table 1
Comparison with other works.

GRNs	[2–4,9,10]	[17–22]	[23]	[33–35]	[47,48]	This paper
Fractional-order	×	√	×	×	×	√
Fuzzy approaches	×	×	×	√	×	√
Adaptive control	×	×	×	×	√	√
LMI criteria	√	×	×	×	×	√
FTS	×	×	√	×	×	√

(1) For the first time, the adaptive synchronization of nonlinear dynamic FOFGRNs with time-varying delay is investigated.
(2) Sufficient conditions are established to ensure that the FOFGRNs achieve asymptotic and finite-time synchronization by designing a set of adaptive control strategies with updated law in the fractional domain.
(3) To account for ambiguity, the fuzzy theory is incorporated into the FOGRNs, and we discovered a novel system, namely fractional-order fuzzy genetic regulatory networks.
(4) Based on fractional Lyapunov stability theory and the LMI technique, the asymptotic and finite-time criteria are derived.
(5) The obtained LMI stability criteria have a simpler form than the algebraic stability criteria proposed in [17–23], which reduces computational complexity.

To better illustrate the innovations and major contributions of this paper, we provide Table 1 for comparison with other research works on GRNs, where fractional-order, fuzzy approaches (fuzzy AND and fuzzy OR), Adaptive control, LMI criteria, and finite-time synchronization (FTS). Moreover, $\surd$ means this item is included in that paper, $\times$ means it is not.

Notation. $\mathcal{R}^q$ represents the q-dimensional real vector space. $\mathcal{R}$, $\mathcal{R}^q$, and $R^{p\times q}$ denote the real numbers, the q-dimensional Euclidean space and the space of $p \times q$ real matrices, respectively. For any $\phi(t) = (\phi_1(t), \phi_2(t), \ldots, \phi_q(t))^T$, the vector norm is given by

$$\|\phi(t)\| = \Big(\sum_{\imath=1}^{q} |\phi_\imath(t)|\Big)^2.$$

2. Problem description and preliminaries

Definition 2.1 (*[11]*)**.** The fractional integral of Riemann–Liouville with a function $\mathfrak{I}(t)$ is defined as

$${}_{t}^{RL}I_0^{\hbar}\mathfrak{I}(t) = \frac{1}{\Gamma(\hbar)}\int_{t_0}^{t}(t-\nu)^{\hbar-1}\mathfrak{I}(\nu)d\nu, \quad t \ge t_0,$$

where $\Gamma(\hbar) = \int_0^{+\infty} \nu^{\hbar-1}e^{-\nu}d\nu$.

The Caputo fractional derivative of a function $\mathfrak{I}(t) \in C^q([t_0,+\infty),\mathcal{R})$ is

$${}_{t_0}^{C}D_t^{\hbar}\mathfrak{I}(t) = \begin{cases} \frac{1}{\Gamma(q-\hbar)}\int_{t_0}^{t}\frac{\mathfrak{I}^{(q)}(\nu)}{(t-\nu)^{\hbar+1-q}}d\nu, & q-1<\hbar<q, \\ \frac{d^q}{dt^q}\mathfrak{I}(t), & \hbar = q. \end{cases}$$

Remark 2.2. In order to take vagueness into consideration, fuzzy theory is incorporated in the GRNs and we found a novel model, it is necessary to add the fuzzy logic in both concentrations of mRNA and protein. The stability analysis of GRNs could be dealt with using fuzzy approaches (fuzzy AND and fuzzy OR), where more biological information can be incorporated. Recently, some results have been reported on the dynamical behaviors of FGRNs (see [33–35] and references therein). To our knowledge, the result of fuzzy approaches (fuzzy AND and fuzzy OR) based on FOGRNs has not been reported. Inspired by the works [33–35], in this study to investigate the both theoretical significance and biological meaning of fuzzy approaches (fuzzy AND and fuzzy OR) based on FOFGRNs. Therefore, our model (2) is more practical than the existing models [17–24].

GRNs with fuzzy dependent can be modeled by the following nonlinear dynamic fractional-order system:

$$\begin{cases} {}_0^C D_t^\hbar \phi_r^\varsigma(t) = & -a_r\phi_r^\varsigma(t) + \sum_{s=1}^q b_{rs} g_s(\psi_s^\varsigma(t-\tau(t))) \\ & + \bigwedge_{s=1}^q \mu_{rs} g_s(\psi_s^\varsigma(t-\tau(t))) \\ & + \bigvee_{s=1}^q \varphi_{rs} g_s(\psi_s^\varsigma(t-\tau(t))) + \mathcal{J}_r, \\ {}_0^C D_t^\hbar \psi_r^\varsigma(t) = & -c_r\psi_r^\varsigma(t) + d_r\phi_r^\varsigma(t-\sigma(t)) + \bigwedge_{s=1}^q \gamma_{rs}\phi_s^\varsigma(t-\sigma(t)) \\ & + \bigvee_{s=1}^q \rho_{rs}\phi_s^\varsigma(t-\sigma(t)) + \widehat{\mathcal{J}}_r, \end{cases} \tag{2}$$

where $\phi_r^\varsigma(t)$ and $\psi_r^\varsigma(t)$ are the concentrations of mRNAs and protein at time t; the parameters a_r and c_r are the decay rates of mRNA and protein, respectively; $d_r > 0$ is the translation rate; the protein on the transcription is defined by the nonlinear function $g_r(\cdot)$; $\tau(t)$, and $\sigma(t)$ are transcriptional, and translational delays; $\bigwedge$ and $\bigvee$ denotes the fuzzy AND and fuzzy OR operations, respectively; μ_{rs} γ_{rs}, and φ_{rs} ρ_{rs} are the elements of fuzzy feedback MIN template and fuzzy feedback MAX template, respectively. The matrix $B = (b_{rs}) \in \mathcal{R}^{q\times q}$ is the coupling matrix of the gene network, which is defined as follows:

$$b_{rs} = \begin{cases} \ell_{rs}, & \text{if transcription factor } s \text{ is an activator of gene } r, \\ -\ell_{rs}, & \text{if transcription factor } s \text{ is a repressor of gene } r, \\ 0, & \text{if there is no link from node } s \text{ to } r. \end{cases}$$

The initial value of system (2) are given in the form

$$\begin{cases} \phi_r^\varsigma(t) = \hat{\alpha}_\kappa(t), & t \in [-\hat{\rho}, 0], \\ \psi_r^\varsigma(t) = \hat{\beta}_\kappa(t), & t \in [-\hat{\rho}, 0], \end{cases}$$

where $\hat{\rho} = \max\{\hat{\alpha}_\kappa, \hat{\beta}_\kappa\}$, $\hat{\alpha}_\kappa(t)$ and $\hat{\beta}_\kappa(t)$ is bounded and continuous function on $[-\hat{\rho}, 0]$.

The nonlinear dynamics of the response system is described by

$$\begin{cases} {}_0^C D_t^\hbar \phi_r^\kappa(t) = & -a_r\phi_r^\kappa(t) + \sum_{s=1}^q b_{rs} g_s(\psi_s^\kappa(t-\tau(t))) \\ & + \bigwedge_{s=1}^q \mu_{rs} g_s(\psi_s^\kappa(t-\tau(t))) \\ & + \bigvee_{s=1}^q \varphi_{rs} g_s(\psi_s^\kappa(t-\tau(t))) + \mathcal{J}_r + u_r^\iota(t), \\ {}_0^C D_t^\hbar \psi_r^\kappa(t) = & -c_r\psi_r^\kappa(t) + d_r\phi_r^\kappa(t-\sigma(t)) + \bigwedge_{s=1}^q \gamma_{rs}\phi_s^\kappa(t-\sigma(t)) \\ & + \bigvee_{s=1}^q \rho_{rs}\phi_s^\kappa(t-\sigma(t)) + \widehat{\mathcal{J}}_r + \hat{u}_r^\jmath(t). \end{cases} \tag{3}$$

The initial value of system (3) are given in the form

$$\begin{cases} \phi_r^\kappa(t) = \hat{\alpha}_\varsigma(t), & t \in [-\hat{\rho}, 0], \\ \psi_r^\kappa(t) = \hat{\beta}_\varsigma(t), & t \in [-\hat{\rho}, 0], \end{cases}$$

where $\hat{\alpha}_\varsigma(t)$ and $\hat{\beta}_\varsigma(t)$ is bounded and continuous function on $[-\hat{\rho}, 0]$.

Before proceeding further, we need the following assumptions:

Assumption 1. The nonlinear functions $g_r(\cdot)$ are continuous, $g_r(0) = 0$ $(r = 1, 2, \dots, q)$ and satisfy the following condition

$$\beta_r^- \leq \frac{g_r(x) - g_r(y)}{x - y} \leq \beta_r^+, \quad r = 1, 2, \dots, q,$$

where $x, y \in \mathcal{R}$, $x \neq y$, and β_r^+, β_r^- are known real scalars.

Definition 2.3 ([16]). If any solutions $\phi^\varsigma(t)$ of (2) and $\psi^\kappa(t)$ of (3) satisfy the condition

$$\lim_{t\to+\infty} \|\phi^\varsigma(t) - \psi^\kappa(t)\| = 0$$

then drive-response systems (2) and (3) are said to be global asymptotic synchronization.

Definition 2.4 ([23]). Drive–response systems (2) and (3) are said to be finite-time synchronized if, for a suitable designed feedback controllers $u^\iota(t)$ and $\hat{u}^\jmath(t)$, there exists a constant $T > 0$ such that

$$\lim_{t\to T}\big(\|\varepsilon(t)\|_2 + \|\delta(t)\|_2\big) = 0, \quad \|\varepsilon(t)\|_2 + \|\delta(t)\|_2 \equiv 0, \quad \forall t \geq T,$$

where $\varepsilon(t)$ and $\delta(t)$ are the solutions of drive–response systems (2) and (3) with initial conditions.

Lemma 2.5 ([32]). *If two states x_s, y_s of the FOFGRNs (2), then we have*

$$|\bigwedge_{s=1}^n \mu_{rs} g_s(y_s) - \bigwedge_{s=1}^n \mu_{rs} g_s(x_s)| \leq \sum_{s=1}^n |\mu_{rs}|\ |g(y_s) - g(x_s)|,$$

$$|\bigvee_{s=1}^n \varphi_{rs} g_s(y_s) - \bigvee_{s=1}^n \varphi_{rs} g_s(x_s)| \leq \sum_{s=1}^n |\varphi_{rs}|\ |g(y_s) - g(x_s)|.$$

Lemma 2.6 ([46]). *Let $\tilde{\omega}(t)$ be a vector-valued differentiable function, $\mathcal{M}$ is a positive definite matrix. Then, the following inequality holds for every $t > 0$ and $0 < \hbar \leq 1$,*

$${}_t^C D_0^\hbar\ \tilde{\omega}^T(t)\mathcal{M}\tilde{\omega}(t) \leq 2\tilde{\omega}^T(t)\mathcal{M}\ {}_t^C D_0^\hbar\ \tilde{\omega}(t).$$

Lemma 2.7 ([23]). *Let $\mho = [0, y]$ be an interval on the real axis $\mathcal{R}$, $q = [\hbar] + 1$ for $\hbar \neq \mathcal{N}_+$ on $q = \hbar$ for $\hbar \in \mathcal{N}_+$, if $\hat{\chi} \in C^q[x, y]$, then*

$${}_t^{RL} I_0^\hbar\ {}_t^C D_0^\hbar\ (\hat{\chi}(t)) = \hat{\chi}(t) - \sum_{\ell=0}^{q-1} \frac{\hat{\chi}^{(\ell)}}{\ell\,!} t^\hbar, \quad q - 1 < \hbar \leq q.$$

In particular, when $0 < \hbar \leq 1$ and $\hat{\chi}(t) \in C^1[0, y]$, then

$${}_t^{RL} I_0^\hbar\ {}_t^C D_0^\hbar(\hat{\chi}(t)) = \hat{\chi}(t) - \hat{\chi}(0).$$

3. Main results

In this section, we derive a sufficient condition for asymptotic and finite-time adaptive synchronization of a class of FOFGRNs drive–response systems (2) and (3).

3.1. Asymptotic adaptive synchronization

The sufficient conditions of asymptotic synchronization of the drive-response FOFGRNs will be given. In order to obtain these synchronization criteria, the synchronization controllers $u_r^\iota(t)$ and $\hat{u}_r^\jmath(t)$ is designed as follows:

$$\begin{cases} u_r^\iota(t) = -\theta_r^\iota \varepsilon_r(t) - \xi_r^\iota \operatorname{sign}(\varepsilon_r(t)), \\ \hat{u}_r^\jmath(t) = -\eta_r^\jmath \delta_r(t) - \zeta_r^\jmath \operatorname{sign}(\delta_r(t)), \end{cases} \tag{4}$$

where $\operatorname{sign}(\cdot)$ is the symbolic function.

Adaptive update law

$$\begin{cases} {}_0^C D_t^\hbar \theta_r^\iota(t) = \omega_r \varepsilon_r^2(t), & {}_0^C D_t^\hbar \xi_r^\iota(t) = -\varpi_r|\varepsilon_r(t)|, \\ {}_0^C D_t^\hbar \eta_r^\jmath(t) = \vartheta_r \delta_r^2(t), & {}_0^C D_t^\hbar \zeta_r^\jmath(t) = -\varrho_r|\delta_r(t)|. \end{cases} \tag{5}$$

In the following, we deal with asymptotic synchronization for a FOFGRNs. Therefore, the error dynamics system of the drive-response systems (2) and (3) can be expressed by the following form:

$$\begin{cases} {}_0^C D_t^\hbar \varepsilon_r(t) = & -a_r\varepsilon_r(t) + \sum_{s=1}^n b_{rs}\wp_s(\delta_s(t-\tau(t))) \\ & + \bigwedge_{s=1}^n \mu_{rs}\wp_s(\delta_s(y-\tau(t))) \\ & + \bigvee_{s=1}^n \varphi_{rs}\wp_s(\delta_s(t-\tau(t))) + u_r^\iota(t), \\ {}_0^C D_t^\hbar \delta_r(t) = & -c_r\delta_r(t) + d_r\varepsilon_s(t-\sigma(t)) + \bigwedge_{s=1}^n \gamma_{rs}\varepsilon_s(t-\sigma(t)) \\ & + \bigvee_{s=1}^n \rho_{rs}\varepsilon_s(t-\sigma(t)) + \hat{u}_r^\jmath(t), \end{cases} \tag{6}$$

where $\varepsilon(t) = \phi_r^\varsigma(t) - \phi_r^\kappa(t)$, $\delta(t) = \psi_r^\varsigma(t) - \psi_r^\kappa(t)$, $\wp_s(\delta_s(t-\tau(t))) = g_s(\psi_s^\varsigma(t-\tau(t))) - g_s(\psi_s^\kappa(t-\tau(t)))$.

The initial condition of (6) is assumed to be

$$\begin{cases} \varepsilon_r(t) = \hat{\alpha}_{\kappa\varsigma}(t), & t \in [-\hat{\rho}, 0], \\ \delta_r(t) = \hat{\beta}_{\kappa\varsigma}(t), & t \in [-\hat{\rho}, 0], \end{cases}$$

where $\hat{\alpha}_{\kappa\varsigma}(t) = \hat{\alpha}_\varsigma(t) - \hat{\alpha}_\kappa$, $\hat{\beta}_{\kappa\varsigma}(t) = \hat{\beta}_\varsigma(t) - \hat{\beta}_\kappa$, $\hat{\alpha}_{\kappa\varsigma}(t)$ and $\hat{\beta}_{\kappa\varsigma}(t)$ is bounded and continuous function on $[-\hat{\rho}, 0]$.

Theorem 3.1. *Suppose that Assumption 1 hold, for θ_r^ι, ξ_r^ι, $\eta_r^\jmath$, $\zeta_r^\jmath$, ω_r, ϖ_r, ϑ_r, ϱ_r, β_1 and β_2 be scalars, the error system (6) is globally asymptotical stable under the adaptive controller (5), if there exist a positive diagonal matrices Ψ, Φ and positive definite matrices $\mathcal{L}$, $\mathcal{H}$, such that the following inequalities are feasible for Eq. (7) (see Box I)*
where $\mho_1 = -\Phi A - A^T\Phi - \aleph - \aleph^T + \mathcal{L}$, $\mho_2 = -\Psi C - C^T\Psi - \widehat{\aleph} - \widehat{\aleph}^T + \mathcal{H}$, $\Phi = \operatorname{diag}\{\hat{\phi}_1, \hat{\phi}_2, \dots, \hat{\phi}_q\}$, $\Psi = \operatorname{diag}\{\hat{\psi}_1, \hat{\psi}_2, \dots, \hat{\psi}_q\}$, $\aleph = \operatorname{diag}\{\aleph_1, \aleph_2, \dots, \aleph_q\}$, *and* $\widehat{\aleph} = \operatorname{diag}\{\widehat{\aleph}_1, \widehat{\aleph}_2, \dots, \widehat{\aleph}_q\}$.

$$\Theta=\begin{bmatrix} \mho_1 & 0 & 0 & 0 & \Phi B & 0 & 0 & \Phi(|\mu|+|\varphi|) & \Psi(|\gamma|+|\rho|)\\ * & \mho_2 & \Psi D & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & \beta_2 I & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \beta_1 I-\Lambda_2\Omega_1 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\Omega_1 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\mathcal{L} & 0 & 0 & 0\\ * & * & * & * & * & * & -\mathcal{H} & 0 & 0\\ * & * & * & * & * & * & * & -\beta_1 n^{-1} I & 0\\ * & * & * & * & * & * & * & * & -\beta_2 n^{-1} I \end{bmatrix}<0, \tag{7}$$

Box I.

Proof. The Lyapunov function candidate is chosen as the following:

$$V(t)=V_x(t)+V_y(t). \tag{8}$$

Consider the first candidate Lyapunov function:

$$V_x(t)=\varepsilon^T(t)\Phi\varepsilon(t)+\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}\hat{\phi}_r(\theta_r^{\iota}-\aleph_r)^2+\frac{1}{\varpi_r}\hat{\phi}_r(\xi_r^{\iota})^2\Big]$$
$$+{}_0^C D_t^{1-\hbar}\Big(\int_{t-\sigma}^{t}\varepsilon^T(y)\mathcal{L}\varepsilon^T(y)dy\Big). \tag{9}$$

Using Lemma 2.7 and calculating the fractional derivative of $V_x(t)$ along the solution of (6), one obtains that

$$\begin{aligned}
{}_0^C D_t^{\hbar}V_x(t)&={}_0^C D_t^{\hbar}\big(\varepsilon^T(t)\Phi\varepsilon(t)\big)+{}_0^C D_t^{\hbar}\Big(\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}\hat{\phi}_r(\theta_r^{\iota}-\aleph_r)^2+\frac{1}{\varpi_r}\hat{\phi}_r(\xi_r^{\iota})^2\Big]\Big)\\
&\quad+{}_0^C D_t^{\hbar}\Big({}_0^C D_t^{1-\hbar}\int_{t-\sigma}^{t}\varepsilon^T(y)\mathcal{L}\varepsilon^T(y)dy\Big)\\
&\le 2\sum_{r=1}^{q}\varepsilon_r^T(t)\ \hat{\phi}_r\ {}_0^C D_t^{\hbar}\varepsilon_r(t)\\
&\quad+\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}\ \hat{\phi}_r\ {}_0^C D_t^{\hbar}(\theta_r^{\iota}-\aleph_r)^2+\frac{1}{\varpi_r}\hat{\phi}_r\ {}_0^C D_t^{\hbar}(\xi_r^{\iota})^2\Big]\\
&\quad+{}_0^C D_t\Big(\int_{t-\sigma}^{t}\varepsilon^T(y)\mathcal{L}\varepsilon^T(y)dy\Big)\\
&\le 2\sum_{r=1}^{q}\varepsilon_r^T(t)\ \hat{\phi}_r\ \Big[-a_r\varepsilon_r(t)+\sum_{s=1}^{q}b_{rs}\wp_s(\delta_s(t-\tau(t)))\\
&\quad+\bigwedge_{s=1}^{q}\mu_{rs}\wp_s(\delta_s(y-\tau(t)))\\
&\quad+\bigvee_{s=1}^{q}\varphi_{rs}\wp_s(\delta_s(t-\tau(t)))+u_r^{\iota}(t)\ \Big]\\
&\quad+\sum_{r=1}^{q}\Big[2\ \hat{\phi}_r\ (\theta_r^{\iota}-\aleph_r)\varepsilon_r^2(t)+2\hat{\phi}_r\ \xi_r^{\iota}\varepsilon_r(t)\mathrm{sign}(\varepsilon_r(t))\Big]\\
&\quad+\varepsilon^T(t)\mathcal{L}\varepsilon(t)-\varepsilon^T(t-\sigma)\mathcal{L}\varepsilon(t-\sigma).
\end{aligned}$$

Then

$$\begin{aligned}
{}_0^C D_t^{\hbar}V_x(t)&\le 2\varepsilon^T(t)\Phi\Big[-A\varepsilon(t)+B\wp(\delta(t-\tau(t)))\Big]\\
&\quad+\beta_1^{-1}n\varepsilon^T(t)\Phi(|\mu|+|\varphi|)(|\mu|+|\varphi|)^T\Phi\varepsilon(t)\\
&\quad+\beta_1\wp^T(\delta(t-\sigma(t)))\wp(\delta(t-\sigma(t)))\\
&\quad+2\sum_{r=1}^{q}\varepsilon_r(t)\hat{\phi}_r\big(-\theta_r^{\iota}\varepsilon_r(t)-\xi_r^{\iota}\mathrm{sign}(\varepsilon_r(t))\big)\\
&\quad+\sum_{r=1}^{q}\Big[2\ \hat{\phi}_r\ (\theta_r^{\iota}-\aleph_r)\varepsilon_r^2(t)+2\hat{\phi}_r\ \xi_r^{\iota}\varepsilon_r(t)\mathrm{sign}(\varepsilon_r(t))\Big]\\
&\quad+\varepsilon^T(t)\mathcal{L}\varepsilon(t)-\varepsilon^T(t-\sigma)\mathcal{L}\varepsilon(t-\sigma)\\
&=-2\varepsilon^T(t)\Phi A\varepsilon(t)+2\varepsilon^T(t)\Phi B\wp(\delta(t-\tau(t)))\\
&\quad+\beta_1^{-1}n\varepsilon^T(t)\Phi(|\mu|+|\varphi|)(|\mu|+|\varphi|)^T\Phi\varepsilon(t)\\
&\quad+\beta_1\wp^T(\delta(t-\sigma(t)))\wp(\delta(t-\sigma(t)))\\
&\quad-\sum_{r=1}^{q}2\ \hat{\phi}_r\aleph_r\varepsilon_r^2(t)+\varepsilon^T(t)\mathcal{L}\varepsilon(t)-\varepsilon^T(t-\sigma)\mathcal{L}\varepsilon(t-\sigma).
\end{aligned}\tag{10}$$

Consider the second candidate Lyapunov function

$$V_y(t)=\delta^T(t)\Psi\delta(t)+\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\hat{\psi}_r(\eta_r^{J}-\widehat{\aleph}_r)^2+\frac{1}{\varrho_r}\hat{\psi}_r(\zeta_r^{J})^2\Big]$$
$$+{}_0^C D_t^{1-\hbar}\Big(\int_{t-\tau}^{t}\delta^T(y)\mathcal{H}\delta^T(y)dy\Big). \tag{11}$$

Taking the time fractional derivative along (6), we have

$$\begin{aligned}
{}_0^C D_t^{\hbar}V_y(t)&={}_0^C D_t^{\hbar}\big(\delta^T(t)\Psi\delta(t)\big)+{}_0^C D_t^{\hbar}\Big(\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\hat{\psi}_r(\eta_r^{J}-\widehat{\aleph}_r)^2+\frac{1}{\varrho_r}\hat{\psi}_r(\zeta_r^{J})^2\Big]\Big)\\
&\quad+{}_0^C D_t^{\hbar}\Big({}_0^C D_t^{1-\hbar}\int_{t-\sigma}^{t}\delta^T(y)\mathcal{H}\delta^T(y)dy\Big)\\
&\le 2\sum_{r=1}^{q}\delta_r^T(t)\ \hat{\psi}_r\ {}_0^C D_t^{\hbar}\delta_r(t)\\
&\quad+\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\ \hat{\psi}_r\ {}_0^C D_t^{\hbar}(\eta_r^{J}-\widehat{\aleph}_r)^2+\frac{1}{\varrho_r}\hat{p}_r\ {}_0^C D_t^{\hbar}(\zeta_r^{J})^2\Big]\\
&\quad+{}_0^C D_t\Big(\int_{t-\sigma}^{t}\delta^T(y)\mathcal{H}\delta^T(y)dy\Big)\\
&\le 2\sum_{r=1}^{q}\delta_r^T(t)\ \hat{\psi}_r\ \Big[-c_r(t)\delta_r(t)+d_r\varepsilon_r(t-\sigma(t))+\bigwedge_{s=1}^{q}\gamma_{rs}\varepsilon_r(t-\sigma(t))\\
&\quad+\bigvee_{s=1}^{q}\rho_{rs}\varepsilon_r(t-\sigma(t))+\hat{u}_r^{J}(t)\ \Big]\\
&\quad+\sum_{r=1}^{q}\Big[2\ \hat{\psi}_r\ (\eta_r^{J}-\widehat{\aleph}_r)\delta_r^2(t)+2\hat{p}_r\ \zeta_r^{J}\delta_r(t)\mathrm{sign}(\delta_r(t))\Big]\\
&\quad+\delta^T(t)\mathcal{H}\delta(t)-\delta^T(t-\sigma)\mathcal{H}\delta(t-\sigma)\\
&\le 2\delta^T(t)\Psi\Big[-C\delta(t)+D\varepsilon(t-\sigma(t))\Big]\\
&\quad+\beta_2^{-1}n\delta^T(t)\Psi(|\gamma|+|\rho|)(|\gamma|+|\rho|)^T\Psi\delta(t)\\
&\quad+\beta_2\varepsilon^T(t-\sigma(t))\varepsilon(t-\sigma(t))\\
&\quad+2\sum_{r=1}^{q}\delta_r(t)\hat{\psi}_r\big(-\eta_r^{J}\delta_r(t)-\zeta_r^{J}\mathrm{sign}(\delta_r(t))\big)\\
&\quad+\sum_{r=1}^{q}\Big[2\ \hat{\psi}_r\ (\eta_r^{J}-\widehat{\aleph}_r)\delta_r^2(t)+2\hat{p}_r\ \zeta_r^{J}\delta_r(t)\mathrm{sign}(\delta_r(t))\Big]\\
&\quad+\delta^T(t)\mathcal{H}\delta(t)-\delta^T(t-\sigma)\mathcal{H}\delta(t-\sigma)\\
&=-2\delta^T(t)\Psi C\delta(t)+2\delta^T(t)\Psi D\varepsilon(t-\sigma(t))\\
&\quad+\beta_2^{-1}q\delta^T(t)\Psi(|\gamma|+|\rho|)(|\gamma|+|\rho|)^T\Psi\delta(t)\\
&\quad+\beta_2\varepsilon^T(t-\sigma(t))\varepsilon(t-\sigma(t))\\
&\quad-\sum_{r=1}^{q}2\ \hat{\psi}_r\widehat{\aleph}_r\delta_r^2(t)+\delta^T(t)\mathcal{H}\delta(t)-\delta^T(t-\sigma)\mathcal{H}\delta(t-\sigma).
\end{aligned}\tag{12}$$

Taking the fractional derivative of $V(t)$ in (8), then (10) and (12), it follows that

$$\begin{aligned}{}^{C}_{0}D^{\hbar}_{t}V(t) \leq & -2\varepsilon^{T}(t)\Phi A\varepsilon(t)+2\varepsilon^{T}(t)\Phi B\wp(\delta(t-\tau(t)))\\
&+\beta_1^{-1}q\varepsilon^{T}(t)\Phi(|\mu|+|\varphi|)(|\mu|+|\varphi|)^{T}\Phi\varepsilon(t)\\
&+\beta_1\wp^{T}(\delta(t-\sigma(t)))\wp(\delta(t-\sigma(t)))\\
&-\sum_{r=1}^{q}2\,\hat{\phi}_r\aleph_r\varepsilon_r^2(t)+\varepsilon^{T}(t)\mathcal{L}\varepsilon(t)-\varepsilon^{T}(t-\sigma)\mathcal{L}\varepsilon(t-\sigma)\\
&-2\delta^{T}(t)\Psi C\delta(t)+2\delta^{T}(t)\Psi D\varepsilon(t-\sigma(t))\\
&+\beta_2^{-1}q\delta^{T}(t)\Psi(|\gamma|+|\rho|)(|\gamma|+|\rho|)^{T}\Psi\delta(t)\\
&+\beta_2\varepsilon^{T}(t-\sigma(t))\varepsilon(t-\sigma(t))\\
&-\sum_{r=1}^{q}2\,\hat{\psi}_r\hat{\aleph}_r\delta_r^2(t)+\delta^{T}(t)\mathcal{H}\delta(t)-\delta^{T}(t-\sigma)\mathcal{H}\delta(t-\sigma).\end{aligned} \tag{13}$$

From Assumption 1, one can easily get

$$\left[\wp_r(\delta_r(t-\tau(t)))-\Lambda_r^{+}\delta_r(t-\tau(t))\right]\left[\wp_r(\delta_r(t-\tau(t)))-\Lambda_r^{-}\delta_r(t-\tau(t))\right]>0.$$

Thus, for any diagonal matrices $\Omega = \mathrm{diag}\{\chi_{11},\chi_{12},\dots,\chi_{1n}\} > 0$, it follows that

$$\sum_{r=1}^{n}\chi_{1r}\begin{bmatrix}\delta(t-\tau(t))\\ \wp(\delta(t-\tau(t)))\end{bmatrix}^{T}\begin{bmatrix}\Lambda_r^{+}\Lambda_r^{-}e_re_r^{T} & -\frac{\Lambda_r^{+}+\Lambda_r^{-}}{2}e_re_r^{T}\\ -\frac{\Lambda_r^{+}+\Lambda_r^{-}}{2}e_re_r^{T} & e_re_r^{T}\end{bmatrix}\times\begin{bmatrix}\delta(t-\tau(t))\\ \wp(\delta(t-\tau(t)))\end{bmatrix}\leq 0,$$

where e_r represents the unit column vector having "1" element on its rth row and zeros elsewhere.

On the other hand, one can write the inequality

$$\begin{bmatrix}\delta(t-\tau(t))\\ \wp(\delta(t-\tau(t)))\end{bmatrix}^{T}\begin{bmatrix}\hat{\Lambda}_1\Omega_1 & -\hat{\Lambda}_2\Omega_1\\ * & \Omega_1\end{bmatrix}\begin{bmatrix}\delta(t-\tau(t))\\ \wp(\delta(t-\tau(t)))\end{bmatrix}\leq 0. \tag{14}$$

Combining (13) and (14), the following inequality can be obtained:

$${}^{C}_{0}D^{\hbar}_{t}V(t)\leq \mathfrak{J}^{T}(t)\Theta\mathfrak{J}(t). \tag{15}$$

Here, $\mathfrak{J}^{T}(t)=\left[\varepsilon^{T}(t)\ \ \delta^{T}(t)\ \ \varepsilon^{T}(t-\sigma(t))\ \ \delta^{T}(t-\sigma(t))\ \ \wp^{T}(\delta(t-\tau(t)))\ \ \varepsilon(t-\sigma)\ \ \varepsilon(t-\tau)\right]$.

It follows from (7) that $\Theta<0$. Then according to Lemma 1 in [46] one has $\lim_{t\to\infty}V(t)=0$, which implies that the FOFGRNs (6) achieve adaptive synchronization asymptotically. It completes the proof.

3.2. Finite-time synchronization

In this section, we present finite-time synchronization sufficient conditions for drive system (2) and response system (3) under controllers $u_r^{\imath}(t)$ and $\hat{u}_r^{\jmath}(t)$ is designed as follow:

$$\begin{cases} u_r^{\imath}(t)=-\theta_r^{\imath}\varepsilon_r(t)-\xi_r^{\imath}\mathrm{sign}(\varepsilon_r(t))-\frac{\hat{\alpha}_r^{\imath}}{2}\frac{\mathrm{sign}(\varepsilon_r(t))}{\|\varepsilon_r(t)\|_1},\\ \hat{u}_r^{\jmath}(t)=-\eta_r^{\jmath}\delta_r(t)-\zeta_r^{\jmath}\mathrm{sign}(\delta_r(t))-\frac{\hat{\gamma}_r^{\imath}}{2}\frac{\mathrm{sign}(\delta_r(t))}{\|\delta_r(t)\|_1},\end{cases} \tag{16}$$

where $\mathrm{sign}(\cdot)$ is the symbolic function.

Adaptive update law

$$\begin{cases} {}^{C}_{0}D^{\hbar}_{t}\theta_r^{\imath}(t)=\omega_r\varepsilon_r^2(t),\ {}^{C}_{0}D^{\hbar}_{t}\xi_r^{\imath}(t)=-\varpi_r|\varepsilon_r(t)|,\\ {}^{C}_{0}D^{\hbar}_{t}\eta_r^{\jmath}(t)=\vartheta_r\delta_r^2(t),\ {}^{C}_{0}D^{\hbar}_{t}\zeta_r^{\jmath}(t)=-\varrho_r|\delta_r(t)|.\end{cases} \tag{17}$$

Theorem 3.2. *Suppose that Assumption 1 hold, for $\theta_r^{\imath}$, $\xi_r^{\imath}$, $\eta_r^{\jmath}$, $\zeta_r^{\jmath}$, ω_r, ϖ_r, ϑ_r, ϱ_r, β_1 and β_2 be scalars, the error system (6) is finite-time stable under the adaptive controller (17), if there exist a positive diagonal matrices Ψ and Φ, such that the following inequalities are feasible for Eq. (18) (see Box II) where $\mho_1=-\Phi A-A^{T}\Phi-\aleph-\aleph^{T}+\mathcal{L}$, $\mho_2=-\Psi C-C^{T}\Psi-\hat{\aleph}-\hat{\aleph}^{T}+\mathcal{H}$, $\Phi=\mathrm{diag}\{\hat{\phi}_1,\hat{\phi}_2,\dots,\hat{\phi}_n\}$, and $\Psi=\mathrm{diag}\{\hat{\psi}_1,\hat{\psi}_2,\dots,\hat{\psi}_n\}$.*

Proof. The Lyapunov function candidate is chosen as the following:

$$V(t)=V_z(t)+V_\pi(t), \tag{19}$$

where

$$\begin{aligned}V_z(t)=&\varepsilon^{T}(t)\Phi\varepsilon(t)+\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}\hat{\phi}_r(\theta_r^{\imath}-\aleph_r)^2+\frac{1}{\varpi_r}p_r(\xi_r^{\imath})^2\Big],\\
V_\pi(t)=&\delta^{T}(t)\Psi\delta(t)+\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\hat{\psi}_r(\eta_r^{\jmath}-\aleph_r)^2+\frac{1}{\varrho_r}\tilde{p}_r(\zeta_r^{\jmath})^2\Big].\end{aligned}$$

By the similar techniques used in Theorem 3.1, we obtain that

$$\begin{aligned}{}^{C}_{0}D^{\hbar}_{t}V(t)\leq&-2\varepsilon^{T}(t)\Phi A\varepsilon(t)+2\varepsilon^{T}(t)\Phi B\wp(\delta(t-\tau(t)))\\
&+\beta_1^{-1}n\varepsilon^{T}(t)\Phi(|\mu|+|\varphi|)(|\mu|+|\varphi|)^{T}\Phi\varepsilon(t)\\
&+\beta_1\wp^{T}(\delta(t-\sigma(t)))\wp(\delta(t-\sigma(t)))-\Phi\hat{\alpha}\\
&-\sum_{r=1}^{q}2\,\hat{\phi}_r\aleph_r\varepsilon_r^2(t)-2\delta^{T}(t)\Psi C\delta(t)\\
&+2\delta^{T}(t)\Psi D\varepsilon(t-\sigma(t))+\beta_2^{-1}n\delta^{T}(t)\Psi(|\gamma|+|\rho|)(|\gamma|+|\rho|)^{T}\Psi\delta(t)\\
&+\beta_2\varepsilon^{T}(t-\sigma(t))\varepsilon(t-\sigma(t))-\sum_{r=1}^{q}2\,\hat{\phi}_r\hat{\aleph}_r\delta_r^2(t)-\Psi\hat{\gamma}\\
\leq&\mathcal{Z}^{T}\hat{\Lambda}\mathcal{Z}-\Phi\hat{\alpha}-\Psi\hat{\gamma},\end{aligned}$$

where $\mathcal{Z}^{T}(t)=\left[\varepsilon^{T}(t)\ \ \delta^{T}(t)\ \ \varepsilon^{T}(t-\sigma(t))\ \ \delta^{T}(t-\sigma(t))\ \ \wp^{T}(\delta(t-\tau(t)))\right]$.

It follows from (18) that $\hat{\Lambda}<0$, then

$${}^{C}_{0}D^{\hbar}_{t}V(t)\leq-\Phi\hat{\alpha}-\Psi\hat{\gamma}\leq-\Xi, \tag{20}$$

where $\Xi=\Upsilon\ (\hat{\alpha}+\hat{\gamma})$, and $\Upsilon=\max\{\lambda_{\max}(\Phi),\lambda_{\min}(\Psi)\}$.

Now, there exists a nonnegative function $\mathcal{F}(t)$ satisfying

$${}^{C}_{0}D^{\hbar}_{t}V(t)+\mathcal{F}(t)\leq-\Xi. \tag{21}$$

By using Lemma 2.7, take $\hbar$ order integral on both sides of (21) from 0 to t, then

$$\begin{aligned}V(t)=&V(0)-{}^{RL}_{t}I^{\hbar}_{0}\mathcal{F}(t)+{}^{RL}_{t}I^{\hbar}_{0}(-\Xi)\\
=&V(0)-\frac{1}{\Gamma(\hbar)}\int_0^t(t-v)^{\hbar-1}\mathcal{F}(v)dv-\frac{\Xi}{\Gamma(\hbar)}\int_0^t(t-v)^{\hbar-1}dv.\end{aligned} \tag{22}$$

$\Gamma(\hbar)>0$, and $(t-v)^{\hbar-1}\mathcal{F}(v)\geq 0$ for $v\in[0,t]$, thus ${}^{RL}_{t}I^{\hbar}_{0}\mathcal{F}(t)\geq 0$, moreover

$$\begin{aligned}V(t)\leq&V(0)-\frac{\Xi}{\Gamma(\hbar)}\int_0^t(t-v)^{\hbar-1}dv\\
=&V(0)-\frac{\Xi t^{\hbar}}{\Gamma(\hbar+1)}.\end{aligned}$$

Let $\hat{\Theta}(t)=V(0)-\frac{\Xi t^{\hbar}}{\Gamma(\hbar+1)}$, obviously $\hat{\Theta}(t)$ is a strictly decreasing function, thus $\hat{\Theta}(t)=0$ iff

$$\begin{aligned}T=&\Big[\frac{V(0)\Gamma(\hbar+1)}{\Xi}\Big]^{\frac{1}{\hbar}}\\
=&\Big[\frac{\big(\Upsilon\ \big(\ \varepsilon^2(0)+\delta^2(0)\big)+\hat{\Omega}_{\imath}+\hat{\Omega}_{\jmath}\big)\Gamma(\hbar+1)}{\Xi}\Big]^{\frac{1}{\hbar}},\end{aligned}$$

where

$$\begin{aligned}\hat{\Omega}_{\imath}=&\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}p_r(\theta_r^{\imath}(0)-\aleph_r)^2+\frac{1}{\varpi_r}p_r(\xi_r^{\imath}(0))^2\Big],\\
\hat{\Omega}_{\jmath}=&\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\tilde{p}_r(\eta_r^{\jmath}(0)-\aleph_r)^2+\frac{1}{\varrho_r}\tilde{p}_r(\zeta_r^{\jmath}(0))^2\Big],\end{aligned}$$

and $\hat{\Theta}(t)\leq 0$ for any $t\geq T$, thus $V(t)\leq\hat{\Theta}(t)\leq 0$ for any $t\geq T$, and $V(t)$ is a nonnegative functions, so $V(t)\equiv 0$ for any $t\geq T$, that is

$$\begin{aligned}&\sum_{r=1}^{q}\varepsilon_r^{T}(t)\Phi\varepsilon_r(t)+\sum_{r=1}^{q}\delta_r^{T}(t)\Psi\delta_r(t)+\sum_{r=1}^{q}\Big[\frac{1}{\omega_r}p_r(\theta_r^{\imath}-\aleph_r)^2+\frac{1}{\varpi_r}p_r(\xi_r^{\imath})^2\Big]\\
&\quad+\sum_{r=1}^{q}\Big[\frac{1}{\vartheta_r}\tilde{p}_r(\eta_r^{\jmath}-\aleph_r)^2+\frac{1}{\varrho_r}\tilde{p}_r(\zeta_r^{\jmath})^2\Big]\equiv 0.\end{aligned}$$

Thus

$$\sum_{r=1}^{q}\varepsilon_r^{T}(t)\Phi\varepsilon_r(t)+\sum_{r=1}^{q}\delta_r^{T}(t)\Psi\delta_r(t)\equiv 0,\quad\forall t\geq T. \tag{23}$$

$$\hat{\Lambda}=\begin{bmatrix} \mho_1 & 0 & 0 & 0 & \Phi B & \Phi(|\mu|+|\varphi|) & \Psi(|\gamma|+|\rho|) \\ * & \mho_2 & \Psi D & 0 & 0 & 0 & 0 \\ * & * & \beta_2 I & 0 & 0 & 0 & 0 \\ * & * & * & \beta_1 I-\Lambda_2\Omega_1 & 0 & 0 & 0 \\ * & * & * & * & -\Omega_1 & 0 & 0 \\ * & * & * & * & * & -\beta_1 n^{-1} I & 0 \\ * & * & * & * & * & * & -\beta_2 n^{-1} I \end{bmatrix}<0, \quad (18)$$

Box II.

By Definition 2.4, drive system (2) synchronizes with response system (3) in a finite-time under adaptive controller (17).

Remark 3.3. The adaptive finite-time synchronization results on FOFGRNs have not yet been seen, hence Theorem 3.2 is new. Compared with the results in [33–35], we deal with FOFGRNs and finite-time synchronization which is better than that of global asymptotical synchronization.

If there are not fuzzy feedback MAX template, fuzzy feedback MIN template and delayed feedback template in the system (2) and (3), i.e. $\mu_{rs}=\gamma_{rs}=\varphi_{rs}=\rho_{rs}=0$, then the corresponding drive–response system can be rewritten as follow

$$\begin{cases} {}_0^C D_t^{\hbar}\phi_r^{\varsigma}(t)=-a_r\phi_r^{\varsigma}(t)+\sum_{s=1}^n b_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t)))+\mathcal{J}_r \\ {}_0^C D_t^{\hbar}\psi_r^{\varsigma}(t)=-c_r\psi_r^{\varsigma}(t)+d_r\phi_r^{\varsigma}(t-\sigma(t))+\hat{\mathcal{J}}_r, \end{cases} \quad (24)$$

and

$$\begin{cases} {}_0^C D_t^{\hbar}\phi_r^{\kappa}(t)=-a_r\phi_r^{\kappa}(t)+\sum_{s=1}^n b_{rs}(t)g_s(\psi_s^{\kappa}(t-\tau(t)))+\mathcal{J}_r+u_r^{\iota}, \\ {}_0^C D_t^{\hbar}\psi_r^{\kappa}(t)=-c_r\psi_r^{\kappa}(t)+d_r\phi_r^{\kappa}(t-\sigma(t))+\hat{\mathcal{J}}_r+\hat{u}_r^{\jmath}. \end{cases} \quad (25)$$

The error system should be defined as follows:

$$\begin{cases} {}_0^C D_t^{\hbar}\varepsilon_r(t)= & -a_r\varepsilon_r(t)+\sum_{s=1}^n b_{rs}\wp_s(\delta_s(t-\tau(t)))+u_r^{\iota}(t), \\ {}_0^C D_t^{\hbar}\delta_r(t)= & -c_r\delta_r(t)+d_r\varepsilon_s(t-\sigma(t))+\hat{u}_r^{\jmath}(t). \end{cases} \quad (26)$$

Corollary 3.4. *Suppose that Assumption 1 hold, for θ_r^{ι}, ξ_r^{ι}, $\eta_r^{\jmath}$, $\zeta_r^{\jmath}$, ω_r, ϖ_r, ϑ_r, ϱ_r, β_1 and β_2 be scalars, the error system (26) is globally asymptotically stable, if there exist a positive diagonal matrices Ψ, Φ and positive definite matrices $\mathcal{L}$, $\mathcal{H}$ such that the following inequalities are feasible for*

$$\Theta=\begin{bmatrix} \mho_1 & 0 & 0 & 0 & \Phi B & 0 & 0 \\ * & \mho_2 & \Psi D & 0 & 0 & 0 & 0 \\ * & * & \beta_2 I & 0 & 0 & 0 & 0 \\ * & * & * & \beta_1 I-\Lambda_2\Omega_1 & 0 & 0 & 0 \\ * & * & * & * & -\Omega_1 & 0 & 0 \\ * & * & * & * & * & -\mathcal{L} & 0 \\ * & * & * & * & * & * & -\mathcal{H} \end{bmatrix}<0, \quad (27)$$

where $\mho_1=-\Phi A-A^T\Phi-\aleph-\aleph^T+\mathcal{L}$, $\mho_2=-\Psi C-C^T\Psi-\hat{\aleph}-\hat{\aleph}^T+\mathcal{H}$, $\Phi=\mathrm{diag}\{\hat{\phi}_1,\hat{\phi}_2,\dots,\hat{\phi}_n\}$, and $\Psi=\mathrm{diag}\{\hat{\psi}_1,\hat{\psi}_2,\dots,\hat{\psi}_n\}$.

Corollary 3.5. *Suppose that Assumption 1 hold, for θ_r^{ι}, ξ_r^{ι}, $\eta_r^{\jmath}$, $\zeta_r^{\jmath}$, ω_r, ϖ_r, ϑ_r, ϱ_r, β_1 and β_2 be scalars, the error system (26) is finite-time stable, if there exist a positive diagonal matrices Ψ, Φ such that the following inequalities are feasible for*

$$\hat{\Lambda}=\begin{bmatrix} \mho_1 & 0 & 0 & 0 & \Phi B \\ * & \mho_2 & \Psi D & 0 & 0 \\ * & * & \beta_2 I & 0 & 0 \\ * & * & * & \beta_1 I-\Lambda_2\Omega_1 & 0 \\ * & * & * & * & -\Omega_1 \end{bmatrix}<0, \quad (28)$$

where $\mho_1=-\Phi A-A^T\Phi-\aleph-\aleph^T+\mathcal{L}$, $\mho_2=-\Psi C-C^T\Psi-\hat{\aleph}-\hat{\aleph}^T+\mathcal{H}$, $\Phi=\mathrm{diag}\{\hat{\phi}_1,\hat{\phi}_2,\dots,\hat{\phi}_n\}$, and $\Psi=\mathrm{diag}\{\hat{\psi}_1,\hat{\psi}_2,\dots,\hat{\psi}_n\}$.

Remark 3.6. Compared with the previous results [17–24], the following are the key aspects and benefits of this paper:
(1) We construct the fuzzy logic theory (fuzzy AND and fuzzy OR) to analyze its dynamic characteristics. This makes our research results more practical significance.
(2) In this article, the controllers designed are simpler and more powerful than.
(3) In Theorems 3.1 and 3.2, sufficient conditions for FOFGRNs are given by constructing proper Lyapunov functions and using the LMI method to guarantee asymptotic and finite-time synchronization, respectively. We can see from the proof that these stability conditions are formulated without algebraic conditions, which may result in less conservative results.

4. Numerical simulation

In this section, an example is provided to demonstrate the theoretical results of the main results.

Example 1. The following nonlinear dynamic FOGRNs fuzzy model

$$\begin{cases} {}_0^C D_t^{\hbar}\phi_r^{\varsigma}(t)= & -a_r\phi_r^{\varsigma}(t)+\sum_{s=1}^3 b_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t))) \\ & +\bigwedge_{s=1}^3 \mu_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t))) \\ & +\bigvee_{s=1}^3 \varphi_{rs}g_s(\psi_s^{\varsigma}(t-\tau(t)))+\mathcal{J}_r, \quad r=1,2,3, \\ {}_0^C D_t^{\hbar}\psi_r^{\varsigma}(t)= & -c_r\psi_r^{\varsigma}(t)+d_r\phi_r^{\varsigma}(t-\sigma(t))+\bigwedge_{s=1}^3 \gamma_{rs}\phi_s^{\varsigma}(t-\sigma(t)) \\ & +\bigvee_{s=1}^3 \rho_{rs}\phi_s^{\varsigma}(t-\sigma(t))+\hat{\mathcal{J}}_r, \quad r=1,2,3, \end{cases} \quad (29)$$

where $\hbar=0.96$, the regulatory function are chosen as $g_s(\psi)=\frac{\psi^2}{1+\psi^2}$, $(s=1,2,3)$, which give $\Lambda_1^-=\Lambda_2^-=\Lambda_3^-=0$ and $\Lambda_1^+=\Lambda_2^+=\Lambda_3^+=0$, i.e., $\hat{\Lambda}_1=\mathrm{diag}\{0,0,0\}$, $\hat{\Lambda}_2=\mathrm{diag}\{0,0,0\}$, $\hat{\Lambda}_3=\mathrm{diag}\{1,1,1\}$, and $\tau(t)=2.0810\ |\sin(t)|$, $\sigma(t)=0.9320\ |\sin(t)|$.

The matrices for system (29) can be selected as

$$A=\begin{bmatrix} 2.3 & 0 & 0 \\ 0 & 2.7 & 0 \\ 0 & 0 & 3.1 \end{bmatrix}, B=\begin{bmatrix} 0 & 0 & -0.2769 \\ -0.2769 & 0 & 0 \\ -0.2769 & 0 & 0 \end{bmatrix},$$

$$C=\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix},$$

$$D=\begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}, \mu=\begin{bmatrix} 0.0210 & -0.0327 & 0.0491 \\ -0.0327 & 0.0491 & 0.0210 \\ 0.0210 & 0.0327 & 0.0491 \end{bmatrix},$$

$$\varphi=\begin{bmatrix} 0.0327 & 0.0501 & 0.0491 \\ -0.0491 & 0.0327 & 0.0501 \\ 0.0210 & 0.0491 & 0.0327 \end{bmatrix},$$

$$\gamma=\begin{bmatrix} -0.0327 & -0.6029 & 0.0210 \\ -0.0327 & 0.0501 & 0.0491 \\ -0.6029 & -0.0491 & 0.0501 \end{bmatrix},$$

$$\rho=\begin{bmatrix} 0.6029 & 0.0491 & 0.0210 \\ -0.0327 & -0.0057 & 0.0491 \\ -0.0210 & -0.0491 & -0.0057 \end{bmatrix}.$$

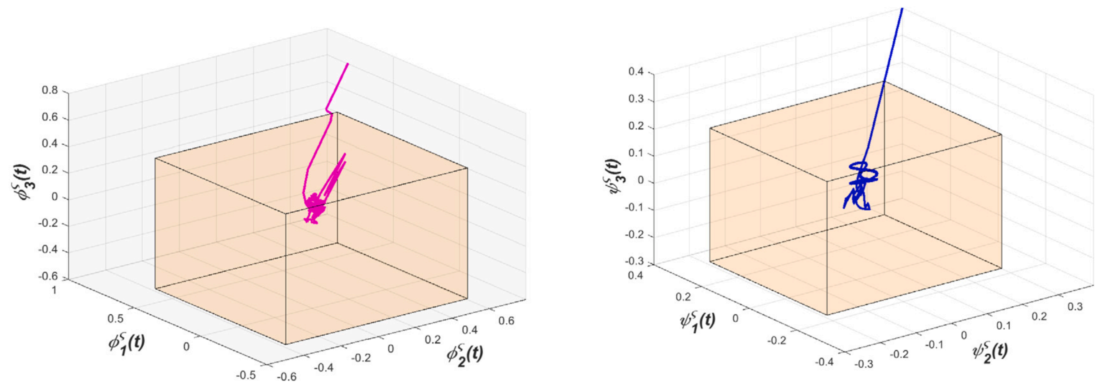

Fig. 1. Phase trajectories of system (29) with $\alpha = 0.97$.

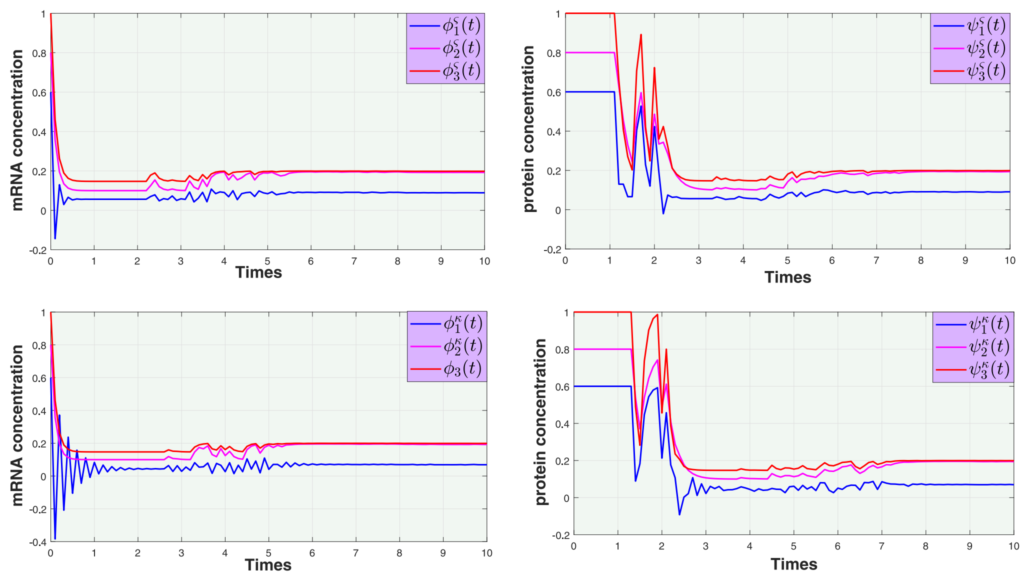

Fig. 2. The drive-response trajectories of the mRNA and protein concentrations without controller.

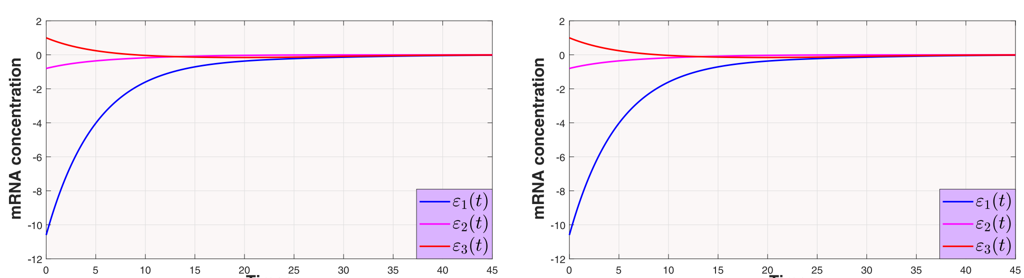

Fig. 3. The errors system of mRNA and protein concentrations with feedback controller (4).

Next, the controlled response system is depict

$$\begin{cases} {}_0^C D_t^{\hbar} \phi_r^{\kappa}(t) = -a_r \phi_r^{\kappa}(t) & + \sum_{s=1}^{3} b_{rs} g_s(\psi_s^{\kappa}(t-\tau(t))) \\ & + \bigwedge_{s=1}^{3} \mu_{rs} g_s(\psi_s^{\kappa}(t-\tau(t))) \\ & + \bigvee_{s=1}^{3} \varphi_{rs} g_s(\psi_s^{\kappa}(t-\tau(t))) + \mathcal{J}_r + u_r^{\iota}(t), \\ {}_0^C D_t^{\hbar} \psi_r^{\kappa}(t) = -c_r \psi_r^{\kappa}(t) & + d_r \phi_r^{\kappa}(t-\sigma(t)) + \bigwedge_{s=1}^{3} \gamma_{rs} \phi_s^{\kappa}(t-\sigma(t)) \\ & + \bigvee_{s=1}^{3} \rho_{rs} \phi_s^{\kappa}(t-\sigma(t)) + \widehat{\mathcal{J}}_r + \hat{u}_r^{\jmath}(t), \end{cases} \tag{30}$$

where the parameters same as in (29).

Case 1. Now, we controller parameters values selected are as follows $\theta_1^{\iota} = 2.1$, $\theta_2^{\iota} = 2.3$, $\theta_3^{\iota} = 2.8$, $\xi_1^{\iota} = 1.2$, $\xi_2^{\iota} = 2.9$, $\xi_3^{\iota} = 1.7$, $\eta_1^{\jmath} = 0.8$, $\eta_2^{\jmath} = 0.9$, $\eta_3^{\jmath} = 1.4$, $\zeta_1^{\jmath} = 3.4$, $\zeta_2^{\jmath} = 2.9$, $\zeta_3^{\jmath} = 3.4$, $\omega_1 = 1.7$, $\omega_2 = 2.3$, $\omega_3 = 1.7$, $\varpi_1 = 4.7$, $\varpi_2 = 3.9$, $\varpi_3 = 3.7$, $\vartheta_1 = 1.9$, $\vartheta_2 = 2.9$, $\vartheta_3 = 2.2$, $\varrho_1 = 0.3$, $\varrho_1 = 0.9$, $\varrho_3 = 0.9$, $\beta_1 = 1$, $\beta_2 = 1$, and $\beta_3 = 1.5$. We now apply Theorem 3.1 to verify the synchronization of this example, we solve (7) and the feasible solutions are given by

$$\Psi = \begin{bmatrix} 0.6442 & 0 & 0 \\ 0 & 0.6438 & 0 \\ 0 & 0 & 0.6439 \end{bmatrix}, \Phi = \begin{bmatrix} 0.7864 & 0 & 0 \\ 0 & 0.7835 & 0 \\ 0 & 0 & 0.7839 \end{bmatrix},$$

$$\mathcal{L} = \begin{bmatrix} 3.0179 & 0 & 0 \\ 0 & 3.0179 & 0 \\ 0 & 0 & 3,0179 \end{bmatrix},$$

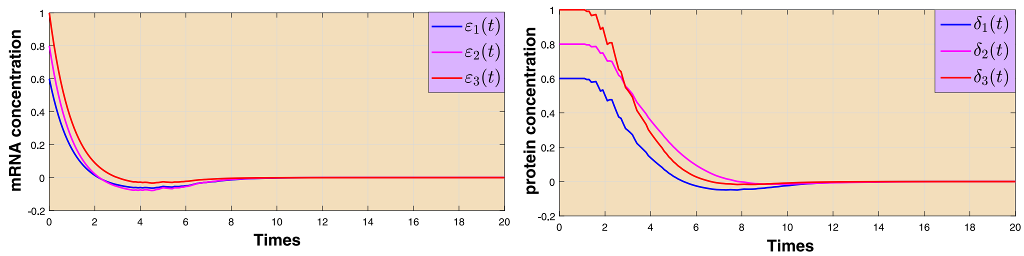

Fig. 4. The errors system of mRNA and protein concentrations with adaptive feedback controllers (4) and (5).

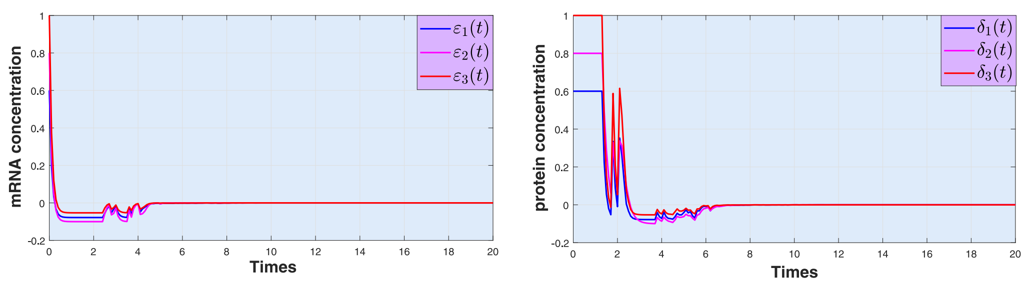

Fig. 5. The errors system of mRNA and protein concentrations with adaptive feedback controllers (16) and (17).

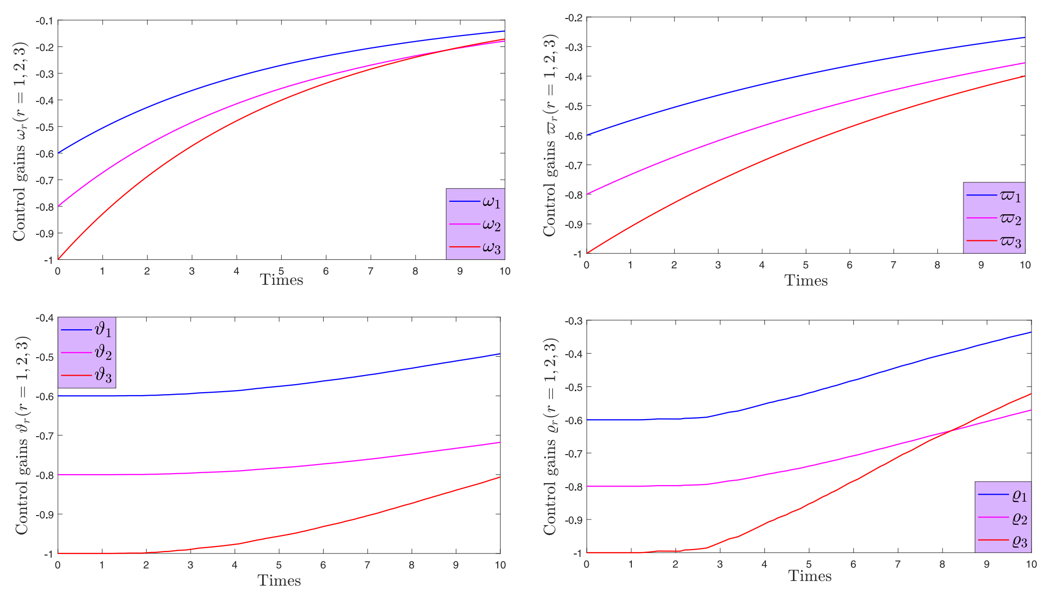

Fig. 6. The trajectory of the adaptive gains ω_r, ϖ_r, ϑ_r, and ϱ_r.

$$\mathcal{H}=\begin{bmatrix}3.7881 & 0 & 0\\ 0 & 3.7881 & 0\\ 0 & 0 & 3.7881\end{bmatrix}, \aleph=\begin{bmatrix}0.0043 & 0 & 0\\ 0 & 0.0043 & 0\\ 0 & 0 & 0.0043\end{bmatrix},$$
$$\hat{\aleph}=\begin{bmatrix}0.0079 & 0 & 0\\ 0 & 0.0079 & 0\\ 0 & 0 & 0.0079\end{bmatrix}.$$

Thus, according to Theorem 3.1, we can substantiate that (21) is asymptotic synchronization. Fig. 1 depicts the systems phase diagrams (29). Fig. 2 depicts the drive-response trajectories of the mRNA and protein concentrations without the controller. The errors of the drive-response of mRNA and protein concentrations with feedback controller (4) are depicts in Fig. 3. Under adaptive feedback controllers, the errors of the drive-response of mRNA and protein concentrations are shows in Fig. 4, which suggests that adaptive laws have a significant impact on the dynamic behaviors of the drive–response system (29) and (30).

Case 2. Now, we controller parameters values selected are as follows $\theta_1^{\imath}=1.8$, $\theta_2^{\imath}=1.9$, $\theta_3^{\imath}=1.7$, $\xi_1^{\imath}=2.4$, $\xi_2^{\imath}=1.2$, $\xi_3^{\imath}=3.9$, $\eta_1^{\jmath}=2.6$, $\eta_2^{\jmath}=1.1$, $\eta_3^{\jmath}=1.2$, $\zeta_1^{\jmath}=1.9$, $\zeta_2^{\jmath}=0.8$, $\zeta_3^{\jmath}=0.6$, $\omega_1=2.1$, $\omega_2=4.1$, $\omega_3=2.9$, $\varpi_1=2.1$, $\varpi_2=0.7$, $\varpi_3=4.1$, $\vartheta_1=1.7$, $\vartheta_2=1.8$, $\vartheta_3=2.6$, $\varrho_1=1.5$, $\varrho_1=3.7$, $\varrho_3=0.9$, $\beta_1=1$, $\beta_2=1$, and $\beta_3=1.5$. To verify the synchronization of this example, we use Theorem 3.2, solve (18), and the feasible solutions are given by

$$\Psi=\begin{bmatrix}0.0961 & 0 & 0\\ 0 & 0.3001 & 0\\ 0 & 0 & 0.0451\end{bmatrix}, \Phi=\begin{bmatrix}0.0481 & 0 & 0\\ 0 & 0.0391 & 0\\ 0 & 0 & 0.0990\end{bmatrix},$$
$$\aleph=\begin{bmatrix}0.0009 & 0 & 0\\ 0 & 0.0009 & 0\\ 0 & 0 & 0.0009\end{bmatrix}, \hat{\aleph}=\begin{bmatrix}0.0129 & 0 & 0\\ 0 & 0.0129 & 0\\ 0 & 0 & 0.0129\end{bmatrix},$$

therefore drive–response system (29) and (30) achieved in finite time under controllers (6). The setting time is calculated as $T\simeq 5.9037$.

Based on the adaptive controller designed, the finite-time synchronization system of mRNAs and protein concentrations and their error estimations are illustrated in Fig. 5. The adaptive gains of the transient behaviors are shown in Fig. 6. From Figs. 3 and 4, it is noted that the controller affects the time to achieve asymptotic synchronization, but it taken the convergence time at largely. However, according to Fig. 4, it suggests that the controller does influence of finite-time synchronization of error system and its reached the convergence time at quickly.

5. Conclusion

A new class of fuzzy modeling of FOGRNs is introduced in this paper. We used an adaptive feedback controller to investigate the synchronization problem for FOFGRNs with time-varying delays in this paper. To deal with the nonlinear dynamics of GRNs whose structure changes, the well-known fractional derivative and fuzzy model have been used as an effective strategy. Furthermore, suitable controllers were designed to ensure asymptotic and finite-time synchronization in terms of LMIs for proposed FOFGRNs. Finally, a numerical example are provided to demonstrate the effectiveness of our theoretical results.

CRediT authorship contribution statement

G. Narayanan: Conceptualization, Methodology, Software. **M. Syed Ali:** Supervision. **Rajagopal Karthikeyan:** Software, Validation. **Grienggrai Rajchakit:** Data curation, Writing – original draft. **Anuwat Jirawattanapanit:** Visualization, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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