

Research Article

Adaptive Event-Triggered Control for Complex Dynamical Network with Random Coupling Delay under Stochastic Deception Attacks

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Received 21 March 2022; Accepted 16 April 2022; Published 11 May 2022

Academic Editor: Fathalla A. Rihan

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This study concentrates on adaptive event-triggered control of complex dynamical networks with unpredictable coupling delays and stochastic deception attacks. The adaptive event-triggered mechanism is used to avoid the wasting of limited bandwidth. The probability of data communicated by the network is established by statistical properties and Bernoulli stochastic variables with an uncertain occurrence probability. Stability analysis based on Lyapunov–Krasovskii functional (LKF) and the stability of the closed-loop system is guaranteed. Using the LMI technique, we obtain triggered parameters. To demonstrate the feasibility and usefulness of the suggested methodology, two examples are shown.

1. Introduction

Complex dynamical network systems (CDNs) are typically made up of multiple nodes spread across a large area, with each node representing a dynamical system and control signals exchanged via a communication network [1]. CDNs have piqued the interest of many researchers in recent decades due to their wide range of applications in fields such as real-world networks, physics, telephone cell graphs, scientific citation webs, metabolic pathways, electrical power grids, biological networks, and food webs [2–5]. As a result, academics have spent considerable time studying the topological structure and dynamic behavior of CDNs [6–9]. As we all know, event-triggered mechanism has been demonstrated to be a good technique to reduce communication burden and preserve bandwidth resources when compared to implementing control systems that operate on a time-triggered scheme, which results in unneeded transmitted signals during the network process [10]. The eventtriggered mechanism means that the control mission is only done when the system state meets specified criteria, which has a number of benefits, including reduced data transmission and improved resource [11–13]. It further reduces the limited bandwidth and optimizes the utilization of communication resources. As an aperiodic scheduling technique, the event-triggered mechanism (ETM) offers a way to avoid duplicate communication transmission 2

[14–18]. To deal with limited communication and processing resources, a learning-based ETM is presented, in which the triggering threshold can be adaptively changed via a vehicle communication network based on the states of the vehicle [19, 20].

The time delay is inevitable due to the amplifier's constrained switching speed and the nodes' inherent communication time. Its presence will have an impact on the stability of complex networks by causing oscillation and instability. In the models, the coupling delays of the huge complex systems are deterministic [10, 21]. Coupling delay is unavoidable in large-scale coupled nonlinear systems, such as CDNs, due to the limiting transmission speed of information between nodes [22]. Time-varying delays are more regular than constant time delays. The use of time-varying coupling delays in complex dynamical network stability has gained a lot of attention [23, 24]. In [25, 26], the synchronization of both continuous and discrete time complex dynamical networks are investigated. In [27], the problem of CDNs with sampled-data control and time-varying coupling delay was studied. To construct CDNs with time-varying coupling delay, the event-triggered mechanism and Jensen inequality were employed to estimate portions of the integral terms of the Lyapunov functional [28, 29].

Networked embedded signals in networked power systems are frequently transmitted through infrastructures, public networks, and devices that are susceptible to potential cyberattacks, and due to the inherent cyber vulnerability, the transmitted data could be exposed to malicious attacks by adversaries [30-32]. Cyberattacks are carried out by malicious attackers, according to several network control system researchers, and different cyberattacks attempt to compromise the data's security or availability [33, 34]. Deception attacks and denial-of service (DoS) attacks are two of the most popular forms of attacks [35, 36]. It is worth noting that DoS attacks can disrupt communication and cause data to become unavailable by interrupting the transmission medium [37, 38]. One of the really popular types of network security risks is deception attacks. Deception attacks, in particular, may undermine information integrity by modifying the content of sent data packets to prohibit the achievement of a predetermined performance index [39-44]. Some recent results about deception attacks are discussed in [36, 45, 46].

Based on the previous discussions, the purpose of this research is to build an adaptive event-triggered technique to address the complex dynamical network with random coupling delay and unknown probability under stochastic deception attacks. The following are the major contributions of this study:

- This study addresses the problem of adaptive eventtriggered control for complex dynamical networks with time-varying coupling delays under stochastic deception attacks.
- (2) Due to the threat of cyber security, the effect of deception attacks is considered. The independent Bernoulli variable is used to determine the probability of deception attacks.





FIGURE 1: A framework of an adaptive event-triggered control (AETC) for CDNSs.

- (3) By constructing Lyapunov-Krasvosikii functional, novel sufficient criteria are established for stochastic stability.
- (4) The deception attacks damage the actuators and sensor signals, changing their value, delaying them, or doing both.

Notations: throughout the study, the symmetry-induced vector term is denoted by the symbol $* \cdot \mathcal{Q} > 0$ denotes \mathcal{Q} is a positive definite matrix. The superscript *T* is the transpose. \mathbb{R}^m signifies the *m*-dimensional Euclidean space, and $\mathbb{R}^{m \times n}$ denotes the set of all $m \times n$ real matrices. Kronecker product is written in the form \otimes . The expectation operator is denoted by **E** and $\|\cdot\|$ refers to Euclidean norm. \Im is an identity matrix with appropriate dimension.

2. Problem Formulation and Preliminaries

Consider the complex dynamical network system determined by the following equations:

$$\begin{split} \dot{\omega}(r) &= (\Im \otimes \mathscr{A})\omega(r) + (\Im \otimes \mathscr{B})h(\omega(r)) + (\Im \otimes \mathscr{D})u(r) \\ &+ (1 - \lambda(r))\sum_{q=1}^{N} o_{pq}\Lambda \varpi_{q}(r) + (\Im \otimes \mathscr{E})\omega(r) \\ &+ \lambda(r)\sum_{q=1}^{N} \widehat{o}_{pq}\widehat{\Lambda} \varpi_{q}(r - \alpha(r)), \end{split}$$
(1)

 $\begin{aligned} z\left(r\right) &= (\Im\otimes \mathcal{C})\varpi\left(r\right),\\ \varpi\left(r\right) &= \varsigma\left(r\right), \quad r \in (-\infty, 0], \end{aligned}$

in which $z(r) \in \mathbb{R}^{n_z}$, $u(r) \in \mathbb{R}^{n_u}$, and $\overline{\omega}(r) \in \mathbb{R}^{n_{\overline{\omega}}}$ denote, respectively, the controlled output vector, the control input vector, and the state vector. The external disturbance vector $\omega(r)$, that is, $\omega(r) \in \mathcal{L}_2[0,\infty)$. $h(\overline{\omega}(r)) = [h_1(\overline{\omega}_1(r)),$ $h_2(\overline{\omega}_2(r)), \dots, h_n(\overline{\omega}_n(r))]^T$ represents the nonlinear vectorvalued function. Delayed and nondelayed inner coupling matrices are $\widehat{\Lambda}$ and Λ , respectively. Delayed and nondelayed outer coupling matrices are $\widehat{O} = \widehat{o}_{pq}$ and $O = o_{pq}$, respectively. $\varsigma(r)$ denotes the initial condition of the state. $\alpha(r)$ is the time-varying coupling delay. Its satisfies $\alpha_1 \leq \alpha(r) \leq \alpha_2$, where $\alpha_1 > 0$ and $\alpha_2 > 0$, which represents minimum and maximum bounds of $\alpha(r)$ and $\dot{\alpha}(r) \leq \varrho < 1$. Furthermore, the given system matrices are $\mathcal{A}, \mathcal{B}, \mathcal{D}, \mathcal{E}$, and \mathcal{C} with proper dimensions.

Remark 1. A coupling delay appears during the transmission of signals or information in many natural systems and also practical systems such as communication channels. The random variable $\lambda(r)$ satisfies the Bernoulli distributed white sequence, where $0 \le \lambda \le 1$. If $\lambda = 0$, the coupling delay does not happen. If $\lambda = 1$, the coupling delay happens, which obeys the following probability distribution laws: $\Pr{\lambda(r) = 0} = 1 - \overline{\lambda}$ and $\Pr{\lambda(r) = 1} = \overline{\lambda}$. $E{\lambda(r) - \overline{\lambda}} = 0$ and $E{(\lambda(r) - \overline{\lambda})^2} = \overline{\lambda}(1 - \overline{\lambda})$.

The configuration of AETC for the complex dynamical network system with deception attack is given in Figure 1. Adaptive event-triggered device is introduced between the sensor and the communication network in each subsystem. This device is responsible for selecting some necessary sampling packets for the control system to transmit over the network. It is assumed that the systems' state variables are periodically measured by a set of sensors with a constant sampling period. The measured state variables are transmitted to the AETM which is located near the sensor.

If the following condition is violated, the AETM transmits the instant to the controller via the communication network:

$$e^{T}(i\hbar)\Phi e(i\hbar) - \eta \omega^{T}(i\hbar)\Phi \omega(i\hbar) \le 0, \qquad (2)$$

where $e(i\hbar) = \mathfrak{O}(r_k\hbar) - \mathfrak{O}(i\hbar)$. Here, $\mathfrak{O}(r_k\hbar)$ and $\mathfrak{O}(i\hbar)$ represent the released instant and initial instant, respectively. \hbar is sampling period; $r_k\hbar, \{r_1, r_2, \ldots\} \subseteq \mathbb{N}$ represents the triggered instant. $0 < \Phi \in \mathbb{R}^{n_0 \times n_0}$ is a matrix to be designed. The time-dependent function $0 < \eta(i\hbar) < 1$ is a triggering threshold function which is revised by the adaption law that follows:

$$\eta\left((i+1)\hbar\right) = Sat_{\left[\underline{\eta},\overline{\eta}\right]} \left[\eta\left(i\hbar\right) + \zeta\left(\overline{\omega}^{T}\left(r_{k}\hbar\right)\Phi\overline{\omega}\left(r_{k}\hbar\right) - \overline{\omega}^{T}\left(i\hbar\right)\Phi\overline{\omega}\left(i\hbar\right)\right)\right], \quad \eta\left(0\right) \in \left[\underline{\eta},\overline{\eta}\right], \tag{3}$$

where $\zeta > 0$ is a design parameter,

$$Sat_{\left[\underline{\eta},\overline{\eta}\right]}[\boldsymbol{\omega}] = \begin{cases} \eta, \quad \boldsymbol{\omega} \ge \eta, \\ \boldsymbol{\omega}, \quad \underline{\eta} \le \boldsymbol{\omega} \le \overline{\eta}, \\ \eta, \quad \boldsymbol{\omega} \le \underline{\eta}. \end{cases}$$
(4)

 $\underline{\eta}$ and $\overline{\eta}$ are chosen as lower and upper bound of the triggering threshold function $\eta(i\hbar)$, respectively, and prescribed by the designer with considering the constraint $0 < \eta \leq \overline{\eta} < 1$.

Remark 2. It is worth noting that the presented eventtriggered mechanism (2) and the adaptation law (3) are completely discrete unlike continuous-time adaptation laws presented in [16]. Hence, the proposed AETM is more practical for implementing on digital hardware. In the event of a disturbance, the value of ρ_{ih} should be decreased properly to improve performance. Some adaptation mechanisms, such as those described in [16, 18], keep the value of ρ_{ih} constant between two release instants, resulting in a delayed controller response to the disturbance. The suggested event-triggered mechanism (2), in contrast to them, uses an adaptive threshold ρ_{ih} that is modified at each sampling instant. As a result, the controller may respond quickly to external disturbances.

The control signal after the zero-order-hold (ZOH), while considering the effect of deception attacks, is regarded as

$$u(r) = \rho(r_k \hbar) K \mathfrak{O}(r_k \hbar) + (1 - \rho(r_k \hbar)) K f(\mathfrak{O}(r_q \hbar)), \quad r \in \Theta_k,$$
(5)

where $\Theta(k) = [r_k \hbar + \beta_k, r_{k+1} \hbar + \beta_k), K \in \mathbb{R}^{n_u \times n_{\overline{\omega}}}$ is the state feedback gain matrix to be designed. The time-varying network-induced delay $\beta_k \ge 0$, which is satisfying

$$0 \le \beta_k \le \beta_a, \quad \forall k \in \mathbb{N},$$
 (6)

where $\beta_a \ge 0$ is known as a constant scalar, and

$$r_{k+1} = inf_{i>r_k} \left\{ i \mid e^T(i\hbar) \Phi e(i\hbar) > \eta(i\hbar) \overline{\omega}^T(i\hbar) \Phi \overline{\omega}(i\hbar) \right\}.$$
(7)

Define $\beta(r) = r - (i\hbar)$ and $\gamma(r) = r - (r_j\hbar)$, $r \in \Theta_k$. It is easy to show that $0 \le \beta_c \le \beta(r) \le \hbar + \beta_a = v_a$ and $0 \le \gamma(r) \le M\hbar = \gamma_M$, where $M \in \mathbb{N}$ is a known constant.

To solve the problem under investigation with all the above challenges, we suggest that the adaptive controller (5) can be written as

$$u(r) = \rho(r_k\hbar) K \overline{\omega}(r - \beta(r)) + \rho(r_k\hbar) K e(i\hbar) + (1 - \rho(r_k\hbar)) K f(\overline{\omega}(r - \gamma(r))), \quad r \in \Theta_k,$$
(8)

where $f: \mathbb{R}^{n_{\omega}} \longrightarrow \mathbb{R}^{n_{\omega}}$ is the function of deception attacks and $\gamma(r)$ is the time delay of the deception attacks. *Remark 3.* The objective of a deception attack is to modify or inject data into communication channels. In contrast to the

attack method in [20], the data injection issue in the sensorto-actuator controller channel is specifically examined in this study. The random variable $\rho(r_k\hbar)$ describes the occurrence of deception attacks on the communication network channel. If $\rho(r_k\hbar) = 0$, the communication network channel suffers from the deception attack, which means the real transmission data are replaced by the deception attack signal. Otherwise, $\rho(r_k\hbar) = 1$ means that there is no attack and the network is working normally. The sequence of data transmission is from the sampler to the controller. Here, $\rho(r_k\hbar)$ is a random variable with Bernoulli distribution which takes the value 1 with probability $\overline{\rho}$ and the value 0 with probability $1 - \overline{\rho}$, that is, $\operatorname{Prob}\{\rho(r_kh) = 1\} = \overline{\rho}$ and $\operatorname{Prob}\{\rho(r_kh) = 0\} = 1 - \overline{\rho}$.

Abovementioned $\overline{\rho}$ is very difficult or impossible in practice. So, it is assumed that this value is accompanied by uncertainty and is described by

$$\mathbb{E}\{\rho(r_k\hbar)\} = \overline{\rho} = \rho_1 + \rho_2 y(r_k\hbar),$$

$$\mathbb{E}\{(\rho(r_k\hbar) - \mathbb{E}\{\rho(r_k\hbar)\})^2\} = \overline{\delta} = \overline{\rho}(1-\overline{\rho}),$$
(9)

where ρ_1 and ρ_2 are known nominal value and known constant scaling of the uncertainty, respectively, and $y(r_k\hbar)$ is an unknown function which satisfies the following condition:

$$y^2(r_k\hbar) \le 1, \quad \forall r_k \in \mathbb{N}.$$
 (10)

Noticed that $\overline{\rho} \in [0, 1]$. If $\rho_1 \in [0, 1]$ and $\rho_2 \in [0, 0.5]$; then, the closer $\overline{\rho}$ to zero, the greater the chances of an attack. The values of ρ_1 and ρ_2 indicate the uncertainty interval on this probability.

Assumption 1 (see [43]). The functions $h: \mathbb{R}^{n_{\omega}} \longrightarrow \mathbb{R}^{n_{h}}$ and $f: \mathbb{R}^{n_{\omega}} \longrightarrow \mathbb{R}^{n_{\omega}}$ are assumed to satisfy the following conditions:

$$\|h(\varpi(r))\|_{2} \le \|H\varpi(r)\|_{2},$$

$$\|f(\varpi(r))\|_{2} \le \|F\varpi(r)\|_{2},$$
(11)

where H and F are known constant matrices.

From (1) and (8), the closed-loop complex dynamical network system can be described by

$$\dot{\varpi}(r) = (\mathfrak{T} \otimes A)\mathfrak{D}(r) + (\mathfrak{T} \otimes B)h(\mathfrak{D}(r)) + (\mathfrak{T} \otimes D)\overline{\rho}K\mathfrak{D}(r - \beta(r)) + (\mathfrak{T} \otimes D)\overline{\rho}Ke(r) + (\mathfrak{T} \otimes D) (1 - \overline{\rho})Kf(\mathfrak{D}(r - \gamma(r))) + (\mathfrak{T} \otimes D)(\rho(r_kh) - \overline{\rho})K[(\mathfrak{D}(r - \beta(r))) + (\mathfrak{T} \otimes E)\omega(r) + (1 - \lambda(r))(O \otimes \Lambda)\mathfrak{D}(r) + \lambda(r)(\widehat{O} \otimes \widehat{\Lambda})\mathfrak{D}(r - \alpha(r)), \quad r \in \Theta_k z(r) = (\mathfrak{T} \otimes C)\mathfrak{D}(r), \quad r \in \Theta_k.$$
(12)

Definition 1 (see [17]). The closed-loop system (12), under AETM and deception attacks, is stochastically stable and satisfies a prescribed \mathcal{H}_{∞} performance index γ if the following conditions hold:

$$\mathbb{E}\left\{\int_{0}^{\infty} z^{T}(r)z(r)\mathrm{d}r\right\} \leq \tilde{\gamma}^{2}\int_{0}^{\infty} \omega^{T}(r)\omega(r)\mathrm{d}r, \qquad (13)$$

for any nonzero $\omega \in \mathscr{L}_2[0,\infty)$ under zero initial condition, where $\tilde{\gamma}$ is prescribed performance level.

Lemma 1 (see [18], improved statement). For any constant matrices U and V, the inequality,

$$U + \eta(r)V < 0, \tag{14}$$

holds, for all $\eta \leq \eta(r) \leq \overline{\eta}$, if and only if

$$U + \overline{\eta}V < 0,$$

$$U + \eta V < 0.$$
(15)

Lemma 2 (see [22]). For $\beta(r) \in [0, \hbar]$ and any matrices \mathcal{Q} and \mathcal{S} with proper dimension, which satisfy $\begin{bmatrix} \mathcal{Q} & \mathcal{S} \\ * & \mathcal{Q} \end{bmatrix} \ge 0$, the following inequality holds:

$$-\hbar \int_{r-\nu_a}^{r} \dot{\varpi}^T(t) \, (\Im \otimes \mathcal{Q}) \dot{\varpi}(t) dt \le \psi^T(r) \Gamma \psi(r), \tag{16}$$

where $\psi(r) = col\{\varpi(r)\varpi(r - \beta(r))\varpi(r - v_a)\}$ and

$$\Gamma = \begin{bmatrix} -(\mathfrak{T} \otimes \mathcal{Q}) & (\mathfrak{T} \otimes \mathcal{Q}) - (\mathfrak{T} \otimes \mathcal{S}) & (\mathfrak{T} \otimes \mathcal{S}) \\ * & -2(\mathfrak{T} \otimes \mathcal{Q}) + (\mathfrak{T} \otimes \mathcal{S}) + (\mathfrak{T} \otimes \mathcal{S}^T) & (\mathfrak{T} \otimes \mathcal{Q}) - (\mathfrak{T} \otimes \mathcal{S}) \\ * & * & -(\mathfrak{T} \otimes \mathcal{Q}) \end{bmatrix}.$$
(17)

3. Main Results

In this section, a sufficient criterion will be established to verify that a complex networked control system with a deception attack is stochastically stable in controlling instants via an adaptive event-triggered mechanism.

Theorem 1. For a given positive constants $\gamma, \overline{\eta}, \mu, \varrho$ and $\overline{\rho}, \overline{\lambda} \in [0, 1]$, the closed-loop system (12) is stochastically stable, presumed the existence of positive definite matrices $\mathcal{V}, \mathcal{T}, \mathcal{W}_1, \mathcal{W}_2, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{R}_1, \backslash \backslash \mathcal{R}_2, \mathcal{R}_3, J_1, \mathcal{S}_1, \text{ and } \mathcal{S}_2$ are any proper dimension matrices and the event-triggered weighting matrix $\Phi > 0$, such that the following conditions hold:

Complexity

$$\begin{bmatrix} \vec{\sigma}_{11} & \vec{\sigma}_{12} & \vec{\sigma}_{13} & \vec{\sigma}_{14} & \vec{\sigma}_{15} & \vec{\sigma}_{15} & \vec{\sigma}_{17} & \vec{\sigma}_{18} & \vec{\sigma}_{19} & \vec{\sigma} & \vec{\sigma}_{1,11} & \vec{\sigma} & \vec{\sigma}_{1,13} \\ + & \vec{\sigma}_{22} & 0 & 0 & 0 & \vec{\sigma}_{36} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ + & * & * & -7^{-1} \cdot 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ + & * & * & * & + & -7^{-1} \cdot 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & + & \vec{\sigma}_{77} & 0 & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & \vec{\sigma}_{77} & 0 & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{79} & 0 & 0 & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & * & * & * & * & * & * & * & \vec{\sigma}_{1,11} & 0 & 0 \\ + & (3 \otimes \vec{\sigma}_{1}) + ((3 \otimes \vec{\sigma}_{1})^{T} + ((1 - \lambda(\tau)))(O \otimes \vec{\sigma}_{1})) \\ + & (3 \otimes \vec{\sigma}_{1}) + ((3 \otimes \vec{\sigma}_{1})^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} \\ + & (3 \otimes \vec{\sigma}_{1}) + ((3 \otimes \vec{\sigma}_{1})^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} + ((3 \otimes \vec{\sigma}_{1}))^{T} \\ - & 2(\vec{\rho}(\tau_{1}) - \vec{\rho})((3 \otimes \vec{\sigma}_{2}))^{T} \\ \vec{\sigma}_{1,1} = (1 - \vec{\rho})(3 \otimes \vec{\sigma}_{2}), \\ \vec{\sigma}_{1,1} = (1 - \vec{\rho})(3 \otimes \vec{\sigma}_{2}), \\ \vec{\sigma}_{1,1} = (3 \otimes \vec{\sigma}_{2}) + ((3 \otimes \vec{\sigma}_{2})) + ((3 \otimes \vec{\sigma}_{2}))^{T} + \vec{\rho}(3 \otimes \vec{\sigma}_{2}), \\ \vec{\sigma}_{1,1} = 2(3 \otimes \vec{\sigma}_{1}) + ((3 \otimes \vec{\sigma}_{2})) + ((3 \otimes \vec{\sigma}_{2})) + ((3 \otimes \vec{\sigma}_$$

(23)

Proof. Choose the Lyapunov–Krasovskii functional candidate as follows:

$$\begin{split} \mathbb{V}_{1}\left(\widehat{\omega}\left(r\right)\right) &= \widehat{\omega}^{T}\left(r\right)\left(\mathfrak{T}\otimes\mathcal{T}\right)\widehat{\omega}\left(r\right),\\ \mathbb{V}_{2}\left(\widehat{\omega}\left(r\right)\right) &= \int_{r-\nu_{a}}^{r} \widehat{\omega}^{T}\left(r\right)\left(\mathfrak{T}\otimes\mathcal{W}_{1}\right)\widehat{\omega}\left(t\right)dt + \int_{r-\gamma_{M}}^{r} \widehat{\omega}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{W}_{2}\right)\widehat{\omega}\left(t\right)dt,\\ \mathbb{V}_{3}\left(\widehat{\omega}\left(r\right)\right) &= \nu_{a}\int_{-\nu_{a}}^{0}\int_{r+\theta}^{r} \dot{\widehat{\omega}}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{Q}_{1}\right)\dot{\widehat{\omega}}\left(t\right)d\theta dt + \gamma_{M}\int_{-\gamma_{M}}^{0}\int_{r+\theta}^{r} \dot{\widehat{\omega}}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{Q}_{2}\right)\dot{\widehat{\omega}}\left(t\right)d\theta dt,\\ \mathbb{V}_{4}\left(\widehat{\omega}\left(r\right)\right) &= \int_{r-\alpha(r)}^{r} \widehat{\omega}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{V}\right)\widehat{\omega}\left(t\right)dt,\\ \mathbb{V}_{5}\left(\widehat{\omega}\left(r\right)\right) &= \int_{r-\alpha_{1}}^{r} \widehat{\omega}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{R}_{1}\right)\widehat{\omega}\left(t\right)dt + \int_{r-\alpha(r)}^{r-\alpha_{1}} \widehat{\omega}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{R}_{2}\right)\widehat{\omega}\left(t\right)dt + \int_{r-\alpha_{2}}^{r-\alpha_{1}} \widehat{\omega}^{T}\left(t\right)\left(\mathfrak{T}\otimes\mathcal{R}_{3}\right)\widehat{\omega}\left(t\right)dt\Big]. \end{split}$$

Let \mathfrak{Q} be the infinitesimal generator of $\mathbb{V}(r)$: $\mathfrak{Q}\mathbb{V}_1(\mathfrak{Q}(r)) = 2\mathfrak{Q}^T(r)(\mathfrak{T}\otimes\mathcal{T})\dot{\mathfrak{Q}}(r),$ $\mathfrak{Q}\mathbb{V}_2(\mathfrak{Q}(r)) = \mathfrak{Q}^T(r)(\mathfrak{T}\otimes\mathcal{W}_1 + \mathfrak{T}\otimes\mathcal{W}_2)\mathfrak{Q}(r)$ (21) $-\mathfrak{Q}^T(r - v_a)(\mathfrak{T}\otimes\mathcal{W}_1)\mathfrak{Q}(r - v_a),$ $-\mathfrak{Q}^T(r - \gamma_M)(\mathfrak{T}\otimes\mathcal{W}_2)\mathfrak{Q}(r - \gamma_M),$ $\mathfrak{Q}\mathbb{V}_3(\mathfrak{Q}(r)) = \dot{\mathfrak{Q}}^T(r)[v_a^2(\mathfrak{T}\otimes\mathfrak{Q}_1) + \gamma_M^2(\mathfrak{T}\otimes\mathfrak{Q}_2)]\dot{\mathfrak{Q}}(r)$ $-v_a \int_{r-v_a}^r \dot{\mathfrak{Q}}^T(t)(\mathfrak{T}\otimes\mathfrak{Q}_1)\dot{\mathfrak{Q}}(t)dt,$

According to Lemma (2), we have

$$-v_{a}\int_{r-v_{a}}^{r}\dot{\boldsymbol{\omega}}^{T}(t)\left(\boldsymbol{\Im}\otimes\boldsymbol{\mathcal{Q}}_{1}\right)\dot{\boldsymbol{\omega}}(t)\mathrm{d}t\leq\boldsymbol{\psi}_{1}^{T}(r)\boldsymbol{\Gamma}_{1}\boldsymbol{\psi}_{1}(r),\qquad(24)$$

 $-\gamma_M \int_{r-\nu_M}^r \dot{\varpi}^T(t) (\mathfrak{T} \otimes \mathcal{Q}_2) \dot{\varpi}(t) dt.$

$$-\gamma_{M} \int_{r-\gamma_{M}}^{r} \dot{\boldsymbol{\omega}}^{T}(t) \left(\mathfrak{T} \otimes \mathcal{Q}_{2} \right) \dot{\boldsymbol{\omega}}(t) dt \leq \boldsymbol{\psi}_{2}^{T}(r) \Gamma_{2} \boldsymbol{\psi}_{2}(r), \quad (25)$$

where $\psi_1(r) = col\{\varpi(r) \ \varpi(r - \beta(r)) \ \varpi(r - v_a)\}, \ \psi_2(r) = col\{\varpi(r) \ \varpi(r - \gamma(r)) \ \varpi(r - \gamma_M)\}^T$, and

$$\Gamma_{i} = \begin{bmatrix} -(\Im \otimes \mathcal{Q}_{i}) & (\Im \otimes \mathcal{Q}_{i}) - (\Im \otimes \mathcal{S}_{i}) & (\Im \otimes \mathcal{S}_{i}) \\ * & -2(\Im \otimes \mathcal{Q}_{i}) + (\Im \otimes \mathcal{S}_{i}) + (\Im \otimes \mathcal{S}_{i}^{T}) & (\Im \otimes \mathcal{Q}_{i}) - (\Im \otimes \mathcal{S}_{i}) \\ * & * & -(\Im \otimes \mathcal{Q}_{i}) \end{bmatrix}, \quad i = 1, 2,$$

$$(26)$$

$$\mathfrak{L}_{4}(\mathfrak{d}(r)) = \mathfrak{d}^{T}(r)(\mathfrak{T} \otimes \mathscr{V})\mathfrak{d}(r) - (1-\varrho)\mathfrak{d}^{T}(r-\alpha(r))(\mathfrak{T} \otimes \mathscr{V})\mathfrak{d}(r-\alpha(r)),$$

$$\mathfrak{L}\mathbb{V}_{5}(\mathfrak{Q}(r)) = \mathfrak{Q}^{T}(r)(\mathfrak{T}\otimes\mathfrak{R}_{1})\mathfrak{Q}(r) - \mathfrak{Q}^{T}(r-\alpha_{1})(\mathfrak{T}\otimes\mathfrak{R}_{1})\mathfrak{Q}(r-\alpha_{1}) + \mathfrak{Q}^{T}(r-\alpha_{1})(\mathfrak{T}\otimes\mathfrak{R}_{2})\mathfrak{Q}(r-\alpha_{1}) - (1-\varrho)\mathfrak{Q}^{T}(r-\alpha(r))(\mathfrak{T}\otimes\mathfrak{R}_{2})\mathfrak{Q}(r-\alpha(r)) + \mathfrak{Q}^{T}(r-\alpha_{1})(\mathfrak{T}\otimes\mathfrak{R}_{3})\mathfrak{Q}(r-\alpha_{1}) - \mathfrak{Q}^{T}(r-\alpha_{2})$$
(27)
$$\times(\mathfrak{T}\otimes\mathfrak{R}_{3})\mathfrak{Q}(r-\alpha_{2}).$$

From Assumption (1),

$$\mu \boldsymbol{\omega}^{T}(r) \boldsymbol{H}^{T} \boldsymbol{H} \boldsymbol{\omega}(r) - \mu \boldsymbol{h}^{T}(\boldsymbol{\omega}(r)) \boldsymbol{h}(\boldsymbol{\omega}(r)) \ge 0,$$
(28)

 $\widehat{\omega}^{T}(r-\gamma(r))F^{T}F\widehat{\omega}(r-\gamma(r))$ $-f^{T}(\widehat{\omega}(r-\gamma(r)))f(\widehat{\omega}(r-\gamma(r))) \ge 0.$ (29)

For any appropriately dimensioned matrices J_1 , the following equations hold:

$$0 = 2\left[(\Im \otimes A)\varpi(r) + (\Im \otimes B)h(\varpi(r)) + (\Im \otimes D)\overline{\rho} \times K\varpi(r - \beta(r)) + (\Im \otimes D)\overline{\rho}Ke(r) + (\Im \otimes D)(\rho(r)) + (\Im \otimes D)(\rho(r_kh) - \overline{\rho})K[(\varpi(r - \beta(r))) + e(r) - f(\varpi(r - \gamma(r)))] + (\Im \otimes E)\omega(r) + (1 - \lambda(r))(O \otimes \Lambda)\varpi(r) + \lambda(r)(\widehat{O} \otimes \widehat{\Lambda})\varpi(r - \alpha(r)) - \dot{\varpi}(r)\right][\varpi(r)J_1 + \dot{\varpi}(r)J_1].$$
(30)

Combining (21)-(30) and taking mathematical expectation, we obtain

$$\begin{split} \mathbf{E}\Big\{\mathscr{D}\mathbb{V}(\widehat{\mathbf{u}}(r)) + z^{T}(r)z(r) - \varphi^{2}\omega^{T}(r)\omega(r)\Big\}, \\ &\leq 2\omega^{T}(r)(\mathfrak{T}\otimes\mathscr{D})\dot{\widehat{\mathbf{u}}}(r) + \omega^{T}(r)\big(\mathfrak{T}\otimes\mathscr{M}_{1} + \mathfrak{T}\otimes\mathscr{M}_{2}\big)\widehat{\mathbf{u}}(r) - \omega^{T}(r - v_{a})\big(\mathfrak{T}\otimes\mathscr{M}_{1}\big)\widehat{\mathbf{u}}(r - v_{a}) - \omega^{T}(r - \gamma_{M}) \\ &(\mathfrak{T}\otimes\mathscr{M}_{2})\widehat{\mathbf{u}}(r - \gamma_{M}) + \dot{\widehat{\mathbf{u}}}^{T}(r)\Big[v_{a}^{2}\big(\mathfrak{T}\otimes\mathfrak{Q}_{1}\big) + \gamma_{M}^{2}\big(\mathfrak{T}\otimes\mathfrak{Q}_{2}\big)\Big]\dot{\widehat{\mathbf{u}}}(r) + \psi_{1}^{T}(r)\Gamma_{1}\psi_{1}(r) + \psi_{2}^{T}(r)\Gamma_{2}\psi_{2}(r) \\ &+ \omega^{T}(r)(\mathfrak{T}\otimes\mathscr{V})\widehat{\mathbf{u}}(r) - (1 - \varrho)\omega^{T}(r - \alpha(r))(\mathfrak{T}\otimes\mathscr{V})\widehat{\mathbf{u}}(r - \alpha(r)) + \omega^{T}(r)\big(\mathfrak{T}\otimes\mathscr{R}_{1}\big)\widehat{\mathbf{u}}(r) - \omega^{T}(r - \alpha_{1})\big(\mathfrak{T}\otimes\mathscr{R}_{1}\big) \\ &\times \widehat{\mathbf{u}}(r - \alpha_{1}) + \omega^{T}(r - \alpha_{1})\big(\mathfrak{T}\otimes\mathscr{R}_{2}\big)\widehat{\mathbf{u}}(r - \alpha_{1}) - (1 - \varrho)\omega^{T}(r - \alpha(r))\big(\mathfrak{T}\otimes\mathscr{R}_{2}\big)\widehat{\mathbf{u}}(r - \alpha(r)) + \omega^{T}(r - \alpha_{1})\big) \\ &\times \widehat{\mathbf{u}}(r - \alpha_{1}) + \omega^{T}(r - \alpha_{1})\big(\mathfrak{T}\otimes\mathscr{R}_{2}\big)\widehat{\mathbf{u}}(r - \alpha_{1}) - (1 - \varrho)\omega^{T}(r - \alpha(r))\big(\mathfrak{T}\otimes\mathscr{R}_{2}\big)\widehat{\mathbf{u}}(r - \alpha(r)) + \omega^{T}(r - \alpha_{1})\big) \\ &\times \widehat{\mathbf{u}}(r - \alpha_{1}) - \omega^{T}(r - \alpha_{2})\big(\mathfrak{T}\otimes\mathscr{R}_{3}\big) + \mu\omega^{T}(r)H^{T}H\widehat{\mathbf{u}}(r) - \mu h^{T}(\widehat{\mathbf{u}}(r)\big)h(\widehat{\mathbf{u}}(r)\big) + \omega^{T}(r - \gamma_{M}) \\ &\times (\mathfrak{T}\otimes\mathscr{R}_{3}\big)\widehat{\mathbf{u}}(r - \alpha_{1}) - \omega^{T}(r - \alpha_{2})\big(\mathfrak{T}\otimes\mathscr{R}_{3}\big) + \mu\omega^{T}(r)H^{T}H\widehat{\mathbf{u}}(r) - \mu h^{T}(\widehat{\mathbf{u}}(r)\big)h(\widehat{\mathbf{u}}(r)\big) + \omega^{T}(r - \gamma_{M}) \\ &\times F^{T}F\widehat{\mathbf{u}}(r - \gamma_{M}) - f^{T}(\widehat{\mathbf{u}}(r - \gamma_{M})\big)f\left(\widehat{\mathbf{u}}(r - \gamma_{M})\big) - e^{T}(r)\Phi e(r) + \eta(i\hbar)\widehat{\mathbf{u}}^{T}(r - \beta(r))\Phi\widehat{\mathbf{u}}(r - \beta(r))\big) \\ &+ 2[(\mathfrak{T}\otimes\mathcal{A})\widehat{\mathbf{u}}(r) + (\mathfrak{T}\otimes\mathcal{B})h(\widehat{\mathbf{u}}(r)\big) + (\mathfrak{T}\otimes\mathcal{D})\overline{p}\times\mathcal{K}\widehat{\mathbf{u}}(r - \beta(r))\big) + (\mathfrak{T}\otimes\mathcal{D})\overline{p}\mathcal{K}e(r) + (\mathfrak{T}\otimes\mathcal{D})\big(1 - \overline{p}) \\ &\times Kf\left(\widehat{\mathbf{u}}(r - \gamma(r))\right) + (\mathfrak{T}\otimes\mathcal{D})\big(\rho(r_{k}h) - \overline{\rho}\big)\mathcal{K}\left((\widehat{\mathbf{u}}(r - \beta(r))\big) + e(r) - f\left(\widehat{\mathbf{u}}(r - \gamma(r))\right)\right) + (\mathfrak{T}\otimes\mathcal{E})\omega(r) \\ &+ (1 - \lambda(r))(O\otimes\Lambda)\widehat{\mathbf{u}}(r) + \lambda(r)(\widehat{O}\otimes\widehat{\Lambda})\widehat{\mathbf{u}}(r - \alpha(r)) - \dot{\widehat{\mathbf{u}}}(r)\big]\big[\widehat{\mathbf{u}}(r)J_{1} + \dot{\widehat{\mathbf{u}}}(r)J_{1}\big] + z^{T}(r)z(r) \\ &- \overline{\gamma}^{2}\omega^{T}(r)\omega(r) \\ &\leq \mathbf{E}\big[\pi^{T}(r)\Upsilon{Y}\pi(r)\big], \end{split}$$

where $\pi(r) = col\{\varpi(r) \quad \varpi(r - \beta \ (r))e(r)f(\varpi(r - \gamma(r)))\omega(r)\varpi(r - v_a)\varpi(r - \gamma_M)h(\varpi(r))\varpi(r - \gamma \ (r))\varpi(r - \alpha_1)\varpi(r - \alpha(r))\varpi(r - \alpha_2)\varpi(r)\}$. By employing Schur complement [4], it can be implied that (18) is equivalent to (31), which means $\Upsilon < 0$ then $\Im < 0$.

Then, substituting $\overline{\eta}$ with $\eta(i\hbar)$. Based on Lemma (1), the sufficient condition for $\Upsilon = \Upsilon_1 + \eta \Upsilon_2 < 0$ holds for

$$Y = Y_1 + \underline{\eta} Y_2 < 0,$$

$$Y = Y_1 + \overline{\eta} Y_2 < 0.$$
(32)

By the fact that $\Upsilon_2 < 0$, the following is always true:

$$\Upsilon_1 + \eta \,\Upsilon_2 \le \Upsilon_1 + \overline{\eta} \,\Upsilon_2. \tag{33}$$

So, $\Upsilon < 0$ is equivalent to $\Upsilon_1 + \overline{\eta} \Upsilon_2 < 0$ and (18) by Schur complements [4]. Therefore,

$$\mathbf{E}\left\{\mathscr{LV}(\boldsymbol{\varpi}(r)) + \boldsymbol{z}^{T}(r)\boldsymbol{z}(r) - \boldsymbol{\gamma}^{2}\boldsymbol{\omega}^{T}(r)\boldsymbol{\omega}(r)\right\} < 0.$$
(34)

By integrating both sides of the inequality from 0 to ∞ under zero initial conditions, one obtains (13). Note that, for $\omega(t) = 0$, the above condition can be expressed as

$$\mathbf{E}\left\{\mathscr{LV}(\mathfrak{Q}(r)) + \mathbf{E}\left\{z^{T}(r)z(r)\right\} < 0.$$
(35)

It can be conclude that closed-loop system (12) with $\omega(r) = 0$ is stochastically stable according to Definition 1. This completes the proof.

Theorem 2. For given positive constants $\gamma, \overline{\eta}, \varrho$ and $\overline{\rho}, \overline{\lambda} \in [0, 1]$, the closed-loop system (12) is stochastically stable, presumed the existence of positive definite matrices $\overline{\mathcal{V}}, \overline{\mathcal{T}}, \overline{\mathcal{W}}_1, \overline{\mathcal{W}}_2, \overline{\mathcal{Q}}_1, \overline{\mathcal{Q}}_2, \overline{\mathcal{R}}_1, \overline{\mathcal{R}}_2, \overline{\mathcal{R}}_3, J_1, \overline{\mathcal{S}}_1$, and $\overline{\mathcal{S}}_2$ are any proper dimension matrices. μ be positive scalars and the event-triggered weighting matrix $\overline{\Phi} > 0$, such that the following conditions hold:

$$\overline{\mathcal{O}} < 0, \tag{36}$$

where

8

$$\begin{split} \left[\overline{\partial_{11}} \ \overline{\partial_{12}} \ \overline{\partial_{13}} \$$

Moreover, the controller gain matrix K is given by

 $K = \overline{\mathcal{T}}^{-1}X.$

Proof. We can calculate
$$\mathcal{O} = \aleph^{T}(r)\mathcal{O}_{i\times j}\aleph(r)(i, j = 1, 2, ..., 15)$$
 with
 $\aleph(r) = \operatorname{diag}\{\overline{\mathcal{T}}, \overline{\mathcal{T}}, \overline{\mathcal{T}},$

where $\overline{\mathcal{T}} = \mathcal{T}^{-1}$. Defining $\overline{\mathcal{W}}_{\Diamond} = \overline{\mathcal{T}}^T \mathcal{W}_{\Diamond} \overline{\mathcal{T}}, \overline{\mathcal{Q}}_{\Diamond} = \overline{\mathcal{T}}^T \mathcal{Q}_{\Diamond} \overline{\mathcal{T}},$ $\overline{\mathcal{R}}_* = \overline{\mathcal{T}}^T \mathcal{R}_* \overline{\mathcal{T}} (* = 1, 2, 3), \qquad \overline{\mathcal{V}}_{\Diamond} = \overline{\mathcal{T}}^T \mathcal{V}_{\Diamond} \overline{\mathcal{T}}, \qquad \overline{\mathcal{S}}_{\Diamond} = \overline{\mathcal{T}}^T \mathcal{S}_{\Diamond} \overline{\mathcal{T}} \quad (\Diamond = 1, 2), \text{ and } \overline{\Phi} = \overline{\mathcal{T}}^T \Phi \overline{\mathcal{T}} \text{ and letting } J_1 = \epsilon \overline{\mathcal{T}},$ we obtain (36).

4. Numerical Example

In this section, we provide some numerical information to ensure the method's effectiveness and applicability.

Example 1. Consider the adaptive event-triggered control for complex dynamical network system (12) with time-varying coupling delays under stochastic deception attacks, with the following parameters:

$$\begin{split} \dot{\varpi}(r) &= (\mathfrak{T} \otimes \mathscr{A}) \widetilde{\varpi}(r) + (\mathfrak{T} \otimes \mathscr{B}) h(\mathfrak{Q}(r)) \\ &+ (\mathfrak{T} \otimes \mathscr{D}) \overline{\rho} \times K \mathfrak{Q}(r - \beta(r)) \\ &+ (\mathfrak{T} \otimes \mathscr{D}) \overline{\rho} K e(r) + (\mathfrak{T} \otimes \mathscr{D}) z \\ (1 - \overline{\rho}) K f(\mathfrak{Q}(r - \gamma(r))) \\ &+ (\mathfrak{T} \otimes \mathscr{D}) \left(\rho(r_k h) - \overline{\rho} \right) K [(\mathfrak{Q}(r - \beta(r)) \\ &+ e(r) - f(\mathfrak{Q}(r - \gamma(r)))] \\ &+ (\mathfrak{T} \otimes \mathscr{C}) \mathfrak{Q}(r) + (1 - \lambda(r)) (O \otimes \Lambda) \mathfrak{Q}(r) \\ &+ \lambda(r) (\widehat{O} \otimes \widehat{\Lambda}) \mathfrak{Q}(r - \alpha(r)), \\ z(r) &= (\mathfrak{T} \otimes \mathscr{C}) \mathfrak{Q}(r), \\ \mathscr{A} &= \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & -0.2 \\ 0 & -4 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.05 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \mathscr{B} &= \begin{bmatrix} 0.2 & -0.2 \\ 0.2 & -0.2 \end{bmatrix}, \\ \mathscr{J} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{split}$$

 $O = \begin{bmatrix} -8.9 & 0.1 & 0.1 \\ 0.1 & -8.9 & 0.1 \\ 0.1 & 0.1 & -8.9 \end{bmatrix},$ $\widehat{O} = \begin{bmatrix} -4.4 & 0.1 & 0.1 \\ 0.1 & -4.4 & 0.1 \\ 0.1 & 0.1 & -4.4 \end{bmatrix}.$ (41)

Choose $\gamma_M = 0.8$, $\lambda(r) = 0.5$, $\mu = 2$, $v_a = 12.5$, $\overline{\rho} = 0.5$, $\eta = 0.4$, $\epsilon = 0.1$, $\varrho = 0.2$, and H_{∞} performance $\overline{\gamma} = 1.4$. The following solutions are achieved by solving LMI in Theorem (2):

$$\begin{split} & \mathcal{W}_{1} = \begin{bmatrix} 85.0696 & 17.6977 \\ 17.6977 & 126.4490 \end{bmatrix}, \\ & \mathcal{W}_{2} = \begin{bmatrix} 99.3898 & 5.6280 \\ 5.6280 & 133.1595 \end{bmatrix}, \\ & \mathcal{Q}_{1} = \begin{bmatrix} 4.2512 & -0.0382 \\ -0.0382 & 4.1737 \end{bmatrix}, \\ & \mathcal{Q}_{2} = \begin{bmatrix} 369.6281 & -4.3094 \\ -4.3094 & 327.1350 \end{bmatrix}, \\ & \mathcal{W} = \begin{bmatrix} 213.2716 & 28.8430 \\ 28.8430 & 160.1214 \end{bmatrix}, \\ & \mathcal{R}_{1} = \begin{bmatrix} 152.2082 & 25.0528 \\ 25.0528 & 145.7121 \end{bmatrix}, \\ & \mathcal{R}_{2} = \begin{bmatrix} 64.3787 & 4.0647 \\ 4.0647 & 13.3491 \end{bmatrix}, \\ & \mathcal{R}_{3} = \begin{bmatrix} 43.9148 & 10.4940 \\ 10.4940 & 66.1815 \end{bmatrix}, \\ & X = \begin{bmatrix} 4.0192 & -3.7714 \\ -3.7714 & 0.4453 \end{bmatrix}, \\ & \mathcal{S}_{1} = \begin{bmatrix} 51.7924 & -9.4347 \\ -9.4347 & 70.5017 \end{bmatrix}, \\ & \mathcal{S}_{2} = \begin{bmatrix} 18.0418 & 6.4145 \\ 6.4145 & 22.5261 \end{bmatrix}, \\ & J_{1} = \begin{bmatrix} 16.59265 & 1.67527 \\ 1.67527 & 8.33000 \end{bmatrix}, \\ & \mathcal{T} = \begin{bmatrix} 165.9265 & 16.7527 \\ 16.7527 & 83.3000 \end{bmatrix}, \\ & \mathcal{P} = \begin{bmatrix} 72.4205 & 13.7013 \\ 13.7013 & 103.9256 \end{bmatrix}, \\ & K = \begin{bmatrix} 0.0294 & -0.0512 \\ -0.0238 & 0.0101 \end{bmatrix}. \end{split}$$

The inner coupling matrices are $\Lambda = \text{diag}\{0.7.0.7\}$ and $\widehat{\Lambda} = \text{diag}\{0.6.0.6\}$. The outer coupling matrices are

Then, it follows from Theorem (2), adaptive eventtriggered mechanism for a complex dynamical network system subject to deception attack (12), is stochastically stable.

(42)

Example 2. Consider the adaptive event-triggered control for complex dynamical network system (12) with time-varying coupling delays under stochastic deception attacks, with the following parameters:

$$\begin{split} \dot{\varpi}(r) &= (\mathfrak{T} \otimes \mathscr{A}) \mathfrak{D}(r) + (\mathfrak{T} \otimes \mathscr{B})h(\varpi(r)) \\ &+ (\mathfrak{T} \otimes \mathscr{D})\overline{\rho} \times K \mathfrak{D}(r - \beta(r)) \\ &+ (\mathfrak{T} \otimes \mathscr{D})\overline{\rho} K e(r) + (\mathfrak{T} \otimes \mathscr{D}) \\ &(1 - \overline{\rho}) K f(\varpi(r - \gamma(r))) \\ &+ (\mathfrak{T} \otimes \mathscr{D}) (\rho(r_k h) - \overline{\rho}) K[(\varpi(r - \beta(r))) \\ &+ (\mathfrak{T} \otimes \mathscr{E}) \omega(r) + (1 - \lambda(r)) (O \otimes \Lambda) \mathfrak{D}(r) \\ &+ \lambda(r) (\widehat{O} \otimes \widehat{\Lambda}) \mathfrak{D}(r - \alpha(r)), \end{split}$$

$$z(r) = (\mathfrak{T} \otimes \mathscr{C})\mathfrak{d}(r),$$

$$\mathcal{A} = \begin{bmatrix} -23 & 0.1 & 0 \\ 0 & -23 & 0.1 \\ 0 & 0.1 & -22 \end{bmatrix},$$
$$\mathcal{B} = \begin{bmatrix} -1.2 & -0.5 & 0.2 \\ -0.5 & 0 & 0.5 \\ 0.1 & 0.5 & -0.5 \end{bmatrix},$$
$$\mathcal{C} = \begin{bmatrix} 0.1 & -0.8 & 0 \\ -0.8 & 0.2 & 0 \\ 0 & -1.4 & 0.6 \end{bmatrix},$$
$$\mathcal{D} = \begin{bmatrix} 1.8 & 0 & -1.2 \\ -1.5 & 0 & 1.8 \\ 1.2 & -0.1 & 0 \end{bmatrix},$$
$$\mathcal{C} = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix},$$
$$\mathcal{F} = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix},$$
$$\mathcal{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\mathcal{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(43)

The inner coupling matrices are $\Lambda = \text{diag}\{1, 1, 1\}$ and $\widehat{\Lambda} = \text{diag}\{1, 1, 1\}$. The outer coupling matrices are

$$O = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix},$$

$$\hat{O} = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix}.$$
(44)

Choose $\gamma_M = 0.82, \lambda(r) = 0.5, \mu = 2, \nu_a = 11.3, \overline{\rho} = 0.5, \eta = 0.4, \varrho = 0.2, \text{ and } \epsilon = 0.1 \text{ with } H_{\infty} \text{ performance } \gamma = 0.9.$ The following solutions are achieved by solving LMI in Theorem (2):

$$\begin{split} &\mathcal{W}_{1} = \begin{bmatrix} 85.0696 & 17.6977 \\ 17.6977 & 126.4490 \end{bmatrix}, \\ &\mathcal{W}_{2} = \begin{bmatrix} 99.3898 & 5.6280 \\ 5.6280 & 133.1595 \end{bmatrix}, \\ &\mathcal{Q}_{1} = \begin{bmatrix} 4.2512 & -0.0382 \\ -0.0382 & 4.1737 \end{bmatrix}, \\ &\mathcal{Q}_{2} = \begin{bmatrix} 369.6281 & -4.3094 \\ -4.3094 & 327.1350 \end{bmatrix}, \\ &\mathcal{V} = \begin{bmatrix} 213.2716 & 28.8430 \\ 28.8430 & 160.1214 \end{bmatrix}, \\ &\mathcal{R}_{1} = \begin{bmatrix} 152.2082 & 25.0528 \\ 25.0528 & 145.7121 \end{bmatrix}, \\ &\mathcal{R}_{2} = \begin{bmatrix} 64.3787 & 4.0647 \\ 4.0647 & 13.3491 \end{bmatrix}, \\ &\mathcal{R}_{3} = \begin{bmatrix} 4.0192 & -3.7714 \\ -3.7714 & 0.4453 \end{bmatrix}, \\ &\mathcal{K}_{1} = \begin{bmatrix} 51.7924 & -9.4347 \\ -9.4347 & 70.5017 \end{bmatrix}, \\ &\mathcal{K}_{2} = \begin{bmatrix} 18.0418 & 6.4145 \\ 6.4145 & 22.5261 \end{bmatrix}, \\ &J_{1} = \begin{bmatrix} 16.59265 & 1.67527 \\ 1.67527 & 8.33000 \end{bmatrix}, \\ &\mathcal{F} = \begin{bmatrix} 72.4205 & 13.7013 \\ 13.7013 & 103.9256 \end{bmatrix}, \\ &K = \begin{bmatrix} 0.0294 & -0.0512 \\ -0.0238 & 0.0101 \end{bmatrix}. \end{split}$$

(45)

Then, it follows from Theorem (2) that adaptive eventtriggered mechanism for a complex dynamical network system subject to deception attack (12) is stochastically stable.

5. Conclusion

The issue of adaptive event-triggered mechanism for a class of complex dynamical networks with random time-varying coupling delays under stochastic deception attacks has been investigated. We established two sets of random stochastic variables $\rho(i_k\hbar)$ and $\lambda(r)$, respectively, to represent the probability of data conveyed by the network being subjected to deception attacks and time-varying coupling delays. Based on the Lyapunov–Krasovskii functional theory some sufficient conditions derived for the closed-loop system that can ensure the system is stochastically stable. Two examples are presented to demonstrate the effectiveness of the presented approach.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Acknowledgments

This work was supported by the Rajamangala University of Technology Suvarnabhumi, Thailand.

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