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Research article

Regularity of Pythagorean neutrosophic graphs with an illustration in MCDM

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Abstract: Pythagorean neutrosophic set is an extension of a neutrosophic set which represents incomplete, uncertain and imprecise details. Pythagorean neutrosophic graphs (PNG) are more flexible than fuzzy, intuitionistic, and neutrosophic models. PNG are similar in structure to fuzzy graphs but the fuzziness is more resilient when compared with other fuzzy models. In this article, regular Pythagorean neutrosophic graphs are studied, where for each element the membership ($\mathfrak{M}$), and non-membership ($\mathfrak{N}\mathfrak{M}$) are dependent and indeterminacy ($\mathfrak{I}$) is independently assigned. The new ideas of regular, full edge regular, edge regular, and partially edge regular Pythagorean Neutrosophic graphs are introduced and their properties are investigated. A new MCDM method has been introduced using the Pythagorean neutrosophic graphs and an illustrative example is given by applying the proposed MCDM method.

Keywords: strongly regular; edge regular; Pythagorean neutrosophic graph; MCDM; totally edge regular
Mathematics Subject Classification: 03E72, 05C72, 94D05

Abbreviations

FS	Fuzzy set
FR	Fuzzy Relation
PN	Pythagorean Neutrosophic
PNN	Pythagorean Neutrosophic Number

PNS	Pythagorean Neutrosophic Set
PNG	Pythagorean Neutrosophic Graph
PFS	Pythagorean Fuzzy Set
PFG	Pythagorean Fuzzy Graph
PFR	Pythagorean Fuzzy Relation
RPNG	Regular Pythagorean Neutrosophic Graph
IFS	Intuitionistic Fuzzy Set
IFG	Intuitionistic Fuzzy Graph
MCDM	Multi Criteria Decision Making
SVNS	Single Valued Neutrosophic Set
SVNG	Single Valued Neutrosophic Graph
$\mathfrak{M}$, $\mathfrak{I}$, $\mathfrak{NM}$	Membership, Indeterminacy, Non-Membership
$\mathfrak{m}$, $\mathfrak{id}$, $\mathfrak{nm}$	membership, indeterminacy, non-Membership
$\mathfrak{ED}$	Edge degree
$\mathfrak{TED}$	Total edge degree
$\mathfrak{Cs}$	Constant
$\mathfrak{CF}$	Constant function
ER	Edge Regular
TER	Totally Edge Regular
RCG	Regular crisp graph
PR	Partially regular
PER	Partially edge regular
FER	Full edge regular
FRPNG	Full regular Pythagorean Neutrosophic Graph
ERPNG	Edge regular Pythagorean Neutrosophic Graph
FERPNG	Full edge regular Pythagorean Neutrosophic Graph
PERPNG	Partially edge regular Pythagorean Neutrosophic Graph
SRPNG	Strongly regular Pythagorean Neutrosophic Graph
ERPNG	Edge Regular Pythagorean Neutrosophic Graph
TERPNG	Totally Edge Regular Pythagorean Neutrosophic Graph
CN	Common neighborhood
CBPNG	Complete bipartite Pythagorean Neutrosophic Graph

1. Introduction

Atanassov established the idea of an Intuitionistic set [1] by introducing a generalization of fuzzy set [2]. Each element in the set is assigned a membership and non-membership degree with the constraint that the addition of these values lies between 0 to 1. Researchers have studied Intuitionistic fuzzy sets (IFS) and have been implemented in various fields, including decision making [3,4], cluster analysis [5], pattern recognition [6], market prediction [7], medical diagnosis [8]. Smarandache [9] initiated the neutrosophic set theory in which each element is independently assigned a truth, indeterminacy, and falsity membership degree in the non-standard interval $]0^-, 1^+[$.

Wang et al. [10] presented the concept of a single valued neutrosophic set (SVNS) as a special case of a neutrosophic set. These sets have been widely used in a various of fields, including image processing [11], medical diagnosis [12], decision making [13], information fusion [14], control theory [15], and graph theory [16, 17] among others.

To deal with complex imprecision and uncertainty Pythagorean fuzzy sets PFS was pioneered by Yager [18–20] such that the addition of the squares of $\mathfrak{M}$ and $\mathfrak{NM}$ degrees lies in 0 and 1. Consequently, in comparison to IFSs, PFSs account for a greater amount of uncertainty. Smarandache introduced and developed the degree of dependence among components of fuzzy sets and neutrosophic sets. One special case with independent indeterminacy and dependent truth and falsity is chosen out of three membership functions of neutrosophic sets with the constraint addition of squares of $\mathfrak{M}$, $\mathfrak{I}$, and $\mathfrak{NM}$ lies between 0 and 2, and it is known as the Pythagorean Neutrosophic set (PNS) [21].

To deal with structural information, graph representations are widely used in domains such as networks, economics, systems analysis, image interpretation, operations research, and pattern recognition. Based on the fuzzy relations Kauffman [22] presented the Fuzzy graphs. Rosenfeld [23] established the structure of fuzzy graphs to derive numerous basic theoretical concepts. Bhattacharya [24] introduced various concepts on fuzzy graphs and Radha and Kumaravel investigated edge regular fuzzy graphs in [25]. Atanassov [26] introduced intuitionistic fuzzy graphs (IFG) with IFS as edge sets and vertex sets, which were later developed by Akram [27].

The idea of edge regular IFGs was defined by Karunambigai et al. [28, 29]. Borzooei et al. [30] recently developed the notion of fuzzy graph regularity to vague graph regularity. Interval-valued IFGs were first pioneered by Mishra and Pal [31]. Kandasamy et al. [32] proposed the notion of neutrosophic graphs in which the edge weights are neutrosophic numbers. Broumi et al. [33, 34] also proved the existence of some of the properties of SVNGs and their extensions.

In [35], the fuzzy graph was extended to Pythagorean fuzzy graphs. By combining concepts of PNSs and fuzzy graphs, the new Pythagorean neutrosophic graph (PNG) [36] was developed. The PNGs are a graphical representation that is the same as the structure of the graphs but the sum of the membership grades of the vertices is less than 2 and the same goes for the edges of the graph.

Decision-making problems are complex to deal with and examining them using a single criterion to take optimum decisions can lead to an unrealistic decision. Simultaneous consideration of all the factors to the problem is a mere good approach. Multicriteria Decision Making (MCDM) is an advancement of operation research which is for the development of decision methodologies to make the decision problems simple involving multiple criteria, goals of conflicting nature [37,38]. Problems with a finite set of alternatives and evaluated to rank them and to select the most appropriate one are called discrete MCDM problems while problems with an infinite set of alternatives are continuous MCDM problems. Discrete MCDM problems are addressed through the multiattribute decision making (MADM) [39–41] methods while continuous MCDM problems are addressed through multiobjective decision making (MODM) [42–45] methods. Fuzzy MCDM methods are used to assess the alternatives through a single or a committee of decision-makers, where the values of alternatives, weights of criteria can be evaluated using linguistic values which can be represented by fuzzy numbers [46–48]. Several approaches have been developed to solve the fuzzy MCDM problems. A review and comparison of various of these methods are presented in [49–51] and [52]. A brief review of the category in fuzzy MCDM and some of its recent applications is presented in [53–56].

The proposed method is dealt with using Pythagorean Neutrosophic graphs which is a recently developed fuzzy set having the advantage of holding a bit more fuzziness when compared with the previously developed fuzzy sets. Although the constrain is restricted to two, this provides independence to two membership values. With all these additional advantages our proposed method is more useful in developing models or methods for real-life problems. The proposed method using Pythagorean Neutrosophic graphs for the decision model is more compatible and at the same time, holds little restrictions to select the suitable criterion. Thus the proposed method, is an advantage in the decision-making field, because of its usage and recent developments.

This paper is arranged as follows: Section 2 discusses the preliminary concepts which are necessary for the work presented in the manuscript. Section 3 proposes the ideas of edge regular, regular, partially edge regular, strongly regular, full edge regular, and bi-regular PNGs, and examines their properties. In Section 4, an illustrative example is given for the newly proposed MCDM method using PNGs and finally, we conclude Section 5. In this article, $\mathfrak{V}$ denotes a crisp universe of generic elements, $\mathbb{G}$ stands for the crisp graph, $\mathfrak{G}$ is the PN. The membership, non-membership, and indeterminacy are represented by $\mathfrak{M}, \mathfrak{NM}, \mathfrak{I}$.

2. Preliminaries

Fuzzy set theory is an effective mathematical concept that is used to deal with uncertain and vague values. The basic definition of the fuzzy subset and its extended fuzzy sets were given for the basic reference. In addition, a few more definitions of the fuzzy graphs were given for the base understanding of the paper. The section holds basic definitions and terminologies for the paper:

Definition 2.1. [2] On a universe $\mathfrak{U}$, $\mathfrak{A} = \{\langle \mathfrak{s}, \mu_{\mathfrak{A}}(\mathfrak{s})\rangle \,|\mathfrak{s} \in \mathfrak{U}\}$ is a fuzzy set (FS) where $\mu_{\mathfrak{A}} : \mathfrak{U} \to [0, 1]$ symbolizes the $\mathfrak{M}$ grade of $\mathfrak{s} \in \mathfrak{U}$.

Definition 2.2. [22] A fuzzy graph is a duo $\mathbb{G} = (\mathfrak{A}, \mathfrak{B})$ on $\mathfrak{U}$ with a FS ($\mathfrak{A}$) and FR ($\mathfrak{B}$) on $\mathfrak{U}$ such that $\mu_{\mathfrak{B}}(\mathfrak{y}\mathfrak{h})) \leq \mu_{\mathfrak{A}}(\mathfrak{y}) \wedge \mu_{\mathfrak{A}}(\mathfrak{h}) \;\forall\; \mathfrak{y}, \mathfrak{h} \in \mathfrak{U}$, where $\mathfrak{A} : \mathfrak{U} \to [0, 1]$ and $\mathfrak{B} : \mathfrak{U} \times \mathfrak{U}$ to $[0, 1]$.

Definition 2.3. [1] An IFS on a universe $\mathfrak{W}$ is $\mathfrak{I} = \{\langle a, \mu_{\mathfrak{I}}(\mathfrak{a}), \vartheta_{\mathfrak{I}}(\mathfrak{a})\rangle | \mathfrak{a} \in \mathfrak{W}\}$, where $\mu_{\mathfrak{I}} : \mathfrak{W} \to [0, 1]$ and $\vartheta_{\mathfrak{I}} : \mathfrak{W} \to [0, 1]$ signify the $\mathfrak{M}$ and $\mathfrak{NM}$ grades of $\mathfrak{I}$, and $\mu_{\mathfrak{I}}, \vartheta_{\mathfrak{I}}$ satisfy $0 \leq \mu_{\mathfrak{I}}(a) + \vartheta_{\mathfrak{I}}(\mathfrak{a}) \leq 1 \;\forall\; \mathfrak{a} \in \mathfrak{W}$.

Definition 2.4. [26] An IFG is $\mathbb{G} = (\mathfrak{A}, \mathfrak{B})$ with a IFS ($\mathfrak{A}$) and an IFR ($\mathfrak{B}$) on $\mathfrak{W}$ such that $\mu_{\mathfrak{B}}(\mathfrak{o}\mathfrak{u}) \leq \mu_{\mathfrak{A}}(\mathfrak{o}) \wedge \mu_{\mathfrak{A}}(\mathfrak{u})$, $\vartheta_{\mathfrak{B}}(\mathfrak{o}\mathfrak{u}) \geq \vartheta_{\mathfrak{A}}(\mathfrak{o}) \vee \vartheta_{\mathfrak{A}}(\mathfrak{u})$ and $0 \leq \mu_{\mathfrak{B}}(\mathfrak{o}\mathfrak{u}) + \vartheta_{\mathfrak{B}}(\mathfrak{o}\mathfrak{u}) \leq 1 \;\forall\; \mathfrak{o}, \mathfrak{u} \in \mathfrak{W}$ where $\mu_{\mathfrak{B}} : \mathfrak{W} \times \mathfrak{W} \to [0, 1]$ and $\vartheta_{\mathfrak{B}} : \mathfrak{W} \times \mathfrak{W} \to [0, 1]$ symbolize the $\mathfrak{M}$ and $\mathfrak{NM}$ grades of $\mathfrak{B}$, correspondingly.

Definition 2.5. [10] A SVNS $\mathfrak{A}$ in $\mathfrak{U}$ is defined as an element with a truth-membership $\mu_{\mathfrak{A}}$, an indeterminacy-membership $\beta_{\mathfrak{A}}$ and a falsity-membership $\gamma_{\mathfrak{A}}$ where $\mu, \beta, \gamma \in [0, 1]$ with the constraint $0 \leq \mu_{\mathfrak{A}}(\mathfrak{x}) + \beta_{\mathfrak{A}}(\mathfrak{x}) + \gamma_{\mathfrak{A}}(\mathfrak{x}) \leq 3$.

Definition 2.6. [57] A neutrosophic fuzzy graph is $\mathbb{G} = (\mathfrak{V}, \mathfrak{E})$, where $\mathfrak{V} = \{v_1, v_2, ..., v_n\}$ such that μ_1, β_1, γ_1 are from $\mathfrak{V}$ to $[0, 1]$ with $0 \leq \mu_1(v_i) + \beta_1(v_i) + \gamma_1(v_i) \leq 3 \;\forall v_i \in \mathfrak{V}$ indicates the $\mathfrak{M}$, $\mathfrak{I}$ and $\mathfrak{NM}$ functions and μ_2, β_2, γ_2 are from $\mathfrak{V} \times \mathfrak{V}$ to $[0, 1]$ such that $\mu_2(v_i v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$, $\beta_2(v_i v_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$ and $\gamma_2(v_i v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$ with $0 \leq \mu_2(v_i v_j) + \beta_2(v_i v_j) + \gamma_2(v_i v_j) \leq 3 \;\forall v_i v_j \in \mathfrak{V} \times \mathfrak{V}$.

Definition 2.7. [18] A PFS on a universe $\mathfrak{W}$ is $\mathfrak{A} = \{\langle \mathfrak{s}, \mu_{\mathfrak{A}}(\mathfrak{s}), \vartheta_{\mathfrak{A}}(\mathfrak{s})\rangle | \mathfrak{s} \in \mathfrak{W}\}$, where $\mu_{\mathfrak{A}} : \mathfrak{W} \to [0, 1]$ and $\vartheta_{\mathfrak{A}} : \mathfrak{W} \to [0, 1]$ signify the $\mathfrak{M}$ and $\mathfrak{NM}$ grades of $\mathfrak{A}$, and $\mu_{\mathfrak{A}}, \vartheta_{\mathfrak{A}}$ satisfy $0 \leq \mu^2_{\mathfrak{A}}(\mathfrak{s}) + \vartheta^2_{\mathfrak{A}}(\mathfrak{s}) \leq 1 \;\forall\; \mathfrak{s} \in \mathfrak{W}$.

Definition 2.8. [35] A PFG is $\mathbb{G} = (\mathfrak{A}, \mathfrak{B})$ with $\mathfrak{A}$ and $\mathfrak{B}$, a PFS and PFR on $\mathfrak{W}$ such that $\mu_{\mathfrak{B}}(\mathfrak{w}\mathfrak{z}) \leq \mu_{\mathfrak{A}}(\mathfrak{w}) \wedge \mu_{\mathfrak{A}}(\mathfrak{z})$, $\vartheta_{\mathfrak{B}}(\mathfrak{w}\mathfrak{z}) \geq \vartheta_{\mathfrak{A}}(\mathfrak{w}) \vee \vartheta_{\mathfrak{A}}(\mathfrak{z})$ and $0 \leq \mu^2_{\mathfrak{B}}(\mathfrak{w}\mathfrak{z}) + \vartheta^2_{\mathfrak{B}}(\mathfrak{w}\mathfrak{z}) \leq 1 \;\forall\; \mathfrak{w}, \mathfrak{z} \in \mathfrak{W}$. where $\mu_{\mathfrak{B}} : \mathfrak{W} \times \mathfrak{W} \to [0, 1]$ and $\vartheta_{\mathfrak{B}} : \mathfrak{W} \times \mathfrak{W} \to [0, 1]$ symbolize the $\mathfrak{M}$ and $\mathfrak{N}\mathfrak{M}$ grades of $\mathfrak{B}$, correspondingly.

Definition 2.9. [36] Pythagorean Neutrosophic Graph (PNG) is $\mathbb{G} = (\mathfrak{V}, \mathfrak{E})$, where $\mathfrak{V} = \{v_1, v_2, ..., v_n\}$ such that μ_1, β_1, σ_1 are from $\mathfrak{V}$ to $[0, 1]$ with $0 \leq \mu_1(v_i)^2 + \beta_1(v_i)^2 + \sigma_1(v_i)^2 \leq 2 \;\forall v_i \in \mathfrak{V}$ and indicate the $\mathfrak{M}$, $\mathfrak{I}$ and $\mathfrak{N}\mathfrak{M}$ functions and μ_2, β_2, σ_2 are from $\mathfrak{V} \times \mathfrak{V}$ to $[0, 1]$ such that $\mu_2(v_i v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$, $\beta_2(v_i v_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$ and $\sigma_2(v_i v_j) \leq \sigma_1(v_i) \vee \sigma_1(v_j)$ with $0 \leq \mu_2(v_i v_j)^2 + \beta_2(v_i v_j)^2 + \sigma_2(v_i v_j)^2 \leq 2 \;\forall v_i v_j \in \mathfrak{V} \times \mathfrak{V}$.

3. Regularity of Pythagorean neutrosophic graphs

In this section, we describe the regularity ideas of PNGs. The concept of degree, total degree, regular, and totally regular were discussed in detail with their characterizations and properties.

Definition 3.1. A regular PNG (RPNG) $\mathfrak{G} = (\sigma, \mu)$ over $\mathbb{G}$ is a PNG in which each vertex has the same degree. A PNG is named $< k_1, k_2, k_3 >$-regular if every vertex has $< k_1, k_2, k_3 >$ as degree, i.e., $\mathfrak{d}_{\mathfrak{G}}(v_i) =< k_1, k_2, k_3 > \;\forall\; v_i \in \mathfrak{V}$ degree.

Example 3.2. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG defined on $\mathbb{G} = (\mathfrak{V}, \mathfrak{E})$, where $\mathfrak{V} = \{a, b, c, d\}$ and $\mathfrak{E} = \{ab, bc, cd, de\}$. The PNG is defined by

$$\sigma = \left\langle \frac{a}{(.4, .3, .2)}, \frac{b}{(.7, .2, .4)}, \frac{c}{(.5, .4, .3)}, \frac{d}{(.6, .2, .4)} \right\rangle$$

$$\mu = \left\langle \frac{ab}{(.6, .1, .3)}, \frac{bc}{(.4, .2, .2)}, \frac{cd}{(.6, .1, .3)}, \frac{da}{(.4, .2, .2)} \right\rangle$$

The PNG in Figure 1 has $\mathfrak{d}_{\mathfrak{G}}(a) = \mathfrak{d}_{\mathfrak{G}}(b) = \mathfrak{d}_{\mathfrak{G}}(c) = \mathfrak{d}_{\mathfrak{G}}(d) =< 1, .3, .5 >$. Thus, $\mathfrak{G}$ is $< 1, .3, .5 >$-RPNG.

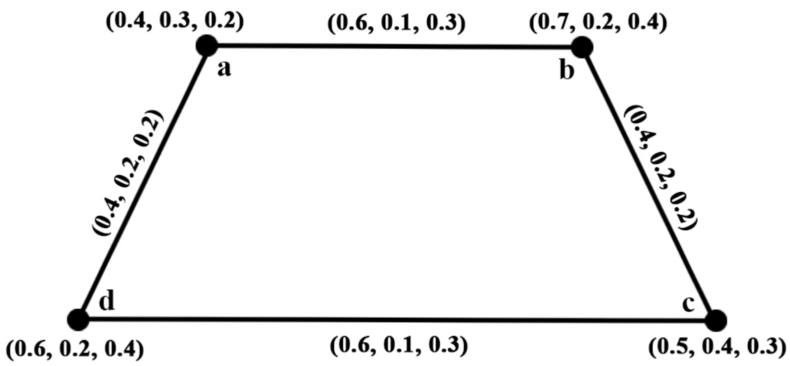

Figure 1. $< 1, .3, .5 >$- regular PNG.

Definition 3.3. The degree ($\mathfrak{D}$) of an edge $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$ of $\mathfrak{G} = (\sigma, \mu)$ is
$\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) \right\rangle$, where

$$\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_k \in \mathfrak{E},\; k \neq j} \mu_1(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k \in \mathfrak{E},\; k \neq i} \mu_1(\mathfrak{h}_j\mathfrak{h}_k),$$

$$\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_k\in\mathfrak{E},\ k\neq j} \mu_2(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k\in\mathfrak{E},\ k\neq i} \mu_2(\mathfrak{h}_j\mathfrak{h}_k),$$

$$\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_k\in\mathfrak{E},\ k\neq j} \mu_3(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k\in\mathfrak{E},\ k\neq i} \mu_3(\mathfrak{h}_j\mathfrak{h}_k).$$

Definition 3.4.

1) $S_{\mathfrak{E}}(\mathfrak{G}) = \langle S_{\mathfrak{M}}(\mathfrak{G}), S_{\mathfrak{I}}(\mathfrak{G}), S_{\mathfrak{N}\mathfrak{M}}(\mathfrak{G})\rangle$ is the minimum edge degree ($\mathfrak{E}\mathfrak{D}$) of $\mathfrak{G}$, where
$S_{\mathfrak{M}}(\mathfrak{G}) = min\left\{\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$,
$S_{\mathfrak{I}}(\mathfrak{G}) = min\left\{\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$,
$S_{\mathfrak{N}\mathfrak{M}}(\mathfrak{G}) = max\left\{\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$.

2) $\Delta_{\mathfrak{E}}(\mathfrak{G}) = (\Delta_{\mathfrak{M}}(\mathfrak{G}), \Delta_{\mathfrak{I}}(\mathfrak{G}), \Delta_{\mathfrak{N}\mathfrak{M}}(\mathfrak{G}))$ is the maximum $\mathfrak{E}\mathfrak{D}$ of $\mathfrak{G}$, where
$\Delta_{\mathfrak{M}}(\mathfrak{G}) = max\left\{\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$,
$\Delta_{\mathfrak{I}}(\mathfrak{G}) = max\left\{\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$,
$\Delta_{\mathfrak{N}\mathfrak{M}}(\mathfrak{G}) = min\left\{\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)/\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}\right\}$.

Definition 3.5. Total edge degree ($\mathfrak{T}\mathfrak{E}\mathfrak{D}$) of $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$ in $\mathfrak{G} = (\sigma, \mu)$ is
$\mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{t}\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{t}\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{t}\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$ where

$\mathfrak{t}\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_j) + \mu_1(\mathfrak{h}_i\mathfrak{h}_j)$
$\mathfrak{t}\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_j) + \mu_2(\mathfrak{h}_i\mathfrak{h}_j)$
$\mathfrak{t}\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_j) + \mu_3(\mathfrak{h}_i\mathfrak{h}_j)$.

Example 3.6. The graph $\mathbb{G} = (\mathfrak{V}, \mathfrak{E})$, with $\mathfrak{V} = \{a, b, c, d\}$ and $\mathfrak{E} = \{ab, ac, ad, bc, cd\}$. PNG $\mathfrak{G} = (\sigma, \mu)$ is defined as

$$\sigma = \left\langle \frac{a}{(.7, .3, .2)}, \frac{b}{(.6, .4, .2)}, \frac{c}{(.5, .3, .4)}, \frac{d}{(.8, .2, .3)} \right\rangle$$

$$\mu = \left\langle \frac{ab}{(.5, .3, .2)}, \frac{ac}{(.5, .3, .4)}, \frac{ad}{(.6, .2, .3)}, \frac{bc}{(.4, .3, .3)}, \frac{cd}{(.5, .2, .4)} \right\rangle$$

In Figure 2 $\mathfrak{d}_{\mathfrak{G}}(ab) = (1.5, .8, 1)$, $\mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(ab) = (2, 1.1, 1.2)$ are the $\mathfrak{D}$ and $\mathfrak{T}\mathfrak{D}$ of ab. Similarly the $\mathfrak{D}$ and $\mathfrak{T}\mathfrak{D}$ of other edges of the graph can be calculated.

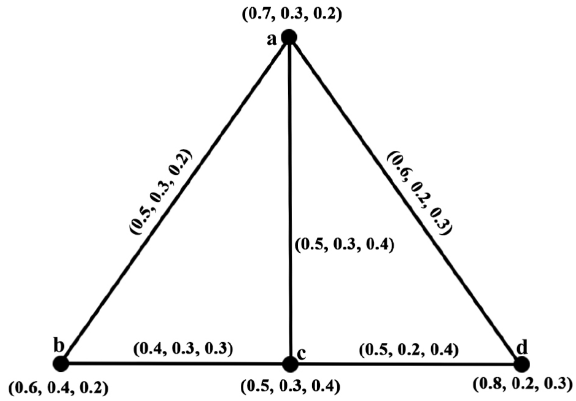

Figure 2. Pythagorean Neutrosophic Graph $\mathfrak{G} = (\sigma, \mu)$.

Definition 3.7. A PNG $\mathfrak{G}$ having same degree for every edge is named as the edge regular PNG (ERPNG). $\mathfrak{G}$ is called $< g_1, g_2, g_3 >$ ER, when each edge has degree $< g_1, g_2, g_3 >$, i.e., $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) =< g_1, g_2, g_3 > \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.

Definition 3.8. A PNG $\mathfrak{G}$ is termed as totally edge regular PNG (TERPNG) when each edge has the same $\mathfrak{TD}$.

Theorem 3.9. $\mathfrak{G} = (\sigma, \mu)$ is a PNG on a cycle $\mathbb{G}$. Then $\sum\limits_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i) = \sum\limits_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j)$.

Proof. Let $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 ... \mathfrak{h}_n$ be the vertices of PNG $\mathfrak{G}$ on a cycle $\mathbb{G}$. Then

$$\sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_{i+1}) = \langle \sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_{i+1}), \sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_{i+1}), \sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_{i+1}) \rangle.$$

Consider, $\sum\limits_{i=1}^{n} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_{i+1}) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_1\mathfrak{h}_2) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_2\mathfrak{h}_3) + ... + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_n\mathfrak{h}_1)$, where $\mathfrak{h}_{n+1} = \mathfrak{h}_1$

$$= \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_1) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_2) - 2\mu_1(\mathfrak{h}_1\mathfrak{h}_2) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_2) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_3) - 2\mu_1(\mathfrak{h}_2\mathfrak{h}_3) + ... + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_n) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_1) - 2\mu_1(\mathfrak{h}_n\mathfrak{h}_1)$$

$$= 2\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_1) + 2\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_2) + 2\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_3) + ... + 2\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_n) - 2[\mu_1(\mathfrak{h}_1\mathfrak{h}_2) + \mu_1(\mathfrak{h}_2\mathfrak{h}_3) + ... + \mu_1(\mathfrak{h}_n\mathfrak{h}_1)]$$

$$= 2\sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) - 2\sum_{i=1}^{n} \mu_1(\mathfrak{h}_i\mathfrak{h}_{i+1})$$

$$= 2\sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) - 2\sum_{i=1}^{n} \mu_1(\mathfrak{h}_i\mathfrak{h}_{i+1})$$

$$= \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) - 2\sum_{i=1}^{n} \mu_1(\mathfrak{h}_i\mathfrak{h}_{i+1})$$

$$= \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + 2\sum_{i=1}^{n} \mu_1(\mathfrak{h}_i\mathfrak{h}_{i+1}) - 2\sum_{i=1}^{n} \mu_1(\mathfrak{h}_i\mathfrak{h}_{i+1}) = \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) \tag{3.1}$$

In a similar way, $$\sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_{i+1}) = \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i) \tag{3.2}$$

and $$\sum_{i=1}^{n} \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_{i+1}) = \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i) \tag{3.3}$$

By using the values in (3.1), (3.2) and (3.3), we get

$$\sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i), \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i), \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i) \right\rangle = \sum_{\mathfrak{h}_i \in \mathfrak{V}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i).$$

Theorem 3.10. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG on a RCG $\mathbb{G}$. Then

$$\sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j) \rangle$$

where $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j) - 2\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.

Proof. Consider the PNG $\mathfrak{G}$ on RCG $\mathbb{G}$. $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j) \rangle$, where $\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)$ are the total of $\mathfrak{M}$, $\mathfrak{I}$ and $\mathfrak{NM}$ values of the adjacent edges, correspondingly.

Every edge in $\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)$ has $\mathfrak{M}$ value exactly of the edge in it's respective crisp graph times.

Thus, $$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j)$$
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j)$$
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j)$$

Hence $$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle.$$

Proposition 3.11. $\mathfrak{G} = (\sigma, \mu)$ be a PNG on a $\mathfrak{k}$-RCG $\mathbb{G}$. Then
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i), (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i), (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i)\rangle.$$

Proof. Let $\mathfrak{G}$ be a PNG on $\mathfrak{k}$-RCG $\mathbb{G}$. Then by theorem 3.10,
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$$
$$= \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} (\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j) - 2)\mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} (\mathfrak{d}_{G}(\mathfrak{h}_i) + \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j) - 2)\mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} (\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) + \mathfrak{d}_{G}(\mathfrak{h}_j) - 2)\mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$$
Since $\mathbb{G}$ is $\mathfrak{k}$-RCG, $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) = \mathfrak{k}$ where $\mathfrak{h}_i \in \mathfrak{V}, \forall\ i$ we have
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle (\mathfrak{k}+\mathfrak{k}-2)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), (\mathfrak{k}+\mathfrak{k}-2)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), (\mathfrak{k}+\mathfrak{k}-2)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$$
$$= \langle 2(\mathfrak{k}-1)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), 2(\mathfrak{k}-1)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), 2(\mathfrak{k}-1)\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$$
$$= \langle (\mathfrak{k}-1)\, 2\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), (\mathfrak{k}-1)\, 2\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), (\mathfrak{k}-1)\, 2\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$$
$$= \langle (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i), (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i), (\mathfrak{k}-1)\sum_{\mathfrak{h}_i\in\mathfrak{V}} \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i)\rangle.$$

Proposition 3.12. $\mathfrak{G} = (\sigma, \mu)$ be a PNG on a RCG $\mathbb{G}$.

Then $$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j),$$
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j),$$
$$\sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j\in\mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle.$$

Theorem 3.13. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG. Then (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$ iff the following statements hold:

1) $\mathfrak{G}$ is an ERPNG.
2) $\mathfrak{G}$ is a TERPNG.

Proof. Let us consider a $\mathfrak{C}\mathfrak{F}$, (μ_1, μ_2, μ_3), then $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$, $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3$ $\forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$, $\mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3$ are $\mathfrak{C}\mathfrak{s}$.
To prove: (1) $\Rightarrow$ (2)

Let $\mathfrak{G}$ be the $< f_1, f_2, f_3 >$-ERPNG. Then $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) =< f_1, f_2, f_3 > \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
$t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)+ \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$
$= \langle f_1 + \mathfrak{y}_1, f_2 + \mathfrak{y}_2, f_3 + \mathfrak{y}_3\rangle \ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
Thus $\mathfrak{G}$ is a TERPNG.
To prove: (2) $\Rightarrow$ (1)
Let $\mathfrak{G}$ be a $< f_1, f_2, f_3 >$-TERPNG.
Then $\mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$
$=< t_1, t_2, t_3 > \quad \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
Now, $\left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle = \left\langle t_1 - \mu_1(\mathfrak{h}_i\mathfrak{h}_j), t_2 - \mu_2(\mathfrak{h}_i\mathfrak{h}_j), t_1 - \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$
$\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) =< t_1 - \mathfrak{y}_1, t_2 - \mathfrak{y}_2, t_3 - \mathfrak{y}_3 >$. Thus $\mathfrak{G}$ is a $< t_1 - \mathfrak{y}_1, t_2 - \mathfrak{y}_2, t_3 - \mathfrak{y}_3 >$-ERPNG.
Conversely, consider that (1) & (2) are equivalent. We claim that (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$.
Let us assume (μ_1, μ_2, μ_3) to be not a $\mathfrak{C}\mathfrak{F}$. Then $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_1(\mathfrak{h}_q\mathfrak{h}_r)$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_2(\mathfrak{h}_q\mathfrak{h}_r)$, $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_3(\mathfrak{h}_q\mathfrak{h}_r)$ for at least one duo of $(\mathfrak{h}_i\mathfrak{h}_j), (\mathfrak{h}_q\mathfrak{h}_r) \in \mathfrak{E}$. Consider $\mathfrak{G}$ is a $< f_1, f_2, f_3 >$-ERPNG. Then $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_q\mathfrak{h}_r) =< f_1, f_2, f_3 >$. Thus
$\mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) + \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\rangle$
$= \left\langle f_1 + \mu_1(\mathfrak{h}_i\mathfrak{h}_j), f_2 + \mu_2(\mathfrak{h}_i\mathfrak{h}_j), f_3 + \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$
$t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_q\mathfrak{h}_r) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_q\mathfrak{h}_r) + \mu_1(\mathfrak{h}_q\mathfrak{h}_r), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_q\mathfrak{h}_r) + \mu_2(\mathfrak{h}_q\mathfrak{h}_r), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_q\mathfrak{h}_r) + \mu_3(\mathfrak{h}_q\mathfrak{h}_r)\rangle$
$= \left\langle f_1 + \mu_1(\mathfrak{h}_q\mathfrak{h}_r), f_2 + \mu_2(\mathfrak{h}_q\mathfrak{h}_r), f_3 + \mu_3(\mathfrak{h}_q\mathfrak{h}_r)\right\rangle$
As $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_1(\mathfrak{h}_q\mathfrak{h}_r), \mu_2(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_2(\mathfrak{h}_q\mathfrak{h}_r), \mu_3(\mathfrak{h}_i\mathfrak{h}_j) \neq \mu_3(\mathfrak{h}_q\mathfrak{h}_r)$, so $t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_q\mathfrak{h}_r)$.
We get $\mathfrak{G}$ is not a TER which is a contradiction, thus (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$.
Likewise, we can prove (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$, if $\mathfrak{G}$ is a TERPNG.

Theorem 3.14. If a PNG $\mathfrak{G}$ is both ER and TER, then (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$.

Theorem 3.15. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG on a $\mathfrak{k}$-RCG $\mathbb{G}$. (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$ iff $\mathfrak{G}$ is both RPNG and TERPNG.

Proof. $\mathfrak{G}$ be a PNG on a $\mathfrak{k}$-RCG $\mathbb{G}$. Consider (μ_1, μ_2, μ_3) is a $\mathfrak{C}\mathfrak{F}$, that is, $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3 \ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$, where $\mathfrak{y}_1, \mathfrak{y}_2$ and $\mathfrak{y}_3$ are $\mathfrak{C}\mathfrak{s}$. From the definition of vertex degree, we have
$\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i), \mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i)\rangle$

$$= \left\langle \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$$

$$= \left\langle \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{y}_1, \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{y}_2, \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{y}_3 \right\rangle$$

$= \langle \mathfrak{k}\mathfrak{y}_1, \mathfrak{k}\mathfrak{y}_2, \mathfrak{k}\mathfrak{y}_3\rangle \ \forall\ \mathfrak{h}_i \in \mathfrak{B}$.
Therefore, $\mathfrak{G}$ is RPNG. Now $\mathfrak{t}\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{t}\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{t}\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{t}\mathfrak{d}_{\mathfrak{N}\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$, where

$$\mathfrak{t}\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_k \in \mathfrak{E},\ k \neq j} \mu_1(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k \in \mathfrak{E},\ k \neq i} \mu_1(\mathfrak{h}_j\mathfrak{h}_k) + \mu_1(\mathfrak{h}_i\mathfrak{h}_j)$$
$$= \sum_{\mathfrak{h}_i\mathfrak{h}_k \in \mathfrak{E},\ k \neq j} \mathfrak{y}_1 + \sum_{\mathfrak{h}_i\mathfrak{h}_k \in \mathfrak{E},\ k \neq i} \mathfrak{y}_1 + \mathfrak{y}_1 = \mathfrak{y}_1(\mathfrak{k} - 1) + \mathfrak{y}_1(\mathfrak{k} - 1) + \mathfrak{y}_1 = \mathfrak{y}_1(2\mathfrak{k} - 1) \ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}.$$

Likewise, $t\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j)= \mathfrak{y}_2(2\mathfrak{k} - 1)$ and $\mathfrak{t}\mathfrak{d}_{\mathfrak{N}\mathfrak{M}} = \mathfrak{y}_3(2\mathfrak{k} - 1) \ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$ can also be expressed.
Thus $\mathfrak{G}$ is a TERPNG.
Conversely, consider $\mathfrak{G}$ is both RPNG and TERPNG. We have to prove that $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{C}\mathfrak{F}$.

Since PNG $\mathfrak{G}$ is RPNG, $t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) =< t_1, t_2, t_3 > \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$. By definition of $\mathfrak{TED}$, we have
$t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle t\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), t\mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j), t\mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$, where, $t\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_j) - \mu_1(\mathfrak{h}_i\mathfrak{h}_j)$

$$t_1 = g_1 + g_1 - \mu_1(\mathfrak{h}_i\mathfrak{h}_j)$$
$$t\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = 2g_1 - t_1\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}.$$

In the same way, we can illustrate that $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = 2g_2 - t_2$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = 2g_3 - t_3\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
Therefore $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$.

Theorem 3.16. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG on a crisp graph $\mathbb{G}$. If (μ_1, μ_2, μ_3) is a $\mathfrak{CF}$, then $\mathfrak{G}$ is an ERPNG iff $\mathbb{G}$ is ER.

Proof. Consider $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$, i.e., $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$, where $\mathfrak{y}_1, \mathfrak{y}_2$ and $\mathfrak{y}_3$ are $\mathfrak{Cs}$. Assume that $\mathfrak{G}$ is an ERPNG. We claim that $\mathbb{G}$ is an ER. On the other hand assume $\mathbb{G}$ is not an ER. i.e., $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_l\mathfrak{h}_m)$ for at least one duo of $\mathfrak{h}_i\mathfrak{h}_j, \mathfrak{h}_l\mathfrak{h}_m \in \mathfrak{E}$. By the definition of $\mathfrak{ED}$ of PNG, $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$, where

$$\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \sum_{\mathfrak{h}_i\mathfrak{h}_k\in\mathfrak{E},\ k\neq j} \mu_1(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k\in\mathfrak{E},\ k\neq i} \mu_1(\mathfrak{h}_j\mathfrak{h}_k) = \sum_{\mathfrak{h}_i\mathfrak{h}_k\in\mathfrak{E},\ k\neq j} \mathfrak{y}_1 + \sum_{\mathfrak{h}_j\mathfrak{h}_k\in\mathfrak{E},\ k\neq i} \mathfrak{y}_1 = \mathfrak{y}_1(\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i)-1)+\mathfrak{y}_1(\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j)-1)$$
$$= \mathfrak{y}_1(\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j) - 2) = \mathfrak{y}_1\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}.$$

Likewise, we can illustrate that $\mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)$ and $\mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$. Therefore $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{y}_1\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{y}_2\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{y}_3\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$, $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_l\mathfrak{h}_m) = \langle \mathfrak{y}_1\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_l\mathfrak{h}_m), \mathfrak{y}_2\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_l\mathfrak{h}_m), \mathfrak{y}_3\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_l\mathfrak{h}_m)\rangle$, $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_l\mathfrak{h}_m)$. so $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_l\mathfrak{h}_m)$. Therefore $\mathfrak{G}$ is not ER, a contradiction. Thus $\mathbb{G}$ is ER.
Conversely, assume $\mathbb{G}$ is an ER graph. We claim $\mathfrak{G}$ is an ERPNG.
Consider $\mathfrak{G}$ is not an ERPNG. i.e., $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_p\mathfrak{h}_q)$ for at least one duo of $\mathfrak{h}_i\mathfrak{h}_j, \mathfrak{h}_p\mathfrak{h}_q \in \mathfrak{E}$,
$\left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle \neq \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_p\mathfrak{h}_q), \mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_p\mathfrak{h}_q), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_p\mathfrak{h}_q)\right\rangle$. Now $\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_p\mathfrak{h}_q)$ implies

$$\sum_{\mathfrak{h}_i\mathfrak{h}_k\in\mathfrak{E},\ k\neq j} \mu_1(\mathfrak{h}_i\mathfrak{h}_k) + \sum_{\mathfrak{h}_j\mathfrak{h}_k\in\mathfrak{E},\ k\neq i} \mu_1(\mathfrak{h}_j\mathfrak{h}_k) \neq \sum_{\mathfrak{h}_p\mathfrak{h}_s\in\mathfrak{E},\ s\neq q} \mu_1(\mathfrak{h}_p\mathfrak{h}_s) + \sum_{\mathfrak{h}_s\mathfrak{h}_q\in\mathfrak{E},\ s\neq p} \mu_1(\mathfrak{h}_s\mathfrak{h}_q),$$

since μ_1 is a $\mathfrak{CF}$, so $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) \neq \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_p\mathfrak{h}_q)$, a contradiction. Therefore $\mathfrak{G}$ is an ERPNG.

Theorem 3.17. Let $\mathfrak{G} = (\sigma, \mu)$ be a RPNG. Then $\mathfrak{G}$ is an ERPNG iff (μ_1, μ_2, μ_3) is a $\mathfrak{CF}$.

Proof. $\mathfrak{G}$ be a $< g_1, g_2, g_3 >$-regular PNG, (ie) $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) =< g_1, g_2, g_3 > \forall\ \mathfrak{h}_i \in \mathfrak{V}$. Consider that $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$. Then $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$, and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$, where $\mathfrak{y}_1, \mathfrak{y}_2$ and $\mathfrak{y}_3$ are $\mathfrak{Cs}$.
By the definition of $\mathfrak{ED}$ of a PNG, $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$,
where $\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_j) - 2\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = g_1 + g_1 - 2\mathfrak{y}_1 = 2(g_1 - \mathfrak{y}_1)\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
In the same way, $\mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j) = 2(g_2 - \mathfrak{y}_2)$ and $\mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j) = 2(g_2 - \mathfrak{y}_3)\ \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
Therefore $\mathfrak{G}$ is an ERPNG.
Conversely, presume that $\mathfrak{G}$ is an ERPNG, i.e., $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) =< f_1, f_2, f_3 > \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
We claim that $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$. Since $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{J}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j)\right\rangle$,
where $\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_j) - 2\mu_1(\mathfrak{h}_i\mathfrak{h}_j)$

$$f_1 = g_1 + g_1 - 2\mu_1(\mathfrak{h}_i\mathfrak{h}_j)$$
$$\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \tfrac{2g_1 - f_1}{2}\ \forall\ (\mathfrak{h}_i\mathfrak{h}_j) \in \mathfrak{E}.$$

In the same way, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \frac{2g_2 - f_2}{2}$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \frac{2g_3 - f_3}{2}\ \forall\ (\mathfrak{h}_i\mathfrak{h}_j) \in \mathfrak{E}$.
Hence $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$.

Definition 3.18. A PNG $\mathfrak{G}$ is said to be a

1) partially regular (PR), if $\mathbb{G}$ is regular.
2) full regular (FR), if $\mathfrak{G}$ is both regular and PR.
3) partially ER (PER), if $\mathbb{G}$ is an ER.
4) full edge regular (FER), if $\mathfrak{G}$ is both ER and PER.

Theorem 3.19. Let $\mathfrak{G} = (\sigma, \mu)$ be a PNG on $\mathbb{G}$ such that (μ_1, μ_2, μ_3) is a $\mathfrak{CF}$. If $\mathfrak{G}$ is a FRPNG, then $\mathfrak{G}$ is a FERPNG.

Proof. Suppose that $< \mu_1, \mu_2, \mu_3 >$ is a $\mathfrak{CF}$. Then $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1, \mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3 \forall$ $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$,
where $\mathfrak{y}_1, \mathfrak{y}_2$ and $\mathfrak{y}_3$ are $\mathfrak{C}\mathfrak{s}$. $\mathfrak{G}$ be a FRPNG, then $\mathfrak{d}_G(\mathfrak{h}_i) = \mathfrak{k}$ and $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i) =< g_1, g_2, g_3 > \forall\ \mathfrak{h}_i \in \mathfrak{V}$,
where $\mathfrak{k}, g_1, g_2, g_3$ are $\mathfrak{C}\mathfrak{s}$. $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_j) - 2 = 2\mathfrak{k} - 2 = \mathfrak{C}$.
Therefore $\mathbb{G}$ is an ER graph. Then, $\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \left\langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j) \right\rangle \forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$,
where $\mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i) + \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_j) - 2\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = g_1 + g_2 - 2\mathfrak{y}_1 = 2g_1 - 2\mathfrak{y}_1 =$ a constant.
Similarly, $\mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i\mathfrak{h}_j) = 2g_2 - 2\mathfrak{y}_2 =$ constant and $\mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i\mathfrak{h}_j) = 2g_3 - 2\mathfrak{y}_2 =$ a constant $\forall\ \mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$.
Therefore $\mathfrak{G}$ is an ERPNG. Thus $\mathfrak{G}$ is a FERPNG.

Theorem 3.20. Let $\mathfrak{G} = (\sigma, \mu)$ be a t-TER and t'-PERPNG. Then $S(\mathfrak{G}) = \frac{rt}{1+t'}$, where $r = |\mathfrak{E}|$.

Proof. The size of $\mathfrak{G}$ is $S(\mathfrak{G}) = \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j) \rangle$.

Meanwhile $\mathfrak{G}$ is t-TER and t'-PERPNG, i.e., $t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = t$ and $\mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j) = t'$, correspondingly.

$$\begin{aligned}
\text{Therefore, } \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} t\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i\mathfrak{h}_j) = \langle &\sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \\
&\sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \\
&\sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j) + \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j) \rangle.
\end{aligned}$$

$$= \langle \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}} \mathfrak{d}_{\mathbb{G}}(\mathfrak{h}_i\mathfrak{h}_j)\mu_3(\mathfrak{h}_i\mathfrak{h}_j) \rangle + S(\mathfrak{G})$$

$rt = t'S(\mathfrak{G}) + S(\mathfrak{G})$.

Definition 3.21. A $g =< g_1, g_2, g_3 >$ RPNG $\mathfrak{G}$ on n vertices is said to be strongly regular PNG (SRPNG), if it holds the following characteristics:

1) Total of $\mathfrak{M}$, $\mathfrak{I}$, and $\mathfrak{NM}$ values in the same neighbourhood of any 2 adjacent vertices of $\mathfrak{G}$ have $\lambda =< \lambda_1, \lambda_2, \lambda_3 >$ weight.
2) Total of $\mathfrak{M}$, $\mathfrak{I}$, and $\mathfrak{NM}$ values in the same neighbourhood of any 2 non-adjacent vertices of $\mathfrak{G}$ have $\chi =< \chi_1, \chi_2, \chi_3 >$ weight.

A SRPNG $\mathfrak{G}$ is represented by $\mathfrak{G} = (\sigma, \mu, \lambda, \chi)$.

Theorem 3.22. Let $\mathfrak{G} = (\sigma, \mu)$ be a complete PNG with $(\sigma_1, \sigma_2, \sigma_3)$ and (μ_1, μ_2, μ_3) as $\mathfrak{CF}$. Then $\mathfrak{G}$ is a SRPNG.

Proof. Assume $\mathfrak{G} = (\sigma, \mu)$ is a complete PNG over n vertices. Since $< \mu_1, \mu_2, \mu_3 >$ and $< \sigma_1, \sigma_2, \sigma_3 >$ are $\mathfrak{CF}$, so $\sigma_1(\mathfrak{h}_i) = \mathfrak{y}_1'$, $\sigma_2(\mathfrak{h}_i) = \mathfrak{y}_2'$ and $\sigma_3(\mathfrak{h}_i) = \mathfrak{y}_3'$ $\forall$ $\mathfrak{h}_i \in \mathfrak{V}$, $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_1$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_2$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = \mathfrak{y}_3$ $\forall$ $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{E}$, where $\mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3', \mathfrak{y}_1, \mathfrak{y}_2$ and $\mathfrak{y}_3$ are $\mathfrak{Cs}$. To verify $\mathfrak{G}$ is a SRPNG, we intend to illustrate $\mathfrak{G}$ is $g =< \sigma_1, \sigma_2, \sigma_3 >$-RPNG and the adjacent and non-adjacent vertices have the same CN $\lambda =< \lambda_1, \lambda_2, \lambda_3 >$ and $\chi =< \chi_1, \chi_2, \chi_3 >$ correspondingly. Since $\mathfrak{G}$ is complete PNG

$$\mathfrak{d}_{\mathfrak{G}}(\mathfrak{h}_i) = \langle \mathfrak{d}_{\mathfrak{M}}(\mathfrak{h}_i), \mathfrak{d}_{\mathfrak{I}}(\mathfrak{h}_i), \mathfrak{d}_{\mathfrak{NM}}(\mathfrak{h}_i) \rangle = \langle \sum_{\mathfrak{h}_i, \mathfrak{h}_j \neq \mathfrak{h}_i \in \mathfrak{V}} \mu_1(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i, \mathfrak{h}_j \neq \mathfrak{h}_i \in \mathfrak{V}} \mu_2(\mathfrak{h}_i\mathfrak{h}_j), \sum_{\mathfrak{h}_i, \mathfrak{h}_j \neq \mathfrak{h}_i \in \mathfrak{V}} \mu_3(\mathfrak{h}_i\mathfrak{h}_j) \rangle$$

$$= \langle (n-1)\mathfrak{y}_1, (n-1)\mathfrak{y}_2, (n-1)\mathfrak{y}_3 \rangle$$

Hence $\mathfrak{G}$ is a $\langle (n-1)\mathfrak{y}_1, (n-1)\mathfrak{y}_2, (n-1)\mathfrak{y}_3 \rangle$-RPNG.
The sum of $\mathfrak{M}$, $\mathfrak{I}$, and $\mathfrak{NM}$ values of CN vertices of any two adjacent vertices.
$\lambda = \left\langle (n-1)\mathfrak{y}_1', (n-1)\mathfrak{y}_2', (n-1)\mathfrak{y}_3' \right\rangle$ are the same and the total of $\mathfrak{M}$, $\mathfrak{I}$ and $\mathfrak{NM}$ values of CN vertices of any two non-adjacent vertices $\chi =< 0, 0, 0 >$ are the same.

Definition 3.23. A PNG $\mathfrak{G} = (\sigma, \mu)$ is bipartite if the vertex set $\mathfrak{V}$ can be separated into two non-empty sets $\mathfrak{V}_1$ & $\mathfrak{V}_2$ such that $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = 0$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = 0$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = 0$ if $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{V}_1$ or $\mathfrak{h}_i\mathfrak{h}_j \in \mathfrak{V}_2$. Further if $\mu_1(\mathfrak{h}_i\mathfrak{h}_j) = min\left\{\sigma_1(\mathfrak{h}_i), \sigma_1(\mathfrak{h}_j)\right\}$, $\mu_2(\mathfrak{h}_i\mathfrak{h}_j) = min\left\{\sigma_2(\mathfrak{h}_i), \sigma_2(\mathfrak{h}_j)\right\}$ and $\mu_3(\mathfrak{h}_i\mathfrak{h}_j) = min\{\sigma_3(\mathfrak{h}_i), \sigma_3(\mathfrak{h}_j)\}$ $\forall$ $\mathfrak{h}_i \in \mathfrak{V}_1$ and $\mathfrak{h}_j \in \mathfrak{V}_2$, then $\mathfrak{G}$ is said to be a complete bipartite PNG (CBPNG).

Definition 3.24. A bipartite PNG $\mathfrak{G}$ is biregular PNG (BRPNG) if each vertex in $\mathfrak{V}_1$ has similar degree $\varphi =< \varphi_1, \varphi_2, \varphi_3 >$ and all the vertex in $\mathfrak{V}_2$ has similar degree $\psi =< \psi_1, \psi_2, \psi_3 >$ where φ and ψ are all $\mathfrak{Cs}$.

4. MCDM method based on the Pythagorean neutrosophic graphs

PN sets have become an interesting topic in research due to their powerful dealing with incomplete, inconsistent information. PNG can illustrate the uncertainty in a real-life context, thus we propose the use of PNG in solving MCDM problems. This newly proposed model is named the PNG-based MCDM method.

At first to frame the algorithm or method, we describe the decision making problem. Consider that $\mathfrak{P} = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, ...\mathfrak{p}_m\}$ is a collection of alternatives and $\mathfrak{B} = \{\alpha_1, \alpha_2, \alpha_3, ...\alpha_n\}$ is a set of criteria, with weight vector $\mathfrak{w} = (\mathfrak{w}_1, \mathfrak{w}_2, \mathfrak{w}_3, ...\mathfrak{w}_n)^T$ fulfilling $\mathfrak{w}_j \in [0, 1]$, $\sum_{j=1}^{n} \mathfrak{w}_j = 1$. If the decision maker provides a PN value for the alternative $\mathfrak{p}_k$ $(k = 1, 2, 3, ..., m)$ under the attribute α_j $(j = 1, 2, .., n)$ and can be characterized by a PN number (PNN) $\mathfrak{d}_{kj} = \mathfrak{m}_{kj}, \mathfrak{id}_{kj}, \mathfrak{nm}_{kj}$ (where $\mathfrak{m}, \mathfrak{id}, \mathfrak{nm}$ represents the membership, indeterminacy and non-membership value) $j = 1, 2, .., n; k = 1, 2, 3..., m$. Imagine that $\mathfrak{D} = [\mathfrak{d}_{kj}]_{m \times n}$ is the decision matrix, where $\mathfrak{d}_{kj}$ is given by PNN. If the decision maker provides a PN value for the alternative $\mathfrak{p}_k(k = 1, 2, ...m)$ under the criteria $\alpha_j(j = 1, 2...n)$, these values are illustrated as $\mathfrak{e}_{kj} = (\mathfrak{m}_{kj}, \mathfrak{id}_{kj}, \mathfrak{nm}_{kj})$, $(j = 1, 2, ...n, k = 1, 2...m)$. If there exists a relation between two criteria $\alpha_i = (\mathfrak{m}_i, \mathfrak{id}_i, \mathfrak{nm}_i)$ and $\alpha_j = (\mathfrak{m}_j, \mathfrak{id}_j, \mathfrak{nm}_j)$, we represent the PN relation as $\beta_{ij} = (\mathfrak{m}_{ij}, \mathfrak{id}_{ij}, \mathfrak{nm}_{ij})$, with the properties: $\mathfrak{m}_{ij} \leq min(\mathfrak{m}_i, \mathfrak{m}_j)$, $\mathfrak{id}_{ij} \leq min(\mathfrak{id}_i, \mathfrak{id}_j)$, $\mathfrak{nm}_{ij} \leq max(\mathfrak{nm}_i, \mathfrak{nm}_j)$ $\forall$ $(i, j = 1, 2, ...m)$; otherwise $\beta_{ij} =< 0, 0, 1 >$. Based on the established PNG structure, we suggest a technique for decision-maker to choose the best alternative with PN information.

The technique has been illustrated in the steps following:
Step 1: Compute the influence co-efficient between the criteria α_i and α_j $(i, j = 1, 2, ...n)$ in decision

process by,

$$\chi_{ij} = \frac{\mathfrak{m}_{ij} + (1 - \mathfrak{id}_{ij})(1 - \mathfrak{nm}_{ij})}{2} \tag{4.1}$$

where $\beta_{ij} = (\mathfrak{m}_{ij}, \mathfrak{id}_{ij}, \mathfrak{nm}_{ij})$ is the PN edge between the verices α_i and α_j $(i, j = 1, 2, ...n)$. we have $\chi_{ij} = 1$ and $\chi_{ij} = \chi_{ji}$ for $i = j$.

Step 2: Obtain the complete criterion value of the alternative $\mathfrak{p}_k(k = 1, 2, ...m)$ by

$$\widetilde{\mathfrak{p}_k} = \sum_{j=1}^{n} \mathfrak{w}_j \left(\sum_{i=1}^{n} \chi_{ki} \mathfrak{d}_{ij} \right) \tag{4.2}$$

where $\mathfrak{e}_{ki} = (\mathfrak{m}_{ki}, \mathfrak{id}_{ki}, \mathfrak{nm}_{ki})$ is a PNN.

Step 3: Compute the score value of the alternative $\mathfrak{p}_k(k = 1, 2, ...m)$ which is expressed by:

$$S(\widetilde{\mathfrak{p}_k}) = \frac{1 + \mathfrak{m} - \mathfrak{id} - \mathfrak{nm}}{2}$$

Step 4: Rank all the alternatives $\mathfrak{p}_k(k = 1, 2, ...m)$ and choose the top one in concordance with $S(\widetilde{\mathfrak{p}_k})$.

4.1. An illustrative example

In this subsection, an illustration of PNG based MCDM problem with PN information is applied to show the application and efficiency of the suggested decision-making method.

An investing company wants to invest a sum of money in the best option. There is a panel of attributes with YouTube channels in which to invest the money: (1) $\mathfrak{p}_1$ is a movie review channel, (2) $\mathfrak{p}_2$ is an educational content channel, (3) $\mathfrak{p}_3$ is a food related channel (food review, cooking guidance) (4) $\mathfrak{p}_4$ is a technical content channel. The investment company must take a decision according to the criteria (1) α_1 is the subscribers (2) α_2 is the content worth (3) α_3 is the growth of the criteria is given by $\mathfrak{w} = (.34, .33, .33)$. The four possible alternatives are to be calculated under these three criteria shown in the form of PN information by decision-maker. The information evaluation is consistent to $\alpha_j(j = 1, 2, 3)$ on the alternative $\mathfrak{p}_k(k = 1, 2, 3, 4)$ under the factors $\alpha_j(j = 1, 2, 3)$ and the resultant PN decision matrix is represented as $\mathfrak{D}$:

$$\mathfrak{D} = \begin{bmatrix} (.8, .7, .6) & (.7, .5, .4) & (.6, .7, .4) \\ (.9, .6, .5) & (.6, .7, .4) & (.7, .4, .2) \\ (.7, .4, .3) & (.8, .4, .3) & (.9, .4, .1) \\ (.8, .5, .4) & (.9, .3, .5) & (.6, .5, .3) \end{bmatrix}$$

The relation among factors α_j is given by the complete graph $\mathfrak{G} = (\mathfrak{V}, \mathfrak{E})$ where $\mathfrak{V} = \{\alpha_1, \alpha_2, \alpha_3\}$ and $\mathfrak{E} = \{\alpha_1\alpha_2, \alpha_1\alpha_3, \alpha_2\alpha_3\}$. We can get the influence coefficients to quantify the relationships among the criteria. Suppose that PN edges denoting the connection among the criteria are described in Figure 3 as: $\mathfrak{l}_{12} = (.4, .5, .5)$, $\mathfrak{l}_{13} = (.5, .6, .2)$, $\mathfrak{l}_{23} = (.5, .3, .2)$. $\mathfrak{G} = (\mathfrak{V}, \mathfrak{E})$ describes the PNG according to relationship among criteria for each alternative. The steps given below are followed to achieve the best alternative.

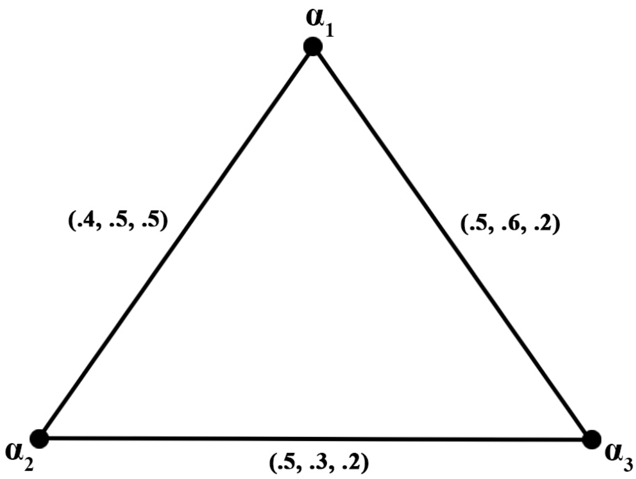

Figure 3. Relation between criteria.

Step 1: The influence coefficients between criteria were computed by using (4.1) and the values are $\chi_{12} = .325, \chi_{13} = .41, \chi_{23} = .53$

Step 2: The overall criterion value of the alternatives $\mathfrak{p}_i, i = 1, 2, 3, 4$ are obtained by using (4.2) are as follows:

$\widetilde{\mathfrak{p}}_1 = \mathfrak{w}_1 \times (\chi_{11}\mathfrak{d}_{11} + \chi_{12}\mathfrak{d}_{21} + \chi_{13}\mathfrak{d}_{31}) + \mathfrak{w}_2 \times (\chi_{11}\mathfrak{d}_{12} + \chi_{12}\mathfrak{d}_{22} + \chi_{13}\mathfrak{d}_{32}) + \mathfrak{w}_3 \times (\chi_{11}\mathfrak{d}_{13} + \chi_{12}\mathfrak{d}_{23} + \chi_{13}\mathfrak{d}_{33})$

$= .34((.8, .7, .6) + (.7, .5, .4) \times .325 + (.6, .7, .4) \times .41) + .33((.8, .7, .6) \times .325 + (.7, .5, .4) + (.6, .7, .4) \times .53) + .33\,((.8, .7, .6) \times .41 + (.7, .5, .4) \times .53 + (.6, .7, .4))$

$= .34((.8, .7, .6) + (.2275, .1625, .13) + (.246, .287, .164)) + .33((.26, .2275, .195) + (.7, .5, .4) + (.318, .371, .212)) + .33((.328, .287, .246) + (.371, .265, .212) + (.6, .7, .4))$

$= .34(1.2735, 1.1495, .894) + .33(1.278, 1.0985, .807) + .33(1.299, 1.252, .858)$

$= (.43299, .39083, .30396) + (.42174, .362505, .26631) + (.42867, .41316, .28314)$

$\widetilde{\mathfrak{p}}_1 = (1.2834, 1.166495, .85341)$

Similarly for $\widetilde{\mathfrak{p}}_2 = (1.344545, 1.03803, .666295)$
$\widetilde{\mathfrak{p}}_3 = (1.47995, .7369, .423815)$
$\widetilde{\mathfrak{p}}_4 = (1.40648, .798045, .73401)$

Step 3: Calculating the score value of the alternatives $\mathfrak{p}_k (k = 1, 2, 3, 4)$,
we get $\mathfrak{S}(\widetilde{\mathfrak{p}}_1) = .1317475$,
$\mathfrak{S}(\widetilde{\mathfrak{p}}_2) = .32011$,
$\mathfrak{S}(\widetilde{\mathfrak{p}}_3) = .6596175$,
$\mathfrak{S}(\widetilde{\mathfrak{p}}_4) = .4372125$.

Step 4: Since $\mathfrak{S}(\widetilde{\mathfrak{p}}_3) > \mathfrak{S}(\widetilde{\mathfrak{p}}_4) > \mathfrak{S}(\widetilde{\mathfrak{p}}_2) > \mathfrak{S}(\widetilde{\mathfrak{p}}_1)$, the ranking of four alternatives is

$$\mathfrak{p}_3 > \mathfrak{p}_4 > \mathfrak{p}_2 > \mathfrak{p}_1$$

Thus, the alternative $\mathfrak{p}_3$ is chosen among the alternatives.

4.2. Comparative analysis

A comparison with neutrosophic graph based decision-making approach in [58] is made to check the effectiveness of the presented decision making-method based on the illustrative example. Applying the technique of [58] and our proposed method, we compute and compare the decision results individually. They are shown in Table 1. The decision outcomes of the method in [58] are consistent with the findings of our suggested method, as shown in Table 1. The results are the same as the ranking order results in [58] and the best alternative is $\mathfrak{p}_3$. Although the ranking principle is different, the two methods derive the same best and worst alternative with the same ranking order for the alternatives.

Table 1. Comparison analysis.

Method	Score Values of Alternatives	Ranking Order
R. Sahin [40]	$\mathfrak{p}_1 = -0.4515$	$\mathfrak{p}_3 > \mathfrak{p}_4 > \mathfrak{p}_2 > \mathfrak{p}_1$
	$\mathfrak{p}_2 = 0.007223$	
	$\mathfrak{p}_3 = 0.2911675$	
	$\mathfrak{p}_4 = 0.03819$	
Our proposed method	$\mathfrak{p}_1 = 0.1317475$	$\mathfrak{p}_3 > \mathfrak{p}_4 > \mathfrak{p}_2 > \mathfrak{p}_1$
	$\mathfrak{p}_2 = 0.32011$	
	$\mathfrak{p}_3 = 0.6596175$	
	$\mathfrak{p}_4 = 0.4372125$	

5. Conclusions

PNS is an extension of fuzzy set to symbolize incomplete, uncertainty and imprecise data that exists in real situations. PN models are more flexible and practical than fuzzy, Intuitionistic fuzzy, and neutrosophic fuzzy models. In this work, we have proposed the ideas of partially edge regular, edge regular, regular, strongly regular graphs, and full edge regular under PN environment and studied their properties. A PNG based MCDM method has been presented and an illustrative example is given using the PN information. In further work, this work is to be extended to the concepts of irregularity, planar PNGs, and their applications, in decision making.

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Conflict of interest

The authors declare no conflict of interest.

References

1. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
2. L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338–353. https://doi.org/10.2307/2272014
3. X. He, Y. Wu, D. Yu, Intuitionistic fuzzy multi-criteria decision making with application to job hunting: A comparative perspective, *J. Intell. Fuzzy Syst.*, **30** (2016), 1935–1946. https://doi.org/10.3233/IFS-151904
4. S. Zeng, Y. Xiao, TOPSIS method for intuitionistic fuzzy multiple-criteria decision making and its application to investment selection, *Kybernetes*, **45** (2016), 282–296. https://doi.org/10.1108/K-04-2015-0093
5. Z. Wang, Z. Xu, S. Liu, J. Tang, A netting clustering analysis method under intuitionistic fuzzy environment, *Appl. Soft Comput.*, **11** (2011), 5558–5564. https://doi.org/10.1016/j.asoc.2011.05.004
6. K. I. Vlachos, G. D. Sergiadis, Intuitionistic fuzzy information-applications to pattern recognition, *Pattern Recogn. Lett.*, **28** (2007), 197–206. https://doi.org/10.1016/j.patrec.2006.07.004
7. Z. Z. Liang, P. F. Shi, Similarity measures on intuitionistic fuzzy sets, *Pattern Recogn. Lett.*, **24** (2003), 2687–2693. https://doi.org/10.1016/S0167-8655(03)00111-9
8. S. K. De, R. Biswas, A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, *Fuzzy Set. Syst.*, **117** (2001), 209–213. https://doi.org/10.1016/S0165-0114(98)00235-8
9. F. Smarandache, *A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic*, Rehoboth: American Research Press, 1999.
10. H. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, *Single valued neutrosophic sets*, 2010.
11. Y. Guo, H. D. Cheng, New neutrosophic approach to image segmentation, *Pattern Recogn.*, **42** (2009), 587–595. https://doi.org/10.1016/j.patcog.2008.10.002
12. J. Ye, J. Fu, Multi-period medical diagnosis method using a single valued neutrosophic similarity measure based on tangent function, *Comput. Meth. Prog. Bio.*, **123** (2016), 142–149. https://doi.org/10.1016/j.cmpb.2015.10.002
13. J. Ye, Multicriteria decision-making method using the correlation coefficient under single valued neutrosophic environment, *Int. J. Gen. Syst.*, **42** (2013), 386–394. https://doi.org/10.1080/03081079.2012.761609
14. S. Bhattacharya, Neutrosophic information fusion applied to the options market, *Investment Management and Financial Innovations*, **1** (2005), 139–145.
15. S. Aggarwal, R. Biswas, A. Q. Ansari, Neutrosophic modeling and control, *2010 International Conference on Computer and Communication Technology (ICCCT)*, 2010, 718–723. https://doi.org/10.1109/ICCCT.2010.5640435
16. S. Y. Wu, Y. M. Kao, The compositions of fuzzy digraphs, *J. Res. Educ. Sci.*, **31** (1986), 603–628.

17. H. L. Yang, Z. L. Guo, Y. She, X. Liao, On single valued neutrosophic relations, *J. Intell. Fuzzy Syst.*, **30** (2016), 1045–1056. https://doi.org/10.3233/IFS-151827

18. R. R. Yager, Pythagorean fuzzy subsets, In: *Proceedings of the Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, AB, Canada, 2013, 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375

19. R. R. Yager, A. M. Abbasov, Pythagorean membership grades, complex numbers and decision making, *Int. J. Intell. Syst.*, **28** (2013), 436–452. https://doi.org/10.1002/int.21584

20. R. R. Yager, Pythagorean membership grades in multi-criteria decision making, *IEEE Trans. Fuzzy Syst.*, **22** (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989

21. R. Jansi, K. Mohana, F. Smarandache, Correlation measure for pythagorean neutrosophic sets with t and f as dependent neutrosophic components, *Neutrosophic Sets and Systems*, **30** (2019), 202–212.

22. A. Kauffman, Introduction a la Theorie des Sous-emsembles Flous, *Masson et Cie.*, **1** (1973).

23. A. Rosenfeld, Fuzzy graphs, In: *Fuzzy sets and their applications to cognitive and decision processes*, New York: Academic Press, 1975, 77–95. https://doi.org/10.1016/B978-0-12-775260-0.50008-6

24. P. Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recogn. Lett.*, **6** (1987), 297–302. https://doi.org/10.1016/0167-8655(87)90012-2

25. K. Radha, N. Kumaravel, On edge regular fuzzy graphs, *International Journal of Mathematical Archive*, **5** (2014), 100–112.

26. K. T. Atanassov, Intuitionistic fuzzy sets, In: *Intuitionistic fuzzy sets*, Heidelberg: Physica, 1999, 1–137. https://doi.org/10.1007/978-3-7908-1870-3_1

27. M. Akram, B. Davvaz, Strong intuitionistic fuzzy graphs, *Filomat*, **26** (2012), 177–196. https://doi.org/10.2298/FIL1201177A

28. M. G. Karunambigai, S. Sivasankar, K. Palanivel, Some properties of a regular intuitionistic fuzzy graph, *International Journal of Mathematics and Computation*, **26** (2015), 53–61.

29. M. G. Karunambigai, K. Palanivel, S. Sivasankar, Edge regular intuitionistic fuzzy graph, *Advances in Fuzzy Sets and Systems*, **20** (2015), 25–46. http://dx.doi.org/10.17654/AFSSSep2015_025_046

30. R. A. Borzooei, H. Rashmanlou, S. Samanta, M. Pal, Regularity of vague graphs, *J. Intell. Fuzzy Syst.*, **30** (2016), 3681–3689. https://doi.org/10.3233/IFS-162114

31. S. N. Mishra, A. Pal, Product of interval-valued intuitionistic fuzzy graph, *Annals of Pure and Applied Mathematics*, **5** (2013), 37–46.

32. W. B. Vasantha Kandasamy, K. Ilanthenral, F. Smarandache, *Neutrosophic graphs: A new dimension to graph theory*, 2015.

33. S. Broumi, M. Talea, A. Bakali, F. Smarandache, On bipolar single valued neutrosophic graphs, *Journal of New Theory*, **2016** (2016), 84–102.

34. S. Broumi, M. Talea, A. Bakali, F. Smarandache, Interval valued neutrosophic graphs, *Critical Review*, **XII** (2016), 5–33.

35. S. Naz, S. Ashraf, M. Akram, A novel approach to decision making with Pythagorean fuzzy information, *Mathematics*, **6** (2018), 95. https://doi.org/10.3390/math6060095

36. D. Ajay, P. Chellamani, Pythagorean neutrosophic fuzzy graphs, *International Journal of Neutrosophic Science*, **11** (2020), 108–114.

37. M. Doumpos, C. Zopounidis, *Multicriteria decision aid classification methods*, Boston, MA: Springer, 2002. https://doi.org/10.1007/b101986

38. P. Agarwal, M. Ramadan, H. S. Osheba, Y. M. Chu, Study of hybrid orthonormal functions method for solving second kind fuzzy Fredholm integral equations, *Adv. Differ. Equ.*, **2020** (2020), 533. https://doi.org/10.1186/s13662-020-02985-3

39. S. Singh, A. H. Ganie, Applications of picture fuzzy similarity measures in pattern recognition, clustering, and MADM, *Expert Syst. Appl.*, **168** (2021), 114–264. https://doi.org/10.1016/j.eswa.2020.114264

40. K. R. Mokarrari, S. A. Torabi, Ranking cities based on their smartness level using MADM methods, *Sustain. Cities Soc.*, **72** (2021), 103030. https://doi.org/10.1016/j.scs.2021.103030

41. H. A. Hammad, M. De la Sen, P. Agarwal, New coincidence point results for generalized graph-preserving multivalued mappings with applications, *Adv. Differ. Equ.*, **2021** (2021), 334. https://doi.org/10.1186/s13662-021-03489-4

42. O. Grigorenko, Fuzzy metrics for solving MODM problems, In: *19th World Congress of the International Fuzzy Systems Association (IFSA), 12th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT), and 11th International Summer School on Aggregation Operators (AGOP)*, Atlantis Press, 2021, 360–366. https://doi.org/10.2991/asum.k.210827.048

43. W. B. Jurkat, H. J. Ryser, Matrix factorizations of determinants and permanents, *J. Algebra*, **3** (1966), 1–27. https://doi.org/10.1016/0021-8693(66)90016-0

44. T. Rasham, M. S. Shabbir, P. Agarwal, S. Momani, On a pair of fuzzy dominated mappings on closed ball in the multiplicative metric space with applications, *Fuzzy Set. Syst.*, in press. https://doi.org/10.1016/j.fss.2021.09.002

45. A. Singh, P. Agarwal, M. Chand, Image encryption and analysis using dynamic AES, In: *2019 5th International Conference on Optimization and Applications (ICOA)*, IEEE, 2019, 1–6. https://doi.org/10.1109/ICOA.2019.8727711

46. S. J. Chen, C. L. Hwang, Fuzzy multiple attribute decision making methods, In: *Fuzzy multiple attribute decision making*, Berlin, Heidelberg: Springer, 1992, 289–486. https://doi.org/10.1007/978-3-642-46768-4_5

47. P. Agarwal, M. Chand, J. Choi, G. Singh, Certain fractional integrals and image formulas of generalized k-Bessel function, *Commun. Korean Math. Soc.*, **33** (2018), 423–436. https://doi.org/10.4134/CKMS.c170056

48. G. Rajchakit, P. Agarwal, S. Ramalingam, *Stability analysis of neural networks*, Singapore: Springer, 2021. https://doi.org/10.1007/978-981-16-6534-9

49. C. Carlsson, R. Fuller, Fuzzy multiple criteria decision making: recent developments, *Fuzzy Set. Syst.*, **78** (1996), 139–153. https://doi.org/10.1016/0165-0114(95)00165-4

50. R. V. Rao, A decision-making framework model for evaluating flexible manufacturing systems using digraph and matrix methods, *Int. J. Adv. Manuf. Technol.*, **30** (2006), 1101–1110. https://doi.org/10.1007/s00170-005-0150-6

51. R. A. Ribeiro, Fuzzy multiple attribute decision making: a review and new preference elicitation techniques, *Fuzzy Set. Syst.*, **78** (1996), 155–181. https://doi.org/10.1016/0165-0114(95)00166-2

52. E. Triantaphyllou, C. T. Lin, Development and evaluation of five fuzzy multiattribute decision-making methods, *Int. J. Approx. Reason.*, **14** (1996), 281–310. https://doi.org/10.1016/0888-613X(95)00119-2

53. P. Chellamani, D. Ajay, Pythagorean neutrosophic Dombi fuzzy graphs with an application to MCDM, *Neutrosophic Sets and Systems*, **47** (2021), 411–431.

54. P. Chellamani, D. Ajay, S. Broumi, T. Ligori, An approach to decision-making via picture fuzzy soft graphs, *Granul. Comput.*, 2021, in press. https://doi.org/10.1007/s41066-021-00282-2

55. L. Abdullah, Fuzzy multi criteria decision making and its applications: A brief review of category, *Procedia-Social and Behavioral Sciences*, **97** (2013), 131–136. https://doi.org/10.1016/j.sbspro.2013.10.213

56. D. Ajay, P. Chellamani, Pythagorean neutrosophic soft sets and their application to decision-making scenario, In: *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021*, Cham: Springer, 2021, 552–560. https://doi.org/10.1007/978-3-030-85577-2_65

57. S. Broumi, M. Talea, A. Bakali, F. Smarandache, Single valued neutrosophic graphs, *Journal of New Theory*, **2016** (2016), 86–101.

58. R. Sahin, An approach to neutrosophic graph theory with applications, *Soft Comput.*, **23** (2019), 569–581. https://doi.org/10.1007/s00500-017-2875-1