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Synchronization of Fractional Order Uncertain BAM Competitive Neural Networks

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Abstract: This article examines the drive-response synchronization of a class of fractional order uncertain BAM (Bidirectional Associative Memory) competitive neural networks. By using the differential inclusions theory, and constructing a proper Lyapunov-Krasovskii functional, novel sufficient conditions are obtained to achieve global asymptotic stability of fractional order uncertain BAM competitive neural networks. This novel approach is based on the linear matrix inequality (LMI) technique and the derived conditions are easy to verify via the LMI toolbox. Moreover, numerical examples are presented to show the feasibility and effectiveness of the theoretical results.

Keywords: fractional order; synchronization; competitive neural network; Lyapunov–Krasovskii functional

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1. Introduction

In recent years, fractional calculus has attracted the attention of researchers because it can describe real phenomenon more accurately. Therefore, both in theory and in application, fractional-order calculus is more applicable in many branches of science and engineering than traditional integer-order calculus, such as artificial intelligence, optimal combination, material science, electronic information, and cognitive science. When the model of a system includes at least one fraction derivative or integral term, we call it a fractional order system. The main advantage of fractional-order models in comparison with their integer-order counterparts is that fractional derivatives provide an excellent instrument in the description of memory and hereditary properties of various materials and process [1–3]. On the other hand, the interest in stability analysis of various fractional differential systems has been growing rapidly due to their successful applications in widespread fields of science and engineering. Different types of stability criteria, such as as the global stability [4], global asymptotic stability [5], the quasi-uniform stability [6], global uniform asymptotic fixed deviation stability [7] are studied.

Meanwhile, the genuine systems are regularly dependent upon external disturbances. However, the outside disturbance cannot be particularly assessable and hard to measure. On the other hand, external disturbance and uncertainties generally exist in almost all industrial systems and speed up unfavorable impacts execution and even strength of control frameworks. One instinctive plan to manage this issue is to access the disturbance or effect of the disturbances from computable variables, and thereafter, a control move can be made, in context of the disturbance estimate, to make up the effect of the disturbances. Parameter uncertainties negatively affect networks dynamical behaviors, including stability and synchronization, hence permit further investigations. To our best knowledge, the impact of parameter uncertainties was not given enough consideration, and was only studied by few authors [8–14].

The synchronization issue has got a continually growing variety of analysts attention due to its expected applications. Pecora and Carrol introduced a method to synchronize two identical chaotic systems with different initial conditions. The study of synchronization can provide research ideas for the analytical method of dynamic behavior, which further extends the study of dynamic behavior. However, few practical network systems can be synchronized directly. Neural networks can arise chaotic behaviors due to an unpredictable disturbance. In other words, in the drive-response (or master-slave) systems, the response (or slave) system is influenced by the behavior of the drive (or master) system, but the drive (or master) system is independent of the response (or slave) one. Recently, the study of fractional-order synchronization has been attracting a host of attention due to emerging excellent applications in biological technology, chemical systems, ecological systems, cryptography, etc. As we know, there are synchronizations among which the NNs have been extensively studied [15–19].

Recently, Meyer-Base et al. [20] proposed the so-called competitive neural networks with different time scales. Generally speaking, competitive neural network (CNN) contains two types of state variable: short-term memory (STM) and long-term memory (LTM). STM describes the rapidly changing behavior of neuronal dynamics, whereas LTM describes the slow behavior of unsupervised neuronal synapses [21,22]. On the other hand, much attention has been devoted to analyzing the stability and synchronization of Competitive neural networks, for example [23,24]. Since fractional calculus provides a better way to show the nature of hereditary and memory in dynamical process, fractional order competitive neural network (FCNN) is more appropriate than integer-order CNN. The multi-stability, global stability, and complete synchronization for fractional-order competitive neural network are researched [25,26].

Bidirectional associative memory neural networks (BAMNNs) are a class of two-layer neural systems, which were first introduced by Kosko in 1987 [27]. There are many types of neural networks modeled inspired by biological neural networks and are used to approximate functions that are generally unknown. Generally speaking, the neurons in one layer are fully incorporated to the neurons in the other layer. Moreover, there may be no interconnection among neurons in the same layer. Owing to these reasons, BAM neural networks have been widely discussed both in theory and applications, due to their wide application in many fields, such as signal processing [28], image processing [29] and pattern recognition [30].

Inspired by above discussions, we propose stability and synchronization criteria for the fractional order uncertain BAM competitive neural networks.

(1) We extend competitive neural networks with different time scales to bidirectional associative memory neural networks with different time scales and proposed a double-layer uncertain BAM competitive neural network.
(2) The uncertainty BAM competitive neural networks are introduced for the first time.
(3) We constructed novel Lyapunov–Krasovskii functionals and applied inequality techniques to achieve synchronization of the considered systems.
(4) The derived conditions are expressed in terms of linear matrix inequalities (LMIs), which can be checked numerically very efficiently via the LMI toolbox.
(5) Lastly, numerical results are given to show the effectiveness of the proposed results.

2. Preliminaries

Definition 1 ([31]). *The fractional-order integral of order $\alpha \in (0,1)$ for an integral function $\mathfrak{f}(t) \in C^m([0,+\infty), R)$ is defined as*

$$I^{\alpha}\mathfrak{f}(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1}\mathfrak{f}(\xi)d\xi,$$

where $\Gamma(\cdot)$ is the Euler's gamma function, which is denominated by

$$\Gamma(i) = \int_0^{\infty} \exp(-\gamma)t^{i-1}d\gamma, (Re(i) > 0),$$

where $Re(i)$ is the real part of i.

Definition 2 ([31]). *The Caputo fractional derivative of order α for a function $\mathfrak{f}(t)$ is defined as*

$$D^{\beta}f(t) = \frac{1}{\Gamma(m-\beta)} \int_0^t \frac{f^m(\gamma)}{(t-\gamma)^{\beta-m+1}}d\gamma,$$

where $t \geq 0$ and $m-1 < \beta < m \in Z^+$. In particular, when $\beta \in (0,1)$,

$$D^{\beta}f(t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{f'(\gamma)}{(t-\gamma)^{\beta}}d\gamma.$$

Definition 3 ([31]). *The one-parameter Mittag–Leffler function is defined as*

$$E_{\phi}(\mathfrak{p}) = \sum_{l=0}^{\infty} \frac{\mathfrak{p}^l}{\Gamma(l\phi+1)}.$$

Definition 4 ([31]). *The two-parameter Mittag–Leffler function is defined as*

$$E_{\phi,\psi}(\mathfrak{p}) = \sum_{l=0}^{\infty} \frac{\mathfrak{p}^l}{\Gamma(l\phi+\psi)}.$$

Lemma 1 ([32]). *Leth $: J \to R$ be continuous function. A function $\varphi \in C(J,R)$ is a solution of fractional integral equation:*

$$\varphi(t) = \varphi_0 - \frac{1}{\Gamma(\alpha)} \int_0^b (b-i)^{\alpha-1}\mathfrak{f}(i)di + \frac{1}{\Gamma(\alpha)} \int_0^b (t-i)^{\alpha-1}\mathfrak{f}(i)di$$

if and only if $\varphi(t)$ is a solution of the following fractional Cauchy problem:

$$D^{\alpha}\varphi(t) = \mathfrak{f}(t,\varphi(t)), t \in J,$$
$$\varphi(b) = \varphi_0, b \in (0,T).$$

Lemma 2 ([32]). *If $\mathfrak{p}(t) \in C^m[0,\infty]$, then*

$$D^{-\alpha}D^{\alpha}\mathfrak{p}(t) = \mathfrak{p}(t) - \sum_{\delta=0}^{m-1} \frac{t^{\delta}}{\delta!}\mathfrak{p}^{\delta}(0), \alpha \geq 0,$$

where $n \in Z^+$. Especially for $0 < \alpha < 1$, $\mathfrak{p}(t) \in C^1[0,\infty)$, then

$$D^{\alpha}I^{\alpha}\mathfrak{p}(t) = \mathfrak{p}(t)$$

and

$$I^{\alpha}D^{\alpha}\mathfrak{p}(t) = \mathfrak{p}(t) - \mathfrak{p}(0).$$

Lemma 3 ([33])**.** *For $\alpha > 0$, assume $\mathfrak{h}(t)$ is a non-negative, non-decreasing function locally integrable on $0 \leq t < R$ $(R \leq +\infty)$, and $d(t) \leq N$ is a non-negative, non-decreasing continuous function defined on $0 \leq t < R$, where N is a constant. $\mathfrak{v}(t)$ is non-negative and locally integrable on $0 \leq t < R$ and satisfying*

$$\mathfrak{v}(t) = \mathfrak{h}(t) + d(t)\int_0^t (t-s)^{\alpha-1}\mathfrak{v}(s)ds$$

then

$$\mathfrak{v}(t) \leq \mathfrak{h}(t)E_{\alpha}(d(t)\Gamma(\alpha)t^{\alpha}).$$

Lemma 4 ([33])**.** *Assume that $\xi(t) \in C^1[\mathfrak{p}, \mathfrak{q}]$ and satisfies,*

$$D^{\alpha}\xi(t) = \mathfrak{f}(t, \xi(t)) \geq 0, 0 < \alpha < 1,$$

for all $t \in [\mathfrak{p}, \mathfrak{q}]$; then, $\xi(t)$ is monotonously nondecreasing for $0 < \alpha < 1$. If

$$D^{\alpha}\xi(t) = \mathfrak{f}(t, \xi(t)) \leq 0, 0 < \alpha < 1,$$

then $\xi(t)$ is monotonously non increasing for $0 < \alpha < 1$.

Lemma 5 ([34])**.** *Let a vector-valued function $\mathfrak{u}(t) \in R^n$ is differentiable. Then, for any $t > 0$, one has*

$$D^{\alpha}\mathfrak{u}^T(t)S\mathfrak{u}(t) \leq 2\mathfrak{u}^T(t)SD^{\alpha}\mathfrak{u}(t), 0 < \alpha < 1.$$

Lemma 6 ([34])**.** *For the given positive scalar $\lambda > 0$, $\mathfrak{w}, \mathfrak{z} \in R^m$ and matrix D, then*

$$\mathfrak{w}^T D\mathfrak{z} \leq \frac{\lambda^{-1}}{2}\mathfrak{w}^T DD^T\mathfrak{w} + \frac{\lambda}{2}\mathfrak{z}^T\mathfrak{z}.$$

Lemma 7 ([35])**.** *If $N > 0$, given matrices are S, Q, N then*

$$\begin{bmatrix} Q & S^T \\ S & -N \end{bmatrix} < 0$$

if and only if

$$Q + S^T N^{-1} S < 0.$$

Lemma 8 ([36])**.** *Let $\pi(\mathfrak{p}) : \bar{\lambda} \to R^n$ be a continuous mapping. If $deg(\pi(\mathfrak{p}), \lambda, y) \neq 0$, then there exists at least one solution of $\pi(\mathfrak{p}) = y \in \lambda$.*

For the neuron activation function g, the following assumption is given.

Assumption 1. *For j = 1, 2, ... m, the nonlinear activation function h_j with $h_j(0) = 0$ satisfies the Lipchitz continuous; namely, there exists a Lipchitz constant $l_j > 0$ such that*

$$|h(p_j) - h(q_j)| \leq l_j|p_j - q_j| for\ all\ p_j, q_j \in R.$$

3. Mian Results

In this section, we will present the synchronization of following fractional order uncertain BAM competitive neural networks with delays,

$$\begin{cases} \epsilon D^{\alpha}\mathfrak{x}_{\mathfrak{i}}(t) & = -(a_{\mathfrak{i}} + \Delta a_{\mathfrak{i}}(t))\mathfrak{x}_{\mathfrak{i}}(t) + \sum_{\mathfrak{j}=1}^{m}(c_{\mathfrak{j}\mathfrak{i}} + \Delta c_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{j}}(y_{\mathfrak{j}}(t)) \\ & \quad + \sum_{\mathfrak{j}=1}^{m}(\mathfrak{p}_{\mathfrak{j}\mathfrak{i}} + \Delta\mathfrak{p}_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{v}_{\mathfrak{j}}(y_{\mathfrak{j}}(t-\sigma)) + (b_{\mathfrak{i}} + \Delta b_{\mathfrak{i}}(t))\sum_{l=1}^{\gamma} r_{\mathfrak{l}\mathfrak{i}}(t)\theta_l + I_{\mathfrak{i}}, \\ D^{\alpha} r_{\mathfrak{l}\mathfrak{i}}(t) & = -(d_{\mathfrak{i}} + \Delta d_{\mathfrak{i}}(t))r_{\mathfrak{l}\mathfrak{i}}(t) + \theta_l(\mathfrak{h}_{\mathfrak{i}} + \Delta\mathfrak{h}_{\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{i}}(y_{\mathfrak{i}}(t)), \\ & \quad \mathfrak{i} = 1,2,..,n, \mathfrak{l} = 1,2,...\gamma, \\ \epsilon D^{\alpha} y_{\mathfrak{j}}(t) & = -(l_{\mathfrak{j}} + \Delta l_{\mathfrak{j}}(t))y_{\mathfrak{j}}(t) + \sum_{\mathfrak{i}=1}^{n}(e_{\mathfrak{i}\mathfrak{j}} + \Delta e_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{g}_{\mathfrak{i}}(\mathfrak{x}_{\mathfrak{i}}(t)) \\ & \quad + \sum_{\mathfrak{i}=1}^{n}(\mathfrak{q}_{\mathfrak{i}\mathfrak{j}} + \Delta\mathfrak{q}_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{u}_{\mathfrak{i}}(\mathfrak{x}_{\mathfrak{i}}(t-\sigma)) + (m_{\mathfrak{j}} + \Delta m_{\mathfrak{j}}(t))\sum_{\mathfrak{o}=1}^{\rho} s_{\mathfrak{o}\mathfrak{j}}(t)\delta_{\mathfrak{o}} + J_{\mathfrak{j}}, \\ D^{\alpha} s_{\mathfrak{o}\mathfrak{j}}(t) & = -(t_{\mathfrak{j}} + \Delta t_{\mathfrak{j}}(t))s_{\mathfrak{o}\mathfrak{j}}(t) + \delta_{\mathfrak{o}}(\bar{\mathfrak{h}}_{\mathfrak{j}} + \Delta\bar{\mathfrak{h}}_{\mathfrak{j}}(t))g_{\mathfrak{j}}(\mathfrak{x}_{\mathfrak{j}}(t)), \\ & \quad \mathfrak{j} = 1,2,...,m, \mathfrak{o} = 1,2,....\rho. \end{cases} \quad (1)$$

where $\mathfrak{x}(t) = (\mathfrak{x}_1(t), \mathfrak{x}_2(t), ..., \mathfrak{x}_n(t))^T, y(t) = (y_1(t), y_2(t), ..., y_m(t))^T$, $\mathfrak{x}_{\mathfrak{i}}(t)$, $y_{\mathfrak{j}}(t)$ are the states of ith and jth neurons in the U-layer and V-layer, respectively. $\mathfrak{f}_{\mathfrak{j}}(\cdot)$, $\mathfrak{g}_{\mathfrak{i}}(\cdot)$ are the neuron activation functions. Where D^{α}, $\alpha(0,1)$ is the Caputo fractional-order derivative operator, which is discussed in the succeeding section; $a_{\mathfrak{i}} > 0$,and $l_{\mathfrak{j}} > 0$ are the time constants; $d_{\mathfrak{i}}$, $\mathfrak{h}_{\mathfrak{i}}$, $t_{\mathfrak{j}}$ and $\bar{\mathfrak{h}}_{\mathfrak{j}}$ represent the disposable scaling positive scalars; $c_{\mathfrak{j}\mathfrak{i}}$ and $e_{\mathfrak{i}\mathfrak{j}}$ represents the connection weights of ith neuron and jth neuron, $\mathfrak{p}_{\mathfrak{j}\mathfrak{i}}$ and $\mathfrak{q}_{\mathfrak{i}\mathfrak{j}}$ represents the synaptic interconnection weight of describing the dynamical efficiency of the synaptic strength between the U-layer and the V-layer, respectively. σ denotes the constant time delay; $r_{\mathfrak{l}\mathfrak{i}}$ and $s_{\mathfrak{o}\mathfrak{j}}$ represents the synaptic efficiency; $b_{\mathfrak{i}}$, $m_{\mathfrak{j}}$ represents the strength of the external stimulus terms; $\epsilon = (\epsilon_1, \epsilon_2, ..., \epsilon_m)^T$ represents the constant external stimulus; and $I_{\mathfrak{i}}$ and $J_{\mathfrak{j}}$ are constant external inputs. Next, we denote $\mathfrak{u}_{\mathfrak{i}}(t) = \sum_{l=1}^{\gamma} r_{\mathfrak{l}\mathfrak{i}}(t)\theta_l = \theta r_i^T(t)$, $\mathfrak{i} = 1,2,..,n$ and $\mathfrak{v}_{\mathfrak{j}}(t) = \sum_{o=1}^{\rho} s_{\mathfrak{o}\mathfrak{j}}(t)\delta_o = \delta s_{\mathfrak{j}}^T(t)$, $\mathfrak{j} = 1,2,..,m$. $\Delta a_{\mathfrak{i}}$, $\Delta c_{\mathfrak{j}\mathfrak{i}}$, $\Delta\mathfrak{p}_{\mathfrak{j}\mathfrak{i}}$, $\Delta b_{\mathfrak{i}}$,$\Delta d_{\mathfrak{i}}$, $\Delta\mathfrak{h}_{\mathfrak{i}}$,$\Delta l_{\mathfrak{j}}$, $\Delta e_{\mathfrak{i}\mathfrak{j}}$, $\Delta\mathfrak{q}_{\mathfrak{i}\mathfrak{j}}$, $\Delta m_{\mathfrak{j}}$, $\Delta t_{\mathfrak{j}}$, $\Delta\bar{\mathfrak{h}}_{\mathfrak{j}}$, are uncertain parameters, which will be defined later.

The state-space form of Equation (1) can be rearranged by the following form:

$$\begin{cases} \epsilon D^{\alpha}\mathfrak{x}_{\mathfrak{i}}(t) & = -(a_{\mathfrak{i}} + \Delta a_{\mathfrak{i}}(t)\mathfrak{x}_{\mathfrak{i}}(t) + \sum_{\mathfrak{j}=1}^{m}(c_{\mathfrak{j}\mathfrak{i}} + \Delta c_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{j}}(y_{\mathfrak{j}}(t)) \\ & \quad + \sum_{\mathfrak{j}=1}^{m}(\mathfrak{p}_{\mathfrak{j}\mathfrak{i}} + \Delta\mathfrak{p}_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{v}_{\mathfrak{j}}(y_{\mathfrak{j}}(t-\sigma)) + (b_{\mathfrak{i}} + \Delta b_{\mathfrak{i}}(t))\mathfrak{u}_{\mathfrak{i}}(t) + I_{\mathfrak{i}}, \\ D^{\alpha}\mathfrak{u}_{\mathfrak{i}}(t) & = -(d_i + \Delta d_i(t))\mathfrak{u}_{\mathfrak{i}}(t) + |\theta_l^2|(\mathfrak{h}_{\mathfrak{i}} + \Delta\mathfrak{h}_{\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{i}}(\mathfrak{u}_{\mathfrak{i}}(t)), \mathfrak{i} = 1,2,..,n, \\ \epsilon D^{\alpha} y_{\mathfrak{j}}(t) & = -(l_{\mathfrak{j}} + \Delta l_{\mathfrak{j}}(t))y_{\mathfrak{j}}(t) + \sum_{\mathfrak{i}=1}^{n}(e_{\mathfrak{i}\mathfrak{j}} + \Delta e_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{g}_{\mathfrak{i}}(x_{\mathfrak{i}}(t)) \\ & \quad + \sum_{\mathfrak{i}=1}^{n}(\mathfrak{q}_{\mathfrak{i}\mathfrak{j}} + \Delta\mathfrak{q}_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{u}_{\mathfrak{i}}(\mathfrak{x}_{\mathfrak{i}}(t-\sigma)) + (m_{\mathfrak{j}} + \Delta m_{\mathfrak{j}}(t))\mathfrak{v}_{\mathfrak{j}}(t) + J_{\mathfrak{j}}, \\ D^{\alpha}\mathfrak{v}_{\mathfrak{j}}(t) & = -(t_{\mathfrak{j}} + \Delta t_{\mathfrak{j}}(t))\mathfrak{v}_{\mathfrak{j}}(t) + |\delta_o^2|(\bar{\mathfrak{h}}_{\mathfrak{j}} + \Delta\bar{\mathfrak{h}}_{\mathfrak{j}}(t))g_{\mathfrak{j}}(\mathfrak{v}_{\mathfrak{j}}(t)), \mathfrak{j} = 1,2,...,m \end{cases} \quad (2)$$

where $|\theta_l|^2 = \theta_1^2 + ... + \theta_m^2$ and $|\delta_o|^2 = \delta_1^2 + ... + \delta_n^2$ are scalars. Without loss of generality, the input stimulus vectors θ and δ are assumed to normalized with unit magnitudes $|\theta_l^2| = 1$ and $|\delta_o^2| = 1$ for l = 1, 2 ,..., n and o = 1, 2, ..., m.

$$\begin{cases} \epsilon D^{\alpha}\mathfrak{x}_{\mathfrak{i}}(t) & = -(a_{\mathfrak{i}} + \Delta a_{\mathfrak{i}}(t))\mathfrak{x}_{\mathfrak{i}}(t) + \sum_{\mathfrak{j}=1}^{m}(c_{\mathfrak{j}\mathfrak{i}} + \Delta c_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{j}}(y_{\mathfrak{j}}(t)) \\ & \quad + \sum_{\mathfrak{j}=1}^{m}(\mathfrak{p}_{\mathfrak{j}\mathfrak{i}} + \Delta\mathfrak{p}_{\mathfrak{j}\mathfrak{i}}(t))\mathfrak{v}_{\mathfrak{j}}(y_{\mathfrak{j}}(t-\sigma)) + (b_{\mathfrak{i}} + \Delta b_{\mathfrak{i}}(t))\mathfrak{u}_{\mathfrak{i}}(t) + I_{\mathfrak{i}}, \\ D^{\alpha}\mathfrak{u}_{\mathfrak{i}}(t) & = -(d_{\mathfrak{i}} + \Delta d_{\mathfrak{i}}(t))\mathfrak{u}_{\mathfrak{i}}(t) + (h_{\mathfrak{i}} + \Delta\mathfrak{h}_{\mathfrak{i}}(t))\mathfrak{f}_{\mathfrak{i}}(\mathfrak{u}_{\mathfrak{i}}(t)), \mathfrak{i} = 1,2,..,n, \\ \epsilon D^{\alpha} y_{\mathfrak{j}}(t) & = -(l_{\mathfrak{j}} + \Delta l_{\mathfrak{j}}(t))y_{\mathfrak{j}}(t) + \sum_{\mathfrak{i}=1}^{n}(e_{\mathfrak{i}\mathfrak{j}} + \Delta e_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{g}_{\mathfrak{i}}(\mathfrak{x}_{\mathfrak{i}}(t)) \\ & \quad + \sum_{\mathfrak{i}=1}^{n}(\mathfrak{q}_{\mathfrak{i}\mathfrak{j}} + \Delta\mathfrak{q}_{\mathfrak{i}\mathfrak{j}}(t))\mathfrak{u}_{\mathfrak{i}}(\mathfrak{x}_{\mathfrak{i}}(t-\sigma)) + (m_{\mathfrak{j}} + \Delta m_{\mathfrak{j}}(t))\mathfrak{v}_{\mathfrak{j}}(t) + J_{\mathfrak{j}}, \\ D^{\alpha}\mathfrak{v}_{\mathfrak{j}}(t) & = -(t_{\mathfrak{j}} + \Delta t_{\mathfrak{j}}(t))\mathfrak{v}_{\mathfrak{j}}(t) + (\bar{\mathfrak{h}}_{\mathfrak{j}} + \Delta\bar{\mathfrak{h}}_{\mathfrak{j}}(t))g_{\mathfrak{j}}(\mathfrak{v}_{\mathfrak{j}}(t)), \mathfrak{j} = 1,2,...,m, \end{cases} \quad (3)$$

The compact form of (3) is given by

$$\begin{cases} D^{\alpha}\mathfrak{x}(t) & = -\frac{1}{\epsilon}(A + \Delta A(t))\mathfrak{x}(t) + \frac{1}{\epsilon}(C + \Delta C(t))\mathfrak{f}(y(t)) \\ & \quad + \frac{1}{\epsilon}(P + \Delta P(t))\mathfrak{v}(y(t-\sigma)) + \frac{1}{\epsilon}(B + \Delta B(t))\mathfrak{u}(t) + \frac{I}{\epsilon}, \\ D^{\alpha}\mathfrak{u}(t) & = -(D + \Delta D(t))\mathfrak{u}(t) + (H + \Delta H(t))\mathfrak{f}(\mathfrak{u}(t)), \\ D^{\alpha} y(t) & = -\frac{1}{\epsilon}(L + \Delta L(t))y(t) + \frac{1}{\epsilon}(E + \Delta E(t))g(\mathfrak{x}(t)) \\ & \quad + \frac{1}{\epsilon}(Q + \Delta Q(t))\mathfrak{u}(\mathfrak{x}(t-\sigma)) + \frac{1}{\epsilon}(M + \Delta M(t))\mathfrak{v}(t) + \frac{J}{\epsilon}, \\ D^{\alpha}\mathfrak{v}(t) & = -(T + \Delta T(t))\mathfrak{v}(t) + (\bar{H} + \Delta\bar{H}(t))\mathfrak{g}(\mathfrak{v}(t)), \end{cases} \quad (4)$$

where $A = diag\{a_1(t), ..., a_n(t)\}$, $P = (\mathfrak{p}_{ji})_{n\times n}$, $C = (c_{ji})_{n\times n}$, $B = diag\{b_1(t), ..., b_n(t)\}$, $D = diag\{d_1(t), ..., d_n(t)\}$, $H = diag\{\mathfrak{h}_1(t), ..., \mathfrak{h}_n(t)\}$, $\Delta A(t) = diag\{\Delta a_1(t), ..., \Delta a_n(t)\}$, $\Delta C(t) = (\Delta c_{\mathfrak{j}\mathfrak{i}})_{n\times n}(t)$, $\Delta P(t) = (\Delta\mathfrak{p}_{\mathfrak{j}\mathfrak{i}})_{n\times n}(t)$, $\Delta B(t) = diag\{\Delta b_1(t), ..., \Delta b_n(t)\}$,

$\Delta D(t) = diag\{\Delta d_1(t), ..., \Delta d_n(t)\}$, $\Delta H(t) = diag\{\Delta \mathfrak{h}_1(t), ..., \Delta \mathfrak{h}_n(t)\}$,
$L = diag\{l_1(t), ..., l_m(t)\}$, $E = (e_{ij})_{m\times m}$, $Q = (\mathfrak{q}_{ij})_{m\times m}$, $M = diag\{m_1(t), ..., m_m(t)\}$,
$T = diag\{t_1(t), ..., t_m(t)\}$, $\bar{H} = diag\{\bar{\mathfrak{h}}_1(t), ..., \bar{\mathfrak{h}}_m(t)\}$, $\Delta L(t) = diag\{\Delta l_1(t), ..., \Delta l_m(t)\}$,
$\Delta E(t) = (\Delta e_{ij})_{m\times m}(t)$, $\Delta Q(t) = (\Delta \mathfrak{q}_{ij})_{m\times m}(t)$, $\Delta M(t) = diag\{\Delta m_1(t), ..., \Delta m_m(t)\}$,
$\Delta T(t) = diag\{\Delta t_1(t), ..., \Delta t_m(t)\}$, $\Delta\bar{H}(t) = diag\{\Delta\bar{\mathfrak{h}}_1(t), ..., \Delta\bar{\mathfrak{h}}_m(t)\}$.

The structure of time-varying parameter uncertain matrices $\Delta A(t), \Delta C(t), \Delta P(t), \Delta B(t)$, $\Delta D(t)$,$\Delta H(t)$ $\Delta L(t), \Delta E(t), \Delta Q(t), \Delta M(t), \Delta T(t)$, and $\Delta\bar{H}(t)$ satisfy the following conditions:

$$\begin{aligned}
&\Delta A(t) = J_a K(t) L_a, \Delta C(t) = J_c K(t) L_c, \Delta P(t) = J_{\mathfrak{p}} K(t) L_{\mathfrak{p}}, \Delta B(t) = J_b K(t) L_b,\\
&\Delta D(t) = J_d K(t) L_d, \Delta H(t) = J_h K(t) L_h, \Delta L(t) = J_l K(t) L_l, \Delta E(t) = J_e K(t) L_e,\\
&\Delta Q(t) = J_{\mathfrak{q}} K(t) L_{\mathfrak{q}}, \Delta M(t) = J_m K(t) L_m, \Delta T(t) = J_t K(t) L_t, and \Delta\bar{H}(t) = J_h K(t) L_h.
\end{aligned}$$

where $I, J_a, J_c, J_p, J_b, J_d, J_h, J_l, J_e, J_{\mathfrak{q}}, J_m, J_h, J_t, L_a, L_c, L_{\mathfrak{p}}, L_b, L_d, L_h, L_l, L_e, L_{\mathfrak{q}}, L_m, L_h$, and L_t are matrices are known constant, and $K(t)$ is an unknown time-varying matrix, satisfying $K^T(t)K(t) \leq 1$. The initial values of system (4) are associated with $\mathfrak{x}(t) = \rho(t) \in C([-\sigma, 0], R^n)$, $\mathfrak{u}(t) = \tilde{\rho}(t) \in C([-\sigma, 0], R^n)$ $y(t) = \Psi(t) \in C([-\sigma, 0], R^m)$ and $\mathfrak{v}(t) = \tilde{\Psi}(t) \in C([-\sigma, 0], R^m)$

The corresponding response (slave) system of master system (4) is

$$\begin{cases}
D^\alpha \check{\mathfrak{x}}(t) &= -\frac{1}{\epsilon}(A + \Delta A(t))\check{\mathfrak{x}}(t) + \frac{1}{\epsilon}(C + \Delta C(t))f(\check{y}(t))\\
&\quad +\frac{1}{\epsilon}(P + \Delta P(t))\mathfrak{v}(\check{y}(t-\sigma)) + \frac{1}{\epsilon}(B + \Delta B(t))\check{\mathfrak{u}}(t) + \frac{I}{\epsilon} + \alpha(t),\\
D^\alpha \check{\mathfrak{u}}(t) &= -(D + \Delta D(t))\check{\mathfrak{u}}(t) + (H + \Delta H(t))\mathfrak{f}(\check{\mathfrak{u}}(t)) + \beta(t),\\
D^\alpha \check{y}(t) &= -\frac{1}{\epsilon}(L + \Delta L(t))\check{y}(t) + \frac{1}{\epsilon}(E + \Delta E(t))\mathfrak{g}(\check{\mathfrak{x}}(t))\\
&\quad +\frac{1}{\epsilon}(Q + \Delta Q(t))\mathfrak{u}(\check{\mathfrak{x}}(t-\sigma)) + \frac{1}{\epsilon}(M + \Delta M(t))\check{\mathfrak{v}}(t) + \frac{I}{\epsilon} + \gamma(t),\\
D^\alpha \check{\mathfrak{v}}(t) &= -(T + \Delta T(t))\check{\mathfrak{v}}(t) + (\bar{H} + \Delta\bar{H}(t))\mathfrak{g}(\check{\mathfrak{v}}(t)) + \delta(t),
\end{cases} \tag{5}$$

Then, the error system can be described as follows: $\check{\mathfrak{x}}(t) - \mathfrak{x}(t) = \mathfrak{w}(t)$, $\check{\mathfrak{u}}(t) - \mathfrak{u}(t) = \theta(t)$, $\check{y}(t) - y(t) = \mathfrak{z}(t)$, $\check{\mathfrak{v}}(t) - \mathfrak{v}(t) = \psi(t)$

$$\begin{cases}
D^\alpha \mathfrak{w}(t) &= -\frac{1}{\epsilon}(A + \Delta A(t)\mathfrak{w}(t) + \frac{1}{\epsilon}(C + \Delta C(t))\mathfrak{f}(\mathfrak{z}(t))\\
&\quad +\frac{1}{\epsilon}(P + \Delta P(t))\mathfrak{v}(\mathfrak{z}(t-\sigma)) + \frac{1}{\epsilon}(B + \Delta B(t))\theta(t) + \alpha(t),\\
D^\alpha \theta(t) &= -(D + \Delta D(t))\theta(t) + (H + \Delta H(t))\mathfrak{f}(\theta(t)) + \beta(t),\\
D^\alpha \mathfrak{z}(t) &= -\frac{1}{\epsilon}(L + \Delta L(t))\mathfrak{z}(t) + \frac{1}{\epsilon}(E + \Delta E(t))\mathfrak{g}(\mathfrak{w}(t))\\
&\quad +\frac{1}{\epsilon}(Q + \Delta Q(t))\mathfrak{u}(\mathfrak{w}(t-\sigma)) + \frac{1}{\epsilon}(M + \Delta M(t))\psi(t) + \gamma(t),\\
D^\alpha \psi(t) &= -(T + \Delta T(t))\psi(t) + (\bar{H} + \Delta\bar{H}(t))\mathfrak{g}(\psi(t)) + \delta(t),
\end{cases} \tag{6}$$

where, $\alpha(t), \beta(t), \gamma(t), \delta(t)$ denotes an feedback controllers:

$$\alpha(t) = \Im_1 \mathfrak{w}(t), \beta(t) = \Im_2 \theta(t), \gamma(t) = \Im_3 \mathfrak{z}(t), \delta(t) = \Im_4 \psi(t).$$

Theorem 1. *For a given scalars, λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, μ_1, μ_2, μ_3, μ_4, μ_5, μ_6, ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6, η_1, η_2, η_3, η_4, η_5, η_6, if there exist a positive definite matrices R_1, M_1, U_1, R_2, M_2, U_2, such that the following LMI holds:*

$$\Upsilon_1 = \begin{bmatrix}
\Psi_1 & R_1C & R_1P & R_1B & R_1 & R_1J_a & R_1J_c & R_1J_p & R_1J_b\\
* & -\epsilon\lambda_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & -\epsilon\lambda_2 I & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & -\epsilon\lambda_3 I & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & -\lambda_4 I & 0 & 0 & 0 & 0\\
* & * & * & * & * & -\epsilon\mu_1 I & 0 & 0 & 0\\
* & * & * & * & * & * & -\epsilon\mu_2 I & 0 & 0\\
* & * & * & * & * & * & * & -\epsilon\mu_3 I & 0\\
* & * & * & * & * & * & * & * & -\epsilon\mu_4 I
\end{bmatrix} < 0 \tag{7}$$

$$\Upsilon_2 = \begin{bmatrix} \Psi_2 & R_2E & R_2Q & R_2M & R_2 & R_2J_l & R_2J_e & R_2J_q & R_2J_m \\ * & -\epsilon\xi_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\epsilon\xi_2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon\xi_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\xi_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\epsilon\eta_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon\eta_2 I & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon\eta_3 I & 0 \\ * & * & * & * & * & * & * & * & -\epsilon\eta_4 I \end{bmatrix} < 0 \quad (8)$$

$$\Upsilon_3 = \begin{bmatrix} \Psi_3 & U_1H & U_1 & U_1J_d & U_1J_h \\ * & -\lambda_5 I & 0 & 0 & 0 \\ * & * & -\lambda_6 I & 0 & 0 \\ * & * & * & -\mu_5 I & 0 \\ * & * & * & * & -\mu_6 I \end{bmatrix} < 0 \quad (9)$$

$$\Upsilon_4 = \begin{bmatrix} \Psi_4 & U_2\bar{H} & U_2 & U_2J_t & U_2J_{\mathfrak{h}} \\ * & -\xi_5 I & 0 & 0 & 0 \\ * & * & -\xi_6 I & 0 & 0 \\ * & * & * & -\eta_5 I & 0 \\ * & * & * & * & -\eta_6 I \end{bmatrix} < 0 \quad (10)$$

$$\begin{aligned} \Psi_1 &= \frac{-2}{\epsilon}R_1A + M_1 + \frac{\mu_1}{\epsilon}L_a^TL_a + \frac{\xi_1}{\epsilon}\phi^T\phi + \frac{\eta_2}{\epsilon}\phi^TL_e^TL_e\phi + 2R_1\Im_1, \\ \Psi_2 &= -\frac{2}{\epsilon}R_2L + M_2 + \frac{\eta_1}{\epsilon}L_l^TL_l + \frac{\lambda_1}{\epsilon}\phi^T\phi + \frac{\mu_2}{\epsilon}\phi^TL_c^TL_c\phi + 2R_2\Im_3, \\ \Psi_3 &= \frac{\lambda_3}{\epsilon} + \frac{\mu_4}{\epsilon}L_b^TL_b - \frac{2}{\epsilon}U_1D + \lambda_5\phi^T\phi + \mu_5L_d^TL_d + \mu_6\phi^TL_{\mathfrak{h}}^TL_{\mathfrak{h}}\phi + 2U_1\Im_2, \\ \Psi_4 &= \frac{\xi_3}{\epsilon} + \frac{\eta_4}{\epsilon}L_m^TL_m - \frac{2}{\epsilon}TU_2 + \xi_5\phi^T\phi + \eta_5L_t^TL_t + \eta_6\phi^TL_{\bar{\mathfrak{h}}}^TL_{\bar{\mathfrak{h}}}\phi + 2U_2\Im_4, \end{aligned}$$

then the drive system (4) is synchronized with the slave system (5). The detailed proof of Theorem 1 can be referred to in Appendix A.

Theorem 2. *For given scalars, λ_1, λ_2, λ_3, λ_5, ξ_1, ξ_2, ξ_3, ξ_5, if there exist positive definite matrices R_1, R_2 U_1, U_2, M_1, M_2, the scalars and the following matrix hold:*

$$\begin{bmatrix} \pi_1 & R_1C & R_1P & R_1B \\ * & -\epsilon\lambda_1 I & 0 & 0 \\ * & * & -\epsilon\lambda_2 I & 0 \\ * & * & * & -\epsilon\lambda_3 I \end{bmatrix} < 0, \quad (11)$$

$$\begin{bmatrix} \pi_2 & R_2E & R_2Q & R_2M \\ * & -\epsilon\xi_1 I & 0 & 0 \\ * & * & -\epsilon\xi_2 I & 0 \\ * & * & * & -\epsilon\xi_3 I \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \pi_3 & U_1H \\ * & -\lambda_5 I \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} \pi_4 & U_2\bar{H} \\ * & -\xi_5 I \end{bmatrix} < 0, \quad (14)$$

$$\pi_1 = \Big[-\frac{2}{\epsilon}R_1A + M_1 + \frac{\xi_1}{\epsilon}\phi^T\phi\Big],$$
$$\pi_2 = \Big[-\frac{2}{\epsilon}R_2L + M_2 + \frac{\lambda_1}{\epsilon}\phi^T\phi\Big],$$
$$\pi_3 = \Big[\frac{\lambda_3}{\epsilon} - 2U_1D + \lambda_5\phi^T\phi\Big],$$
$$\pi_4 = \Big[\frac{\xi_3}{\epsilon} - 2U_2T + \xi_5\phi^T\phi\Big].$$

then the drive system (4) is synchronized with the slave system (5).

Proof. Neglecting uncertain, controls terms in system (6),

$$\begin{cases} D^\alpha\mathfrak{w}(t) &= \frac{-A}{\epsilon}\mathfrak{w}(t) + \frac{C}{\epsilon}\mathfrak{f}(\mathfrak{z}(t)) + \frac{P}{\epsilon}\mathfrak{v}(\mathfrak{z}(t-\sigma)) + \frac{B}{\epsilon}\theta(t), \\ D^\alpha\theta(t) &= -D\theta(t) + H\mathfrak{f}(\theta(t)), \\ D^\alpha\mathfrak{z}(t) &= \frac{-L}{\epsilon}\mathfrak{z}(t) + \frac{E}{\epsilon}\mathfrak{g}(\mathfrak{w}(t)) + \frac{Q}{\epsilon}\mathfrak{u}(\mathfrak{w}(t-\sigma)) + \frac{M}{\epsilon}\psi(t), \\ D^\alpha\psi(t) &= -T\psi(t) + \bar{H}\mathfrak{g}(\psi(t)), \end{cases} \tag{15}$$

We select the following Lyapunov–Krasovskii functional:

$$\begin{aligned} V(t) = {} & \mathfrak{w}^T(t)R_1\mathfrak{w}(t) + \theta^T(t)U_1\theta(t) + \frac{1}{\Gamma(\beta)}\int_0^t (t-\gamma)\mathfrak{w}^T(\gamma)M_1\mathfrak{w}(\gamma)d\gamma \\ & - \frac{1}{\Gamma(\beta)}\int_0^{t-\sigma}(t-\sigma-\gamma)\mathfrak{w}^T(\gamma)M_1\mathfrak{w}(\gamma)d\gamma + \mathfrak{z}^T(t)R_2\mathfrak{z}(t) + \psi^T(t)U_2\psi(t) \\ & + \frac{1}{\Gamma(\beta)}\int_0^t (t-\gamma)\mathfrak{z}^T(\gamma)M_2\mathfrak{z}(\gamma)d\gamma - \frac{1}{\Gamma(\beta)}\int_0^{t-\sigma}(t-\sigma-\gamma)\mathfrak{z}^T(\gamma)M_2\mathfrak{z}(\gamma)d\gamma \end{aligned} \tag{16}$$

Lemmas 2 and 5, the following estimation can be derived:

$$\begin{aligned} D^\alpha V(t) \le {} & 2\mathfrak{w}^T(t)R_1D^\alpha\mathfrak{w}(t) + 2\theta^T(t)U_1\theta(t) + \mathfrak{w}^T(t)M_1\mathfrak{w}(t) \\ & - \mathfrak{w}^T(t-\sigma)M_1\mathfrak{w}(t-\sigma) + 2\mathfrak{z}^T(t)R_2D^\alpha\mathfrak{z}(t) + 2\psi^T(t)U_2\psi(t) \\ & + \mathfrak{z}^T(t)M_2\mathfrak{z}(t) - \mathfrak{z}^T(t-\sigma)M_2\mathfrak{z}(t-\sigma). \end{aligned} \tag{17}$$

According to Lemma 6, we obtain

$$\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1C\mathfrak{f}(\mathfrak{z}(t)) \le \frac{\lambda_1^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1CC^TR_1^T\mathfrak{w}(t) + \frac{\lambda_1}{\epsilon}\mathfrak{z}^T(t)\phi^T\phi\mathfrak{z}(t) \tag{18}$$
$$\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1P\mathfrak{v}(\mathfrak{z}(t-\sigma)) \le \frac{\lambda_2^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1PP^TR_1^T\mathfrak{w}(t) + \frac{\lambda_2}{\epsilon}\mathfrak{z}^T(t-\sigma)\phi^T\phi\mathfrak{z}(t-\sigma) \tag{19}$$
$$\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1B\theta(t) \le \frac{\lambda_3^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1BB^TR_1^T\mathfrak{w}(t) + \frac{\lambda_3}{\epsilon}\theta^T(t)\theta(t) \tag{20}$$
$$2\theta^T(t)U_1H\mathfrak{f}(\theta(t)) \le \lambda_5^{-1}\theta^T(t)U_1HH^TU_1^T\theta(t) + \lambda_5\theta^T(t)\phi^T\phi\theta(t) \tag{21}$$
$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2E\mathfrak{g}(\mathfrak{w}(t)) \le \frac{\xi_1^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2EE^TR_2^T\mathfrak{z}(t) + \frac{\xi_1}{\epsilon}\mathfrak{w}^T(t)\phi^T\phi\mathfrak{w}(t) \tag{22}$$
$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2Q\mathfrak{u}(\mathfrak{w}(t-\sigma)) \le \frac{\xi_2^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2QQ^TR_2^T\mathfrak{z}(t) + \frac{\xi_2}{\epsilon}\mathfrak{w}^T(t-\sigma)\phi^T\phi\mathfrak{w}(t-\sigma) \tag{23}$$
$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2M\psi(t) \le \frac{\xi_3^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2MM^TR_2^T\mathfrak{z}(t) + \frac{\xi_3}{\epsilon}\psi^T(t)\psi(t) \tag{24}$$
$$2\psi^T(t)U_2\bar{H}g(\psi(t)) \le \xi_5^{-1}\psi^T(t)U_2\bar{H}\bar{H}^TU_2^T\psi(t) + \xi_5\psi^T(t)\phi^T\phi\psi(t). \tag{25}$$

From (17)–(25) we get,

$$
\begin{aligned}
D^{\alpha}V(t) \leq{}& \mathfrak{w}^T(t)\Big[-\frac{2}{\epsilon}R_1A+\frac{\lambda_1^{-1}}{\epsilon}R_1CC^TR_1^T+\frac{\lambda_2^{-1}}{\epsilon}R_1PP^TR_1^T+\frac{\lambda_3^{-1}}{\epsilon}R_1BB^TR_1^T\\
&+M_1+\frac{\xi_1}{\epsilon}\phi^T\phi\Big]\mathfrak{w}(t)+\mathfrak{z}^T(t)\Big[-\frac{2}{\epsilon}R_2L+\frac{\xi_1^{-1}}{\epsilon}R_2EE^TR_2^T+\frac{\xi_2^{-1}}{\epsilon}R_2QQ^TR_2^T\\
&+\frac{\xi_3^{-1}}{\epsilon}R_2MM^TR_2^T+M_2+\frac{\lambda_1}{\epsilon}\phi^T\phi\Big]\mathfrak{z}(t)+\mathfrak{w}^T(t-\sigma)\Big[\frac{\xi_2}{\epsilon}\phi^T\phi-M_1\Big]\mathfrak{w}(t-\sigma)\\
&+\mathfrak{z}^T(t-\sigma)\Big[\frac{\lambda_2}{\epsilon}\phi^T\phi-M_2\Big]\mathfrak{z}(t-\sigma)+\theta^T(t)\Big[\frac{\lambda_3}{\epsilon}-2U_1D+\lambda_5^{-1}U_1HH^TU_1^T\\
&+\lambda_5\phi^T\phi\Big]\theta(t)+\psi^T(t)\Big[\frac{\xi_3}{\epsilon}-2U_2T+\xi_5^{-1}U_2\bar{H}\bar{H}^TU_2^T+\xi_5\phi^T\phi\Big]\psi(t)
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\Theta_1 &= \pi_1+\Big[\frac{\lambda_1^{-1}}{\epsilon}R_1CC^TR_1^T+\frac{\lambda_2^{-1}}{\epsilon}R_1PP^TR_1^T+\frac{\lambda_3^{-1}}{\epsilon}R_1BB^TR_1^T\Big],\\
\Theta_2 &= \pi_2+\Big[\frac{\xi_1^{-1}}{\epsilon}R_2EE^TR_2^T+\frac{\xi_2^{-1}}{\epsilon}R_2QQ^TR_2^T+\frac{\xi_3^{-1}}{\epsilon}R_2MM^TR_2^T\Big],\\
\Theta_3 &= \pi_3+\lambda_5^{-1}U_1HH^TU_1^T,\\
\Theta_4 &= \pi_4+\xi_5^{-1}U_2\bar{H}\bar{H}^TU_2^T.
\end{aligned}
$$

□

System (6) is globally asymptotic stable. As a result, master system (4) is globally synchronized with slave system (5). This completes the proof of the theorem.

4. Numerical Examples

Example 1. *Consider the following fractional order uncertain BAM competitive neural networks,*

$$
\begin{cases}
D^{\alpha}\mathfrak{w}(t) &= -\frac{1}{\epsilon}(A+\Delta A(t)\mathfrak{w}(t)+\frac{1}{\epsilon}(C+\Delta C(t))\mathfrak{f}(\mathfrak{z}(t))\\
&\quad+\frac{1}{\epsilon}(P+\Delta P(t))\mathfrak{v}(\mathfrak{z}(t-\sigma))+\frac{1}{\epsilon}(B+\Delta B(t))\theta(t)+\alpha(t),\\
D^{\alpha}\theta(t) &= -(D+\Delta D(t))\theta(t)+(H+\Delta H(t))\mathfrak{f}(\theta(t))+\beta(t),\\
D^{\alpha}\mathfrak{z}(t) &= -\frac{1}{\epsilon}(L+\Delta L(t))\mathfrak{z}(t)+\frac{1}{\epsilon}(E+\Delta E(t))\mathfrak{g}(\mathfrak{w}(t))\\
&\quad+\frac{1}{\epsilon}(Q+\Delta Q(t))\mathfrak{u}(\mathfrak{w}(t-\sigma))+\frac{1}{\epsilon}(M+\Delta M(t))\psi(t)+\gamma(t),\\
D^{\alpha}\psi(t) &= -(T+\Delta T(t))\psi(t)+(\bar{H}+\Delta\bar{H}(t))\mathfrak{g}(\psi(t))+\delta(t),
\end{cases}
\tag{27}
$$

where $\epsilon = 1.589, \alpha = 0.91$

$$
A=\begin{bmatrix}1.324&0&0&0\\0&1.324&0&0\\0&0&1.324&0\\0&0&0&1.324\end{bmatrix}, D=\begin{bmatrix}1.567&0&0&0\\0&1.567&0&0\\0&0&1.567&0\\0&0&0&1.567\end{bmatrix},
$$

$$
H=\begin{bmatrix}1.876&0&0&0\\0&1.876&0&0\\0&0&1.876&0\\0&0&0&1.876\end{bmatrix}, L=\begin{bmatrix}1.456&0&0&0\\0&1.456&0&0\\0&0&1.456&0\\0&0&0&1.456\end{bmatrix},
$$

$$
T=\begin{bmatrix}2.654&0&0&0\\0&2.654&0&0\\0&0&2.654&0\\0&0&0&2.654\end{bmatrix}, \bar{H}=\begin{bmatrix}1.787&0&0&0\\0&1.787&0&0\\0&0&1.787&0\\0&0&0&1.787\end{bmatrix},
$$

$$C = \begin{bmatrix} 1.567 & 1.677 & 1.765 & 1.654 \\ 1.987 & 1.567 & 1.765 & 1.345 \\ 1.234 & 1.987 & 1.567 & 1.977 \\ 1.677 & 1.979 & 1.098 & 1.567 \end{bmatrix}, P = \begin{bmatrix} 1.876 & 1.0872 & 1.876 & 1.567 \\ 1.678 & 1.876 & 1.678 & 1.098 \\ 1.876 & 1.557 & 1.876 & 1.654 \\ 1.987 & 1.987 & 1.456 & 1.876 \end{bmatrix},$$

$$B = \begin{bmatrix} 1.456 & 0 & 0 & 0 \\ 0 & 1.456 & 0 & 0 \\ 0 & 0 & 1.456 & 0 \\ 0 & 0 & 0 & 1.456 \end{bmatrix}, E = \begin{bmatrix} 2.654 & 1.987 & 1.678 & 1.987 \\ 1.987 & 2.654 & 1.678 & 1.765 \\ 1.876 & 1.987 & 2.654 & 1.876 \\ 1.098 & 1.567 & 1.765 & 2.654 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.567 & 1.876 & 1.876 & 1.872 \\ 1.569 & 1.567 & 1.788 & 1.555 \\ 1.987 & 1.678 & 1.567 & 1.765 \\ 1.098 & 1.876 & 1.778 & 1.567 \end{bmatrix}, M = \begin{bmatrix} 1.876 & 0 & 0 & 0 \\ 0 & 1.876 & 0 & 0 \\ 0 & 0 & 1.876 & 0 \\ 0 & 0 & 0 & 1.876 \end{bmatrix}.$$

$$\begin{aligned}
L_a &= diag\{1.324, 1.324, 1.324, 1.324\}, L_e = diag\{1.567, 1.567, 1.567, 1.567\}, \\
L_l &= diag\{1.876, 1.876, 1.876, 1.876\}, L_c = diag\{1.456, 1.456, 1.456, 1.456\}, \\
L_b &= diag\{2.654, 2.654, 2.654, 2.654\}, L_d = diag\{1.787, 1.787, 1.787, 1.787\}, \\
L_m &= diag\{1.806, 1.806, 1.806, 1.806\}, L_h = diag\{1.876, 1.876, 1.876, 1.876\}, \\
L_t &= diag\{2.654, 2.654, 2.654, 2.654\}, J_c = diag\{2.456, 2.456, 2.456, 2.456\}, \\
J_p &= diag\{0.654, 0.654, 0.654, 0.654\}, J_b = diag\{1.087, 1.087, 1.087, 1.087\}, \\
J_l &= diag\{1.006, 1.006, 1.006, 1.006\}, J_q = diag\{1.806, 1.806, 1.806, 1.806\}, \\
J_m &= diag\{1.654, 1.654, 1.654, 1.654\}, J_d = diag\{1.054, 1.054, 1.054, 1.054\}, \\
J_{\bar{h}} &= diag\{1.767, 1.767, 1.767, 1.767\}, I = diag\{1, 1, 1, 1\}, \\
\phi &= diag\{1.786, 1.786, 1.786, 1.786\}, L_{\bar{h}} = diag\{1.787, 1.787, 1.787, 1.787\}, \\
J_a &= diag\{1.654, 1.654, 1.654, 1.654\}, J_h = diag\{0.597, 0.597, 0.597, 0.597\}, \\
J_e &= diag\{0.517, 0.517, 0.517, 0.517\}, J_l = diag\{1.876, 1.876, 1.876, 1.876
\end{aligned}$$

After using the LMI solver in the hypothesis of Theorem 1, we get the following feasible solutions:

$$T_1 = \begin{bmatrix} -9.0768 & 0.0002 & 0.0002 & 0.0002 \\ 0.0002 & -9.0766 & 0.0001 & 0.0002 \\ 0.0002 & 0.0001 & -9.0766 & 0.0002 \\ 0.0002 & 0.0002 & 0.0002 & -9.0768 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} -7.9371 & -1.0713 & -1.0679 & -1.0340 \\ -1.0713 & -7.9555 & -0.9906 & -1.0916 \\ -1.0679 & -0.9906 & -7.8274 & -1.0083 \\ -1.0340 & -1.0916 & -1.0083 & -7.8004 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} -9.0580 & -0.0057 & -0.0057 & -0.0055 \\ -0.0057 & -9.0581 & -0.0053 & -0.0058 \\ -0.0057 & -0.0053 & -9.0574 & -0.0054 \\ -0.0055 & -0.0058 & -0.0054 & -9.0573 \end{bmatrix},$$

$$T_4 = \begin{bmatrix} -3.1877 & 1.4322 & 1.4272 & 1.5078 \\ 1.4322 & -3.5551 & 1.5254 & 1.2934 \\ 1.4272 & 1.5254 & -3.5077 & 1.5386 \\ 1.5078 & 1.2934 & 1.5386 & -3.1431 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} -0.0166 & 0.0052 & 0.0052 & 0.0050 \\ 0.0052 & -0.0165 & 0.0048 & 0.0053 \\ 0.0052 & 0.0048 & -0.0171 & 0.0049 \\ 0.0050 & 0.0053 & 0.0049 & -0.0172 \end{bmatrix},$$

$$U_1 = 10e^4 * \begin{bmatrix} 0.8253 & 0.0557 & 0.0555 & 0.0538 \\ 0.0557 & 0.8272 & 0.0515 & 0.0568 \\ 0.0555 & 0.0515 & 0.8139 & 0.0524 \\ 0.0538 & 0.0568 & 0.0524 & 0.8111 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -4.5384 & 0.0001 & 0.0001 & 0.0001 \\ 0.0001 & -4.5383 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & -4.5383 & 0.0001 \\ 0.0001 & 0.0001 & 0.0001 & -4.5384 \end{bmatrix},$$

$$R_2 = 10e^4 * \begin{bmatrix} 0.4032 & -0.1389 & -0.1194 & -0.0996 \\ -0.1389 & 0.4255 & -0.1363 & -0.1391 \\ -0.1194 & -0.1363 & 0.4203 & -0.1301 \\ -0.0996 & -0.1391 & -0.1301 & 0.4073 \end{bmatrix},$$

$$U_2 = \begin{bmatrix} 0.2943 & 1.6937 & 1.1419 & 1.8154 \\ 1.6937 & 0.3751 & 1.4464 & 1.7804 \\ 1.1419 & 1.4464 & 1.1290 & 1.1535 \\ 1.8154 & 1.7804 & 1.1535 & 0.1751 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -4.5290 & -0.0029 & -0.0028 & -0.0028 \\ -0.0029 & -4.5290 & -0.0026 & -0.0029 \\ -0.0028 & -0.0026 & -4.5287 & -0.0027 \\ -0.0028 & -0.0029 & -0.0027 & -4.5286 \end{bmatrix}.$$

$\mu_1 = 14.2799, \mu_2 = 14.2799, \mu_3 = 14.2799, \mu_4 = 14.2799, \eta_1 = 14.2799, \eta_2 = 14.2799, \eta_3 = 14.2799, \eta_4 = 14.2799, \xi_1 = 14.2799, \xi_2 = 14.2799, \xi_3 = 14.2799, \xi_4 = 22.6901, \lambda_1 = 14.2799, \lambda_2 = 14.2799, \lambda_3 = 14.2799, \lambda_4 = 22.6901,$

The gain matrices of the desired controller gains can be obtained as follows

$$\Im_1 = 10e^3 * \begin{bmatrix} 1.7840 & 1.3669 & 1.3025 & 1.3118 \\ 1.3670 & 1.7859 & 1.2955 & 1.3172 \\ 1.3025 & 1.2955 & 1.6490 & 1.2479 \\ 1.3118 & 1.3172 & 1.2479 & 1.6703 \end{bmatrix},$$

$$\Im_2 = 10e^4 * \begin{bmatrix} -9.5017 & -0.5810 & -0.5826 & -0.5566 \\ -0.5797 & -9.5032 & -0.5284 & -0.5959 \\ -0.5908 & -0.5370 & -9.5087 & -0.5511 \\ -0.5663 & -0.6077 & -0.5530 & -9.5033 \end{bmatrix},$$

$$\Im_3 = 10e^5 * \begin{bmatrix} -8.0427 & -6.7558 & -6.4348 & -6.3297 \\ -6.7568 & -8.7789 & -6.8866 & -6.8507 \\ -6.4360 & -6.8868 & -8.2460 & -6.5603 \\ -6.3306 & -6.8505 & -6.5600 & -8.2057 \end{bmatrix},$$

$$\Im_4 = \begin{bmatrix} 15.2067 & 6.4515 & -34.2196 & 12.0679 \\ 4.9513 & 6.0147 & -15.9783 & 5.2256 \\ -31.9806 & -19.2653 & 84.7198 & -31.9213 \\ 11.2761 & 6.2499 & -32.0500 & 14.0781 \end{bmatrix}.$$

Hence, the master system (4) is synchronized with the slave system (5).

Example 2. *Consider the following fractional order uncertain BAM competitive neural networks:*

$$\begin{cases} D^{\alpha}\mathfrak{w}(t) &= \frac{-A}{\epsilon}\mathfrak{w}(t) + \frac{C}{\epsilon}\mathfrak{f}(\mathfrak{z}(t)) + \frac{P}{\epsilon}\mathfrak{v}(\mathfrak{z}(t-\sigma)) + \frac{B}{\epsilon}\theta(t), \\ D^{\alpha}\theta(t) &= -D\theta(t) + H\mathfrak{f}(\theta(t)), \\ D^{\alpha}\mathfrak{z}(t) &= \frac{-L}{\epsilon}\mathfrak{z}(t) + \frac{E}{\epsilon}\mathfrak{g}(\mathfrak{w}(t)) + \frac{Q}{\epsilon}\mathfrak{u}(\mathfrak{w}(t-\sigma)) + \frac{M}{\epsilon}\psi(t), \\ D^{\alpha}\psi(t) &= -T\psi(t) + \bar{H}\mathfrak{g}(\psi(t)), \end{cases} \tag{28}$$

where $\alpha = 0.98$, $\epsilon = 0.87$.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2.304 & 0 & 0 & 0 \\ 0 & 2.304 & 0 & 0 \\ 0 & 0 & 2.304 & 0 \\ 0 & 0 & 0 & 2.304 \end{bmatrix},$$

$$D = \begin{bmatrix} 2.567 & 0 & 0 & 0 \\ 0 & 2.567 & 0 & 0 \\ 0 & 0 & 2.567 & 0 \\ 0 & 0 & 0 & 2.567 \end{bmatrix}, H = \begin{bmatrix} 1.826 & 0 & 0 & 0 \\ 0 & 1.826 & 0 & 0 \\ 0 & 0 & 1.826 & 0 \\ 0 & 0 & 0 & 1.826 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.654 & 0 & 0 & 0 \\ 0 & 0.654 & 0 & 0 \\ 0 & 0 & 0.654 & 0 \\ 0 & 0 & 0 & 0.654 \end{bmatrix}, L = \begin{bmatrix} 1.453 & 0 & 0 & 0 \\ 0 & 1.453 & 0 & 0 \\ 0 & 0 & 1.453 & 0 \\ 0 & 0 & 0 & 1.453 \end{bmatrix},$$

$$\bar{H} = \begin{bmatrix} 3.787 & 0 & 0 & 0 \\ 0 & 3.787 & 0 & 0 \\ 0 & 0 & 3.787 & 0 \\ 0 & 0 & 0 & 3.787 \end{bmatrix}, C = \begin{bmatrix} 1.067 & 1.177 & 2.765 & 1.654 \\ 0.987 & 1.561 & 1.765 & 1.345 \\ 1.234 & 1.982 & 1.564 & 1.977 \\ 1.677 & 1.979 & 1.098 & 1.567 \end{bmatrix},$$

$$P = \begin{bmatrix} 2.876 & 1.0872 & 1.876 & 1.567 \\ 1.678 & 4.876 & 0.678 & 1.098 \\ 1.876 & 1.557 & 1.176 & 1.654 \\ 1.987 & 1.987 & 1.456 & 1.876 \end{bmatrix}, B = \begin{bmatrix} 1.436 & 0 & 0 & 0 \\ 0 & 1.426 & 0 & 0 \\ 0 & 0 & 1.436 & 0 \\ 0 & 0 & 0 & 1.436 \end{bmatrix},$$

$$E = \begin{bmatrix} 2.154 & 1.987 & 1.678 & 1.987 \\ 1.917 & 2.654 & 1.678 & 1.765 \\ 1.872 & 1.987 & 2.654 & 1.876 \\ 1.098 & 1.567 & 1.765 & 2.654 \end{bmatrix}, Q = \begin{bmatrix} 1.562 & 1.176 & 1.876 & 1.872 \\ 1.569 & 1.557 & 1.788 & 1.555 \\ 1.987 & 1.678 & 1.567 & 1.765 \\ 1.098 & 1.876 & 1.778 & 1.567 \end{bmatrix},$$

$$M = \begin{bmatrix} 1.816 & 0 & 0 & 0 \\ 0 & 1.816 & 0 & 0 \\ 0 & 0 & 1.816 & 0 \\ 0 & 0 & 0 & 1.846 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\phi = \begin{bmatrix} 1.486 & 0 & 0 & 0 \\ 0 & 1.486 & 0 & 0 \\ 0 & 0 & 1.486 & 0 \\ 0 & 0 & 0 & 1.486 \end{bmatrix}.$$

Using MATLAB LMI control toolbox, we solve the LMI in Theorem 2, and get the following feasible solutions:

$$R_1 = \begin{bmatrix} -0.7438 & 0.0443 & 0.0358 & 0.0387 \\ 0.0443 & -0.7197 & 0.0368 & 0.0419 \\ 0.0358 & 0.0368 & -0.7475 & 0.0370 \\ 0.0387 & 0.0419 & 0.0370 & -0.7485 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -24.8735 & 0.0714 & 0.0577 & 0.0623 \\ 0.0714 & -24.8346 & 0.0593 & 0.0676 \\ 0.0577 & 0.0593 & -24.8794 & 0.0597 \\ 0.0623 & 0.0676 & 0.0597 & -24.8811 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.0018 & -0.0009 & -0.0008 & 0.0000 \\ -0.0009 & 0.0017 & -0.0003 & -0.0006 \\ -0.0008 & -0.0003 & 0.0017 & -0.0007 \\ 0.0000 & -0.0006 & -0.0007 & 0.0013 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -23.3004 & -0.0232 & -0.0189 & -0.0193 \\ -0.0232 & -23.3126 & -0.0188 & -0.0217 \\ -0.0189 & -0.0188 & -23.2987 & -0.0194 \\ -0.0193 & -0.0217 & -0.0194 & -23.2986 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.8519 & 0.1685 & 0.1363 & 0.1470 \\ 0.1685 & 0.9438 & 0.1399 & 0.1595 \\ 0.1363 & 0.1399 & 0.8382 & 0.1409 \\ 0.1470 & 0.1595 & 0.1409 & 0.8341 \end{bmatrix},$$

$$U_2 = \begin{bmatrix} 0.0260 & 0.0591 & 0.0478 & 0.0516 \\ 0.0591 & 0.0583 & 0.0491 & 0.0560 \\ 0.0478 & 0.0491 & 0.0212 & 0.0495 \\ 0.0516 & 0.0560 & 0.0495 & 0.0198 \end{bmatrix}.$$

$\xi_1 = 14.9022, \xi_2 = 14.9022, \xi_3 = 14.9022, \xi_5 = 14.9022, \lambda_1 = 14.9022, \lambda_2 = 14.9022, \lambda_3 = 14.9022, \lambda_5 = 14.9022.$

Hence, the master system (4) is synchronized with the slave system (5).

5. Conclusions

This article investigated the stability of fractional order uncertain BAM competitive neural networks. By using Lyapunov–Krasovskii functional, we explore the synchronization problem of this kind of neural network. The time delays is designed to achieve the synchronization of competitive BAM neural networks and novel conditions to ensure synchronization of the drive system and the corresponding response system are given. By using Lyapunov functional synchronization, the criteria are demonstrated in the terms of LMI approach and we check the feasibility of obtaining results by using the LMI MATLAB control toolbox. Moreover, the controller gain matrices can be obtained by solving the LMIs. Finally, two numerical examples were provided to demonstrate the effectiveness of our theoretical results. In the future, we will furthermore discuss stochastic competitive BAM neural networks with additive time varying delays, complex valued competitive BAM neural networks.

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Appendix A

Proof. We construct the following Lyapunov–Krasovskii functional:

$$\begin{aligned} V(t) &= \mathfrak{w}^T(t)R_1\mathfrak{w}(t) + \theta^T(t)U_1\theta(t) + \frac{1}{\Gamma(\beta)}\int_0^t (t-\gamma)\mathfrak{w}^T(\gamma)M_1\mathfrak{w}(\gamma)d\gamma \\ &- \frac{1}{\Gamma(\beta)}\int_0^{t-\sigma}(t-\sigma-\gamma)\mathfrak{w}^T(\gamma)M_1\mathfrak{w}(\gamma)d\gamma + \mathfrak{z}^T(t)R_2\mathfrak{z}(t) + \psi^T(t)U_2\psi(t) \\ &+ \frac{1}{\Gamma(\beta)}\int_0^t (t-\gamma)\mathfrak{z}^T(\gamma)M_2\mathfrak{z}(\gamma)d\gamma - \frac{1}{\Gamma(\beta)}\int_0^{t-\sigma}(t-\sigma-\gamma)\mathfrak{z}^T(\gamma)M_2\mathfrak{z}(\gamma)d\gamma \end{aligned} \tag{A1}$$

Lemmas 2 and 5, the following estimation can be derived

$$\begin{aligned} D^\alpha V(t) \le\ & 2\mathfrak{w}^T(t)R_1D^\alpha\mathfrak{w}(t) + 2\theta^T(t)U_1D^\alpha\theta(t) + \mathfrak{w}^T(t)M_1\mathfrak{w}(t) \\ &- \mathfrak{w}^T(t-\sigma)M_1\mathfrak{w}(t-\sigma) + 2\mathfrak{z}^T(t)R_2D^\alpha\mathfrak{z}(t) + 2\psi^T(t)U_2D^\alpha\psi(t) \\ &+ \mathfrak{z}^T(t)M_2\mathfrak{z}(t) - \mathfrak{z}^T(t-\sigma)M_2\mathfrak{z}(t-\sigma), \end{aligned} \tag{A2}$$

According to Lemma 6, we obtain

$$\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1C\mathfrak{f}(\mathfrak{z}(t)) \le \frac{\lambda_1^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1CC^TR_1^T\mathfrak{w}(t) + \frac{\lambda_1}{\epsilon}\mathfrak{z}^T(t)\phi^T\phi\mathfrak{z}(t) \tag{A3}$$

$$\begin{aligned} \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1P\mathfrak{v}(\mathfrak{z}(t-\sigma)) \le\ & \frac{\lambda_2^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1PP^TR_1^T\mathfrak{w}(t) \\ &+ \frac{\lambda_2}{\epsilon}\mathfrak{z}^T(t-\sigma)\phi^T\phi\mathfrak{z}(t-\sigma) \end{aligned} \tag{A4}$$

$$\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1B\theta(t) \le \frac{\lambda_3^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1BB^TR_1^T\mathfrak{w}(t) + \frac{\lambda_3}{\epsilon}\theta^T(t)\theta(t) \tag{A5}$$

$$2\mathfrak{w}^T(t)R_1\alpha(t) \le \lambda_4^{-1}\mathfrak{w}^T(t)R_1R_1^T\mathfrak{w}(t) + \lambda_4\alpha^T(t)\alpha(t) \tag{A6}$$

$$\begin{aligned} -\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1\Delta A(t)\mathfrak{w}(t) &\le \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1J_aK(t)L_a\mathfrak{w}(t) \\ &\le \frac{\mu_1^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1J_aJ_a^TR_1^T\mathfrak{w}(t) + \frac{\mu_1}{\epsilon}\mathfrak{w}^T(t)L_a^TL_a\mathfrak{w}(t) \end{aligned} \tag{A7}$$

$$\begin{aligned} \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1\Delta C(t)\mathfrak{f}(\mathfrak{z}(t)) &\le \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1J_cK(t)L_c\mathfrak{f}(\mathfrak{z}(t)) \\ &\le \frac{\mu_2^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1J_cJ_c^TR_1^T\mathfrak{w}(t) + \frac{\mu_2}{\epsilon}\mathfrak{z}^T(t)\phi^TL_c^TL_c\phi\mathfrak{z}(t) \end{aligned} \tag{A8}$$

$$\begin{aligned} \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1\Delta P(t)\mathfrak{v}(\mathfrak{z}(t-\sigma)) &\le \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1J_pK(t)L_p\mathfrak{v}(\mathfrak{z}(t-\sigma)) \\ &\le \frac{\mu_3^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1J_pJ_p^TR_1^T\mathfrak{w}(t) \\ &+ \frac{\mu_3}{\epsilon}\mathfrak{z}^T(t-\sigma)\phi^TL_p^TL_p\phi\mathfrak{z}(t-\sigma) \end{aligned} \tag{A9}$$

$$\begin{aligned}\frac{2}{\epsilon}\mathfrak{w}^T(t)R_1\Delta B(t)\theta(t) &\leq \frac{2}{\epsilon}\mathfrak{w}^T(t)R_1J_bK(t)L_b\theta(t)\\ &\leq \frac{\mu_4^{-1}}{\epsilon}\mathfrak{w}^T(t)R_1J_bJ_b^TR_1^T\mathfrak{w}(t)+\frac{\mu_4}{\epsilon}\theta^T(t)L_b^TL_b\theta(t) \end{aligned} \tag{A10}$$

$$2\theta^T(t)U_1H\mathfrak{f}(\theta(t)) \leq \lambda_5^{-1}\theta^T(t)U_1HH^TU_1^T\theta(t)+\lambda_5\theta^T(t)\phi^T\phi\theta(t) \tag{A11}$$

$$2\theta^T(t)U_1\beta(t) \leq \lambda_6^{-1}\theta^T(t)U_1U_1^T\theta(t)+\lambda_6\beta^T(t)\beta(t) \tag{A12}$$

$$\begin{aligned}2\theta^T(t)U_1\Delta D(t)\theta(t) &\leq 2\theta^T(t)U_1J_dK(t)L_d\theta(t)\\ &\leq \mu_5^{-1}\theta^T(t)U_1J_dJ_d^TU_1^T\theta(t)+\mu_5\theta^T(t)L_d^TL_d\theta(t)\end{aligned} \tag{A13}$$

$$\begin{aligned}2\theta^T(t)U_1\Delta H(t)\mathfrak{f}(\theta(t)) &\leq 2\theta^T(t)U_1J_hK(t)L_{\mathfrak{h}}\mathfrak{f}(\theta(t))\\ &\leq \mu_6^{-1}\theta^T(t)U_1J_{\mathfrak{h}}J_h^TU_1^T\theta(t)+\mu_6\theta^T(t)\phi^TL_{\mathfrak{h}}^TL_{\mathfrak{h}}\phi\theta(t)\end{aligned} \tag{A14}$$

$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2E\mathfrak{g}(\mathfrak{w}(t)) \leq \frac{\xi_1^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2EE^TR_2^T\mathfrak{z}(t)+\frac{\xi_1}{\epsilon}\mathfrak{w}^T(t)\phi^T\phi\mathfrak{w}(t) \tag{A15}$$

$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2Q\mathfrak{u}(\mathfrak{w}(t-\sigma)) \leq \frac{\xi_2^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2QQ^TR_2^T\mathfrak{z}(t)+\frac{\xi_2}{\epsilon}\mathfrak{w}^T(t-\sigma)\phi^T\phi\mathfrak{w}(t-\sigma) \tag{A16}$$

$$\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2M\psi(t) \leq \frac{\xi_3^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2MM^TR_2^T\mathfrak{z}(t)+\frac{\xi_3}{\epsilon}\psi^T(t)\psi(t) \tag{A17}$$

$$2\mathfrak{z}^T(t)R_2\gamma(t) \leq \xi_4^{-1}\mathfrak{z}^T(t)R_2R_2^T\mathfrak{z}(t)+\xi_4\gamma^T(t)\gamma(t) \tag{A18}$$

$$\begin{aligned}-\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2\Delta L(t)\mathfrak{z}(t) &\leq \frac{2}{\epsilon}\mathfrak{z}^T(t)R_2J_lK(t)L_l\mathfrak{z}(t)\\ &\leq \frac{\eta_1^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2J_lJ_l^TR_2^T\mathfrak{z}(t)+\frac{\eta_1}{\epsilon}\mathfrak{z}^T(t)L_l^TL_l\mathfrak{z}(t)\end{aligned} \tag{A19}$$

$$\begin{aligned}\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2\Delta E(t)\mathfrak{g}(\mathfrak{w}(t)) &\leq \frac{2}{\epsilon}\mathfrak{z}^T(t)R_2J_eK(t)L_e\mathfrak{g}(\mathfrak{w}(t))\\ &\leq \frac{\eta_2^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2J_eJ_e^TR_2^T\mathfrak{z}(t)+\frac{\eta_2}{\epsilon}\mathfrak{w}^T(t)\phi^TL_e^TL_e\phi\mathfrak{w}(t)\end{aligned} \tag{A20}$$

$$\begin{aligned}\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2\Delta Q(t)\mathfrak{u}(w(t-\sigma)) &\leq \frac{2}{\epsilon}\mathfrak{z}^T(t)R_2J_qK(t)L_q\mathfrak{u}(\mathfrak{w}(t-\sigma))\\ &\leq \frac{\eta_3^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2J_qJ_q^TR_2^T\mathfrak{z}(t)\\ &+\frac{\eta_3}{\epsilon}\mathfrak{w}^T(t-\sigma)\phi^TL_q^TL_q\phi\mathfrak{w}(t-\sigma)\end{aligned} \tag{A21}$$

$$\begin{aligned}\frac{2}{\epsilon}\mathfrak{z}^T(t)R_2\Delta M(t)\psi(t) &\leq \frac{2}{\epsilon}\mathfrak{z}^T(t)R_2J_mK(t)L_m\psi(t)\\ &\leq \frac{\eta_4^{-1}}{\epsilon}\mathfrak{z}^T(t)R_2J_mJ_m^TR_2^T\mathfrak{z}(t)+\frac{\eta_4}{\epsilon}\psi^T(t)L_m^TL_m\psi(t)\end{aligned} \tag{A22}$$

$$2\psi^T(t)U_2\bar{H}\mathfrak{g}(\psi(t)) \leq \xi_5^{-1}\psi^T(t)U_2\bar{H}\bar{H}^TU_2^T\psi(t)+\xi_5\psi^Tt)\phi^T\phi\psi(t) \tag{A23}$$

$$2\psi^T(t)U_2\delta(t) \leq \xi_6^{-1}\psi^T(t)U_2U_2^T\psi(t)+\xi_6\delta^T(t)\delta(t) \tag{A24}$$

$$\begin{aligned}2\psi^T(t)U_2\Delta T(t)\psi(t) &\leq 2\psi^T(t)U_2J_tK(t)L_t\psi(t)\\ &\leq \eta_5^{-1}\psi^T(t)U_2J_tJ_t^TU_2^T\psi(t)+\eta_5\psi^T(t)L_t^TL_t\psi(t)\end{aligned} \tag{A25}$$

$$\begin{aligned}2\psi^T(t)U_2\Delta\bar{H}(t)\mathfrak{g}(\psi(t)) &\leq 2\psi^T(t)U_2J_{\bar{\mathfrak{h}}}K(t)L_{\bar{\mathfrak{h}}}\mathfrak{g}(\psi(t))\\ &\leq \eta_6^{-1}\psi^T(t)U_2J_{\bar{\mathfrak{h}}}J_{\bar{\mathfrak{h}}}^TU_2^T\psi(t)+\eta_6\psi^T(t)\phi^TL_{\bar{\mathfrak{h}}}^TL_{\bar{\mathfrak{h}}}\phi\psi(t)\end{aligned} \tag{A26}$$

From (30)–(54), we have

$$
\begin{aligned}
D^{\alpha}V(t) \leq\ & \mathfrak{w}^T(t)\Big[-\frac{2}{\epsilon}R_1A + \frac{\lambda_1^{-1}}{\epsilon}R_1CC^TR_1^T + \frac{\lambda_2^{-1}}{\epsilon}R_1PP^TR_1^T + \frac{\lambda_3^{-1}}{\epsilon}R_1BB^TR_1^T \\
&+ \lambda_4^{-1}R_1R_1^T + \frac{\mu_1^{-1}}{\epsilon}R_1J_aJ_a^TR_1^T + \frac{\mu_1}{\epsilon}L_a^TL_a + \frac{\mu_2^{-1}}{\epsilon}R_1J_cJ_c^TR_1^T + \frac{\mu_3^{-1}}{\epsilon}R_1J_pJ_p^TR_1^T \\
&+ \frac{\mu_4^{-1}}{\epsilon}R_1J_bJ_b^TR_1^T + M_1 + \frac{\xi_1}{\epsilon}\phi^T\phi + \frac{\eta_2}{\epsilon}\phi^TL_e^TL_e\phi + 2R_1\Im_1\Big]\mathfrak{w}(t) \\
&+ \mathfrak{z}^T(t)\Big[-\frac{2}{\epsilon}R_2L + \frac{\xi_1^{-1}}{\epsilon}R_2EE^TR_2^T + \frac{\xi_2^{-1}}{\epsilon}R_2QQ^TR_2^T + \frac{\xi_3^{-1}}{\epsilon}R_2MM^TR_2^T \\
&+ \xi_4^{-1}R_2R_2^T + \frac{\eta_1^{-1}}{\epsilon}R_2J_lJ_l^TR_2^T + \frac{\eta_1}{\epsilon}L_l^TL_l + \frac{\eta_2^{-1}}{\epsilon}R_2J_eJ_e^TR_2^T + \frac{\eta_3^{-1}}{\epsilon}R_2J_qJ_q^TR_2^T \\
&+ \frac{\eta_4^{-1}}{\epsilon}R_2J_mJ_m^TR_2^T + M_2 + \frac{\lambda_1}{\epsilon}\phi^T\phi + \frac{\mu_2}{\epsilon}\phi^TL_c^TL_c\phi + 2R_2\Im_3\Big]\mathfrak{z}(t) \\
&+ \theta^T(t)\Big[\frac{\lambda_3}{\epsilon} + \frac{\mu_4}{\epsilon}L_b^TL_b - \frac{2}{\epsilon}U_1D + \lambda_5^{-1}U_1HH^TU_1^T + \lambda_5\phi^T\phi + \lambda_6^{-1}U_1U_1^T \\
&+ \mu_5^{-1}U_1J_dJ_d^TU_1^T + \mu_5L_d^TL_d + \mu_6^{-1}U_1J_{\mathfrak{h}}J_{\mathfrak{h}}^{-1}U_1^T + \mu_6\phi^TL_{\mathfrak{h}}^TL_{\mathfrak{h}}\phi + 2U_1\Im_2\Big]\theta(t) \\
&+ \psi^T(t)\Big[\frac{\xi_3}{\epsilon} + \frac{\eta_4}{\epsilon}L_m^TL_m - \frac{2}{\epsilon}TU_2 + \xi_5^{-1}U_2\bar{H}\bar{H}^TU_2^T + \xi_5\phi^T\phi + \xi_6^{-1}U_2U_2^T \\
&+ \eta_5^{-1}U_2J_tJ_t^TU_2^T + \eta_5L_t^TL_t + \eta_6^{-1}U_2J_{\bar{\mathfrak{h}}}J_{\bar{\mathfrak{h}}}^{-1}U_2^T + \eta_6\phi^TL_{\bar{\mathfrak{h}}}^TL_{\bar{\mathfrak{h}}}\phi + 2U_2\Im_4\Big]\psi(t) \\
&+ \mathfrak{w}^T(t-\tau)\Big[\frac{\xi_2}{\epsilon}\phi^T\phi + \frac{\eta_3}{\epsilon}\phi^TL_{\mathfrak{q}}^TL_{\mathfrak{q}}\phi - M_1\Big]\mathfrak{w}(t-\sigma) \\
&+ \mathfrak{z}^T(t-\tau)\Big[\frac{\lambda_2}{\epsilon}\phi^T\phi + \frac{\mu_3}{\epsilon}\phi^TL_{\mathfrak{p}}^TL_{\mathfrak{p}}\phi - M_2\Big]\mathfrak{z}(t-\sigma), \\
D^{\alpha}V(t) \leq\ & \mathfrak{w}^T(t)\Upsilon_1\mathfrak{w}(t) + \mathfrak{z}^T(t)\Upsilon_2\mathfrak{z}(t) + \theta^T(t)\Upsilon_3\theta(t) + \psi^T(t)\Upsilon_4\psi(t). \\
\Upsilon_1 =\ & \Psi_1 + \frac{\lambda_1^{-1}}{\epsilon}R_1CC^TR_1^T + \frac{\lambda_2^{-1}}{\epsilon}R_1PP^TR_1^T + \frac{\lambda_3^{-1}}{\epsilon}R_1BB^TR_1^T + \lambda_4^{-1}R_1R_1^T \\
&+ \frac{\mu_1^{-1}}{\epsilon}R_1J_aJ_a^TR_1^T + \frac{\mu_2^{-1}}{\epsilon}R_1J_cJ_c^TR_1^T + \frac{\mu_3^{-1}}{\epsilon}R_1J_pJ_p^TR_1^T + \frac{\mu_4^{-1}}{\epsilon}R_1J_bJ_b^TR_1^T, \\
\Upsilon_2 =\ & \Psi_2 + \frac{\xi_1^{-1}}{\epsilon}R_2EE^TR_2^T + \frac{\xi_2^{-1}}{\epsilon}R_2QQ^TR_2^T + \frac{\xi_3^{-1}}{\epsilon}R_2MM^TR_2^T + \xi_4^{-1}R_2R_2^T \\
&+ \frac{\eta_1^{-1}}{\epsilon}R_2J_lJ_l^TR_2^T + \frac{\eta_2^{-1}}{\epsilon}R_2J_eJ_e^TR_2^T + \frac{\eta_3^{-1}}{\epsilon}R_2J_{\mathfrak{q}}J_{\mathfrak{q}}^TR_2^T + \frac{\eta_4^{-1}}{\epsilon}R_2J_mJ_m^TR_2^T, \\
\Upsilon_3 =\ & \Psi_3 + \lambda_5^{-1}U_1HH^TU_1^T + \lambda_6^{-1}U_1U_1^T + \mu_5^{-1}U_1J_dJ_d^TU_1^T + \mu_6^{-1}U_1J_{\mathfrak{h}}J_{\mathfrak{h}}^{-1}U_1^T, \\
\Upsilon_4 =\ & \Psi_4 + \xi_5^{-1}U_2\bar{H}\bar{H}^TU_2^T + \xi_6^{-1}U_2U_2^T + \eta_5^{-1}U_2J_tJ_t^TU_2^T + \eta_6^{-1}U_2J_{\bar{\mathfrak{h}}}J_{\bar{\mathfrak{h}}}^{-1}U_2^T.
\end{aligned}
\tag{A27}
$$

System (6) is globally asymptotic stable. As a result, master system (4) is globally synchronized with slave system (5). This completes the proof of theorem. □

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