IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received February 2, 2022, accepted February 19, 2022, date of publication February 24, 2022, date of current version March 4, 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3154392

Stabilization of Fuzzy Hydraulic Turbine Governing System With Parametric Uncertainty and Membership Function Dependent H_∞ Performance

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This work was supported by the Rajamangala University of Technology Suvarnabhumi.

ABSTRACT This paper examines the stabilization problem and membership function dependent H_∞ performance analysis for uncertain hydraulic turbine governing systems with stochastic actuator faults and time-varying delays via sampled-data control. At first, the nonlinear hydraulic turbine systems are modeled as Takagi-Sugeno (T-S) fuzzy systems with time-varying delay and bounded external disturbance through membership functions. Then, a novel delay-dependent looped Lyapunov-Krasovskii functional (LKF) is formulated with complete information throughout the sampling interval. In the meantime, a membership function dependent H_∞ performance index is suggested to diminish the impact of disturbances on the uncertain fuzzy system. Based on the robust control and novel LKF, new delay-dependent stability conditions for the closed-loop system are attained in the framework of linear matrix inequalities (LMIs). At last, the numerical example validates the proposed theoretical contributions in terms of achieving robust stability and minimizing disturbance attenuation levels.

INDEX TERMS Hydraulic turbine governing system, linear matrix inequality, sampled-data control, stochastic actuator fault, T-S fuzzy model.

I. INTRODUCTION

Due to the exponential rate of the population worldwide, hydropower generation is essential to satisfying clean and renewable energy demand. It has safer, economical, and low-carbon emission operations than other renewable energies like solar and wind [1]–[3]. With the rapid development of power systems, the hydropower station plays a significant role in peak regulation and frequency modulation [4], [5]. In this regard, the hydraulic turbine governing systems (HTGSs), as a critical component of any hydropower plant, is finely researched and designed to ensure the operation's safety and proper response. Many studies on the modeling and dynamic analysis of the hydraulic turbine system have been conducted in recent decades [6]–[9]. For example, modeling and dynamic response control have been investigated for HTGSs with surge tanks [7]. In a real-life situation, the HTGS is a complex time-variant nonlinear system since it couples together with hydraulic, electric, and mechanical systems. Also, any unplanned shutdowns that occur in the actual process due to the complexity, load disturbance, and parameter fluctuations usually lead to massive security issues and economic losses [10], [11]. Therefore, the stable operation of hydro-turbines plays a vital role in the safety of large-scale power stations and power grids.

So far, many researchers have effectively carried out the stability analysis and control issues in the HTGSs [10]–[13]. But it is difficult and more complicated because of its nonlinearity and complexity. At that time, the T-S fuzzy

The associate editor coordinating the review of this manuscript and approving it for publication was Dazhong Ma.

approach dealt with intrinsic nonlinear systems because it could depict complex systems into linear subsystems with fuzzy rules [14]–[16]. This approach has the advantage that the local properties of the nonlinear system are retained in the T-S fuzzy model. Recently, some researchers have obtained fruitful results in the HTGS by the fuzzy technique [17]–[20]. For example, finite-time stability has been studied for the HTGS via T-S fuzzy modeling [20]. In the meantime, time delay and uncertainty are unavoidable in real-time, affecting the system's performance or stability [20]–[22]. However, to our knowledge, both the time-varying delays and uncertainties are not yet considered in the hydraulic turbine system. From this motivation, in this study, we will investigate the stability problems for HTGSs with parameter uncertainty and time-varying delays via the T-S fuzzy approach.

In the meantime, several control approaches have been used to examine the stability behavior of HTGSs, such as adaptive control [11], finite-time H_∞ control [20], proportional integral derivative control [23], sliding mode control [24], etc. Unlike these control techniques [11], [20], [23], [24], sampled-data control has attracted much attention due to the advantage of low-cost maintenance, easy installation, and digital technology development. It updates the control signals only at the sampling time, not the entire time domain [25]–[27]. Due to the aforementioned salient feature, this study will tackle stability issues in HTGSs via sampled-data control. Moreover, when implementing the control to the plant, faults or failures may occur in the actuator due to system components aging or damages, which leads the system can be unstable [28], [29]. For the requirement of safety and reliability, recent researchers pay more observation to study various control problems with actuator faults [29]–[32]. Just to name a few, the authors [29] derived asynchronous adaptive tracking control for the leader-following multi-agents systems with stochastic actuator fault. The resilient reliable load frequency control problem has been investigated in [32] for the power system subject to stochastic actuator failure. However, the stability analysis via sampled-data control design is still open for HTGSs with stochastic actuator fault, which is another motivation for this work.

On the other hand, robust control is essential when exogenous disturbances appear in the dynamical systems will also play a significant contribution to the system's stability. Owing to this reason, H_∞ performance-based control/filter has been established for various stability problems in the recent literature (see [32]–[36] and references therein). In particular, H_∞ control is effectively applied for the HTGS, which diminishes the disturbance attenuation level for bounded external disturbances [5], [20], [37]. To optimize the performance index, a novel membership function dependent (MFD) performance level has been introduced for fuzzy discrete systems in [38]. Unlike the above literature [5], [20], [32], [33], [36], [37], the performance index for the fuzzy system depends on each local linear subsystem, which is clearly described in [38]. Furthermore, the authors in [38] proved that the minimum H_∞ index was obtained via the MFD H_∞ technique compared with the fixed H_∞ approach. However, still, it could not be considered for continuous-time delayed fuzzy systems. Hence, this study will ensure robust stability for the fuzzy-model-based HTGS via MFD H_∞ control technique.

Based on the above motivations, this paper presents a T-S fuzzy-model-based robust sampled-data controller for HTGSs with stochastic actuator faults and time-varying delays. The fundamental aspects and contributions of the work are as follows:

(i). The nonlinear HTGSs are modeled as T-S fuzzy linear subsystems with bounded external disturbances based on the membership functions and fuzzy if-then rules.

(ii). The time-delays, uncertainties, and actuator faults are unavoidable in a practical situation. Due to this, the time-varying delays, parameter additive uncertainties, and stochastic actuator faults are simultaneously considered in the designed fuzzy hydraulic system.

(iii). The aperiodic sampled-data control technique is first time applied to the fuzzy HTGS. It reduces the amount of transmitted data and effectively minimizes the communication bandwidth.

(iv). Unlike the traditional H_∞ performance index, a more general MFD H_∞ index is introduced for the continuous-time fuzzy system to minimize disturbance attenuation.

(v). From the novel delay-dependent looped LKF, sufficient stability conditions and robust performance index are obtained in terms of LMIs. Finally, the theoretical findings are illustrated by the numerical example.

Paper Structure: The state-space form of hydraulic turbine system and its fuzzy modeling are presented in Section 2. Also, the sampled-data controller is introduced with the actuator fault. Sufficient stability conditions with desired MFD H_∞ performance are given in Section 3. Simulation results for the proposed theoretical works are combined in Section 4. The conclusion is provided in Section 5.

Notations: In this paper, I and 0 stand for the identity and zero matrices with proper dimensions, respectively. $\mathrm{Sym}\{L\} = L + L^T$. $X > 0$ means that X is a positive definite matrix. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation operator. Z^T and Z^{-1} stand for the transpose and inverse of the matrix Z, respectively. $\mathrm{diag}\{\cdots\}$ and $\mathrm{col}\{\cdots\}$ indicate the block diagonal matrix and column vector, respectively. $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. $\mathbb{R}^r$ is the r-dimensional Euclidean space.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. T-S FUZZY MODEL

The i^{th} rule of T-S fuzzy systems with time-delay are represented by the following IF-THEN rules:

Plant Rule i: IF $\theta_1(t)$ is ψ_{i1}, $\theta_2(t)$ is ψ_{i2}, $\cdots$, $\theta_l(t)$ is ψ_{il}, THEN,

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{\tau i} x(t-\tau) + B_i u(t) + D_i w(t) \\ z(t) = C_i x(t) \end{cases} \tag{1}$$

where $i = \{1, 2, \cdots, r\}$, r denotes the number of fuzzy IF-THEN rules. $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^m$ represent the state and measured output vectors, respectively. $u(t) \in \mathbb{R}^p$ and $w(t) \in \mathbb{R}^q$ are the control input vector and external disturbance, respectively. τ is the time-delay. A_i, $A_{\tau i}$, B_i, C_i, D_i are the constant appropriate dimensional matrices. $\theta(t) = [\theta_1(t),\ \theta_2(t),\ \cdots, \theta_l(t)]$ is the premise variables with fuzzy set $\psi_{i1},\ \psi_{i2},\ \cdots, \psi_{il}$.

The overall fuzzy system can be represented as follows

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\{A_i x(t) + A_{\tau i} x(t-\tau) + B_i u(t) \\ \quad + D_i w(t)\} \\ z(t) = \sum_{i=1}^{r} \lambda_i(\theta(t))\{C_i x(t)\} \end{cases} \tag{2}$$

where $\lambda_i(\theta(t)) = \frac{\prod_{h=1}^{l} \psi_{ih}(\theta_h(t))}{\sum_{i=1}^{r}(\prod_{h=1}^{l} \psi_{ih}(\theta_h(t)))} \geq 0,\ \sum_{i=1}^{r} \lambda_i(\theta(t)) = 1$. $\psi_{ih}(\theta_h(t))$ is the grade of membership of $\theta_h(t)$ in ψ_{ih}.

B. FUZZY MODELING OF HTGS

The mathematical model of the HTGS is represented as follows [18], [20]:

$$\begin{cases} \dot{\delta} = \omega_0 \omega \\ \dot{\omega} = \frac{1}{T_{ab}}(m_t - D\omega - \frac{E_q V_s}{x_{d\Sigma}} \sin\delta - \frac{V_s^2}{2}\frac{x_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma} x_{q\Sigma}} \sin 2\delta) \\ \dot{m}_t = \frac{1}{\rho_{qh} T_\omega}(-m_t + \rho_y y + \frac{\rho \rho_y T_\omega}{T_y} y) \\ \dot{y} = -\frac{1}{T_y} y \end{cases} \tag{3}$$

where δ, ω, m_t and y are the rotor angle deviation, generator rotor speed deviation, output increment torque deviation and the incremental deviation of the guide vane opening, respectively. $\omega_0 = 2\pi f_0$ is the rated angular speed of the generator with $f_0 = 50Hz$. T_{ab} and D are the unit inertia time constant and damping factor. E_q is the q-axis transient electromotive force. V_s denotes the infinite system bus voltage. $x_{q\Sigma}$ and $x_{d\Sigma}$ represent the d-axis transient reactant and q-axis synchronous reactant, respectively. ρ_{qy} and ρ_y are the turbine flow and torque on the master servomotor stroke transfer coefficient. Similarly, ρ_{qh} and ρ_h denote the turbine flow and torque on the heat transfer coefficients. T_ω and T_y are respectively the flowing water inertia time constant and servo response time constant.

Taking into account the time-delay effect caused by the mechanical inertia of a hydraulic servo system. We consider the hydraulic servo system with time-varying delay as follows:

$$\dot{y}(t) = -\frac{1}{T_y} y(t - \tau(t)) \tag{4}$$

where $\tau(t)$ is the time-varying delay with $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leq \mu < 1$. The random mechanical vibrations and generator load fluctuations always change the original operation of the HTGS. So, we consider exogenous disturbance and control input.

From (3) and (4) we can obtain

$$\begin{cases} \dot{\delta} = \omega_0 \omega + w_1(t) + u_1(t) \\ \dot{\omega} = \frac{1}{T_{ab}}(m_t - D\omega - \frac{E_q V_s}{x_{d\Sigma}} \sin\delta - \frac{V_s^2}{2}\frac{x_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma} x_{q\Sigma}} \sin 2\delta) \\ \quad + w_2(t) + u_2(t) \\ \dot{m}_t = \frac{1}{\rho_{qh} T_\omega}(-m_t + \rho_y y(t-\tau(t)) + \frac{\rho \rho_y T_\omega}{T_y} y(t-\tau(t))) \\ \quad + w_3(t) + u_3(t) \\ \dot{y} = -\frac{1}{T_y} y(t-\tau(t)) + w_4(t) + u_4(t) \end{cases} \tag{5}$$

Now, we define $x_1(t) = \delta(t)$, $x_2(t) = \omega(t)$, $x_3(t) = m_t(t)$, $x_4(t) = y(t)$, then we have the following state space form

$$\begin{aligned} \dot{x}(t) &= F(x(t)) + G(x(t-\tau(t))) + Bu(t) + Dw(t) \\ z(t) &= Cx(t) \end{aligned} \tag{6}$$

where $x(t) = [x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)]$, $w(t) = [w_1(t),\ w_2(t),\ w_3(t),\ w_4(t)]$ is a bounded disturbance satisfies the condition for $T > 0$, $\int_0^T w^T(t)w(t)dt < \kappa^2$, κ is a known scalar. $u(t) = [u_1(t),\ u_2(t),\ u_3(t),\ u_4(t)]$ and B, C, D are the coefficient matrices. $F(x(t))$, $G(x(t-\tau(t)))$, as shown at the bottom of the next page. To construct the fuzzy model for the delayed system (6), we have consider the boundedness of the nonlinear system. Let $x_1(t) \in [-\alpha,\ \alpha]$ with $\alpha = 2$. Then we have obtain the following two fuzzy rule:

Rule 1: IF $x_1(t)$ is $\psi_{11}(x_1(t))$ (near 0), THEN

$$\begin{cases} \dot{x}(t) = \bar{A}_1 x(t) + \bar{A}_{\tau 1} x(t-\tau(t)) + B_1 u^f(t) + D_1 w(t) \\ z(t) = C_1 x(t) \end{cases}$$

Rule 2: IF $x_1(t)$ is $\psi_{21}(x_1(t))$ (near $\pm\alpha$), THEN

$$\begin{cases} \dot{x}(t) = \bar{A}_2 x(t) + \bar{A}_{\tau 2} x(t-\tau(t)) + B_2 u^f(t) + D_2 w(t) \\ z(t) = C_2 x(t) \end{cases}$$

where $\bar{A}_i = A_i + \Delta A_i(t)$, $\bar{A}_{\tau i} = A_{\tau i} + \Delta A_{\tau i}(t)$ be the coefficient matrices with the uncertainties satisfies $[\Delta A_i(t),\ \Delta A_{\tau i}(t)] = \mathcal{M}_i \mathcal{L}_i(t)[\mathcal{N}_{i1}\ \ \mathcal{N}_{i2}]$ with $\mathcal{L}_i^T(t)\mathcal{L}_i(t) \leq I$. A_i, $A_{\tau i}$, $\mathcal{M}_i$, $\mathcal{N}_{i1}$, $\mathcal{N}_{i2}$, B_i, D_i, C_i are the known matrices. $u^f(t)$ is the control input with actuator fault. The membership functions are consider as $\psi_{11}(x_1(t)) = 1 - \frac{|x_1(t)|}{\alpha}$, $\psi_{21}(x_1(t)) = 1 + \frac{|x_1(t)|}{\alpha}$. The overall fuzzy system for the nonlinear system (6) is given as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \lambda_i(\theta(t))\{\bar{A}_i x(t) + \bar{A}_{\tau i} x(t-\tau(t)) \\ \quad + B_i u^f(t) + D_i w(t)\} \\ z(t) = \sum_{i=1}^{2} \lambda_i(\theta(t))\{C_i x(t)\} \end{cases} \tag{7}$$

For the requirements of robust control, we define the controller with actuator fault as follows

$$u^f(t) = \Lambda(t) u(t) \tag{8}$$

where $u(t)$ is the control input and $\Lambda(t) = \text{diag}\{\beta_1(t), \beta_2(t), \cdots, \beta_q(t)\}$ with $0 \leq \beta_\iota(t)(\iota = 1, 2, \cdots, q)$ is a random variable that describes the stochastic failure of the actuator and the mathematical expectation of $\beta_\iota(t)$ is $\mathbb{E}\{\beta_\iota(t)\} = \beta_\iota$. We define $\Lambda = \text{diag}\{\beta_1, \beta_2, \cdots, \beta_q\}$.

Remark 1: In this paper, the actuator fault considered in (8) can be described in the following three classifications:

- If $\beta_q(t) = 1$, the q^{th} actuator has no fault and it work normally.
- If $0 < \beta_q(t) < 1$ or $\beta_q(t) > 1$, the q^{th} actuator work with partial failure.
- If $\beta_q(t) = 0$, the q^{th} actuator is completely failure.

In this work, the control signal is produced by a zero-order hold function at the hold time $0 = t_0 \leq t_1 \leq \cdots \leq t_k \leq \lim_{k\to\infty} t_k = \infty$. Moreover, the aperiodic sampling interval is taken by $0 < t_{k+1} - t_k = d_k \leq d$, $d > 0$, $\forall k \geq 0$. By the parallel distributed compensation technique [14] and from the two fuzzy IF-THEN rules for system (7), we have the following sampled-data control

$$u(t) = \sum_{j=1}^{2} \lambda_j(\theta(t_k)) K_j x(t_k), \quad t_k \leq t < t_{k+1}. \tag{9}$$

Then from the above discussions, the final closed-loop system with actuator fault can be written as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \lambda_i(\theta(t))\lambda_j(\theta(t_k))\{\bar{A}_i x(t) \\ \quad + \bar{A}_{\tau i} x(t-\tau(t)) + B_i \Lambda(t) K_j x(t_k) + D_i w(t)\} \\ z(t) = \sum_{i=1}^{2} \lambda_i(\theta(t))\{C_i x(t)\}, \quad t_k \leq t < t_{k+1}. \end{cases} \tag{10}$$

The main objective of this paper is to frame a reliable sampled-data controller that confirms the stabilization of HTGSs against the stochastic actuator faults and time-varying delays, which can be revealed as follows:

For the proposed system (10), the following requirements are achieved to investigate the stability and stabilization conditions:

- The closed-loop system (10) is globally asymptotically stable with $w(t) = 0$.
- Under the initial condition, for any nonzero $w(t) \in l_2[0, \infty)$ the system (10) satisfies the following condition

$$\|z(t)\|_2 < \bar{\gamma}\|w(t)\|_2,$$

where $\bar{\gamma} = \sqrt{\zeta\lambda_1(\theta(t)) + \lambda_2(\theta(t))}\gamma$, $0 < \zeta < 1$.

Remark 2: In this paper, we consider the novel MFD H_∞ performance index for proposed fuzzy-model-based HTGSs, which is the general case of the traditional H_∞ index. In other words, if $\zeta = 1$, the proposed H_∞ technique is retained to conventional fixed H_∞ performance, which has been studied in the literature [5], [20], [37]. In contrast to the traditional method, the considered disturbance attenuation level for the fuzzy system is different for each local linear subsystem. Also, it significantly minimizes the disturbance attenuation level, which is demonstrated in discrete-time fuzzy systems [38]. From this inspiration, we propose the MFD H_∞ performance level for the HTGS, which is verified in the numerical section.

The lemma stated as in the following is very helpful to derive the main result of this paper.

$$F(x(t)) = \begin{bmatrix} \omega_0 x_2(t) \\ \frac{1}{T_{ab}}(x_3(t) - Dx_2(t) - \frac{E_q V_s}{x_{d\Sigma}}\sin x_1(t) - \frac{V_s^2}{2}\frac{x_{d\Sigma} - x_{q\Sigma}}{x_{d\Sigma}x_{q\Sigma}}\sin 2x_1(t)) \\ -\frac{1}{\rho_{qh}T_\omega}x_3(t) \\ 0 \end{bmatrix},$$

$$G(x(t-\tau(t))) = \begin{bmatrix} 0 \\ 0 \\ -\frac{\rho_y}{\rho_{qh}T_\omega}x_4(t-\tau(t)) + \frac{\rho\rho_y}{\rho_{qh}}x_4(t-\tau(t)) \\ -\frac{1}{T_y}x_4(t-\tau(t)) \end{bmatrix}.$$

Lemma 1: [33] For given scalar $\varpi \in (0,1)$, matrix $G \in \mathbb{R}^{n\times n} > 0$, two matrices H_1 and $H_2 \in \mathbb{R}^{n\times m}$. Define for all vectors $\varsigma \in \mathbb{R}^m$, the function $\Xi(\varpi, G)$ given by:

$$\Xi(\varpi, G) = \frac{1}{\varpi}\varsigma^T H_1^T G H_1 \varsigma + \frac{1}{1-\varpi}\varsigma^T H_2^T G H_2 \varsigma.$$

If there exists a matrix $Z \in \mathbb{R}^{n\times n}$ such that $\begin{bmatrix} G & Z \\ \star & G\end{bmatrix} > 0$, then

$$\min_{\varpi\in(0,1)} \Xi(\varpi, G) \geq \begin{bmatrix} H_1\varsigma \\ H_2\varsigma \end{bmatrix}^T \begin{bmatrix} G & Z \\ \star & G\end{bmatrix}\begin{bmatrix} H_1\varsigma \\ H_2\varsigma \end{bmatrix}.$$

III. MAIN RESULTS

Theorem 1: For known control gain values K_j and the given scalars $d_k \in (0, d]$, $\tau > 0$, $\mu < 1$, ϵ, the closed-loop system (10) is mean square asymptotically stable with the desired performance index $\bar{\gamma}$ if there exist $\sigma > 0$, symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $W > 0$, R, U, X and any matrices M, N such that the following matrix inequalities hold for $i, j = 1, 2$,

$$\begin{bmatrix} Q_3 & X \\ \star & Q_3\end{bmatrix} > 0 \quad (11)$$

$$\begin{bmatrix} \Sigma_{1ij} + d_k\Omega_2 & \sigma\Pi_1^T\mathcal{M}_i & \mathcal{N}_i^T & \frac{d}{2}\Gamma \\ \star & -\sigma I & 0 & 0 \\ \star & \star & -\sigma I & 0 \\ \star & \star & \star & -W \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} \Sigma_{1ij} + d_k\Omega_3 & \sigma\Pi_1^T\mathcal{M}_i & \mathcal{N}_i^T & \frac{d}{2}\Gamma \\ \star & -\sigma I & 0 & 0 \\ \star & \star & -\sigma I & 0 \\ \star & \star & \star & -W \end{bmatrix} < 0 \quad (13)$$

where

$$\begin{aligned}
\Sigma_{1ij} &= \mathrm{Sym}\{e_1^T P e_5\} + e_1^T(Q_1+Q_2)e_1 - (1-\mu)e_3^T Q_1 e_3 \\
&\quad - e_4^T Q_2 e_4 + \tau^2 e_5^T Q_3 e_5 + e_1^T C_i^T C_i e_1 + e_6^T \Upsilon_i I e_6 \\
&\quad + \frac{d^2}{4} e_5^T W e_5 - (e_1-e_2)^T U (e_1 - e_2) \\
&\quad - \begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix}^T \begin{bmatrix} Q_3 & X \\ \star & Q_3 \end{bmatrix}\begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix} \\
&\quad + \mathrm{Sym}\{\Pi_1^T \Pi_{2ij}\}, \quad R = \begin{bmatrix} R_{11} & R_{12} \\ \star & R_{22}\end{bmatrix}, \\
\Omega_2 &= [e_1^T \; e_2^T]R[e_1, \; e_2] + \mathrm{Sym}\{(e_1-e_2)^T U e_5\}, \\
\Omega_3 &= -[e_1^T \; e_2^T]R[e_1, \; e_2], \\
\Pi_1^T &= [N^T, \; 0, \; 0, \; 0, \; M^T, \; 0], \\
\Pi_{2ij} &= [A_i \;\; B_i\Lambda K_j \;\; A_{\tau i} \;\; 0 \;\; -I \;\; D_i], \quad \Upsilon_1 = \zeta\gamma^2, \quad \Upsilon_2 = \gamma^2, \\
\mathcal{N}_i &= [\mathcal{N}_{i1} \;\; 0 \;\; \mathcal{N}_{i2} \;\; 0 \;\; 0 \;\; 0], \quad \Gamma = [R_{11} \;\; R_{12} \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T, \\
e_\vartheta &= [0_{n\times(\vartheta-1)n} \;\; I_n \;\; 0_{n\times(5-\vartheta)n} \;\; 0_{n\times q}] \;\; (\vartheta = 1,2,3,4,5.), \\
e_6 &= [0_{q\times 5n} \;\; I_{q\times q}].
\end{aligned}$$

Proof: Constructing the LKF

$$V(t) = \sum_{b=1}^{4} V_b(t) \quad (14)$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t) \\
V_2(t) &= \int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)ds + \int_{t-\tau}^{t} x^T(s)Q_2x(s)ds \\
&\quad + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Q_3\dot{x}(s)dsd\theta \\
V_3(t) &= (t_{k+1}-t)(t-t_k)\phi^T(t)R\phi(t), \;\; \phi^T(t) = [x^T(t) \; x^T(t_k)] \\
V_4(t) &= (t_{k+1}-t)(x(t)-x(t_k))^T U (x(t)-x(t_k)).
\end{aligned}$$

Taking the derivative of (14) along with the system (10), we get

$$\dot{V}_1(t) = 2x^T(t)P\dot{x}(t) \quad (15)$$

$$\begin{aligned}
\dot{V}_2(t) &= x^T(t)(Q_1+Q_2)x(t) - (1-\mu)x^T(t-\tau(t)) \\
&\quad \times Q_1 x(t-\tau(t)) - x^T(t-\tau)Q_2x(t-\tau) \\
&\quad + \tau^2\dot{x}^T(t)Q_3\dot{x}(t) - \tau\int_{t-\tau}^{t}\dot{x}^T(s)Q_3\dot{x}(s)ds
\end{aligned} \quad (16)$$

$$\begin{aligned}
\dot{V}_3(t) &= [(t_{k+1}-t)-(t-t_k)]\phi^T(t)R\phi(t) \\
&\quad + 2(t_{k+1}-t)(t-t_k)\eta^T(t)\Gamma\dot{x}(t)
\end{aligned} \quad (17)$$

$$\begin{aligned}
\dot{V}_4(t) &= -(x(t)-x(t_k))^T U (x(t)-x(t_k)) \\
&\quad + 2(t_{k+1}-t)(x(t)-x(t_k))^T U\dot{x}(t).
\end{aligned} \quad (18)$$

with $\eta^T(t) = [x^T(t) \;\; x^T(t_k) \;\; x^T(t-\tau(t)) \;\; x^T(t-\tau) \;\; \dot{x}^T(t) \;\; w^T(t)]$. Applying Lemma 1 to the integral term in (16), we obtain

$$\begin{aligned}
-\tau&\int_{t-\tau}^{t}\dot{x}^T(s)Q_3\dot{x}(s)ds \\
&= -\tau\int_{t-\tau(t)}^{t}\dot{x}^T(s)Q_3\dot{x}(s)ds - \tau\int_{t-\tau}^{t-\tau(t)}\dot{x}^T(s)Q_3\dot{x}(s)ds \\
&\leq -\begin{bmatrix} \int_{t-\tau(t)}^{t}\dot{x}(s)ds \\ \int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds \end{bmatrix}^T \begin{bmatrix} Q_3 & X \\ \star & Q_3\end{bmatrix}\begin{bmatrix} \int_{t-\tau(t)}^{t}\dot{x}(s)ds \\ \int_{t-\tau}^{t-\tau(t)}\dot{x}(s)ds \end{bmatrix} \\
&= -\eta^T(t)\begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix}^T \begin{bmatrix} Q_3 & X \\ \star & Q_3\end{bmatrix}\begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix}\eta(t).
\end{aligned} \quad (19)$$

where X is any matrix with compatible dimensions, and it satisfies (11).

From $\dot{V}_3(t)$ and any positive definite matrix W, we get

$$\begin{aligned}
2&(t_{k+1}-t)(t-t_k)\eta^T(t)\Gamma\dot{x}(t) \\
&\leq \frac{d^2}{4}\left(\eta^T(t)\Gamma W^{-1}\Gamma^T\eta(t) + \dot{x}^T(t)W\dot{x}(t)\right).
\end{aligned} \quad (20)$$

For any appropriate dimensional matrices M, N and from (10), we have

$$\mathbb{E}\Big\{2\Big[\dot{x}^T(t)M^T + x^T(t)N^T\Big]\Big[-\dot{x}(t) + \sum_{i=1}^{2}\sum_{j=1}^{2}\lambda_i(\theta(t)) \times \lambda_j(\theta(t_k))\{\bar{A}_i x(t) + \bar{A}_{\tau i}x(t-\tau(t)) + B_i\Lambda(t)K_j x(t_k) + D_i w(t)\}\Big]\Big\} = 0$$

$$2\sum_{i=1}^{2}\sum_{j=1}^{2}\lambda_i(\theta(t))\lambda_j(\theta(t_k))\eta^T(t)\Pi_1^T\Big[\Pi_{2ij} + \mathcal{M}_i\mathcal{L}_i(t)\mathcal{N}_i\Big]\eta(t) = 0. \tag{21}$$

From (15)-(21), we get

$$\mathbb{E}\Big\{\dot{V}(t) + z^T(t)z(t) - \bar{\gamma}^2 w^T(t)w(t)\Big\} \leq \sum_{j=1}^{2}\lambda_1(\theta(t))\lambda_j(\theta(t_k))\eta^T(t)\{\Omega_{1j}(t)\}\eta(t) + \sum_{j=1}^{2}\lambda_2(\theta(t))\lambda_j(\theta(t_k))\eta^T(t)\{\Omega_{2j}(t)\}\eta(t) \tag{22}$$

where $\Omega_{ij}(t) = \Omega_{1ij} + (t_{k+1} - t)\Omega_2 + (t - t_k)\Omega_3$ and $\Omega_{1ij} = \Sigma_{1ij} + \mathrm{Sym}\{\Pi_1^T\mathcal{M}_i^T\mathcal{L}_i(t)\mathcal{N}_i\} + \frac{d^2}{4}\Gamma W^{-1}\Gamma^T$. By convex combination technique, we obtain $\Omega_{ij}(t) < 0$ for any $t \in [t_k,\ t_{k+1})$ if and only if

$$\Omega_{1ij} + d_k\Omega_2 < 0 \tag{23}$$

and

$$\Omega_{1ij} + d_k\Omega_3 < 0. \tag{24}$$

By Schur complement lemma, we obtain (23) and (24) from (12) and (13). Then (22) becomes

$$\mathbb{E}\Big\{\dot{V}(t) + z^T(t)z(t) - \bar{\gamma}^2 w^T(t)w(t)\Big\} \leq 0. \tag{25}$$

Integrating with the limits 0 to ∞, we get

$$\mathbb{E}\Big\{V(x(\infty)) - V(x(0))\Big\} \leq \mathbb{E}\Big\{\int_0^\infty (z^T(t)z(t) - \bar{\gamma}^2 w^T(t)w(t))dt\Big\}. \tag{26}$$

For any $w(t) \neq 0$,

$$\mathbb{E}\{||y(t)||_2\} \leq \bar{\gamma}\mathbb{E}\{||w(t)||_2\}. \tag{27}$$

Suppose that $w(t) = 0$, $\exists\ \varepsilon > 0$ such that

$$\mathbb{E}\{\dot{V}(t)\} \leq -\varepsilon||x(t)||^2. \tag{28}$$

From this, the proposed system (10) with the designed control is mean square asymptotically stable with the desired H_∞ performance level. ■

Remark 3: In recent years, the looped LKF has been constructed to derive the stability and stabilization conditions for the sampled-data control systems via the linear matrix inequalities technique [32], [33]. Inspired from the above, we have considered the looped LKF $V_b(t)$ in this present study as in (14) satisfies $V_b(t_k) = V_b(t_{k+1}) = 0$, $(b = 3, 4)$. Therefore, $V(t)$ is continuous in time and at the sampling instants $t = t_k$, $V(t) = V_1(t_k) + V_2(t_k)$. Thus, it should be mentioned the great strength of the looped functionals are not required to be positive definite at between the sampling times, and involves the full information of $x(t)$ to $x(t_k)$ and $x(t)$ to $x(t_{k+1})$. So, it can lead to the less conservative results.
The design conditions of the proposed sampled-data controller are obtained from the following theorem.

Theorem 2: For given scalars $d_k \in (0, d]$, $\tau > 0$, ϵ, $\mu < 1$, the uncertain fuzzy system (10) is mean square asymptotically stable with the desired performance index $\bar{\gamma}$ if there exist $\sigma > 0$, symmetric matrices $\tilde{P} > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{Q}_3 > 0$, $\tilde{W} > 0$, $\tilde{R}$, $\tilde{U}$, $\tilde{X}$ and appropriate dimensional matrix Y such that the following LMIs hold for $i, j = 1, 2$,

$$\begin{bmatrix} \tilde{Q}_3 & \tilde{X} \\ \star & \tilde{Q}_3 \end{bmatrix} > 0 \tag{29}$$

$$\begin{bmatrix} \tilde{\Sigma}_{1ij} + d_k\tilde{\Omega}_2 & \sigma\tilde{\Pi}_1^T\tilde{\mathcal{M}}_i & \tilde{\mathcal{N}}_i^T & \frac{d}{2}\tilde{\Gamma} & e_1^T Y^T C_i^T \\ \star & -\sigma I & 0 & 0 & 0 \\ \star & \star & -\sigma I & 0 & 0 \\ \star & \star & \star & -\tilde{W} & 0 \\ \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} \tilde{\Sigma}_{1ij} + d_k\tilde{\Omega}_3 & \sigma\tilde{\Pi}_1^T\tilde{\mathcal{M}}_i & \tilde{\mathcal{N}}_i^T & \frac{d}{2}\tilde{\Gamma} & e_1^T Y^T C_i^T \\ \star & -\sigma I & 0 & 0 & 0 \\ \star & \star & -\sigma I & 0 & 0 \\ \star & \star & \star & -\tilde{W} & 0 \\ \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{31}$$

where

$$\begin{aligned}
\tilde{\Sigma}_{1ij} &= \mathrm{Sym}\{e_1^T\tilde{P}e_5\} + e_1^T(\tilde{Q}_1 + \tilde{Q}_2)e_1 - (1-\mu)e_3^T\tilde{Q}_1 e_3 \\
&\quad - e_4^T\tilde{Q}_2 e_4 + \tau^2 e_5^T\tilde{Q}_3 e_5 + \frac{d^2}{4}e_5^T\tilde{W}e_5 + e_6^T\Upsilon_i I e_6 \\
&\quad - \begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix}^T \begin{bmatrix} \tilde{Q}_3 & \tilde{X} \\ \star & \tilde{Q}_3 \end{bmatrix} \begin{bmatrix} e_1 - e_3 \\ e_3 - e_4 \end{bmatrix} \\
&\quad + \mathrm{Sym}\{\tilde{\Pi}_1^T\tilde{\Pi}_{2ij}\} - (e_1 - e_2)^T\tilde{U}(e_1 - e_2), \\
\tilde{\Omega}_2 &= [e_1^T\ \ e_2^T]\tilde{R}[e_1,\ \ e_2] + \mathrm{Sym}\{(e_1 - e_2)^T\tilde{U}e_5\}, \\
\Omega_3 &= -[e_1^T\ \ e_2^T]\tilde{R}[e_1,\ \ e_2], \\
\tilde{\Pi}_1^T &= [\epsilon I,\ 0,\ 0,\ 0,\ I,\ 0], \\
\tilde{\Pi}_{2ij} &= [A_i Y\ B_i\Lambda L_j\ A_{\tau i}Y\ 0\ -Y\ D_i], \\
\Upsilon_1 &= \zeta\gamma^2, \quad \Upsilon_2 = \gamma^2, \\
\tilde{\mathcal{N}}_i &= [\mathcal{N}_{i1}Y\ 0\ \mathcal{N}_{i2}Y\ 0\ 0\ 0], \quad \tilde{\Gamma} = [\tilde{R}_{11}\ \tilde{R}_{12}\ 0\ 0\ 0\ 0]^T, \\
e_\vartheta &= [0_{n\times(\vartheta-1)n}\ I_n\ 0_{n\times(5-\vartheta)n}\ 0_{n\times q}]\ (\vartheta = 1, 2, 3, 4, 5.), \\
\tilde{R} &= \begin{bmatrix} \tilde{R}_{11} & \tilde{R}_{12} \\ \star & \tilde{R}_{22} \end{bmatrix}, \quad e_6 = [0_{q\times 5n}\ I_{q\times q}].
\end{aligned}$$

Furthermore, the corresponding controller gain inputs can be obtained as $K_j = L_j Y^{-1}$.

Proof: Define $M = Y^{-1}$, $N = \epsilon Y^{-1}$, $L_j = K_j Y$, $\widetilde{P} = Y^T P Y$, $\widetilde{Q}_1 = Y^T Q_1 Y$, $\widetilde{Q}_2 = Y^T Q_2 Y$, $\widetilde{Q}_3 = Y^T Q_3 Y$, $\widetilde{R}_{cd} = Y^T R_{cd} Y$ $(c, d = 1, 2)$, $\widetilde{U} = Y^T U Y$, $\widetilde{X} = Y^T X Y$, $\widetilde{W} = Y^T W Y$. Now pre and post multiplying (12) and (13) with diag$\{Y^T,\ Y^T,\ Y^T,\ Y^T,\ Y^T,\ I,\ I,\ I,\ Y^T\}$ and its transpose and utilizing the Schur complement, we obtain (30) and (31), respectively. This completes the proof. ■

When the delay is constant and the uncertainties are not considered in the system (10), we have

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} \lambda_i(\theta(t))\lambda_j(\theta(t_k))\{A_i x(t) \\ \quad + A_{\tau i} x(t-\tau) + B_i \Lambda(t) K_j x(t_k) + D_i w(t)\} \\ z(t) = \sum_{i=1}^{2} \lambda_i(\theta(t))\{C_i x(t)\} \qquad t_k \le t < t_{k+1}. \end{cases} \tag{32}$$

The sufficient stability condition for the system (32) are given in the following corollary.

Corollary 1: For given scalars $d_k \in (0, d]$, $\tau > 0$, ϵ, the system (32) is mean square asymptotically stable with the desired performance index $\bar{\gamma}$ if there exist symmetric matrices $\widetilde{P} > 0$, $\widetilde{Q} > 0$, $\widetilde{W} > 0$, $\widetilde{R}$, $\widetilde{U}$, and appropriate dimensional matrix Y such that the following LMIs hold for $i, j = 1, 2$,

$$\begin{bmatrix} \widetilde{\Sigma}_{1ij} + d_k \widetilde{\Omega}_2 & \frac{d}{2}\widetilde{\Gamma} & e_1^T Y^T C_i^T \\ \star & -\widetilde{W} & 0 \\ \star & \star & -I \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} \widetilde{\Sigma}_{1ij} + d_k \widetilde{\Omega}_3 & \frac{d}{2}\widetilde{\Gamma} & e_1^T Y^T C_i^T \\ \star & -\widetilde{W} & 0 \\ \star & \star & -I \end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned} \widetilde{\Sigma}_{1ij} &= \mathrm{Sym}\{e_1^T \widetilde{P} e_4\} + e_1^T \widetilde{Q} e_1 - (1-\tau) e_3^T \widetilde{Q} e_3 \\ &\quad - (e_1 - e_2)^T \widetilde{U}(e_1 - e_2) + \frac{d^2}{4} e_4^T \widetilde{W} e_4 + e_5^T \Upsilon_i I e_5 \\ &\quad + \mathrm{Sym}\{\widetilde{\Pi}_1^T \widetilde{\Pi}_{2ij}\}, \\ \widetilde{\Omega}_2 &= [e_1^T\ \ e_2^T]\widetilde{R}[e_1,\ \ e_2] + \mathrm{Sym}\{(e_1 - e_2)^T \widetilde{U} e_4\}, \\ \Omega_3 &= -[e_1^T\ \ e_2^T]\widetilde{R}[e_1,\ \ e_2], \quad \Upsilon_1 = \zeta\gamma^2, \ \Upsilon_2 = \gamma^2, \\ \widetilde{\Pi}_1^T &= [\epsilon I,\ 0,\ 0,\ I,\ 0], \quad \widetilde{\Gamma} = [\widetilde{R}_{11}\ \ \widetilde{R}_{12}\ \ 0\ \ 0\ \ 0]^T, \\ \widetilde{\Pi}_{2ij} &= [A_i Y\ \ B_i \Lambda L_j\ \ A_{\tau i} Y\ \ -Y\ \ D_i], \\ e_\vartheta &= [0_{n\times(\vartheta-1)n}\ \ I_n\ \ 0_{n\times(4-\vartheta)n}\ \ 0_{n\times q}]\ (\vartheta = 1, 2, 3, 4.), \\ e_5 &= [0_{q\times 4n}\ \ I_{q\times q}], \quad \widetilde{R} = \begin{bmatrix} \widetilde{R}_{11} & \widetilde{R}_{12} \\ \star & \widetilde{R}_{22} \end{bmatrix}, \\ \eta^T(t) &= \begin{bmatrix} x^T(t) & x^T(t_k) & x^T(t-\tau) & \dot{x}^T(t) & w^T(t) \end{bmatrix}. \end{aligned}$$

Furthermore, the corresponding controller gain inputs can be obtained as $K_j = L_j Y^{-1}$.

Proof: We have consider the following LKF from (11) for the corresponding system (32)

$$V(t) = \sum_{b=1}^{4} V_b(t)$$

where $V_2(t) = \int_{t-\tau}^{t} x^T(s) Q x(s) ds$. $V_1(t)$, $V_3(t)$ and $V_4(t)$ are defined in Theorem 1 as in (11). The remaining proof can be obtained directly from Theorem 2. ■

IV. NUMERICAL EXAMPLE

This section ensures the reliability and applicability of the proposed control scheme for the HTGS. First, we consider parameter values similar to [20] as follows $\omega_0 = 314$rad/s, $T_{ab} = 9.0$s, $D = 2.0$, $E_q = 1.35$, $T_\omega = 0.8$s, $T_y = 0.1$s, $x_{d\Sigma} = 1.15$, $x_{q\Sigma} = 1.474$, $V_s = 1.0$, $\rho = 0.7$, $\rho_{qh} = 0.5$, $\rho_y = 1.0$.

Based on this parameter values, the corresponding system matrices for the fuzzy linear sub-model are obtained as follows

$$A_1 = \begin{bmatrix} 0 & 314 & 0 & 0 \\ \frac{17231}{16951} & -\frac{2}{9} & \frac{1}{9} & 0 \\ 0 & 0 & -\frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{M}_i = \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.3 \end{bmatrix}^T$$

$$A_2 = \begin{bmatrix} 0 & 314 & 0 & 0 \\ \frac{1577}{16951} & -\frac{2}{9} & \frac{1}{9} & 0 \\ 0 & 0 & -\frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_i = C_i = I_{4\times 4} (i = 1, 2)$$

$$A_{\tau 1} = A_{\tau 2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{33}{2} \\ 0 & 0 & 0 & -10 \end{bmatrix},$$

$$\mathcal{N}_{i1} = \mathcal{N}_{i2} = \begin{bmatrix} 0.6 & 0.2 & 0.4 & 0.4 \end{bmatrix}, \quad D_1 = D_2 = I_{4\times 4}.$$

Now, consider the time-varying delay and the external disturbance are $\tau(t) = 0.01\sin(0.5t)$ and $w(t) = \mathrm{col}\{0.1\sin(1.5t),\ 0.1\sin(1.5t),\ 0.1\sin(1.5t),\ 0.1\sin(1.5t)\}$, respectively. Also, we take the output vector as $z(t) = \mathrm{col}\{x_1(t),\ x_2(t),\ x_3(t),\ x_4(t)\}$. Based on the parameter values, the state responses for the system (10) without control are displayed in Figure 1. From this figure, one can easily observe that the system is unstable and also the suitable controller must need to stabilize the system (10). For the values $\epsilon = 0.69$, $\Lambda = \mathrm{diag}\{0.62,\ 0.56,\ 0.5,\ 0.5\}$, $\zeta = 0.9$, $\gamma = 0.191$ and solving the LMIs (30) and (31), we can obtain the control gain values for the sampling period

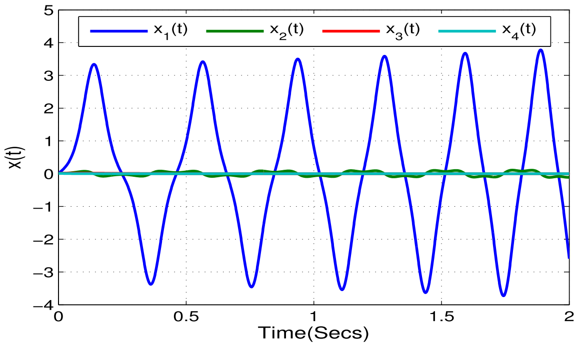

FIGURE 1. State curves of the system without control input.

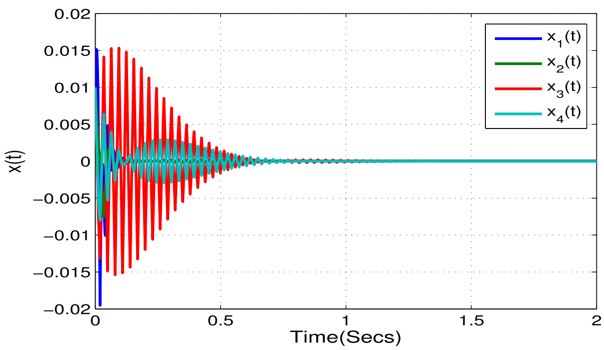

FIGURE 2. State curves of the closed-loop system.

$d = 0.0015$ are given as follows

$$K_1 = \begin{bmatrix} -275.6854 & -446.7547 & -0.2387 & 0.0348 \\ 43.2571 & -159.7939 & 0.4458 & 2.7845 \\ -0.7199 & 2.4537 & -263.8446 & -25.1511 \\ -0.9868 & 8.9832 & 6.9344 & -244.3795 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -275.6854 & -446.7547 & -0.2387 & 0.0348 \\ 43.2571 & -159.7939 & 0.4458 & 2.7845 \\ -0.7199 & 2.4537 & -263.8446 & -25.1511 \\ -0.9868 & 8.9832 & 6.9344 & -244.3795 \end{bmatrix}.$$

Based on the obtained control gain values with the initial condition $x(0) = [0.01 \quad 0.01 \quad 0.01 \quad 0.01]^T$, the state response of the closed-loop system is plotted in Figure 2. The corresponding control input and measured output trajectories are given in Figure 3 and Figure 4, respectively. These figures ensure that the designed controller stabilizes the fuzzy HTGS (10) within a finite time interval.

The disturbance attenuation level $\bar{\gamma} = \sqrt{(0.9\lambda_1 + \lambda_2) \times 0.03648} \in [0.1812, \quad 0.1910]$ can be obtained by using the concept of MFD H_∞ performance index. Furthermore, based on Theorem 2, the minimum perturbation attenuation level is calculated by MFD H_∞ performance index for various ζ and fixed H_∞ index. The calculated H_∞ bound results are listed in Table 1, and it ensures that the proposed H_∞ performance provides a minimum perturbation attenuation level than the traditional techniques. The graphical representation of the disturbance attenuation level discussed in Table 1 is shown in Figure 5.

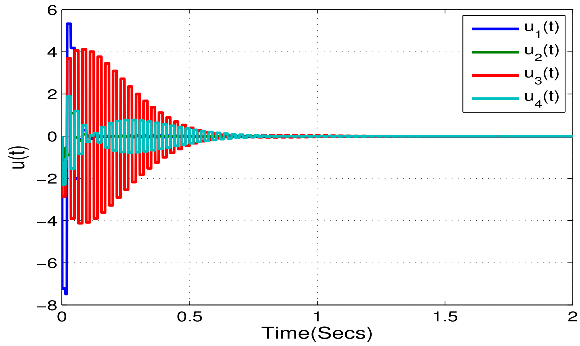

FIGURE 3. Responses of the control input.

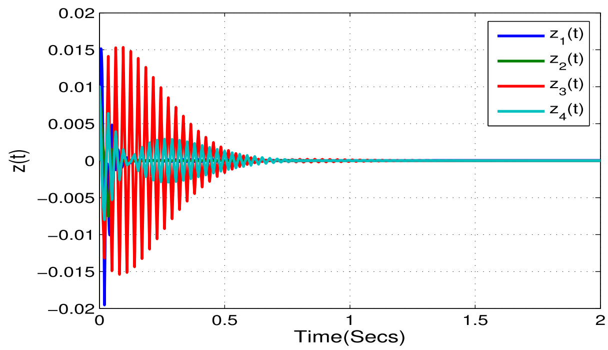

FIGURE 4. Responses of the measured output.

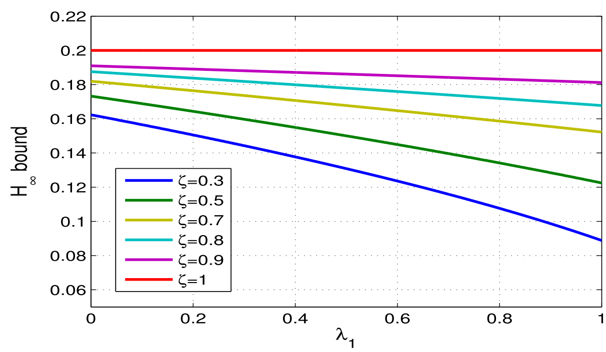

FIGURE 5. H_∞ bound for different membership values and ζ.

Next, we illustrate the validity of Corollary 1. For this, we have consider the system (32) with constant delay $\tau = 0.1$ and other parameters are the same as above. Assume that $\epsilon = 1.66$, $\zeta = 0.7$ and $\gamma = 0.1836$. Then solving the LMIs (33) and (34) in Corollary 1, we can achieve the maximum sampling period $d = 0.002$ with the controller gain values as follows

$$K_1 = \begin{bmatrix} -326.9645 & -392.7387 & -0.0130 & -0.0100 \\ 65.2672 & -224.8066 & -0.1109 & 0.0240 \\ -0.0134 & 0.1769 & -320.8026 & -62.5577 \\ -0.0050 & -0.0027 & -24.8906 & -335.3584 \end{bmatrix},$$

TABLE 1. Calculated values of H_∞ performance index $\bar{\gamma}$.

ζ	0.3	0.5	0.7	0.9	Fixed H_∞
H_∞ bound ($\bar{\gamma}$)	$[0.0889, 0.1624]$	$[0.1225, 0.1733]$	$[0.1523, 0.1820]$	$[0.1812, 0.1910]$	0.2

TABLE 2. Calculated values of H_∞ performance index $\bar{\gamma}$ for Corollary 1.

ζ	0.3	0.5	0.7	0.9	Fixed H_∞
H_∞ bound ($\bar{\gamma}$)	$[0.09892, 0.1802]$	$[0.1288, 0.1821]$	$[0.1536, 0.1836]$	$[0.1772, 0.1868]$	0.1879

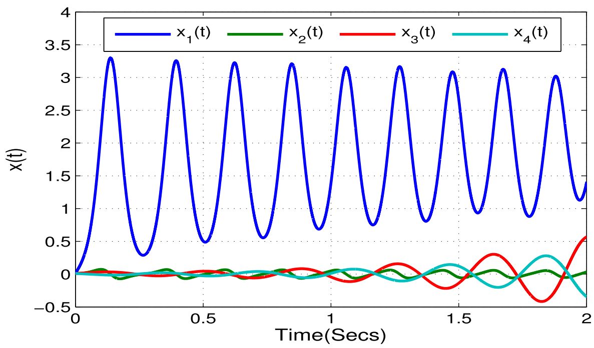

FIGURE 6. State curves of the open-loop system in Corollary 1.

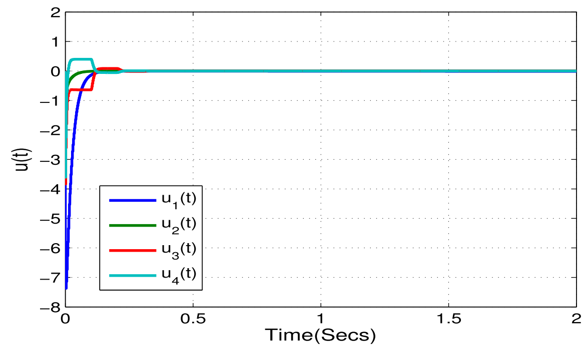

FIGURE 8. Control input responses in Corollary 1.

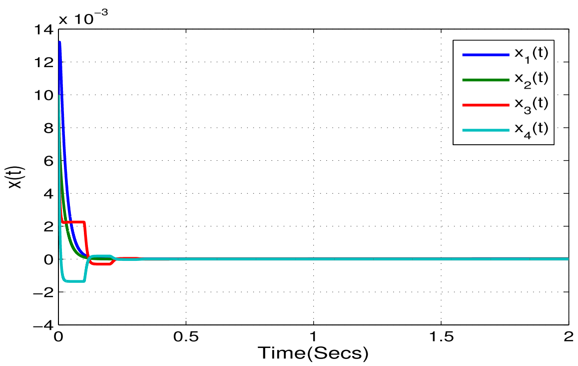

FIGURE 7. State responses of the closed-loop system in Corollary 1.

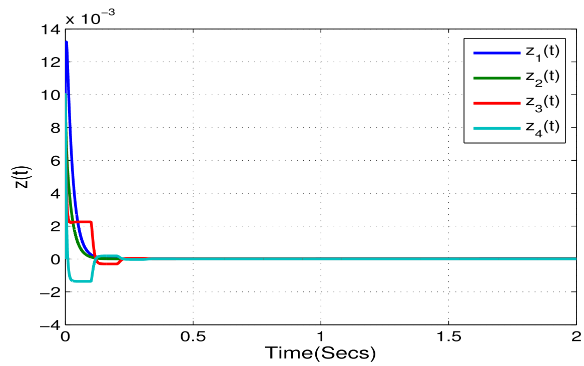

FIGURE 9. Measured output responses in Corollary 1.

$$K_2 = \begin{bmatrix} -326.9645 & -392.7387 & -0.0130 & -0.0100 \\ 65.2672 & -224.8066 & -0.1109 & 0.0240 \\ -0.0134 & 0.1769 & -320.8026 & -62.5577 \\ -0.0050 & -0.0027 & -24.8906 & -335.3584 \end{bmatrix}.$$

For the obtained control gain values and the initial condition $x(0) = [0.01 \ \ 0.01 \ \ 0.01 \ \ 0.01]^T$, the state responses of the system (32) with and without control signal is shown in Figure 7 and 6, respectively. The corresponding input and measured output responses are plotted in Figure 8 and Figure 9, respectively. Table 2 represents that the obtained H_∞ performance index for various values of ζ, including the classical H_∞ index. This table clearly ensures that the MFD H_∞ performance attains the minimum disturbance perturbation level compared with the traditional fixed H_∞ performance. This comparative H_∞ bound is graphically displayed in Figure 10. From this Figure 10 and Table 2 conclude that the membership function plays a vital role in obtaining the minimum attenuation level for the proposed uncertain fuzzy system.

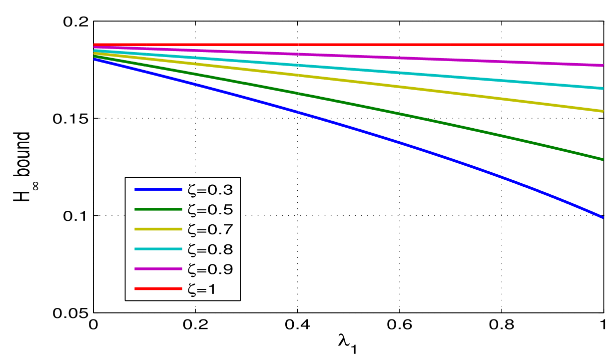

FIGURE 10. H_∞ bound for different membership values and ζ in Corollary 1.

Hence, these numerical findings illustrate the validity and reliability of the suggested theoretical work on the HTGS with the robust sampled-data controller.

V. CONCLUSION

This paper has examined the problem of robust sampled-data control design for nonlinear HTGSs via the T-S fuzzy approach. The sampled-data control has been applied to overcome the stochastic actuator faults and time-varying delays in the uncertain fuzzy system. Unlike the traditional H_∞ index, a novel MFD H_∞ performance index has been proposed for the designed fuzzy model to minimize disturbance attenuation level. Based on Lyapunov theory, sufficient delay-dependent robust stability conditions have been derived in terms of LMIs. Then, the corresponding control gain values have been obtained by solving the LMIs. In the end, the numerical example illustrated the applicability and reliability of the proposed theoretical results.

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