

Research Article

A Sampling Load Frequency Control Scheme for Power Systems with Time Delays

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In this study, we investigate the effectiveness of a robust sampled-data H_∞ load frequency control (LFC) scheme for power systems with randomly occurring time-varying delays. By using the input-delay technique, the sampled-data LFC model is reformulated as a continuous time-delay representation. Then, Bernoulli-distributed white noise sequences are used to describe randomly occurring time-varying delays in the sampled-data LFC model. Some less conservative conditions are achieved by utilizing the Lyapunov–Krasovskii functional (LKF) and employing Jensen inequality and reciprocal convex combination lemma to ensure the considered power system has mean-square asymptotic stability under the designed control scheme. The derived results are based on linear matrix inequalities (LMIs) that can readily be solved using the MATLAB LMI toolbox. The criteria obtained are used to analyze the upper bounds for time delays, and a comparison study to validate the efficacy of the presented method is presented.

1. Introduction

In recent years, research on power systems has advanced tremendously due to increased public demands for electricity, which promotes the incorporation of renewable energy into power systems [1, 2]. When a power system suffers from load disturbance or fluctuation, the system operating point can change, and deviations in the frequency of the system and the planned power exchanges can influence system performance in multiple contexts. Therefore, LFC has been proposed to ensure successful power system operations by maintaining an equilibrium among power supply and demand, thereby restoring the frequency of power systems to a proper range. Recently, a number of

research studies have concentrated on the development of suitable controllers based on LFC for power systems [3–5]. In this respect, LFC schemes including proportional-integral (PI) control [2, 3], sliding mode control [6, 7], adaptive control [8], robust control [9, 10], sampled-data control [11], event-triggered control [12–14], and H_∞ control [15] have been well defined in the literature.

However, most LFC controllers operate in the continuous case, while practical controllers are often operated using a sampling period in the discrete case. In this context, the above-mentioned controllers do not always perform optimally in practice [16]. In power systems, control error signals are first received by sensors and sent to controllers via zero-order holders (ZOHs), before being transmitted to actuators

through ZOHs [17]. This sensor-controller-actuator (SCA) technique has a discrete link design, implying that the power system can be considered a continuous-discrete sampled-data model. In recent years, many studies have introduced sampled-data control designs for LFC-based systems [18–20]. Based on the LMI theory, discrete-time multivariable PID controllers were discussed in reference [19]. The study in reference [20] devised an input-delay strategy to convert a sampled-data model to a continuous model, while the studies in references [21, 22] revealed useful additional results for LFC-based power systems.

In real-world applications, random abrupt events are a common issue in sampled-data models due to environmental changes. Furthermore, owing to the appearance of time delays in dynamic systems, data transmission is frequently affected by random delays from sensors and remote receivers, which can significantly slow down the transmission of information [23, 24]. Therefore, investigations into sampled-data control systems with random delays have recently increased [11, 25, 26]. As an example, random delays are important for the application of the H_∞ algorithm performance, as reported in reference [25]. In reference [26], type-2 fuzzy systems with probabilistic delays and actuator failures were explored under nonfragile sampled-data control. A novel method for the finite-time stability for sampled-data control systems with random delays was proposed in reference [11].

On the other hand, many physical systems, including power systems, are susceptible to uncertainty under various conditions. It is essential to investigate power systems with uncertain parameters. There have been a few studies on robust LFC schemes for power systems [27–31]. In reference [29], the study examined LFC design for delayed power systems integrating a robust decentralized PI control. Applying Riccati equation, the study in reference [30] introduced a robust LFC method for one-area power systems. In reference [31], the potential of a physical system comprehension for robust controller designs in power systems was examined. On the other hand, the classic H_∞ control theory defines a control rule that yields the minimal value of the measured performance under the assumption of zero initial values. In this regard, the delay-dependent H_∞ LFC scheme for power systems has recently received significant interest from researchers [32–35]. For instance, in references [36, 37], based on the Lyapunov functional theory, some sufficient conditions are attained for H_∞ performance of uncertain systems, which gives the minimum value of the measured performance. In reference [32], the issue of robust H_∞ LFC methods for delayed multiarea power systems was examined. In reference [33], decentralized H_∞ LFC strategy for multiarea power systems including communication uncertainties was studied. Some relevant studies have explored the issue of H_∞ LFC methods for power systems with communication delays [34, 35]. To the best of our knowledge, the robust sampled-data H_∞ LFC problem for power systems with randomly occurring time-varying delays has never been completely addressed. As such, there is still potential for further investigation and development in the stability and stabilization analysis of power systems by

addressing the robust sampled-data H_∞ LFC scheme, which motivated the present study.

On the basis of the above discussions, our primary goal in this study is to address the problem of robust sampled-data LFC for stability and stabilization of power systems with randomly occurring time delays by tackling the robust sampled-data H_∞ LFC scheme. The following are the major contributions of our research:

- (i) Different from the traditional analyses, this article presents the robust sampled-data H_∞ LFC for power systems with randomly occurring time-varying delays. By taking the probability distribution characteristic of communication delays into consideration for LFC design, the power systems with PI controllers are modelled as stochastic time-delay systems. Moreover, the input-delay strategy converts the sampled-data model into a continuous time-delay representation.
- (ii) The LMI-based mean-square asymptotic stability and stabilization criteria for power systems are established under the designed sampled-data H_∞ LFC scheme by constructing an appropriate LKF and employing Jensen inequality and the reciprocal convex combination inequality.
- (iii) Two examples are offered to demonstrate that the presence of maximum allowable upper bounds of time delays by our approach is much better than the most recent results. A comparison study is also performed, demonstrating the low computational efficiency of the obtained criteria.

This study is organized as follows: the designed sampled-data LFC scheme for power systems is formally specified in Section 2. In Section 3, the sufficient criteria for stability and stabilization of the considered model are presented. In Section 4, the application of the robust sampled-data H_∞ LFC scheme to power systems with uncertainties is discussed. The case studies in Section 5 show the potential of the criteria presented. Conclusions are given in Section 6.

Notations. In the following sections, the superscripts " T " and " -1 " indicate matrix transposition and inverse, respectively. I is any matrix with identity. Any matrix $Q > 0$ ($Q < 0$) denotes the positive definite (negative definite) matrix. The block diagonal matrix is represented by $\text{diag}\{ \dots \}$, while \mathbb{R}^n and $\mathbb{R}^{n \times n}$ denote the Euclidian n space and $n \times n$ real matrices, respectively. In a symmetric matrix, $*$ denotes symmetric terms.

2. Dynamic Model of Sampled-Data-Based LFC for Power System

In this section, the LFC scheme using sampled-data control and randomly occurring time-varying delays is detailed.

2.1. One-Area LFC Model. A block structure for a one-area LFC-based model with communication delays is shown in Figure 1, which can be represented in the following formula:

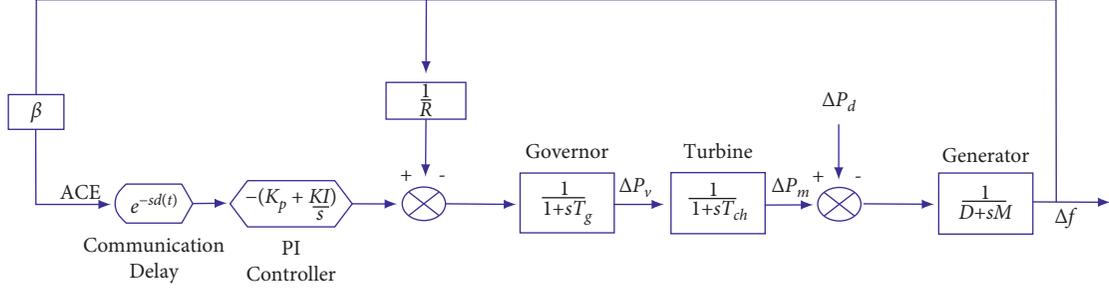


FIGURE 1: Dynamic model of one-area LFC scheme.

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{\mathcal{A}}\tilde{x}(t) + \tilde{\mathcal{B}}u(t) + \tilde{\mathcal{D}}w(t), \\ \tilde{y}(t) = \tilde{\mathcal{C}}\tilde{x}(t). \end{cases} \quad (1)$$

The notations of model (1) are standard and they are available in Appendix.

The area control error (ACE) is clearly illustrated in Figure 1 as the model output with no net tie-line power exchange. It follows that $\tilde{y}(t) = ACE = \beta\Delta f$, where $\beta > 0$ is the frequency bias factor. Furthermore, ACE is utilized as a control input to develop the PI-based controller that follows

$$u(t) = -\mathcal{K}_p ACE - \mathcal{K}_I \int ACE, \quad (2)$$

where $\int ACE$ is the integration of ACE, while bot integral and proportional gains are denoted as \mathcal{K}_I and \mathcal{K}_p , respectively.

2.2. Sampled-Data LFC Scheme. In the continuous-time mode, state vectors are employed directly to generate the control signal. However, the sampled-data LFC scheme can only handle discrete measurements of state vectors in discrete time. Meanwhile, sampled output measurements $\tilde{y}(t)$ can only be used in the sampled-data control loop in the LFC scheme. As a result, we only use the measurement $\tilde{y}(t_k)$ at sampling instant t_k . It is obvious that in Figure 2, network-induced delays affect the communication network, which is defined as $d_{t_k} = d_{t_k}^{sc} + d_{t_k}^{ea} \leq d_M < +\infty$, where d_M is constant. The sampling instants $t_k (k = 0, 1, 2, \dots)$ are supposed to satisfy

$$0 = t_0 < t_1 < t_2 < \dots < t_k < \dots < \lim_{k \rightarrow \infty} t_k = +\infty. \quad (3)$$

The sampling interval $d_k = t_{k+1} - t_k$ is set to satisfy $0 < d_k = t_{k+1} - t_k = d_M$, where d_M denotes the largest upper bound of d_k . Then, taking account of the influence of sampling and communication network delays, the possible LFC control signal $\tilde{y}(t)$ can be articulated as follows:

$$\tilde{y}(t) = \tilde{y}(t_k), \quad t \in [t_k, t_{k+1}). \quad (4)$$

Then, the sampled-data LFC scheme in the network system is drawn out according to equations (2) and (4) as follows:

$$u(t) = u(t_k) = -\mathcal{K}_p \tilde{y}(t_k) - \mathcal{K}_I \int \tilde{y}(t_k) dt. \quad (5)$$

By setting the following new vectors of virtual state and output measurement $x(t) = [\Delta f \Delta P_m \Delta P_v \int ACE]^T$ and $y(t) = [ACE \int ACE]^T$ and incorporating equations (1) and (5), the sampled-data LFC model can be obtained for $t \in [t_k, t_{k+1})$ as follows:

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) - \mathcal{B}x(t_k) + \mathcal{D}w(t), \\ y(t) = \mathcal{C}x(t). \end{cases} \quad (6)$$

The expression of model (6) is given in Appendix. Then, we define $d(t) = t - t_k$ for $t \in [t_k, t_{k+1})$. As such, we can denote the sampling instant as $t_k = t - (t - t_k) = t - d(t)$. It is supposed that $d(t) \leq t_{k+1} - t_k = d_k \leq d_M$ for all t_k . Based on the technique of input delay [20], we can further define the sampled-data control in equation (6) as follows:

$$u(t) = -\mathcal{K}x(t - d(t)). \quad (7)$$

It is noteworthy that time delays often occur in a probabilistic form in many practical control systems. Thus, the impact of random delays in this study is important. Therefore, $d(t)$ is expected to satisfy the following two assumptions:

A1. Consider the probability distribution of the time-varying delay $d(t)$ that takes values in the range $[0, d_1]$ or $(d_1, d_2]$ and define the two sets as follows:

$$\begin{aligned} \Pi_1 &= \{t: d(t) \in [0, d_1]\}, \\ \Pi_2 &= \{t: d(t) \in (d_1, d_2]\}. \end{aligned} \quad (8)$$

We also consider the mapping functions as follows:

$$\begin{aligned} d_1(t) &= \begin{cases} d(t), & \text{for } t \in \Pi_1, \\ 0, & \text{forelse,} \end{cases} \\ d_2(t) &= \begin{cases} d(t), & \text{for } t \in \Pi_2, \\ d_1, & \text{forelse.} \end{cases} \end{aligned} \quad (9)$$

It has been inferred that if $t \in \Pi_1$ appears in the case of $d(t) \in [0, d_1]$, that is, $d(t) = d_1(t)$, and $t \in \Pi_2$ appears in the case of $d(t) \in (d_1, d_2]$, that is, $d(t) = d_2(t)$. New time-varying delays are $d_1(t)$ and $d_2(t)$, such that $0 \leq d_1(t) \leq d_1$ and $d_1 < d_2(t) \leq d_2$. A stochastic variable $\sigma(t)$ is defined as follows:

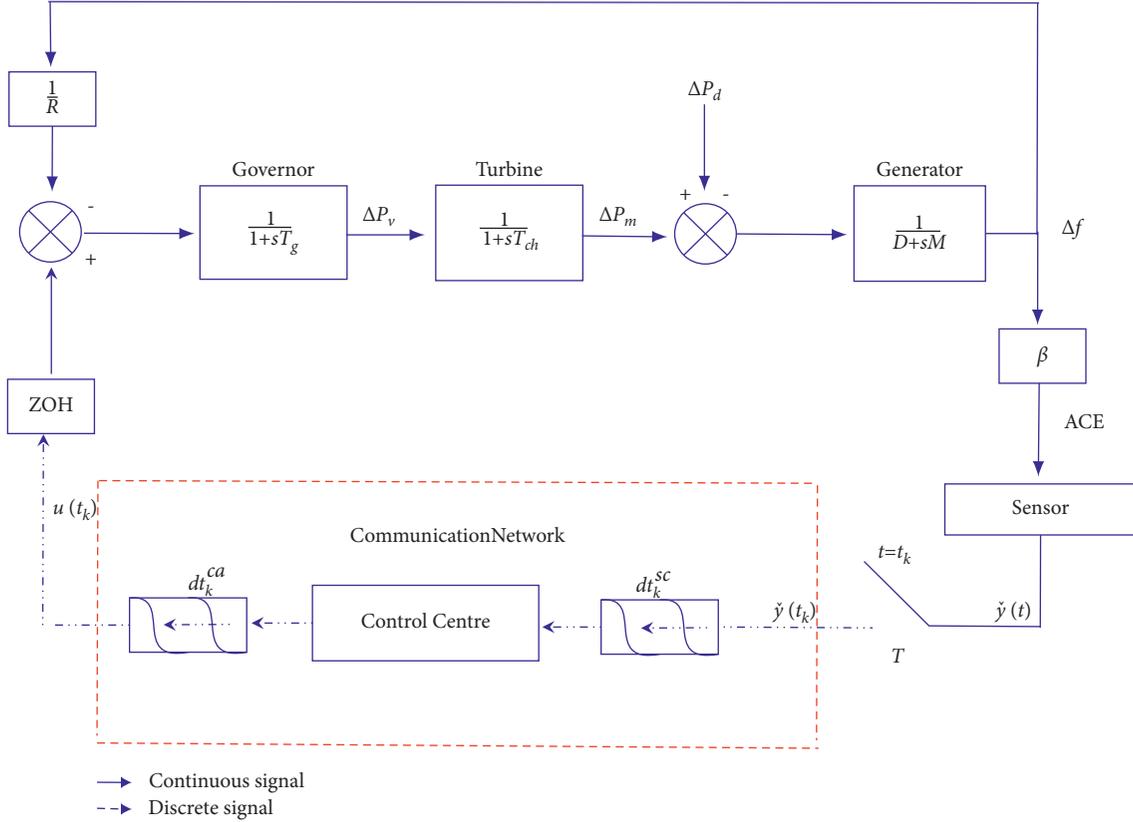


FIGURE 2: Dynamic model of one-area LFC scheme over the communication network and the transmission delay from SCA.

$$\sigma(t) = \begin{cases} 1, & \text{for } t \in \Pi_1, \\ 0, & \text{for } t \in \Pi_2. \end{cases} \quad (10)$$

A2. The variable $\sigma(t)$ is a Bernoulli-distributed sequence with the following properties: $\text{Prob}\{\sigma(t) = 1\} = \mathbb{E}\{\sigma(t)\} = \sigma_0$ and $\text{Prob}\{\sigma(t) = 0\} = 1 - \mathbb{E}\{\sigma(t)\} = 1 - \sigma_0$, where $0 \leq \sigma_0 \leq 1$ and $\mathbb{E}\{\sigma(t)\}$ is the expectation of $\sigma(t)$.

Remark 1. In practice, power systems often need broad open communication networks to supply relevant information. The operations of these networks are subject to undesirable events, including latent faults, packet loss, time delays, and others. Such nonlinear disturbances can appear in random ways as a result of different environmental conditions. As a result, the stochastic variable $\sigma(t)$ is used to represent this randomly occurring phenomenon.

By incorporating $d_1(t)$, $d_2(t)$, $\sigma(t)$, and equation (7) into model (6), we have

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) - \sigma(t)\mathcal{B}\mathcal{K}\mathcal{C}x(t - d_1(t)) \\ -(1 - \sigma(t))\mathcal{B}\mathcal{K}\mathcal{C}x(t - d_2(t)) + \mathcal{D}w(t), \\ y(t) = \mathcal{C}x(t). \end{cases} \quad (11)$$

The initial condition of model (11) is given by $x(t) = \psi(t)$, $t \in [-d_2, 0]$, where $\psi(t)$ is continuous on $[-d_2, 0]$.

The results presented below need the following definition and lemmas.

Definition 1. Given a prescribed scalar $\eta > 0$, model (11) is considered asymptotically stable with H_∞ performance if the conditions below hold [38]:

- (i) Closed-loop model (11) with $w(t) = 0$ is asymptotically stable.
- (ii) For every nonzero $w(t) \in \mathcal{L}_2[0, \infty)$ with a prescribed $\eta > 0$, the following inequality is true under the zero initial condition: $\mathbb{E}\{\|y(t)\|_2\} \leq \eta \mathbb{E}\{\|w(t)\|_2\}$.

Lemma 1. Given matrix $\mathcal{O} = \mathcal{O}^T > 0$, two matrices $\mathcal{X}_1, \mathcal{X}_2 \in \mathbb{R}^{n \times m}$, positive integers n and m , scalar $\theta \in (0, 1)$, and any vector $\zeta \in \mathbb{R}^m$ denote the function $\Xi(\theta, \mathcal{O})$ with the following form [39]:

$$\Xi(\theta, \mathcal{O}) = \frac{1}{\theta} \zeta^T \mathcal{X}_1^T \mathcal{O} \mathcal{X}_1 \zeta + \frac{1}{1 - \theta} \zeta^T \mathcal{X}_2^T \mathcal{O} \mathcal{X}_2 \zeta. \quad (12)$$

There exists a matrix $\mathcal{Y} \in \mathbb{R}^{n \times m}$ satisfying $\begin{bmatrix} \mathcal{O} & \mathcal{Y} \\ \mathcal{Y}^T & \mathcal{O} \end{bmatrix} > 0$, then

$$\min_{\theta \in (0, 1)} \Xi(\theta, \mathcal{O}) \geq \begin{bmatrix} \mathcal{X}_1 \zeta \\ \mathcal{X}_2 \zeta \end{bmatrix}^T \begin{bmatrix} \mathcal{O} & \mathcal{Y} \\ \mathcal{Y}^T & \mathcal{O} \end{bmatrix} \begin{bmatrix} \mathcal{X}_1 \zeta \\ \mathcal{X}_2 \zeta \end{bmatrix}. \quad (13)$$

Lemma 2. Given matrix $\mathcal{O} = \mathcal{O}^T > 0$, the following inequality is true for all continuously differentiable function $x(t)$ in $[\xi_1, \xi_2] \in \mathbb{R}^n$ as follows [40]:

$$\begin{aligned} & -(\xi_2 - \xi_1) \int_{t-\xi_2}^{t-\xi_1} x^T(\alpha) \mathcal{O} x(\alpha) d\alpha \\ & \leq - \left[\int_{t-\xi_2}^{t-\xi_1} x(\alpha) d\alpha \right]^T \mathcal{O} \left[\int_{t-\xi_2}^{t-\xi_1} x(\alpha) d\alpha \right]. \end{aligned} \quad (14)$$

Lemma 3. Let $\Xi = \Xi^T$, \mathcal{H} and \mathcal{E} be real matrices, $\mathcal{F}(t)$ satisfies $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$. Then, $\Xi + \mathcal{H}\mathcal{F}(t)\mathcal{E} + (\mathcal{H}\mathcal{F}(t)\mathcal{E})^T < 0$, iff there exists a scalar $\varepsilon > 0$ such that $\Xi + \varepsilon^{-1}\mathcal{H}\mathcal{H}^T + \varepsilon\mathcal{E}^T\mathcal{E} < 0$ or equivalently [41]:

$$\begin{bmatrix} \Xi & \mathcal{H} & \varepsilon\mathcal{E} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \quad (15)$$

In this study, the main aim is to obtain LMI-based sufficient criteria and ensure model (11) has mean-square

stability and stabilization under the designed sampled-data robust H_∞ LFC scheme.

Problem 1. The following conditions are derived, in order to attain the aim of this study:

- (i) Delay-dependent LKF with full model information is constructed to obtain the stability and stabilization criteria of model (11) based on Definition 1.
- (ii) Sufficient gain matrices for control $\mathcal{K} = [\mathcal{K}_p \ \mathcal{K}_I]$ are calculated from the solution of LMIs to ensure stabilization of model (11) via the robust sampled-data H_∞ LFC scheme.

3. Design of Sampled-Data H_∞ LFC Scheme

We present a delay-dependent stability analysis in Section 3.1 and a stabilization analysis in Section 3.2 for model (11) using Jensen integral inequality, reciprocally convex combination lemma, and LMI methodology. The following notations simplify the remaining presentation:

$$\zeta(t) = \text{col}\{x(t), x(t - d_1(t)), x(t - d_1), x(t - d_2(t)),$$

$$x(t - d_2), \int_{t-d_1}^{t-d_1(t)} x(\alpha) d\alpha, \int_{t-d_1(t)}^t x(\alpha) d\alpha,$$

$$\int_{t-d_2}^{t-d_2(t)} x(\alpha) d\alpha, \int_{t-d_2(t)}^t x(\alpha) d\alpha, \dot{x}(t), w(t)\},$$

$$\int_{t-d_1(t)}^t \dot{x}(\alpha) d\alpha = [x(t) - x(t - d_1(t))],$$

$$\int_{t-d_1}^{t-d_1(t)} \dot{x}(\alpha) d\alpha = [x(t - d_1(t)) - x(t - d_1)],$$

$$\int_{t-d_2(t)}^t \dot{x}(\alpha) d\alpha = [x(t) - x(t - d_2(t))],$$

$$\int_{t-d_2}^{t-d_2(t)} \dot{x}(\alpha) d\alpha = [x(t - d_2(t)) - x(t - d_2)].$$

3.1. Sampled-Data LFC-Based H_∞ Stability Analysis. This subsection focuses on obtaining the sufficient criteria for establishing mean-square asymptotic stability of model (11), which is stated in Theorem 1.

Theorem 1. Under given control gain \mathcal{K} with H_∞ performance index η , model (11) is mean-square asymptotically stable for given positive scalars d_1 , d_2 , and η , if there exist matrices \mathcal{P} , \mathcal{Q}_i , \mathcal{R}_i , \mathcal{S}_i , \mathcal{Y}_i ($i = 1, 2$), and \mathcal{M} , which satisfy the following criteria:

$$\mathcal{P} > 0, \mathcal{Q}_i > 0, \mathcal{R}_i > 0, \mathcal{S}_i > 0, i = 1, 2, \quad (17)$$

$$\begin{bmatrix} \mathcal{S}_i \mathcal{Y}_i \\ \mathcal{Y}_i^T \mathcal{S}_i \end{bmatrix} \geq 0, i = 1, 2, \quad (18)$$

$$[\Xi]_{11 \times 11} < 0, \quad (19)$$

where $\Xi_{1,1} = \mathcal{Q}_1 + \mathcal{Q}_2 + d_1^2 \mathcal{R}_1 + d_2^2 \mathcal{R}_2 - \mathcal{S}_1 - \mathcal{S}_2 + \mathcal{M}\mathcal{A} + (\mathcal{M}\mathcal{A})^T + \mathcal{E}^T \mathcal{E}$, $\Xi_{1,2} = \mathcal{S}_1 - \mathcal{Y}_1 - \sigma_0 \mathcal{M}\mathcal{B}\mathcal{K}\mathcal{K}$, $\Xi_{1,3} = \mathcal{Y}_1$, $\Xi_{1,4} = \mathcal{S}_2 - \mathcal{Y}_2 - (1 - \sigma_0) \mathcal{M}\mathcal{B}\mathcal{K}\mathcal{K}$, $\Xi_{1,5} = \mathcal{Y}_2$, $\Xi_{1,10} = \mathcal{C} - \mathcal{M} + (\mathcal{M})^T$, $\Xi_{1,11} = \mathcal{M}\mathcal{A}$, $\Xi_{2,2} = -\mathcal{S}_1 - \mathcal{S}_1 + \mathcal{Y}_1^T + \mathcal{Y}_1$, $\Xi_{2,3} = \mathcal{S}_1 - \mathcal{Y}_1$, $\Xi_{2,10} = -\sigma_0 (\mathcal{M}\mathcal{B}\mathcal{K}\mathcal{K})^T$, $\Xi_{3,3} = -\mathcal{Q}_1 - \mathcal{S}_1$, $\Xi_{4,4} = -\mathcal{S}_2 - \mathcal{S}_2 + \mathcal{Y}_2^T + \mathcal{Y}_2$, $\Xi_{4,5} = \mathcal{S}_2 - \mathcal{Y}_2$, $\Xi_{4,10} = -(1 - \sigma_0) (\mathcal{M}\mathcal{B}\mathcal{K}\mathcal{K})^T$, $\Xi_{5,5} = -\mathcal{Q}_2 - \mathcal{S}_2$, $\Xi_{6,6} = -\mathcal{R}_1$, $\Xi_{7,7} = -\mathcal{R}_1$, $\Xi_{8,8} = -\mathcal{R}_2$, $\Xi_{9,9} = -\mathcal{R}_2$, $\Xi_{10,10} = d_1^2 \mathcal{S}_1 + d_2^2 \mathcal{S}_2 - \mathcal{M} - \mathcal{M}^T$, $\Xi_{10,11} = \mathcal{M}\mathcal{A}$, and $\Xi_{11,11} = -\eta^2 I$.

Proof. Construct the LKF candidate as follows: $\mathcal{V}(x(t)) = \mathcal{V}_1(x(t)) + \mathcal{V}_2(x(t)) + \mathcal{V}_3(x(t)) + \mathcal{V}_4(x(t))$, where

$$\begin{aligned}
\mathcal{V}_1(x(t)) &= x^T(t)Cx(t), \\
\mathcal{V}_2(x(t)) &= \sum_{i=1}^2 \int_{t-d_i}^t x^T(\alpha)Q_i x(\alpha)d\alpha, \\
\mathcal{V}_3(x(t)) &= \sum_{i=1}^2 d_i \int_{t-d_i}^t \int_{\alpha}^t x^T(\beta)R_i x(\beta)d\beta d\alpha, \\
\mathcal{V}_4(x(t)) &= \sum_{i=1}^2 d_i \int_{t-d_i}^t \int_{\alpha}^t \dot{x}^T(\beta)S_i \dot{x}(\beta)d\beta d\alpha.
\end{aligned} \tag{20}$$

Obtaining the derivatives of $\mathcal{V}(x(t))$ and taking the mathematical expectation, we have

$$\mathbb{E}\{\dot{\mathcal{V}}_1(x(t))\} = \mathbb{E}\{2x^T(t)P\dot{x}(t)\}, \tag{21}$$

$$\mathbb{E}\{\dot{\mathcal{V}}_2(x(t))\} = \mathbb{E}\left\{\sum_{i=1}^2 [x^T(t)Q_i x(t) - x^T(t-d_i)Q_i x(t-d_i)]\right\}, \tag{22}$$

$$\mathbb{E}\{\dot{\mathcal{V}}_3(x(t))\} = \mathbb{E}\left\{\sum_{i=1}^2 x^T(t)d_i^2 R_i x(t) - \sum_{i=1}^2 d_i \int_{t-d_i}^t x^T(\alpha)R_i x(\alpha)d\alpha\right\}, \tag{23}$$

$$\mathbb{E}\{\dot{\mathcal{V}}_4(x(t))\} = \mathbb{E}\left\{\sum_{i=1}^2 \dot{x}^T(t)d_i^2 S_i \dot{x}(t) - \sum_{i=1}^2 d_i \int_{t-d_i}^t \dot{x}^T(\alpha)S_i \dot{x}(\alpha)d\alpha\right\}. \tag{24}$$

According to A1, the integral term in equation (23) can be employed by utilizing Lemma 2 in the form of

$$\begin{aligned}
& - \sum_{i=1}^2 d_i \int_{t-d_i}^{t-d_i(t)} x^T(\alpha)R_i x(\alpha)d\alpha \\
& \leq - \sum_{i=1}^2 \left(\left[\int_{t-d_i}^{t-d_i(t)} x(\alpha)d\alpha \right]^T R_i \left[\int_{t-d_i}^{t-d_i(t)} x(\alpha)d\alpha \right] \right),
\end{aligned} \tag{25}$$

$$\begin{aligned}
& - \sum_{i=1}^2 d_i \int_{t-d_i(t)}^t x^T(\alpha)R_i x(\alpha)d\alpha \\
& \leq - \sum_{i=1}^2 \left(\left[\int_{t-d_i(t)}^t x(\alpha)d\alpha \right]^T R_i \left[\int_{t-d_i(t)}^t x(\alpha)d\alpha \right] \right).
\end{aligned} \tag{26}$$

According to A1, the integral term in equation (24) can be employed by utilizing Lemma 2 in the form of

$$\begin{aligned}
& - \sum_{i=1}^2 d_i \int_{t-d_i}^t \dot{x}^T(\alpha) \mathcal{S}_i \dot{x}(\alpha) d\alpha \\
& = - \sum_{i=1}^2 \left(\frac{d_i}{d_i - d_i(t)} \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\frac{d_i}{d_i(t)} \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& = - \sum_{i=1}^2 \left(\left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\frac{d_i(t)}{d_i - d_i(t)} \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\frac{d_i - d_i(t)}{d_i(t)} \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right).
\end{aligned} \tag{27}$$

If $\begin{bmatrix} \mathcal{S}_i & \mathcal{Y}_i \\ \mathcal{Y}_i^T & \mathcal{S}_i \end{bmatrix} \geq 0, i = 1, 2$ by Lemma 1, the following

inequality is true:

$$\sum_{i=1}^2 \begin{bmatrix} \sqrt{\frac{d_i(t)}{d_i - d_i(t)}} \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \\ \sqrt{\frac{d_i - d_i(t)}{d_i(t)}} \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \end{bmatrix} \begin{bmatrix} \mathcal{S}_i & \mathcal{Y}_i \\ \mathcal{Y}_i^T & \mathcal{S}_i \end{bmatrix} \tag{28}$$

$$\begin{aligned}
& \times \sum_{i=1}^2 \begin{bmatrix} \sqrt{\frac{d_i(t)}{d_i - d_i(t)}} \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \\ \sqrt{\frac{d_i - d_i(t)}{d_i(t)}} \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \end{bmatrix} \geq 0,
\end{aligned}$$

which implies

$$\begin{aligned}
& - \sum_{i=1}^2 \left(\frac{d_i(t)}{d_i - d_i(t)} \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\frac{d_i - d_i(t)}{d_i(t)} \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& \leq - \sum_{i=1}^2 \left(\left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{Y}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{Y}_i^T \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right).
\end{aligned} \tag{29}$$

From equations (27) and (29), we can obtain that

$$\begin{aligned}
& - \sum_{i=1}^2 d_i \int_{t-d_i}^t \dot{x}^T(\alpha) \mathcal{S}_i \dot{x}(\alpha) d\alpha \\
& \leq - \sum_{i=1}^2 \left(\left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{S}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right]^T \mathcal{Y}_i \left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right] \right) \\
& \quad - \sum_{i=1}^2 \left(\left[\int_{t-d_i(t)}^t \dot{x}(\alpha) d\alpha \right]^T \mathcal{Y}_i^T \left[\int_{t-d_i}^{t-d_i(t)} \dot{x}(\alpha) d\alpha \right] \right).
\end{aligned} \tag{30}$$

In addition, for any matrix \mathcal{M} with a suitable size, the following inequalities are valid:

$$0 = 2[x(t) + \dot{x}(t)]\mathcal{M}[-\dot{x}(t) + \mathcal{A}x(t) - \sigma_0\mathcal{B}\mathcal{K}\mathcal{C}x \\ \cdot (t - d_1(t)) - (1 - \sigma_0)\mathcal{B}\mathcal{K}\mathcal{C}x(t - d_2(t)) + \mathcal{D}w(t)]. \quad (31)$$

From equations (21)–(31), we can deduce that for all nonzero $w(t) \in \mathcal{L}_2[0, \infty)$,

$$\mathbb{E}\left\{\mathcal{V}(x(t)) + y^T(t)y(t) - \eta^2 w^T(t)w(t)\right\} \\ \leq \mathbb{E}\left\{\zeta^T(t)([\bar{\mathbb{E}}]_{11 \times 11})\zeta(t)\right\}. \quad (32)$$

Under zero conditions, we have $\mathcal{V}(0) = 0$ and $\mathcal{V}(\infty) \geq 0$. Integrating both sides of equation (30) yields $\mathbb{E}\{\|y(t)\|_2\} \leq \eta \mathbb{E}\{\|w(t)\|_2\}$ for every $w(t) \in \mathcal{L}_2[0, \infty)$. As such, model (11) is mean-square asymptotically stable under Definition 1. The proof is completed. \square

3.2. Sampled-Data LFC-Based H_∞ Stabilization Analysis. According to the stability results developed in Section 3.1, we focus on obtaining stabilization results through the use of sampled-data LFC scheme for model (11), which is stated in Theorem 2.

Theorem 2. *Under sampled-data LFC with H_∞ performance index η , model (11) is mean-square asymptotically stable for given positive scalars d_1, d_2, η , and $\varsigma \rightarrow 0$, if there exist matrices $\mathcal{P}, \mathcal{Q}_i, \mathcal{R}_i, \mathcal{S}_i, \mathcal{Y}_i (i = 1, 2), \mathcal{L}, \mathcal{W}$, and \mathcal{L} , which satisfy the following criteria:*

$$\check{\mathcal{P}} > 0, \check{\mathcal{Q}}_i > 0, \check{\mathcal{R}}_i > 0, \check{\mathcal{S}}_i > 0, (i = 1, 2), \quad (33)$$

$$\begin{bmatrix} \check{\mathcal{S}}_i \check{\mathcal{Y}}_i \\ \check{\mathcal{Y}}_i^T \check{\mathcal{S}}_i \end{bmatrix} \geq 0, \quad i = 1, 2, \quad (34)$$

$$\begin{bmatrix} [\bar{\mathbb{E}}]_{11 \times 11} \bar{\mathcal{Y}} \\ * - I \end{bmatrix} < 0, \quad (35)$$

$$\begin{bmatrix} -\varsigma I (\mathcal{W}\mathcal{C} - \mathcal{C}\mathcal{L})^T \\ * - I \end{bmatrix} < 0, \quad (36)$$

where $\bar{\mathbb{E}}_{1,1} = \check{\mathcal{Q}}_1 + \check{\mathcal{Q}}_2 + d_1\check{\mathcal{R}}_1 + d_2\check{\mathcal{R}}_2 - \check{\mathcal{S}}_1 - \check{\mathcal{S}}_2 + \mathcal{A}\mathcal{L} + (\mathcal{A}\mathcal{L})^T$, $\bar{\mathbb{E}}_{1,2} = \check{\mathcal{S}}_1 - \check{\mathcal{Y}}_1 - \sigma_0\mathcal{L}\mathcal{K}\mathcal{C}$, $\bar{\mathbb{E}}_{1,3} = \check{\mathcal{Y}}_1$, $\bar{\mathbb{E}}_{1,4} = \check{\mathcal{S}}_2 - \check{\mathcal{Y}}_2 - (1 - \sigma_0)\mathcal{L}\mathcal{B}\mathcal{K}\mathcal{C}$, $\bar{\mathbb{E}}_{1,5} = \check{\mathcal{Y}}_2$, $\bar{\mathbb{E}}_{1,10} = \check{\mathcal{P}} - \mathcal{L} + (\mathcal{L}\mathcal{A})^T$, $\bar{\mathbb{E}}_{1,11} = \mathcal{D}$, $\bar{\mathbb{E}}_{2,2} = -\check{\mathcal{S}}_1 - \check{\mathcal{S}}_1 + \check{\mathcal{Y}}_1 + \check{\mathcal{Y}}_1^T$, $\bar{\mathbb{E}}_{2,3} = \check{\mathcal{S}}_1 - \check{\mathcal{Y}}_1$, $\bar{\mathbb{E}}_{2,10} = -\sigma_0(\mathcal{B}\mathcal{K}\mathcal{C})^T$, $\bar{\mathbb{E}}_{3,3} = -\check{\mathcal{Q}}_1 - \check{\mathcal{S}}_1$, $\bar{\mathbb{E}}_{4,4} = -\check{\mathcal{S}}_2 - \check{\mathcal{S}}_2 + \check{\mathcal{Y}}_2 + \check{\mathcal{Y}}_2^T$, $\bar{\mathbb{E}}_{4,5} = \check{\mathcal{S}}_2 - \check{\mathcal{Y}}_2$, $\bar{\mathbb{E}}_{4,10} = -(1 - \sigma_0)(\mathcal{B}\mathcal{K}\mathcal{C})^T$, $\bar{\mathbb{E}}_{5,5} = -\check{\mathcal{Q}}_2 - \check{\mathcal{S}}_2$, $\bar{\mathbb{E}}_{6,6} = -\check{\mathcal{R}}_1$, $\bar{\mathbb{E}}_{7,7} = -\check{\mathcal{R}}_1$, $\bar{\mathbb{E}}_{8,8} = -\check{\mathcal{R}}_2$, $\bar{\mathbb{E}}_{9,9} = -\check{\mathcal{R}}_2$, $\bar{\mathbb{E}}_{10,10} = d_1^2\check{\mathcal{S}}_1 + d_2^2\check{\mathcal{S}}_2 - \mathcal{L} - \mathcal{L}^T$, $\bar{\mathbb{E}}_{10,11} = \mathcal{D}$, $\bar{\mathbb{E}}_{11,11} = -\eta^2 I$, $\bar{\mathcal{Y}} = [\mathcal{A}\mathcal{L} \ 0_{1 \times 10}]^T$, and $\mathcal{K} = \mathcal{L}\mathcal{W}^{-1}$.

Proof. We can define the change of variables in order to achieve the controller gain as follows: $\mathcal{L} = \mathcal{M}^{-1}$, $\check{\mathcal{P}} = \mathcal{L}\mathcal{L}$, $\check{\mathcal{Q}}_1 = \mathcal{L}\mathcal{Q}_1\mathcal{L}$, $\check{\mathcal{Q}}_2 = \mathcal{L}\mathcal{Q}_2\mathcal{L}$, $\check{\mathcal{R}}_1 = \mathcal{L}\mathcal{R}_1\mathcal{L}$, $\check{\mathcal{R}}_2 = \mathcal{L}\mathcal{R}_2\mathcal{L}$, $\check{\mathcal{S}}_1 = \mathcal{L}\mathcal{S}_1\mathcal{L}$, $\check{\mathcal{S}}_2 = \mathcal{L}\mathcal{S}_2\mathcal{L}$, $\check{\mathcal{Y}}_1 = \mathcal{L}\mathcal{Y}_1\mathcal{L}$, and

$\check{\mathcal{Y}}_2 = \mathcal{L}\mathcal{Y}_2\mathcal{L}$. Post and premultiplying equations (18) and (19) by $\text{diag}\{\mathcal{L}, \mathcal{L}\}$ and $\text{diag}\{\mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, I\}$, we can obtain that

$$[\bar{\mathbb{E}}]_{11 \times 11} < 0, \quad (37)$$

where $\bar{\mathbb{E}}_{1,1} = \check{\mathcal{Q}}_1 + \check{\mathcal{Q}}_2 + d_1\check{\mathcal{R}}_1 + d_2\check{\mathcal{R}}_2 - \check{\mathcal{S}}_1 - \check{\mathcal{S}}_2 + \mathcal{A}\mathcal{L} + (\mathcal{A}\mathcal{L})^T + \mathcal{L}\mathcal{C}^T\mathcal{C}\mathcal{L}$, $\bar{\mathbb{E}}_{1,2} = \check{\mathcal{S}}_1 - \check{\mathcal{Y}}_1 - \sigma_0\mathcal{L}\mathcal{K}\mathcal{C}$, $\bar{\mathbb{E}}_{1,3} = \check{\mathcal{Y}}_1$, $\bar{\mathbb{E}}_{1,4} = \check{\mathcal{S}}_2 - \check{\mathcal{Y}}_2 - (1 - \sigma_0)\mathcal{L}\mathcal{B}\mathcal{K}\mathcal{C}$, $\bar{\mathbb{E}}_{1,5} = \check{\mathcal{Y}}_2$, $\bar{\mathbb{E}}_{1,10} = \check{\mathcal{P}} - \mathcal{L} + (\mathcal{L}\mathcal{A})^T$, $\bar{\mathbb{E}}_{1,11} = \mathcal{D}$, $\bar{\mathbb{E}}_{2,2} = -\check{\mathcal{S}}_1 - \check{\mathcal{S}}_1 + \check{\mathcal{Y}}_1 + \check{\mathcal{Y}}_1^T$, $\bar{\mathbb{E}}_{2,3} = \check{\mathcal{S}}_1 - \check{\mathcal{Y}}_1$, $\bar{\mathbb{E}}_{2,10} = -\sigma_0(\mathcal{L}\mathcal{B}\mathcal{K}\mathcal{C})^T$, $\bar{\mathbb{E}}_{3,3} = -\check{\mathcal{Q}}_1 - \check{\mathcal{S}}_1$, $\bar{\mathbb{E}}_{4,4} = -\check{\mathcal{S}}_2 - \check{\mathcal{S}}_2 + \check{\mathcal{Y}}_2 + \check{\mathcal{Y}}_2^T$, $\bar{\mathbb{E}}_{4,5} = \check{\mathcal{S}}_2 - \check{\mathcal{Y}}_2$, $\bar{\mathbb{E}}_{4,10} = -(1 - \sigma_0)(\mathcal{L}\mathcal{B}\mathcal{K}\mathcal{C})^T$, $\bar{\mathbb{E}}_{5,5} = -\check{\mathcal{Q}}_2 - \check{\mathcal{S}}_2$, $\bar{\mathbb{E}}_{6,6} = -\check{\mathcal{R}}_1$, $\bar{\mathbb{E}}_{7,7} = -\check{\mathcal{R}}_1$, $\bar{\mathbb{E}}_{8,8} = -\check{\mathcal{R}}_2$, $\bar{\mathbb{E}}_{9,9} = -\check{\mathcal{R}}_2$, $\bar{\mathbb{E}}_{10,10} = d_1^2\check{\mathcal{S}}_1 + d_2^2\check{\mathcal{S}}_2 - \mathcal{L} - \mathcal{L}^T$, $\bar{\mathbb{E}}_{10,11} = \mathcal{D}$, and $\bar{\mathbb{E}}_{11,11} = -\eta^2 I$. Because of the existence of nonlinear terms in equation (37) and \mathcal{C} is not invertible, let $\mathcal{Y} = \mathcal{K}\mathcal{C}\mathcal{L}$, then $\mathcal{Y}\mathcal{C}^{-1}\mathcal{L}^{-1} = \mathcal{K}$ is not applicable to directly find \mathcal{K} . Therefore, by defining $\mathcal{L}\mathcal{C} = \mathcal{K}\mathcal{C}\mathcal{L}$, $\mathcal{W}\mathcal{C} = \mathcal{C}\mathcal{L}$ and $\mathcal{L}\mathcal{W}^{-1} = \mathcal{K}$.

By using Schur complement, equation (37) is equivalent to equation (35) and the nonlinear functions are changed to W problem as in reference [42]. Moreover, we can formulate inequality (36) for every small scalar ς and use Schur complement in $(\mathcal{W}\mathcal{C} - \mathcal{C}\mathcal{L})^T(\mathcal{W}\mathcal{C} - \mathcal{C}\mathcal{L}) = 0$. Here, $\mathcal{L} > 0$ and \mathcal{W} is an invertible matrix. The proof is completed. \square

4. Design of Robust Sampled-Data H_∞ LFC Scheme

In practical, uncertainties are commonly encountered in modelling of practical systems due to environmental imperfections and changes. As such, this study takes into account norm-bounded uncertainty, and we can describe the power system as follows:

$$\begin{cases} \dot{x}(t) = (\mathcal{A} + \Delta\mathcal{A}(t))x(t) - \sigma(t)\mathcal{B}\mathcal{K}\mathcal{C}x(t - d_1(t)) \\ -(1 - \sigma(t))\mathcal{B}\mathcal{K}\mathcal{C}x(t - d_2(t)) + \mathcal{D}w(t), \\ y(t) = \mathcal{C}x(t), \end{cases} \quad (38)$$

where $\Delta\mathcal{A}(t)$ is the parametric uncertainty satisfying $\Delta\mathcal{A}(t) = \mathcal{H}\mathcal{F}(t)\mathcal{E}$, where \mathcal{H} and \mathcal{E} are known matrices, and $\mathcal{F}(t)$ is time-varying unknown matrix that satisfies $\mathcal{F}(t)^T\mathcal{F}(t) \leq I$.

Theorem 3. *Under sampled-data LFC with H_∞ performance index η , model (38) is mean-square robust asymptotically stable for given positive scalars d_1, d_2, η , and $\varsigma \rightarrow 0$, if there exist matrices $\check{\mathcal{P}}, \check{\mathcal{Q}}_i, \check{\mathcal{R}}_i, \check{\mathcal{S}}_i, \mathcal{Y}_i (i = 1, 2), \mathcal{L}, \mathcal{W}$, and \mathcal{L} , which satisfy the following criteria:*

$$\check{\mathcal{P}} > 0, \check{\mathcal{Q}}_i > 0, \check{\mathcal{R}}_i > 0, \check{\mathcal{S}}_i > 0, \quad (i = 1, 2), \quad (39)$$

$$\begin{bmatrix} \check{\mathcal{S}}_i \check{\mathcal{Y}}_i \\ \check{\mathcal{Y}}_i^T \check{\mathcal{S}}_i \end{bmatrix} \geq 0, \quad (i = 1, 2), \quad (40)$$

$$\begin{bmatrix} [\bar{\Xi}]_{11 \times 11} & \bar{\varepsilon} \bar{\Lambda}_1 \bar{\Lambda}_2 \bar{Y} \\ * & -\bar{\varepsilon} I 0 0 \\ * & * & -\bar{\varepsilon} I 0 \\ 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} -\zeta I (\mathcal{W} \mathcal{C} - \mathcal{C} \mathcal{L})^T \\ * & -I \end{bmatrix} < 0, \quad (42)$$

where $[\bar{\Xi}]_{11 \times 11}$ and \bar{Y} are given in Theorem 2, and $\bar{\Lambda}_1 = [\mathcal{K}^T 0_{1 \times 8} \mathcal{K}^T 0]^T$, $\Lambda_2 = [\mathcal{E} \mathcal{L}^T 0_{1 \times 10}]$, $\bar{\varepsilon} = \varepsilon^{-1}$, and $\mathcal{K} = \mathcal{L} \mathcal{W}^{-1}$.

Proof. First, replacing \mathcal{A} by $\mathcal{A} + \Delta \mathcal{A}(t)$ in equation (19), we have

$$[\Xi]_{11 \times 11} + \Lambda_1 \mathcal{F}(t) \Lambda_2 + (\Lambda_1 \mathcal{F}(t) \Lambda_2)^T, \quad (43)$$

It follows from Lemma 3, there exists a scalar $\varepsilon > 0$, such that

$$[\Xi]_{11 \times 11} + \frac{1}{\varepsilon} \Lambda_1 \Lambda_1^T + \varepsilon \Lambda_2^T \Lambda_2 < 0, \quad (44)$$

where $\Lambda_1 = [(\mathcal{M} \mathcal{H})^T 0_{1 \times 8} (\mathcal{M} \mathcal{H})^T 0]^T$ and $\Lambda_2 = [\mathcal{E} 0_{1 \times 10}]$, and applying Lemma 3, it is easy to obtain

$$\begin{bmatrix} [\Xi]_{11 \times 11} & \Lambda_1 & \varepsilon \Lambda_2 \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \quad (45)$$

Post- and premultiplying equation (45) by $\text{diag}\{\mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, \mathcal{L}, I, \varepsilon^{-1} I, \varepsilon^{-1} I\}$ and by applying a similar procedure of Theorem 2 through the use of Schur complement lemma, equation (41) can be obtained. The proof is completed. \square

Remark 2. In comparison with prior publications [21, 22], the proposed method is novel for model (1), which not only

covers the model with sampled-data control but also considers random time-varying delays in the controller implementation that is characterized by Bernoulli-distributed sequences. It has many advantages over regular power system analysis.

For $\sigma(t) = 1$, since only one delay interval with $0 \leq d_1(t) \leq d_1$ exists, model (11) reduces to

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{A}_d x(t - d_1(t)) + \mathcal{D}w(t), \\ y(t) = \mathcal{C}x(t). \end{cases} \quad (46)$$

Corollary 1. Model (46) with $w(t) = 0$ is asymptotically stable for given positive scalars d_1 , if there exist matrices \mathcal{P} , \mathcal{Q}_1 , \mathcal{R}_1 , \mathcal{S}_1 , \mathcal{Y}_1 , which satisfy the following criteria:

$$\mathcal{P} > 0, \mathcal{Q}_1 > 0, \mathcal{R}_1 > 0, \mathcal{S}_1 > 0,$$

$$\begin{bmatrix} \mathcal{S}_1 \mathcal{Y}_1 \\ \mathcal{Y}_1^T \mathcal{S}_1 \end{bmatrix} \geq 0,$$

$$\Gamma = \begin{bmatrix} \Gamma_{1,1} & * & * & * & * & * \\ \Gamma_{2,1} & \Gamma_{2,2} & * & * & * & * \\ \mathcal{Y}_1^T & \Gamma_{3,2} & \Gamma_{3,3} & * & * & * \\ 0 & 0 & 0 & -\mathcal{R}_1 & * & * \\ 0 & 0 & 0 & 0 & -\mathcal{R}_1 & * \\ d_1 \mathcal{A}^T \mathcal{S}_1 & d_1 \mathcal{A}_d^T \mathcal{S}_1 & 0 & 0 & 0 & -\mathcal{S}_1 \end{bmatrix} < 0, \quad (47)$$

where $\Gamma_{1,1} = \mathcal{Q}_1 + d_1^2 \mathcal{R}_1 - \mathcal{S}_1 + \mathcal{P} \mathcal{A} + (\mathcal{P} \mathcal{P})^T$, $\Gamma_{2,1} = \mathcal{S}_1^T - \mathcal{Y}_1^T + \mathcal{A}_d^T \mathcal{P}^T$, $\Xi_{2,2} = -\mathcal{S}_1 - \mathcal{S}_1 + \mathcal{Y}_1 + \mathcal{Y}_1^T$, $\Xi_{3,2} = \mathcal{S}_1^T - \mathcal{Y}_1^T$, and $\Xi_{3,3} = -\mathcal{Q}_1 - \mathcal{S}_1$.

Proof. Choosing the same LKF as in Theorem 1 with $i = 1$ and computing the derivative of $\mathcal{V}(x(t))$, we have

$$\begin{aligned} \dot{\mathcal{V}}(x(t)) &= 2x^T(t) \mathcal{P} (\mathcal{A}x(t) + \mathcal{A}_d x(t - d(t))) + x^T(t) \mathcal{Q}_1 x(t) \\ &\quad - x^T(t - d_1) \mathcal{Q}_1 x(t - d_1) + d_1^2 x^T(t) \mathcal{R}_1 x(t) - d_1 \int_{t-d_1}^t x^T(\alpha) \mathcal{R}_1 x(\alpha) d\alpha + d_1^2 \\ &\quad ((\mathcal{A}x(t) + \mathcal{A}_d x(t - d(t)))^T \mathcal{S}_1 (\mathcal{A}x(t) + \mathcal{A}_d x(t - d(t))) \\ &\quad - d_1 \int_{t-d_1}^t \dot{x}^T(\alpha) \mathcal{S}_1 \dot{x}(\alpha) d\alpha. \end{aligned} \quad (48)$$

By applying Lemma 2 and using Theorem 1, the remaining proof can be easily obtained. \square

Remark 3. The following constraint optimization issue is presented to solve the stability criterion in Corollary 1:

Optimization problem is

$$\min_{\mathcal{P}, \mathcal{Q}_1, \mathcal{R}_1, \mathcal{S}_1} d_1, \quad (49)$$

s.t

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{Y}_1 \\ \mathcal{Y}_1^T & \mathcal{S}_1 \end{bmatrix} \geq 0, \mathcal{P} = \mathcal{P}^T > 0, \mathcal{Q}_1 = \mathcal{Q}_1^T > 0, \quad (50)$$

$$\mathcal{R}_1 = \mathcal{R}_1^T > 0, \mathcal{S}_1 = \mathcal{S}_1^T > 0, \Gamma < 0.$$

The convex optimization issues are be easily solved by using certain numerical packages [39–43].

Remark 4. There have been a number of recent studies concentrating on developing suitable LFC-based

controllers for power systems. For example, the sliding mode LFC for hybrid power system is based on disturbance observer [8], an adaptive control scheme for LFC of multiarea power systems [9], robust H_∞ LFC of delayed multi-area power systems with stochastic disturbances [10, 37, 41], decentralized H_∞ LFC for multi area power systems with communication uncertainties [11, 42], and resilient reliable H_∞ LFC of power system with random gain fluctuations [44]. However, no work has yet been reported regarding the topic of robust sampled-data LFC schemes for power systems with randomly occurring time-varying delays. So, in order to fill these gaps, we introduced new sufficient conditions for ensuring mean-square asymptotic stability and stabilization criteria for power systems by addressing robust sampling of data H_∞ LFC schemes. Thus, the main results of this study are novel and distinct from those of the existing literature.

5. Case Studies

This section presents case studies including the MAUBs of time delays, H_∞ performance index, and efficiency of proposed sampled-data LFC scheme. In addition, the simulation results are given to demonstrate how the control scheme supports the achievement of system performance.

5.1. The Case of a One-Area LFC-Based Power System.

Table 1 lists the parameters of one-area LFC-based power system taken from references [35, 45]. Correlations between gain of the sampled-data LFC scheme and effects of time delays on stability margins and H_∞ performance based on the proposed results are discussed in this subsection.

Case 1. MAUBs: based on Theorem 1, an MAUB of d_2 is calculated for model (11) without load disturbance. For different settings of σ_0 and fixed $\mathcal{K}_p = 0$, $\mathcal{K}_I = 0.05$, $d_1 = 0.1$, and $\eta = 6.0512$, the MAUB values of d_2 are given in Table 2.

Following that, the MAUB of d_1 is calculated and displayed in Table 3 for various gains of \mathcal{K}_p and \mathcal{K}_I , in order to demonstrate the merit of the results obtained. Furthermore, the results presented in reference [45] are provided for comparison, in order to account for the advantage of the method presented in Corollary 1. According to Tables 3 and 4, our proposed method in Corollary 1 can achieve better MAUBs than those in reference [45] with respect to identical control gain.

The results presented in Tables 3 and 4 demonstrate that d_1 can significantly larger when \mathcal{K}_p and \mathcal{K}_I are smaller, and that d_1 is significantly lower for larger \mathcal{K}_p and \mathcal{K}_I . With a lower $\mathcal{K}_I \leq 0.05$, d_1 drops as \mathcal{K}_p increases. With the growth of $\mathcal{K}_I \geq 0.1$, d_1 initially increases and then decreases with the increase of \mathcal{K}_p . As such, there is a range of \mathcal{K}_p that can achieve the MAUBs of d_1 for a given \mathcal{K}_I .

Case 2. Minimum H_∞ Performance Index: the acceptable minimum H_∞ performance index η_{\min} that uses Theorem 1 in this study is noted in Table 5 for various σ_0 , fixed

TABLE 1: Parameter values of one-area LFC-based model (1).

\mathcal{T}_g	\mathcal{T}_{ch}	β	\mathcal{R}	\mathcal{D}	\mathcal{M}
0.1	0.3	21.0	0.05	1.0	10

TABLE 2: The MAUBs of d_2 for various settings of σ_0 and fixed \mathcal{K}_p , \mathcal{K}_I , d_1 , and η .

Methods	σ_0			
	0.2	0.4	0.6	0.8
Theorem 1	4.2874	4.0964	3.1925	3.0028

TABLE 3: The MAUB of d_1 for various control gains in reference [45].

d_1	\mathcal{K}_I							
	\mathcal{K}_p	0.05	0.1	0.15	0.2	0.4	0.6	1.0
0	27.92	13.77	9.05	6.69	3.12	1.91	0.88	0.88
0.05	27.87	14.06	9.28	6.86	3.21	1.97	0.92	0.92
0.10	27.03	13.68	9.22	6.94	3.29	2.02	0.96	0.96
0.20	25.11	12.76	8.61	6.53	3.32	2.10	1.01	1.01
0.40	20.36	10.42	7.06	5.38	2.83	1.91	1.01	1.01
0.60	14.61	7.47	5.15	3.95	2.13	1.47	0.82	0.82
1.0	0.54	0.53	0.53	0.52	0.48	0.43	0.34	0.34

TABLE 4: The MAUB of d_1 for various control gains in Corollary 1.

d_1	\mathcal{K}_I							
	\mathcal{K}_p	0.05	0.1	0.15	0.2	0.4	0.6	1.0
0	33.92	15.77	12.05	8.69	4.12	2.91	1.88	1.88
0.05	31.87	16.06	9.28	6.86	3.21	1.97	0.92	0.92
0.10	28.03	15.68	10.22	7.94	4.29	2.82	1.16	1.16
0.20	25.11	14.76	8.61	6.53	3.32	2.10	1.10	1.10
0.40	23.36	12.42	8.06	6.38	3.83	2.91	1.08	1.08
0.60	15.61	10.47	6.15	4.95	4.13	3.47	1.02	1.02
1.0	0.84	0.72	0.64	0.61	0.57	0.55	0.52	0.52

TABLE 5: The allowable minimum η_{\min} for given values of d_1 , d_2 , \mathcal{K}_p , and \mathcal{K}_I and different σ_0 .

Methods	σ_0			
	0.2	0.4	0.6	0.8
Theorem 1	4.2013	4.5108	6.0512	6.9512

maximum delay bounds $d_1 = 0.2$ and $d_2 = 0.8$, and given gains $\mathcal{K}_p = 0.05$ and $\mathcal{K}_I = 0.1$. It is found that the designed control strategy in the power system is effective for managing load disturbances.

5.2. Sampled-Data H_∞ LFC Controller Design. To compute the control gain in this example, first let $d_1 = 0.1$, $d_2 = 0.3$, $\eta = 8.5$, $\sigma = 0.8$, and $\zeta = 0.01$, then the inequalities (33)–(36) in Theorem 2 are solved, and the control gain is obtained, as presented in Table 6.

Let us use the disturbance to graphically verify the given results:

TABLE 6: The control gain \mathcal{K} for fixed d_1 , d_2 , η , σ_0 , and ζ in Theorem 2.

d_1	d_2	σ_0	ζ	η	\mathcal{K}
0.1	0.3	0.8	0.01	8.5	[0.0309 0.1343]

$$w(t) = \begin{cases} 0.1\text{pu, if } t = 10\text{s, } 20\text{s, } 30\text{s} \\ 0, \text{ otherwise.} \end{cases} \quad (51)$$

The time-varying delays for the simulation results are chosen as $d_1(t) = 0.05 \sin(t) + 0.05$ and $d_2(t) = 0.10 \sin(t) + 0.20$ satisfying $d_1 = 0.1$ and $d_2 = 0.3$. State trajectories of model (11) are displayed in Figures 3 and 4. These simulation results indicate that model (11) is stabilized with load disturbance and the presence of a stochastic variable and the control gain is listed in Table 6.

5.3. Sampled-Data Robust H_∞ LFC Controller Design. For this scenario, model (38) is considered with $\mathcal{H} = [0.20 \ 0.20 \ 0.20 \ 0.20]^T$, $\mathcal{E} = \text{diag}[0.10 \ 0.10 \ 0.10 \ 0.10]$, and $\mathcal{F}(t) = \text{diag}[\cos(t) \cos(t) \cos(t) \cos(t)]$. In addition, we assume that $d_1 = 0.1$, $d_2 = 0.3$, $\eta = 6.5$, $\sigma_0 = 0.4$, and $\zeta = 0.01$. Then, inequalities (39)–(42) in Theorem 3 are solved, and the control gain is obtained, as given in Table 7.

Delays $d_1(t)$ and $d_2(t)$ and disturbance signal $w(t)$ are selected to be the same as in above case. To show the effect of the established control design scheme, state trajectories of model (38) are depicted in Figures 5 and 6 under the proposed sampled-data robust LFC control gain in Table 7.

5.4. Comparative Analysis

Case 3. Calculation of $MAUBd_1$: the benefit of the proposed criterion in Corollary 1 is proven numerically in the following example. Model (44) is considered to obtain the following equation:

$$\mathcal{A} = \begin{bmatrix} -2.0 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad (52)$$

$$\mathcal{A}_d = \begin{bmatrix} -1.0 & 0 \\ -1.0 & -1 \end{bmatrix}.$$

With the above example, the comparison results are analyzed in detail. The MAUB values of d_1 are shown in Figure 7. They are compared with those published in references [39, 43, 44, 46, 47] for the case of $\mu = 0$. The results in Figure 7 clearly reveal the advantages of the proposed method.

Case 4. Computational Efficiency: the results presented in this study have been achieved on the basis of the LMI methodology. The total number of decision variables in LMI leads to computational issues of the main results. Consequently, the goal of this study is to establish new stability criteria that are less conservative and have a small number of decision variables. To do this, we have leveraged the Jensen-based integral inequality and reciprocal convex combination

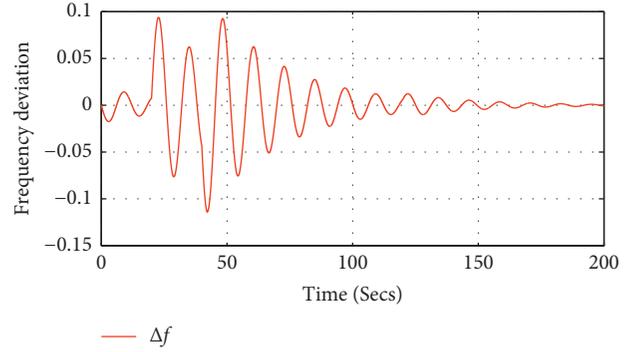


FIGURE 3: Evaluation of Δf in equation (11) in Theorem 2.

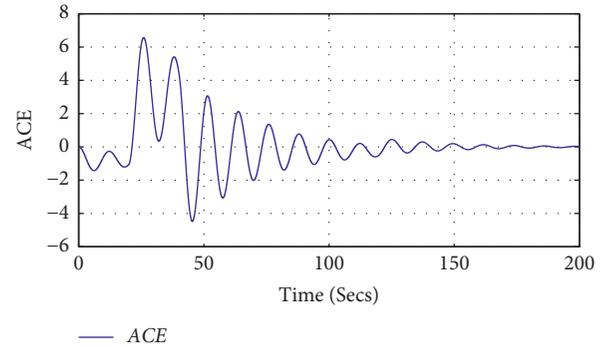


FIGURE 4: Evaluation of ACE in equation (11) in Theorem 2.

TABLE 7: The control gain \mathcal{K} for fixed d_1 , d_2 , η , ζ , and σ_0 in Theorem 3.

d_1	d_2	σ_0	ζ	η	\mathcal{K}
0.1	0.3	0.4	0.01	8.5	[0.0330 0.0275]

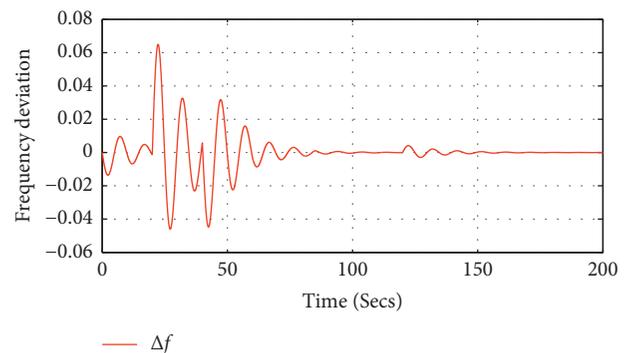


FIGURE 5: Evaluation of Δf in equation (38) in Theorem 3.

methods to reduce conservatism without adding new variables to the derivation of the main results. In addition, we have calculated the number of decision variables and compared them with those in prior studies [39, 43, 44, 46, 47], as listed in Table 8. In reference [43], enhanced stability criteria based on novel LKFs were discussed, which required $79.5n^2 + 4.5n$ in decision variables. With the derived results in Corollary 1, only a limited

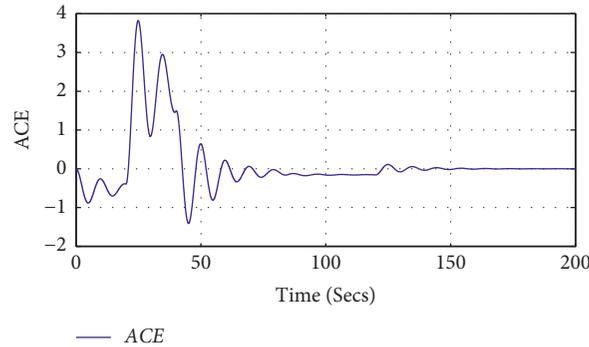


FIGURE 6: Evaluation of ACE in equation (38) in Theorem 3.

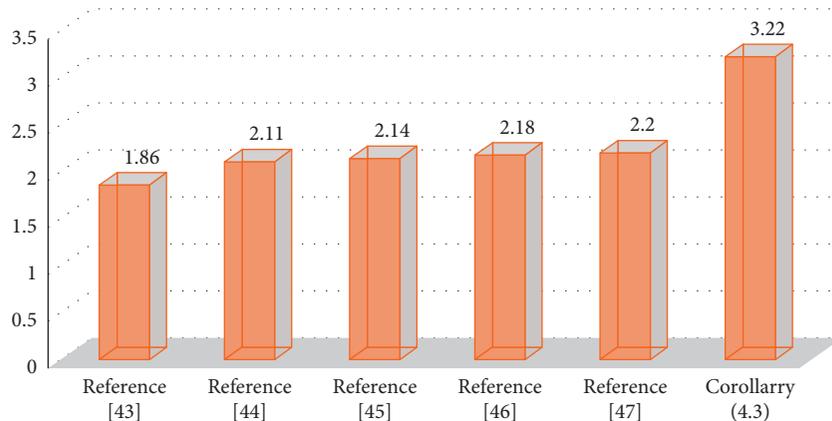


FIGURE 7: Comparative analysis of MAUB d_1 of Corollary 1 with references [39, 43, 44, 44, 46].

TABLE 8: Comparative analysis of number of decision variables in Corollary 1.

Methods	Number of decision variables
[39]	$3.5n^2 + 2.5n$
[44]	$10.5n^2 + 3.5n$
[47]	$21n^2 + 6n$
[46]	$54.5n^2 + 9.5n$
[43]	$79.5n^2 + 4.5n$
Corollary 1	$3n^2 + 2n$

number of decision variables $3n^2 + 2n$ are required. Therefore, it is evident that the stability criteria formulated in this study yields less conservatism with a smaller computational burden.

6. Conclusion

In this article, robust sampled-data H_∞ LFC scheme for power systems with randomly occurring time-varying delays has been considered. To better reflect the actual demands of practical dynamics, a generalized framework of the robust

sampled-data H_∞ LFC scheme has been studied. By leveraging the input-delay technique, the sampled-data model is converted into a continuous representation. Bernoulli-distributed sequences are used to characterize random time-varying delays in the sampled-data LFC scheme. Less conservative conditions are achieved by utilizing the LKF and employing Jensen inequality and reciprocal convex combination lemma to ensure the considered power system is mean-square asymptotic stability under the designed control strategy. The results derived in this study are based on LMIs that can be easily solved using the MATLAB LMI toolbox. The criteria obtained have been used to analyze the upper bounds of time delays, and a comparison study has been presented to validate the efficacy of the designed control method.

It is worth noting that Markov processes are widely used for modelling complex systems that undergo unpredictable changes. Therefore, LFC for power systems with Markov processes is essential. As a result, we intend to analyze observer-based sliding mode LFC of power systems under deception attacks using the proposed Markovian jump approaches in references [48, 49]. The corresponding results will be carried out in the near future.

Appendix

In equation (1), the following matrices are defined:

$$\begin{aligned} \dot{\tilde{x}}(t) &= [\Delta f \Delta P_m \Delta P_v]^T, w(t) = \Delta P_d, \dot{\tilde{y}}(t) = ACE, \\ \tilde{\mathcal{A}} &= \begin{bmatrix} -\frac{\mathcal{D}}{\mathcal{M}} & \frac{1}{\mathcal{M}} & 0 \\ 0 & -\frac{1}{\mathcal{T}_{ch}} & \frac{1}{\mathcal{T}_{ch}} \\ -\frac{1}{\mathcal{R}\mathcal{T}_g} & 0 & \frac{1}{\mathcal{T}_g} \end{bmatrix}, \tilde{\mathcal{B}} = \begin{bmatrix} 0_{2 \times 1} \\ \frac{1}{\mathcal{T}_g} \end{bmatrix}, \\ \tilde{\mathcal{D}} &= \begin{bmatrix} 1 \\ -\frac{1}{\mathcal{M}} \\ 0_{2 \times 1} \end{bmatrix}, \tilde{\mathcal{E}} = [\beta \ 0_{1 \times 2}], \end{aligned} \quad (\text{A.1})$$

where \mathcal{R} , speed droop; \mathcal{T}_g , time constant of the governor; ΔP_m , mechanical output from generator; \mathcal{D} , generator damping coefficient; \mathcal{T}_{ch} , time constant of the turbine; Δf , frequency deviation; ΔP_d , load disturbance; β , frequency bias factor; ΔP_v , deviation of the position valve; \mathcal{M} , moment of inertia of the generator.

In equation (6), the following matrices are defined:

$$\begin{aligned} \mathcal{A} &= \begin{bmatrix} -\frac{\mathcal{D}}{\mathcal{M}} & \frac{1}{\mathcal{M}} & 0 & 0 \\ 0 & -\frac{1}{\mathcal{T}_{ch}} & \frac{1}{\mathcal{T}_{ch}} & 0 \\ -\frac{1}{\mathcal{R}\mathcal{T}_g} & 0 & -\frac{1}{\mathcal{T}_g} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0_{2 \times 1} \\ \frac{1}{\mathcal{T}_g} \\ 0 \end{bmatrix}, \\ \mathcal{C} &= \begin{bmatrix} \beta & 0_{1 \times 3} \\ 0_{1 \times 3} & 1 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} 1 \\ -\frac{1}{\mathcal{M}} \\ 0_{3 \times 1} \end{bmatrix}, \mathcal{H} = [\mathcal{H}_p \ \mathcal{H}_I]. \end{aligned} \quad (\text{A.2})$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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