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Event-Triggered $L_2 - L_\infty$ Filtering for Network-Based Neutral Systems With Time-Varying Delays via T-S Fuzzy Approach

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ABSTRACT This article examines the issue of event-triggered $L_2 - L_{\infty}$ filtering for network-based neutral systems via Takagi-Sugeno (T-S) fuzzy approach. A dynamic discrete event-triggered scheme (ETS) is introduced to save the limited communication resource. Based on the T-S fuzzy model, the consider neutral type system with networked induced delays are represented as a class of T-S fuzzy system. In addition, by considering a suitable Lyapunov-Krasovskii functional (LKF) and by using the Wirtinger inequality technique, the stability conditions with respect to linear matrix inequalities (LMIs) are presented to guarantee the considered filtering systems are asymptotically stable with $L_2 - L_{\infty}$ performance index γ . To the end, numerical examples are given to illustrate the effectiveness of the proposed result.

INDEX TERMS Event-triggered scheme, $L_2 - L_\infty$ filter, networked control systems, T-S fuzzy systems.

I. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy model [1] has been widely used design and analysis of fuzzy control systems. Combining a set of IF-THEN rules with some fuzzy sets, makes it possible to approximate nonlinear systems with high precision using a series of linear subsystems. As a result, recent years have seen an increase in research interest in the control issues associated with T-S fuzzy systems [2]-[6]. The authors in [5] have discussed the stability of a T-S fuzzy system with state quantization under exponential dissipation using a non-fragile sampled-data control. For estimation of state variables in a digital sampled system, network-based estimation/ filtering is required. The standard Kalman filtering does not provide adequate results when the Gaussian noises with known statistics are not satisfied in real-world scenarios [7]–[10]. Thus, network-based filtering has gained much importance [11]–[15]. However, the nonlinear system filter design issue remains unsolvable due to the difficulty of analyzing nonlinear system stability. For this reason, over the last two decades, an increasing number of academics have dedicated themselves to move on to filtering. In addition,

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as a special case of time delay, the neutral delay occurs in the derivative of the state, not in the state itself. Recently, many researchers have focused on the neutral type delay with different controller techniques in [16], [17]. In [16], the authors discussed the H_{∞} filtering technique for T-S fuzzy neutral-type stochastic system.

Generally, the majority of the control tasks are executed periodically in numerous digital control applications. Considering the limited network resources, event-triggered control scheme (ETCS) has emerged as a successful technique to address this issue, which provides for limited communication frequency between the components of networked environments which also regulates the unwanted waste of computation and communication resources in conventional intelligent controls [18]–[21]. So far, the literature has mainly described three types of event-triggered schemes such as absolute event-triggered scheme [22]-[24], relative event-triggered scheme [25]-[27] and mixed event-triggered scheme [28], [29]. The mixed ETCS is more advantageous as it provides for the most suitable properties for networked systems by combining the benefits of both relative and absolute ETCS. But, with mixed ETCS, the design and analysis methods are more complex. ETCS was shown to affect the amount of information that had to be transmitted, which helps to reduce

network bandwidth occupation compared to a conventional periodic sampling technique. Lately, event-triggered filtering has gained more popularity, and key findings have been published [30]–[34]. The authors in [31] has been discussed the event-triggered fault detection filter method for nonlinear networked systems. Event-triggered filtering for nonlinear networked control systems with T-S fuzzy approach has been examined in [34].

On the other hand, the estimation of the system states and filtering is a popular issue in signal processing and control applications. Over the previous decade, the issue of filtering for the networked system has become a focal point of consideration, and numerous powerful methodologies have been created; see, for example, H_{∞} , $L_2 - L_{\infty}$, and passive filtering. Among them, H_{∞} and $L_2 - L_{\infty}$ filtering are good options for disturbances with unknown characteristics [35]–[39]. Based on the above filtering methods, the L_2 – L_{∞} filtering can ensure that the error system is asymptotically stable and possesses a predefined $L_2 - L_\infty$ disturbance attenuation performance in the case when the disturbance is energy bounded. $L_2 - L_\infty$ filtering is aiming to make the estimation error's peak value minimal for all distractions satisfying energy bound ability. Therefore, $L_2 - L_{\infty}$ filtering is preferred on the condition that the filtering error's peak value is expected to be considered minimal. The filtering design techniques were introduced for different T-S fuzzy models in the literature's [31], [34], [40]. To the author's knowledge, the event-triggered $L_2 - L_{\infty}$ filtering for networkbased neutral systems with time-varying delays via T-S fuzzy has not been fully investigated.

As a result of the preceding discussions, this article discusses event-triggered $L_2 - L_{\infty}$ filtering for network-based neutral T-S fuzzy systems. Furthermore, based on the suitable Lyapunov-Krasovskii functional (LKF), the delay stability conditions are derived from linear matrix inequalities (LMIs).

This article mainly focuses on the following points:

- i). A new model of fuzzy filtering error system is provided under the consideration of dynamic discrete ETCS, which can save network resources.
- ii). A novel dynamic discrete ETCS with different triggered thresholds is proposed for different fuzzy rules in terms of the considered error system. Compared with the existing work [19], [41], the proposed one in this paper, which can more effectively save the limited communication resources on the network while achieving good performance.
- iii). The logical Zero Order Holder (ZOH) is used to actively discard packet failures and select the latest packet to drive the filter.
- iv). By choosing the appropriate LKF, Wirtinger integral inequality approach, and the sufficient conditions ensure that the desired filtering system is asymptotically stable with respect to $L_2 L_{\infty}$ performance; as a result, which plays a vital role in achieving less conservative results than [19], which can be evaluated in terms of LMIs.

v). Finally, various numerical examples are given to show the feasibility of the results with a practical application of the proposed method to a tunnel diode circuit model. This implies the merit of derived delay-dependent conditions.

Notation: For a matrix Q, Q^{-1} noted as inverse and Q^T means the transpose of Q, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ indicates the *n*-dimensional Euclidean space and set of $n \times m$ real matrix, respectively. For \mathcal{Z} is a positive (negative) definite matrix, such that $\mathcal{Z} > 0$, ($\mathcal{Z} < 0$), and I_n represents the identity matrix of dimension n, * is used to represent the term that is induced by symmetry. \mathbb{N} represents the set of positive integers. Maximum allowable upper bound (MAUB).

II. SYSTEM DESCRIPTION

Consider the following T-S fuzzy neutral system with disturbance:

Rule *i*: IF $s_1(t)$ is F_{i1} and $s_2(t)$ is $F_{i2}, \dots,$ and $s_n(t)$ is F_{in} , THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_{0i}\hat{x}(t) + A_{1i}\hat{x}(t - d(t)) \\ +A_{2i}\dot{\hat{x}}(t - h(t)) + B_{i}w(t) \\ \hat{y}(t) = C_{i}\hat{x}(t) \\ z(t) = E_{i}\hat{x}(t) \end{cases}$$
(1)

where $s_1(t), s_2(t), \dots, s_n(t)$ are the premise variables that has been measurable, and each $F_{ij}(i = 1, 2, \dots, q, j = 1, 2, \dots, n)$ is a fuzzy set. $i = 1, 2, \dots, q, q$ noted as the number of IF-THEN rules. $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^m$ noted as state variables and measured output of the system. $z(t) \in \mathbb{R}^p$ represents the signal to be estimated. $w(t) \in \mathbb{R}^r$ means the disturbance which refers to $L_2[0, \infty)$. $A_{0i}, A_{1i}, A_{2i}, B_i, C_i$ and E_i are known matrices with adjustable dimensions. Also d(t)and h(t) are time-varying delays, which satisfies $d_1 \leq d(t) \leq d_2$, $\dot{d}(t) \leq \mu_1$ and $h_1 \leq h(t) \leq h_2$, $\dot{h}(t) \leq \mu_2$, $h_{12} = h_2 - h_1$.

Utilizing center-average defuzzifier, product interference and singleton fuzzifier the dynamic fuzzy model (1) can be represented as follows

$$\dot{\hat{x}}(t) = \sum_{i=1}^{q} u_i(s(t)) \Big[A_{0i} \hat{x}(t) + A_{1i} \hat{x}(t - d(t)) \\ + A_{2i} \dot{\hat{x}}(t - h(t)) + B_i w(t) \Big],$$
$$\hat{y}(t) = \sum_{i=1}^{q} u_i(s(t)) \Big[C_i \hat{x}(t) \Big],$$
$$z(t) = \sum_{i=1}^{q} u_i(s(t)) \Big[E_i \hat{x}(t) \Big],$$
(2)

with

$$u_i(s(t)) = \frac{\beta_i(s(t))}{\sum_{i=1}^q \beta_i(s(t))}, \ \beta_i(s(t)) = \prod_{j=1}^n F_{ij}(s_j(t)), \quad (3)$$

in which $F_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in F_{ij} . It is assumed that $\beta_i(s(t)) \ge 0$, i = 1, ..., q, $\sum_{i=1}^q \beta_i(s(t)) > 0$ for all *t*. Therefore, $u_i(s(t)) \ge 0$ and $\sum_{i=1}^q u_i(s(t)) = 1$ for all *t*.



FIGURE 1. Framework of $L_2 - L_\infty$ filtering based on event-triggered mechanism.

A. DISCRETE EVENT-TRIGGERED SCHEME

Consider the sample of measurement output $\hat{y}(t)$ with sampling period h > 0. Moreover, the sampling instant is $kh(k \in \mathbb{N})$, the sampled signal noted as $\hat{y}(kh)$ and also k compressed into a data packet $(k, \hat{y}(kh))$.

To design a filter in a time-triggered scheme (TTS), every sampled-data packets (SDP) are sent to ZOH. Indeed, it is unnecessary to transmit certain data packets that contain no new information. Thus, the ETCS is an efficient way to avoid transmitting unwanted data packets. Assume $t_k h(k \in \mathbb{N})$ is the triggered instant. Using the following threshold condition, the event detector determines if the newly SDP ($t_k + l$, $\hat{y}(t_k h + lh)$) sent to the filter:

$$\begin{aligned} [\hat{y}(t_kh+lh) - \hat{y}(t_kh)]^T \phi(r(t_kh))[\hat{y}(t_kh+lh) - \hat{y}(t_kh)] \\ &\leq \delta(r(t_kh))\hat{y}^T(t_kh)\phi(r(t_kh))\hat{y}(t_kh), \quad (4) \end{aligned}$$

where $0 \le \delta(r(t_k h)) < 1$, $\phi(r(t_k h)) > 0$ will be evaluated later. The data will be stored and transmitted to the filter at the same time if the sampling instant fails to meet the ETCS threshold condition (4). The next triggered instant $t_{k+1}h$ defined as

$$t_{k+1}h = \min_{l \ge 1} \left\{ t_k h + lh | [\hat{y}(t_k h + lh) - \hat{y}(t_k h)]^T \\ \times \phi(r(t_k h)) [\hat{y}(t_k h + lh) - \hat{y}(t_k h)] \\ > \delta(r(t_k h)) \hat{y}^T(t_k h) \phi(r(t_k h)) \hat{y}(t_k h) \right\}.$$
(5)

B. FILTERING DESIGN

The ZOH receives a data packet $(t_k, \hat{y}(t_k h))$, it automatically activates the filter using the signal $\hat{y}(t_k h)$. With respect to the transmission delay and properties of ZOH, we have:

$$\tilde{y}(t) = \hat{y}(t_k h), \ t \in [t_k h + \upsilon_{t_k}, t_{k+1} h + \upsilon_{t_{k+1}}),$$
 (6)

where $v_{t_k} \in (0, \overline{v}]$ and $\overline{v} = \max\{v_{t_k}\}$. The issue of the event-triggered filtering is to estimate the state of system (1) based on the complete information exchange among filters. Here, we will design a discrete ETCS fuzzy filter. This paper assumes that the filter's premise and the plant's premise variables are the same. Based on the parallel distributed

compensation, the fuzzy-rule-dependent filter is conducted. The following full-order filter is designed for system (2).

Rule *i*: IF $s_1(t)$ is F_{i1} and $s_2(t)$ is F_{i2}, \dots , and $s_n(t)$ is F_{in} , THEN

$$\dot{x}_{f}(t) = \sum_{i=1}^{q} u_{i}(s(t)) \Big[A_{fi} x_{f}(t) + B_{fi} \tilde{y}(t) \Big],$$

$$z_{f}(t) = \sum_{i=1}^{q} u_{i}(s(t)) \Big[C_{fi} x_{f}(t) \Big],$$
 (7)

where A_{fi} , B_{fi} and C_{fi} are filter parameters to be determined. Substituting (6) into (7) yields

$$\dot{x}_{f}(t) = \sum_{i=1}^{q} u_{i}(s(t)) \Big[A_{fi} x_{f}(t) + B_{fi} \hat{y}(t_{k}h) \Big],$$

$$z_{f}(t) = \sum_{i=1}^{q} u_{i}(s(t)) \Big[C_{fi} x_{f}(t) \Big].$$
 (8)

C. TIME-DELAY MODELING OF THE FILTERING ERROR SYSTEM

By the preceding discussion, the filtering error system can be modeled using an interval time delay.

Let $\xi(t) = [\hat{x}(t) \ x_f(t)]^T$ and $e(t) = z(t) - z_f(t)$. Then, the resulting error system can be expressed as follows:

$$\dot{\xi}(t) = \sum_{i=1}^{q} u_i(s(t)) \sum_{j=1}^{q} u_j(s(t)) \left\{ \begin{bmatrix} A_{0i} & 0\\ 0 & A_{fj} \end{bmatrix} \xi(t) + \begin{bmatrix} A_{1i} \\ 0 \end{bmatrix} \hat{x}(t - d(t)) + \begin{bmatrix} A_{2i} \\ 0 \end{bmatrix} \dot{x}(t - h(t)) + \begin{bmatrix} B_i \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ B_{fj} \end{bmatrix} \hat{y}(t_k h) \right\},$$

$$e(t) = \sum_{i=1}^{q} u_i(s(t)) \sum_{j=1}^{q} u_j(s(t)) \left\{ \begin{bmatrix} E_i & -C_{fj} \end{bmatrix} \xi(t) \right\}.$$
 (9)

The filtering error system (9) is subject to constraints

$$[\hat{y}(t_kh+lh) - \hat{y}(t_kh)]^T \phi[\hat{y}(t_kh+lh) - \hat{y}(t_kh)] \le \delta \hat{y}^T(t_kh)\phi \hat{y}(t_kh), \quad (10)$$

where $l = 1, 2, \cdots, t_{k+1} - t_{k-1}$.

With reference to [40] and [42], the following two conditions hold: (i) If $t_k h + h + \overline{\upsilon} \ge t_{k+1}h + \upsilon_{t_{k+1}}$ and function $\upsilon(t)$ defined as

$$\upsilon(t) = t - t_k h, t \in [t_k h + \upsilon_{t_k}, t_{k+1} h + \upsilon_{t_{k+1}}).$$
(11)

Obviously,

$$\upsilon_{t_k} \le \upsilon(t) \le (t_{k+1} - t_k)h + \upsilon_{t_{k+1}} \le h + \overline{\upsilon}, \qquad (12)$$

(ii) $t_k h + h + \overline{v} < t_{k+1} h + v_{t_{k+1}}$, considering the following two intervals

 $[t_kh+v_{t_k}, t_kh+h+\overline{v})$ and $[t_kh+\iota h+\overline{v}, t_kh+h+\overline{v}+\iota h)$.

Since $v_{t_k} \leq \overline{v}$ and integer $a \geq 1$ we get

$$t_kh + ah + \overline{\upsilon} < t_{k+1}h + \upsilon_{t_{k+1}} \le t_kh + ah + h + \overline{\upsilon}.$$

Let

$$\begin{aligned} T_0 &= [t_k h + \upsilon_{t_k}, t_k h + \overline{\upsilon} + h) \\ T_\iota &= [t_k h + \iota h + \overline{\upsilon}, t_k h + \overline{\upsilon} + \iota h + h) \\ T_a &= [t_k h + ah + \overline{\upsilon}, t_{k+1} h + \upsilon_{t_{k+1}}), \end{aligned}$$
(13)

where $\iota = 1, 2, \cdots, a - 1$. Then, we obtain

$$[t_k h + v_{t_k}, t_{k+1} h + v_{t_{k+1}}) = \bigcup_{\iota=0}^{\iota=a} T_\iota.$$
 (14)

Define

$$\upsilon(t) = \begin{cases} t - t_k h, t \in T_0 \\ t - t_k h - \iota h, t \in T_\iota \\ t - t_k h - ah, t \in T_a. \end{cases}$$
(15)

Clearly, we have

$$\begin{cases}
\upsilon_{t_k} \leq \upsilon(t) < h + \overline{\upsilon}, t \in T_0 \\
\upsilon_{t_k} \leq \overline{\upsilon} \leq \upsilon(t) < h + \overline{\upsilon}, t \in T_t \\
\upsilon_{t_k} \leq \overline{\upsilon} \leq \upsilon(t) < h + \overline{\upsilon}, t \in T_a.
\end{cases}$$
(16)

As a result,

$$0 \le \upsilon_{t_k} \le \upsilon(t) < h + \overline{\upsilon} = \upsilon,$$

$$t \in [t_k h + \upsilon_{t_k}, t_{k+1} h + \upsilon_{t_{k+1}}).$$
(17)

Furthermore, the following two cases are considered:

• Define $e_k(t) = 0$ for $t \in [t_k h + v_{t_k}, t_{k+1} h + v_{t_{k+1}})$

• Denote

$$e_{k}(t) = \begin{cases} 0, & t \in T_{0} \\ \hat{y}(t_{k}h) - \hat{y}(t_{k}h + \iota h), & t \in T_{\iota}, \\ \hat{y}(t_{k}h) - \hat{y}(t_{k}h + ah), & t \in T_{a} \end{cases}$$
(18)

we obtain

$$\hat{y}(t_k h) = e_k(t) + \hat{y}(t - \upsilon(t)).$$
 (19)

Combining (9)-(19), the filtering error system can be written as

$$\begin{split} \dot{\xi}(t) &= \sum_{i=1}^{q} u_i(s(t)) \sum_{j=1}^{q} u_j(s(t)) \bigg\{ \tilde{\tilde{A}}_0 \xi(t) + \tilde{\tilde{A}}_1 \hat{x}(t - d(t)) \\ &+ \tilde{\tilde{A}}_2 \dot{\tilde{x}}(t - h(t)) + \tilde{\tilde{B}} w(t) + \tilde{\tilde{B}}_1 e_k(t) + \tilde{\tilde{C}} \hat{x}(t - \upsilon(t)) \bigg\}, \\ e(t) &= \sum_{i=1}^{q} u_i(s(t)) \sum_{j=1}^{q} u_j(s(t)) \overline{E} \xi(t), \end{split}$$

where

$$\begin{split} \tilde{\tilde{A}}_0 &= \begin{bmatrix} A_{0i} & 0\\ 0 & A_{fj} \end{bmatrix}, \quad \tilde{\tilde{A}}_1 = \begin{bmatrix} A_{1i}\\ 0 \end{bmatrix}, \\ \tilde{\tilde{A}}_2 &= \begin{bmatrix} A_{2i}\\ 0 \end{bmatrix}, \quad \tilde{\tilde{B}} = \begin{bmatrix} B_i\\ 0 \end{bmatrix}, \quad \tilde{\tilde{B}}_1 = \begin{bmatrix} 0\\ B_{fj} \end{bmatrix}, \\ \tilde{\tilde{C}} &= \begin{bmatrix} 0\\ B_{fj}C_i \end{bmatrix}, \quad \tilde{\tilde{E}} = \begin{bmatrix} E_i & -C_{fj} \end{bmatrix}. \end{split}$$

For our convenience, the above filtering error system can be written as follows:

$$\dot{\xi}(t) = \overline{A}_0 \xi(t) + \overline{A}_1 \hat{x}(t - d(t)) + \overline{A}_2 \dot{\hat{x}}(t - h(t)) + \overline{B}w(t) + \overline{B}_1 e_k(t) + \overline{C} \hat{x}(t - \upsilon(t)), e(t) = \overline{E} \xi(t),$$
(20)

where

$$\overline{A}_{0} = \begin{bmatrix} A_{0} & 0 \\ 0 & A_{f} \end{bmatrix}, \quad \overline{A}_{1} = \begin{bmatrix} A_{1} \\ 0 \end{bmatrix},$$
$$\overline{A}_{2} = \begin{bmatrix} A_{2} \\ 0 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \overline{B}_{1} = \begin{bmatrix} 0 \\ B_{f} \end{bmatrix},$$
$$\overline{C} = \begin{bmatrix} 0 \\ B_{f}C \end{bmatrix}, \quad \overline{E} = \begin{bmatrix} E & -C_{f} \end{bmatrix}.$$

Before proceeding, we introduce the following definition and lemmas, which will help to obtain our key results.

Definition 1 [40]: For a given scalar $\gamma > 0$, the filtering error system (20) is asymptotically stable in terms of $L_2 - L_{\infty}$ performance γ and $w(t) \in L_2[0, \infty)$, if for initial condition, the following inequality hold:

$$\|e(t)\|_{\infty} \le \gamma \|w(t)\|_2 \tag{21}$$

where

$$\|e(t)\|_{\infty} = \sqrt{\sup_{t} \{e^{T}(t)e(t)\}} \\ \|w(t)\|_{2} = \sqrt{\int_{0}^{\infty} w^{T}(t)w(t)dt}.$$

Lemma 1 [43]: Given a matrix $M_1 > 0$, the subsequent inequality satisfies for every continuously differentiable function φ in $[b, c] \rightarrow \mathbb{R}^n$

$$(c-b)\int_{b}^{c} \varphi^{T}(s)M_{1}\varphi(s)ds$$

$$\geq \left(\int_{b}^{c} \varphi(s)ds\right)^{T}M_{1}\left(\int_{b}^{c} \varphi(s)ds\right) + 3\Theta^{T}M_{1}\Theta,$$

where $\Theta = \int_{b}^{c} \varphi(s)ds - \frac{2}{c-b}\int_{b}^{c}\int_{b}^{s} \varphi(u)duds.$

Lemma 2 [43]: For a given matrix P, and vector function $\zeta : [b_1, b_2] \rightarrow \mathbb{R}^n$, the subsequent condition holds:

$$\int_{b_1}^{b_2} \dot{\zeta}^T(s) P \dot{\zeta}(s) ds \ge \frac{1}{b_2 - b_1} \Xi^T \begin{bmatrix} P & 0\\ 0 & 3P \end{bmatrix} \Xi,$$

where

$$\Xi = \begin{bmatrix} \zeta(b_2) - \zeta(b_1) \\ \zeta(b_2) + \zeta(b_1) - \frac{2}{b_2 - b_1} \int_{b_1}^{b_2} \zeta(s) ds \end{bmatrix}.$$

III. MAIN RESULTS

A. $L_2 - L_\infty$ FILTERING PERFORMANCE ANALYSIS FOR THE NEUTRAL SYSTEM

This section presents new delay-dependent conditions for the $L_2 - L_{\infty}$ filtering performance analysis based on Definition 1, which ensure that the neutral system (20) is asymptotically

stable for a defined $L_2 - L_\infty$ performance γ . For clarity, we represent the matrix as follows:

$$A_{0} = \sum_{i=1}^{q} u_{i}(s(t))A_{0i}, A_{1} = \sum_{i=1}^{q} u_{i}(s(t))A_{1i},$$

$$A_{2} = \sum_{i=1}^{q} u_{i}(s(t))A_{2i}, B = \sum_{i=1}^{q} u_{i}(s(t))B_{i},$$

$$C = \sum_{i=1}^{q} u_{i}(s(t))C_{i}, E = \sum_{i=1}^{q} u_{i}(s(t))E_{i},$$

$$A_{f} = \sum_{j=1}^{q} u_{j}(s(t))A_{fj}, B_{f} = \sum_{j=1}^{q} u_{j}(s(t))B_{fj},$$

$$C_{f} = \sum_{j=1}^{q} u_{j}(s(t))C_{fj}.$$

Theorem 1: For given positive scalars d_1, d_2, h_1, h_2, v , $\mu_1, \gamma > 0, \delta$, and μ_2 , the filtering error system (20) can reach asymptotically stable under the $L_2 - L_{\infty}$ performance index γ and event-triggered control (10) if there exist matrices $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0, R_l > 0, U_l > 0, P_m > 0, \phi > 0, l = 1, 2, 3, 4, 5, 6, m = 1, 2, 3$ and positive diagonal matrix F_1 with proper dimensions, such that

$$\Pi_{ij} < 0, \, i, j = 1, 2, \dots, q \tag{22}$$

$$\begin{bmatrix} Q & \overline{E}^T \\ * & \gamma^2 I \end{bmatrix} > 0, \tag{23}$$

where $(\Pi_{ij})_{24\times 24}$,

$$\begin{aligned} \Pi_{ij}^{(1,1)} &= 2Q\overline{A}_0 + R_1 + R_2 + R_3 + d_1U_1 + d_2U_2 \\ &+ (d_2 - d_1)U_3 - \frac{4}{h_1}U_4 - \frac{4}{h_2}U_5 - \frac{4}{v}P_3 \\ &+ P_1 + vP_2 + 2F_1A_0, \end{aligned}$$

$$\begin{aligned} \Pi_{ij}^{(1,4)} &= 2Q\overline{A}_1 + 2F_1A_1, \ \Pi_{ij}^{(1,5)} &= -\frac{2}{h_1}U_4, \\ \Pi_{ij}^{(1,6)} &= -\frac{2}{h_2}U_5, \ \Pi_{ij}^{(1,7)} &= 2Q\overline{C} - \frac{2}{v}P_3, \end{aligned}$$

$$\begin{aligned} \Pi_{ij}^{(1,14)} &= \frac{6}{h_1^2}U_4, \ \Pi_{ij}^{(1,15)} &= \frac{6}{h_2^2}U_5, \\ \Pi_{ij}^{(1,17)} &= \frac{6}{v^2}P_3, \ \Pi_{ij}^{(1,19)} &= -2F_1 + (F_1A_0)^T, \end{aligned}$$

$$\begin{aligned} \Pi_{ij}^{(1,22)} &= 2Q\overline{A}_2 + 2F_1A_2, \ \Pi_{ij}^{(1,23)} &= 2Q\overline{B}_1, \\ \Pi_{ij}^{(1,24)} &= 2QB + 2F_1B, \ \Pi_{ij}^{(2,2)} &= -R_1, \ \Pi_{ij}^{(3,3)} &= -R_2, \\ \Pi_{ij}^{(4,4)} &= -(1 - \mu_1)R_3, \ \Pi_{ij}^{(4,19)} &= (F_1A_1)^T, \end{aligned}$$

$$\begin{aligned} \Pi_{ij}^{(5,5)} &= -\frac{4}{h_1}U_4 - 4(h_{12})^2U_6, \ \Pi_{ij}^{(5,6)} &= -2(h_{12})^2U_6, \\ \Pi_{ij}^{(5,14)} &= \frac{6}{h_1^2}U_4, \ \Pi_{ij}^{(5,16)} &= 6(h_{12})U_6, \\ \\ \Pi_{ij}^{(6,6)} &= -\frac{4}{h_2}U_5 - 4(h_{12})^2U_6, \ \Pi_{ij}^{(6,15)} &= \frac{6}{h_2^2}U_5, \end{aligned}$$

$$\begin{split} \Pi_{ij}^{(6,16)} &= 6(h_{12})U_6, \\ \Pi_{ij}^{(7,7)} &= -P_1 - \frac{4}{\upsilon}P_3 + \delta C^T \phi C, \\ \Pi_{ij}^{(7,17)} &= \frac{6}{\upsilon^2}P_3, \ \Pi_{ij}^{(7,23)} &= \delta C^T \phi, \ \Pi_{ij}^{(8,8)} &= -\frac{4}{d_1}U_1, \\ \Pi_{ij}^{(8,9)} &= \frac{6}{d_1^2}U_1, \ \Pi_{ij}^{(9,9)} &= -\frac{12}{d_1^3}U_1, \ \Pi_{ij}^{(10,10)} &= -\frac{4}{d_2}U_2, \\ \Pi_{ij}^{(10,11)} &= \frac{6}{d_2^2}U_2, \ \Pi_{ij}^{(11,11)} &= -\frac{12}{d_2^3}U_2, \\ \Pi_{ij}^{(12,12)} &= -\frac{4}{d_2 - d_1}U_3, \ \Pi_{ij}^{(12,13)} &= \frac{6}{(d_2 - d_1)^2}U_3, \\ \Pi_{ij}^{(13,13)} &= -\frac{12}{(d_2 - d_1)^3}U_3, \ \Pi_{ij}^{(14,14)} &= -\frac{12}{h_1^3}U_4, \\ \Pi_{ij}^{(15,15)} &= -\frac{12}{h_2^3}U_5, \ \Pi_{ij}^{(16,16)} &= -12U_6, \\ \Pi_{ij}^{(17,17)} &= -\frac{4}{\upsilon}P_2 - \frac{12}{\upsilon^3}P_3, \ \Pi_{ij}^{(17,18)} &= \frac{6}{\upsilon^2}P_2, \\ \Pi_{ij}^{(18,18)} &= -\frac{12}{\upsilon^3}P_2, \ \Pi_{ij}^{(19,19)} &= R_4 + R_5 + R_6 + h_1U_4 \\ &\quad + h_2U_5 + (h_{12})^4U_6 + \upsilon P_3 - 2F_1, \\ \Pi_{ij}^{(19,22)} &= 2F_1A_2, \ \Pi_{ij}^{(12,21)} &= -R_5, \\ \Pi_{ij}^{(22,22)} &= -(1 - \mu_2)R_6, \ \Pi_{ij}^{(23,23)} &= -(1 - \delta)\phi, \\ \Pi_{ij}^{(24,24)} &= -I. \end{split}$$

Proof: Construct a Lyapunov–Krasovskii functional (LKF) candidate as,

$$V(t) = \sum_{l=1}^{6} V_l(t),$$
(24)

where

$$\begin{split} V_{1}(t) &= \xi^{T}(t)Q\xi(t), \\ V_{2}(t) &= \int_{t-d_{1}}^{t} \hat{x}^{T}(s)R_{1}\hat{x}(s)ds + \int_{t-d_{2}}^{t} \hat{x}^{T}(s)R_{2}\hat{x}(s)ds \\ &+ \int_{t-d(t)}^{t} \hat{x}^{T}(s)R_{3}\hat{x}(s)ds, \\ V_{3}(t) &= \int_{t-h_{1}}^{t} \dot{x}^{T}(s)R_{4}\dot{x}(s)ds + \int_{t-h_{2}}^{t} \dot{x}^{T}(s)R_{5}\dot{x}(s)ds \\ &+ \int_{t-h(t)}^{t} \dot{x}^{T}(s)R_{6}\dot{x}(s)ds, \\ V_{4}(t) &= \int_{-d_{1}}^{0} \int_{t+\theta}^{t} \hat{x}^{T}(s)U_{1}\hat{x}(s)dsd\theta + \int_{-d_{2}}^{0} \int_{t+\theta}^{t} \hat{x}^{T}(s) \\ &\times U_{2}\hat{x}(s)dsd\theta + \int_{-d_{2}}^{-d_{1}} \int_{t+\theta}^{t} \hat{x}^{T}(s)U_{3}\hat{x}(s)dsd\theta, \\ V_{5}(t) &= \int_{-h_{1}}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s)U_{4}\dot{x}(s)dsd\theta \\ &+ \int_{-h_{2}}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s)U_{5}\dot{x}(s)dsd\theta \end{split}$$

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$$+h_{12}^{3} \int_{-h_{2}}^{-h_{1}} \int_{t+\theta}^{t} \dot{\hat{x}}^{T}(s) U_{6} \dot{\hat{x}}(s) ds d\theta,$$

$$V_{6}(t) = \int_{t-\upsilon}^{t} \hat{x}^{T}(s) P_{1} \dot{\hat{x}}(s) ds + \int_{-\upsilon}^{0} \int_{t+\theta}^{t} \hat{x}^{T}(s) P_{2} \dot{\hat{x}}(s) ds d\theta$$

$$+ \int_{-\upsilon}^{0} \int_{t+\theta}^{t} \dot{\hat{x}}^{T}(s) P_{3} \dot{\hat{x}}(s) ds d\theta.$$

Taking the time derivative as follows:

$$\begin{split} \dot{V}_{1}(t) &= 2\xi^{T}(t)Q\dot{\xi}(t), \end{split} (25) \\ \dot{V}_{2}(t) &\leq \hat{x}^{T}(t)[R_{1}+R_{2}+R_{3}]\hat{x}(t) \\ &-\hat{x}^{T}(t-d_{1})R_{1}\hat{x}(t-d_{1}) \\ &-\hat{x}^{T}(t-d_{2})R_{2}\hat{x}(t-d_{2}) \\ &-(1-\mu_{1})\hat{x}^{T}(t-d(t))R_{3}\hat{x}(t-d(t)), \end{aligned} (26) \\ \dot{V}_{3}(t) &\leq \dot{\hat{x}}^{T}(t)[R_{4}+R_{5}+R_{6}]\dot{\hat{x}}(t) \\ &-\dot{\hat{x}}^{T}(t-h_{1})R_{4}\dot{\hat{x}}(t-h_{1}) \\ &-\dot{\hat{x}}^{T}(t-h_{2})R_{5}\dot{\hat{x}}(t-h_{2}) \\ &-(1-\mu_{2})\dot{\hat{x}}^{T}(t-h(t))R_{6}\dot{\hat{x}}(t-h(t)), \end{aligned} (27) \\ \dot{V}_{4}(t) &= \hat{x}^{T}(t)[d_{1}U_{1}+d_{2}U_{2}+(d_{2}-d_{1})U_{3}]\hat{x}(t) \\ &-\int_{t-d_{1}}^{t}\hat{x}^{T}(s)U_{1}\hat{x}(s)ds - \int_{t-d_{2}}^{t}\hat{x}^{T}(s)U_{2}\hat{x}(s)ds \\ &-\int_{t-d_{2}}^{t-d_{1}}\hat{x}^{T}(s)U_{3}\hat{x}(s)ds. \end{aligned} (28)$$

By Lemma 1, then

$$-\int_{t-d_{1}}^{t} \hat{x}^{T}(s)U_{1}\hat{x}(s)ds$$

$$\leq -\frac{1}{d_{1}} \left(\int_{t-d_{1}}^{t} \hat{x}(s)ds\right)^{T} \times U_{1} \left(\int_{t-d_{1}}^{t} \hat{x}(s)ds\right)$$

$$-\frac{3}{d_{1}} \left(\int_{t-d_{1}}^{t} \hat{x}(s)ds - \frac{2}{d_{1}}\int_{t-d_{1}}^{t}\int_{s}^{t} \hat{x}(u)duds\right)^{T}$$

$$\times U_{1} \left(\int_{t-d_{1}}^{t} \hat{x}(s)ds - \frac{2}{d_{1}}\int_{t-d_{1}}^{t}\int_{s}^{t} \hat{x}(u)duds\right), \quad (29)$$

$$-\int_{t-d_{2}}^{t} \hat{x}^{T}(s)U_{2}\hat{x}(s)ds$$

$$\leq -\frac{1}{d_{2}} \left(\int_{t-d_{2}}^{t} \hat{x}(s)ds\right)^{T} \times U_{2} \left(\int_{t-d_{2}}^{t} \hat{x}(s)ds\right)$$

$$-\frac{3}{d_{2}} \left(\int_{t-d_{2}}^{t} \hat{x}(s)ds - \frac{2}{d_{2}}\int_{t-d_{2}}^{t}\int_{s}^{t} \hat{x}(u)duds\right), \quad (30)$$

$$-\int_{t-d_{2}}^{t-d_{1}} \hat{x}^{T}(s)U_{3}\hat{x}(s)ds \leq -\frac{1}{d_{2}-d_{1}} \left(\int_{t-d_{2}}^{t-d_{1}} \hat{x}(s)ds\right)^{T}$$

$$\times U_{3} \left(\int_{t-d_{2}}^{t-d_{1}} \hat{x}(s)ds\right)$$

$$-\frac{3}{d_{2}-d_{1}} \left(\int_{t-d_{2}}^{t-d_{1}} \hat{x}(s)ds - \frac{2}{d_{2}-d_{1}}\int_{t-d_{2}}^{t}\int_{s}^{t} \hat{x}(u)duds\right), \quad (31)$$

 $\dot{V}_5(t)$

$$= \dot{\hat{x}}^{T}(t)[h_{1}U_{4} + h_{2}U_{5} + h_{12}^{4}U_{6}]\dot{\hat{x}}(t) - \int_{t-h_{1}}^{t} \dot{\hat{x}}^{T}(s)U_{4}\dot{\hat{x}}(s)ds - \int_{t-h_{2}}^{t} \dot{\hat{x}}^{T}(s)U_{5}\dot{\hat{x}}(s)ds - h_{12}^{3}\int_{t-h_{2}}^{t-h_{1}} \dot{\hat{x}}^{T}(s)U_{6}\dot{\hat{x}}(s)ds.$$
(32)

By Lemma 2, then

$$-\int_{t-h_{1}}^{t} \dot{\hat{x}}^{T}(s) U_{4} \dot{\hat{x}}(s) ds \leq -\frac{1}{h_{1}} \Xi_{1}^{T} \begin{bmatrix} U_{4} & 0\\ 0 & 3U_{4} \end{bmatrix} \Xi_{1},$$
(33)
$$-\int_{t-h_{2}}^{t} \dot{\hat{x}}^{T}(s) U_{5} \dot{\hat{x}}(s) ds \leq -\frac{1}{h_{2}} \Xi_{2}^{T} \begin{bmatrix} U_{5} & 0\\ 0 & 3U_{5} \end{bmatrix} \Xi_{2},$$
(34)
$$-h_{12}^{3} \int_{t-h_{2}}^{t-h_{1}} \dot{\hat{x}}^{T}(s) U_{6} \dot{\hat{x}}(s) ds \leq -h_{12}^{2} \Xi_{3}^{T} \begin{bmatrix} U_{6} & 0\\ 0 & 3U_{6} \end{bmatrix} \Xi_{3},$$
(35)

where

$$\Xi_{1} = \begin{bmatrix} \hat{x}(t) - \hat{x}(t-h_{1}) \\ \hat{x}(t) + \hat{x}(t-h_{1}) - \frac{2}{h_{1}} \int_{t-h_{1}}^{t} \hat{x}(s) ds \end{bmatrix},$$

$$\Xi_{2} = \begin{bmatrix} \hat{x}(t) - \hat{x}(t-h_{2}) \\ \hat{x}(t) + \hat{x}(t-h_{2}) - \frac{2}{h_{2}} \int_{t-h_{2}}^{t} \hat{x}(s) ds \end{bmatrix},$$

$$\Xi_{3} = \begin{bmatrix} \hat{x}(t-h_{1}) - \hat{x}(t-h_{2}) \\ \hat{x}(t-h_{1}) + \hat{x}(t-h_{2}) - \frac{2}{h_{12}} \int_{t-h_{2}}^{t-h_{1}} \hat{x}(s) ds \end{bmatrix},$$

$$\dot{V}_{6}(t) = \hat{x}^{T}(t) [P_{1} + \upsilon P_{2}] \hat{x}(t) + \dot{x}^{T}(t) [\upsilon P_{3}] \dot{x}(t) \\ - \hat{x}^{T}(t-\upsilon) P_{1} \hat{x}(t-\upsilon) - \int_{t-\upsilon}^{t} \hat{x}^{T}(s) P_{2} \hat{x}(s) ds \\ - \int_{t-\upsilon}^{t} \dot{x}^{T}(s) P_{3} \dot{x}(s) ds.$$
(36)

Using Lemma 1 and 2, then

$$-\int_{t-\upsilon}^{t} \hat{x}^{T}(s) P_{2} \hat{x}(s) ds$$

$$\leq -\frac{1}{\upsilon} \Big(\int_{t-\upsilon}^{t} \hat{x}(s) ds \Big)^{T} P_{2} \Big(\int_{t-\upsilon}^{t} \hat{x}(s) ds \Big)$$

$$-\frac{3}{\upsilon} \Big(\int_{t-\upsilon}^{t} \hat{x}(s) ds - \frac{2}{\upsilon} \int_{t-\upsilon}^{t} \int_{s}^{t} \hat{x}(u) du ds \Big)^{T}$$

$$\times P_{2} \Big(\int_{t-\upsilon}^{t} \hat{x}(s) ds - \frac{2}{\upsilon} \int_{t-\upsilon}^{t} \int_{s}^{t} \hat{x}(u) du ds \Big). \quad (37)$$

Similarly,

$$-\int_{t-\upsilon}^{t} \dot{\hat{x}}^{T}(s) P_{3} \dot{\hat{x}}(s) ds \leq -\frac{1}{\upsilon} \Xi_{4}^{T} \begin{bmatrix} P_{3} & 0\\ 0 & 3P_{3} \end{bmatrix} \Xi_{4}, \quad (38)$$

where

$$\Xi_4 = \begin{bmatrix} \hat{x}(t) - \hat{x}(t-\upsilon) \\ \hat{x}(t) + \hat{x}(t-\upsilon) - \frac{2}{\upsilon} \int_{t-\upsilon}^t \hat{x}(s) ds \end{bmatrix}.$$

Furthermore, the following condition is satisfied for any properly dimensioned matrix F_1 :

$$2[\hat{x}^{T}(t)F_{1} + \dot{x}^{T}(t)F_{1}][-\dot{x}(t) + A_{0}\hat{x}(t) + A_{1}\hat{x}(t - d(t)) + A_{2}\dot{x}(t - h(t)) + Bw(t)] = 0.$$
(39)

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Based on ETCS (10) and w(t) = 0, we obtain

$$e_{k}^{T}(t)\phi e_{k}(t) \leq \delta[e_{k}(t) + C\hat{x}(t - \upsilon(t))]^{T} \\ \phi[e_{k}(t) + C\hat{x}(t - \upsilon(t))], \quad (40)$$

which corresponds to

$$\begin{bmatrix} \hat{x}(t-\upsilon(t))\\ e_k(t) \end{bmatrix}^T \begin{bmatrix} \delta C^T \phi C & \delta C^T \phi\\ * & (\delta-1)\phi \end{bmatrix} \times \begin{bmatrix} \hat{x}(t-\upsilon(t))\\ e_k(t) \end{bmatrix} \ge 0.$$
(41)

Combining (25)-(41) yields

$$\dot{V}(t) \le \zeta_1^T(t)\psi_{ij}\zeta_1(t),\tag{42}$$

in which

$$\begin{split} \boldsymbol{\zeta}_{1}^{T}(t) &= \left[\boldsymbol{\xi}^{T}(t), \hat{\boldsymbol{x}}^{T}(t-d_{1}(t)), \hat{\boldsymbol{x}}^{T}(t-d_{2}(t)), \\ \hat{\boldsymbol{x}}^{T}(t-d(t)), \hat{\boldsymbol{x}}^{T}(t-h_{1}(t)), \hat{\boldsymbol{x}}^{T}(t-h_{2}(t)), \\ \hat{\boldsymbol{x}}^{T}(t-\upsilon), \int_{t-d_{1}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \int_{t-d_{1}}^{t} \int_{s}^{t} \hat{\boldsymbol{x}}^{T}(u) du ds, \\ \int_{t-d_{2}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \int_{t-d_{2}}^{t} \int_{s}^{t} \hat{\boldsymbol{x}}^{T}(u) du ds, \int_{t-d_{2}}^{t-d_{1}} \hat{\boldsymbol{x}}^{T}(s) ds, \\ \int_{t-d_{2}}^{t-d_{1}} \int_{s}^{t} \hat{\boldsymbol{x}}^{T}(u) du ds, \int_{t-h_{1}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \int_{t-h_{2}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \\ \int_{t-d_{2}}^{t-d_{1}} \int_{s}^{t} \hat{\boldsymbol{x}}^{T}(u) du ds, \int_{t-h_{1}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \int_{t-h_{2}}^{t} \hat{\boldsymbol{x}}^{T}(s) ds, \\ \hat{\boldsymbol{x}}^{T}(t), \hat{\boldsymbol{x}}^{T}(t-h_{1}), \hat{\boldsymbol{x}}^{T}(t-h_{2}), \hat{\boldsymbol{x}}^{T}(t-h(t)), \boldsymbol{e}_{k}^{T}(t) \right], \end{split}$$

and $(\psi_{ij})_{23 \times 23}$,

$$\begin{split} \psi_{ij}^{(1,1)} &= 2Q\overline{A}_0 + R_1 + R_2 + R_3 + d_1U_1 + d_2U_2 \\ &+ (d_2 - d_1)U_3 - \frac{4}{h_1}U_4 - \frac{4}{h_2}U_5 - \frac{4}{\upsilon}P_3 \\ &+ P_1 + \upsilon P_2 + 2F_1A_0, \ \psi_{ij}^{(1,4)} &= 2Q\overline{A}_1 + 2F_1A_1, \\ \psi_{ij}^{(1,5)} &= -\frac{2}{h_1}U_4, \ \psi_{ij}^{(1,6)} &= -\frac{2}{h_2}U_5, \\ \psi_{ij}^{(1,7)} &= 2Q\overline{C} - \frac{2}{\upsilon}P_3, \ \psi_{ij}^{(1,14)} &= \frac{6}{h_1^2}U_4, \ \psi_{ij}^{(1,15)} &= \frac{6}{h_2^2}U_5, \\ \psi_{ij}^{(1,17)} &= \frac{6}{\upsilon^2}P_3, \ \psi_{ij}^{(1,19)} &= -2F_1 + (F_1A_0)^T, \\ \psi_{ij}^{(1,22)} &= 2Q\overline{A}_2 + 2F_1A_2, \ \psi_{ij}^{(1,23)} &= 2Q\overline{B}_1, \\ \psi_{ij}^{(2,2)} &= -R_1, \ \psi_{ij}^{(3,3)} &= -R_2, \\ \psi_{ij}^{(4,4)} &= -(1 - \mu_1)R_3, \ \psi_{ij}^{(4,19)} &= (F_1A_1)^T, \\ \psi_{ij}^{(5,5)} &= -\frac{4}{h_1}U_4 - 4(h_{12})^2U_6, \ \psi_{ij}^{(5,6)} &= -2(h_{12})^2U_6, \\ \psi_{ij}^{(5,14)} &= \frac{6}{h_1^2}U_4, \ \psi_{ij}^{(5,16)} &= 6(h_{12})U_6, \\ \psi_{ij}^{(6,6)} &= -\frac{4}{h_2}U_5 - 4(h_{12})^2U_6, \ \psi_{ij}^{(6,15)} &= \frac{6}{h_2^2}U_5, \\ \psi_{ij}^{(6,16)} &= 6(h_{12})U_6, \ \psi_{ij}^{(7,7)} &= -P_1 - \frac{4}{\upsilon}P_3 + \delta C^T\phi C, \end{split}$$

$$\begin{split} \psi_{ij}^{(7,17)} &= \frac{6}{\upsilon^2} P_3, \ \psi_{ij}^{(7,23)} = \delta C^T \phi, \ \psi_{ij}^{(8,8)} = -\frac{4}{d_1} U_1, \\ \psi_{ij}^{(8,9)} &= \frac{6}{d_1^2} U_1, \ \psi_{ij}^{(9,9)} = -\frac{12}{d_1^3} U_1, \ \psi_{ij}^{(10,10)} = -\frac{4}{d_2} U_2, \\ \psi_{ij}^{(10,11)} &= \frac{6}{d_2^2} U_2, \ \psi_{ij}^{(11,11)} = -\frac{12}{d_2^3} U_2, \\ \psi_{ij}^{(12,12)} &= -\frac{4}{d_2 - d_1} U_3, \ \psi_{ij}^{(12,13)} = \frac{6}{(d_2 - d_1)^2} U_3, \\ \psi_{ij}^{(13,13)} &= -\frac{12}{(d_2 - d_1)^3} U_3, \ \psi_{ij}^{(14,14)} = -\frac{12}{h_1^3} U_4, \\ \psi_{ij}^{(15,15)} &= -\frac{12}{h_2^3} U_5, \ \psi_{ij}^{(16,16)} = -12 U_6, \\ \psi_{ij}^{(17,17)} &= -\frac{4}{\upsilon} P_2 - \frac{12}{\upsilon^3} P_3, \ \psi_{ij}^{(17,18)} = \frac{6}{\upsilon^2} P_2, \\ \psi_{ij}^{(18,18)} &= -\frac{12}{\upsilon^3} P_2, \ \psi_{ij}^{(19,19)} = R_4 + R_5 + R_6 + h_1 U_4 \\ &\quad + h_2 U_5 + (h_{12})^4 U_6 + \upsilon P_3 - 2F_1, \\ \psi_{ij}^{(19,22)} &= 2F_1 A_2, \\ \psi_{ij}^{(20,20)} &= -R_4, \ \psi_{ij}^{(21,21)} = -R_5, \\ \psi_{ij}^{(22,22)} &= -(1 - \mu_2) R_6, \ \psi_{ij}^{(23,23)} = -(1 - \delta) \phi. \end{split}$$

In view of the condition (42) $\psi_{ij} < 0$, indicates that $\dot{V}(t) < 0$, with the end that the filtering error system (20) for ETCS (10) with w(t) = 0 is asymptotically stable.

In addition, When $w(t) \neq 0$, we calculate the $L_2 - L_{\infty}$ performance of the filtering error system (20) as follows:

Consider the index

$$J = V(t) - \int_0^t w^T(s)w(s)ds.$$
 (43)

For all nonzero $w(t) \in L_2[0, +\infty)$, we get

$$J = \int_0^t (\dot{V}(s) - w^T(s)w(s))ds$$

$$\leq \zeta_2^T(t)\Pi_{ij}\zeta_2(t), \qquad (44)$$

where $\zeta_2^T = \left[\zeta_1^T(t), w^T(t)\right]$. Applying the Schur complement to (44), we know that (22) guarantees J < 0, implying

$$\xi^{T}(t)Q\xi(t) \le V(t) < \int_{0}^{t} w^{T}(s)w(s)ds.$$
(45)

Meanwhile, using Schur complement to (23), we can know that $\overline{E}^T \overline{E} < \gamma^2 Q$. Then, it is simple to see that for every $t \ge 0$

$$e^{T}(t)e(t) = \xi^{T}(t)\overline{E}^{T}\overline{E}\xi(t)$$

$$< \gamma^{2}\xi^{T}(t)Q\xi(t)$$

$$< \gamma^{2}\int_{0}^{t}w^{T}(s)w(s)ds$$

$$< \gamma^{2}\int_{0}^{\infty}w^{T}(s)w(s)ds. \qquad (46)$$

Therefore, by Definition 1, the filtering error system (20) is asymptotically stable with an $L_2 - L_{\infty}$ performance γ .

B. $L_2 - L_\infty$ FILTER DESIGN FOR THE NEUTRAL SYSTEM In reference to the $L_2 - L_\infty$ performance analysis in Theorem 1. In this part, we will provide a sufficient condition to derive the existence of event-triggered $L_2 - L_\infty$ filter of the form (8) is presented in the subsequent Theorem 2.

Theorem 2: For given positive scalars $d_1, d_2, h_1, h_2, \upsilon$, $\mu_1, \gamma > 0, \delta$, and μ_2 , the $L_2 - L_\infty$ filtering system (20) is solvable if there exist matrices $S > 0, Q_1 > 0, R_l > 0, U_l > 0$, $P_m > 0, \phi > 0, l = 1, 2, 3, 4, 5, 6, m = 1, 2, 3$ and positive diagonal matrix F_1 and $\overline{A}_f, \overline{B}_f, \overline{C}_f$ with adjustable dimensions, such that the following LMIs hold:

$$\varphi_{ij} < 0, \ i, j = 1, 2, \dots, q$$

$$\begin{bmatrix} Q_1 & S & E^T \\ * & S & -C_f^T \\ * & * & \gamma^2 I \end{bmatrix} > 0,$$
(47)
(47)
(47)
(47)

where

$$\begin{split} \varphi_{ij}^{(1,1)} &= R_1 + R_2 + R_3 + d_1 U_1 + d_2 U_2 \\ &+ (d_2 - d_1) U_3 - \frac{4}{h_1} U_4 - \frac{4}{h_2} U_5 - \frac{4}{\upsilon} P_3 + P_1 \\ &+ \upsilon P_2 + 2F_1 A_0 + 2Q_1 A_0, \\ \varphi_{ij}^{(1,2)} &= A_0^T S + \overline{A}_f, \ \varphi_{ij}^{(1,7)} &= -\frac{2}{h_2} U_5, \\ \varphi_{ij}^{(1,6)} &= -\frac{2}{\upsilon} P_3 + \overline{B}_f C, \ \varphi_{ij}^{(1,15)} &= \frac{6}{h_1^2} U_4, \\ \varphi_{ij}^{(1,16)} &= \frac{6}{h_2^2} U_5, \ \varphi_{ij}^{(1,18)} &= \frac{6}{\upsilon^2} P_3, \\ \varphi_{ij}^{(1,20)} &= -2F_1 + (F_1 A_0)^T, \ \varphi_{ij}^{(1,23)} &= 2F_1 A_2 + Q_1 A_2, \\ \varphi_{ij}^{(1,22)} &= \overline{B}_f, \ \varphi_{ij}^{(2,5)} &= 2F_1 B + Q_1 B, \\ \varphi_{ij}^{(2,22)} &= 2\overline{A}_f, \ \varphi_{ij}^{(2,5)} &= Q_2^T A_1, \ \varphi_{ij}^{(2,8)} &= \overline{B}_f C, \\ \varphi_{ij}^{(2,23)} &= Q_2^T A_2, \ \varphi_{ij}^{(2,24)} &= \overline{B}_f, \ \varphi_{ij}^{(2,25)} &= Q_2^T B \\ \varphi_{ij}^{(3,3)} &= -R_1, \ \varphi_{ij}^{(4,4)} &= -R_2, \ \varphi_{ij}^{(5,5)} &= -(1 - \mu_1) R_3, \\ \varphi_{ij}^{(5,20)} &= (F_1 A_1)^T, \ \varphi_{ij}^{(6,6)} &= -\frac{4}{h_1} U_4 - 4(h_{12})^2 U_6, \\ \varphi_{ij}^{(6,7)} &= -2(h_{12})^2 U_6, \ \varphi_{ij}^{(6,15)} &= \frac{6}{h_1^2} U_4, \\ \varphi_{ij}^{(6,17)} &= 6(h_{12}) U_6, \\ \varphi_{ij}^{(7,16)} &= \frac{6}{h_2^2} U_5, \ \varphi_{ij}^{(7,17)} &= 6(h_{12}) U_6, \\ \varphi_{ij}^{(8,8)} &= -P_1 - \frac{4}{\upsilon} P_3 + \delta C^T \phi C, \\ \varphi_{ij}^{(8,8)} &= \frac{6}{\upsilon^2} P_3, \ \varphi_{ij}^{(8,24)} &= \delta C^T \phi, \\ \varphi_{ij}^{(9,9)} &= -\frac{4}{d_1} U_1, \ \varphi_{ij}^{(9,10)} &= \frac{6}{d_1^2} U_1, \\ \varphi_{ij}^{(10,10)} &= -\frac{12}{d_1^3} U_1, \ \varphi_{ij}^{(11,11)} &= -\frac{4}{d_2} U_2, \ \varphi_{ij}^{(11,12)} &= \frac{6}{d_2^2} U_2, \end{split}$$

$$\begin{split} \varphi_{ij}^{(12,12)} &= -\frac{12}{d_2^3} U_2, \ \varphi_{ij}^{(13,13)} = -\frac{4}{d_2 - d_1} U_3, \\ \varphi_{ij}^{(13,14)} &= \frac{6}{(d_2 - d_1)^2} U_3, \ \varphi_{ij}^{(14,14)} = -\frac{12}{(d_2 - d_1)^3} U_3, \\ \varphi_{ij}^{(15,15)} &= -\frac{12}{h_1^3} U_4, \ \varphi_{ij}^{(16,16)} = -\frac{12}{h_2^3} U_5, \\ \varphi_{ij}^{(17,17)} &= -12 U_6, \ \varphi_{ij}^{(18,18)} = -\frac{4}{v} P_2 - \frac{12}{v^3} P_3, \\ \varphi_{ij}^{(18,19)} &= \frac{6}{v^2} P_2, \ \varphi_{ij}^{(19,19)} = -\frac{12}{v^3} P_2, \\ \varphi_{ij}^{(20,20)} &= R_4 + R_5 + R_6 + h_1 U_4 + h_2 U_5 \\ &\quad + (h_{12})^4 U_6 + v P_3 - 2 F_1, \\ \varphi_{ij}^{(20,23)} &= 2 F_1 A_2, \ \varphi_{ij}^{(20,25)} &= 2 F_1 B, \ \varphi_{ij}^{(21,21)} = -R_4, \\ \varphi_{ij}^{(22,22)} &= -R_5, \ \varphi_{ij}^{(23,23)} = -(1 - \mu_2) R_6, \\ \varphi_{ii}^{(24,24)} &= -(1 - \delta) \phi, \ \varphi_{ij}^{(25,25)} &= -I. \end{split}$$

Moreover the desired filter of the form (7) is given by

$$A_f = S^{-1}\overline{A}_f, B_f = S^{-1}\overline{B}_f, C_f = \overline{C}_f.$$
(49)

Proof: Since S > 0, there exists a real matrix Q_2 and $Q_3 > 0$, such that $S = Q_2 Q_3^{-1} Q_2^T$. Defining H = 23 times

 $diag\{I, Q_2Q_3^{-1}, \overline{I,I,I}\}$ and denoting $\overline{A}_f = Q_2A_fQ_3^{-1}Q_2^T$, $\overline{B}_f = Q_2B_f, \overline{C}_f = C_f$. Multiplying (22) and (23) by both sides on H and H^T , then we get (47) and (48). According to Theorem 1, if (47) and (48) are feasible, the ETCS $L_2 - L_{\infty}$ problem is solvable, the filtering parameters are described by (49).

Remark 1: Consider the following filtering system from (20) without neutral delay as follows:

$$\dot{\xi}(t) = \overline{A}_0 \xi(t) + \overline{A}_1 \hat{x}(t - d(t)) + \overline{B} w(t) + \overline{B}_1 e_k(t) + \overline{C} \hat{x}(t - \upsilon(t)),$$

$$e(t) = \overline{E} \xi(t), \qquad (50)$$

Using the similar methods in Theorem 2, we can get the following results.

Corollary 1: For given positive scalars $d_1, d_2, \upsilon, \mu_1, \gamma > 0$ and δ , the $L_2 - L_{\infty}$ filtering system (50) is solvable if there exist positive definite matrices $S > 0, Q_1 > 0, R_l > 0, U_l > 0, P_m > 0, \phi > 0, l = 1, 2, 3, m = 1, 2, 3$ and diagonal matrix $F_1 > 0$ and $\overline{A}_f, \overline{B}_f, \overline{C}_f$ with appropriate dimensions such that

$$\zeta_{ij} < 0, \ i, j = 1, 2, \dots, q$$
 (51)

$$\begin{bmatrix} Q_1 & S & E^T \\ * & S & -C_f^T \\ * & * & \gamma^2 I \end{bmatrix} > 0,$$
(52)

where

$$\begin{aligned} \zeta_{ij}^{(1,1)} &= R_1 + R_2 + R_3 + d_1 U_1 + d_2 U_2 + (d_2 - d_1) U_3 \\ &- \frac{4}{\upsilon} P_3 + P_1 + \upsilon P_2 + 2F_1 A_0 + 2Q_1 A_0, \\ \zeta_{ij}^{(1,2)} &= A_0^T S + \overline{A}_f, \ \zeta_{ij}^{(1,5)} = 2F_1 A_1 + Q_1 A_1, \end{aligned}$$

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$$\begin{split} \zeta_{ij}^{(1,6)} &= -\frac{2}{\upsilon} P_3 + \overline{B}_f C, \ \zeta_{ij}^{(1,13)} = \frac{6}{\upsilon^2} P_3, \\ \zeta_{ij}^{(1,15)} &= -2F_1 + (F_1A_0)^T, \ \zeta_{ij}^{(1,16)} = \overline{B}_f \\ \zeta_{ij}^{(1,17)} &= 2F_1B + Q_1B \ \zeta_{ij}^{(2,2)} = 2\overline{A}_f, \\ \zeta_{ij}^{(2,5)} &= Q_2^T A_1, \ \zeta_{ij}^{(2,6)} = \overline{B}_f C, \ \zeta_{ij}^{(2,16)} = \overline{B}_f, \\ \zeta_{ij}^{(2,17)} &= Q_2^T B, \ \zeta_{ij}^{(3,3)} = -R_1, \ \zeta_{ij}^{(4,4)} = -R_2, \\ \zeta_{ij}^{(5,5)} &= -(1 - \mu_1)R_3, \ \zeta_{ij}^{(5,15)} = (F_1A_1)^T \\ \zeta_{ij}^{(6,6)} &= -P_1 - \frac{4}{\upsilon} P_3 + \delta C^T \phi C, \\ \zeta_{ij}^{(6,6)} &= \frac{6}{\upsilon^2} P_3, \ \zeta_{ij}^{(6,16)} = \delta C^T \phi, \ \zeta_{ij}^{(7,7)} = -\frac{4}{d_1} U_1, \\ \zeta_{ij}^{(7,8)} &= \frac{6}{d_1^2} U_1, \ \zeta_{ij}^{(8,8)} = -\frac{12}{d_1^3} U_1, \ \zeta_{ij}^{(9,9)} = -\frac{4}{d_2} U_2, \\ \zeta_{ij}^{(9,10)} &= \frac{6}{d_2^2} U_2, \ \zeta_{ij}^{(10,10)} = -\frac{12}{d_2^3} U_2, \\ \zeta_{ij}^{(12,12)} &= -\frac{12}{(d_2 - d_1)^3} U_3, \ \zeta_{ij}^{(13,13)} = -\frac{4}{\upsilon} P_2 - \frac{12}{\upsilon^3} P_3, \\ \zeta_{ij}^{(13,14)} &= \frac{6}{\upsilon^2} P_2, \ \zeta_{ij}^{(14,14)} = -\frac{12}{\upsilon^3} P_2, \\ \zeta_{ij}^{(15,15)} &= \upsilon P_3 - 2F_1, \ \zeta_{ij}^{(15,17)} &= 2F_1 B, \\ \zeta_{ij}^{(16,16)} &= -(1 - \delta)\phi, \ \zeta_{ij}^{(17,17)} &= -I. \end{split}$$

Moreover the desired filter of the form (7) is given by

$$A_f = S^{-1}\overline{A}_f, B_f = S^{-1}\overline{B}_f, C_f = \overline{C}_f.$$
 (53)

Proof: The proof is similar process in Theorem 2.

Remark 2: Consider the following linear system (similar as in [17]), which is given by:

$$\dot{\hat{x}}(t) = A_0 \hat{x}(t) + A_1 \hat{x}(t - d(t)) + A_2 \dot{\hat{x}}(t - h(t)), \quad (54)$$

According to Theorem 1, we can derive the following Corollary 2 for the asymptotic stability of the neutral type system (54).

Corollary 2: For given positive scalars $d_1, d_2, h_1, h_2, \mu_1$ and μ_2 , the system (54) is solvable if there exist matrices $Q_1 > 0, R_l > 0, U_l > 0, l = 1, 2, 3, 4, 5, 6, and positive$ $diagonal matrix <math>F_1$, such that the following LMI holds:

$$\Phi < 0, \tag{55}$$

where
$$\Phi = (\Phi^{ij})_{19 \times 19}$$

 $\Phi^{(1,1)} = R_1 + R_2 + R_3 + d_1U_1 + d_2U_2$
 $+ (d_2 - d_1)U_3 - \frac{4}{h_1}U_4 - \frac{4}{h_2}U_5 + 2F_1A_0 + 2Q_1A_0,$
 $\Phi^{(1,4)} = 2F_1A_1 + Q_1A_1, \ \Phi^{(1,5)} = -\frac{2}{h_1}U_4,$
 $\Phi^{(1,6)} = -\frac{2}{h_2}U_5, \ \Phi^{(1,13)} = \frac{6}{h_1^2}U_4, \ \Phi^{(1,14)} = \frac{6}{h_2^2}U_5,$
 $\Phi^{(1,16)} = -2F_1 + (F_1A_0)^T, \ \Phi^{(1,19)} = 2F_1A_2 + Q_1A_2,$
 $\Phi^{(2,2)} = -R_1, \ \Phi^{(3,3)} = -R_2, \ \Phi^{(4,4)} = -(1 - \mu_1)R_3,$

$$\begin{split} \Phi^{(4,16)} &= (F_1A_1)^T, \ \Phi^{(5,5)} = -\frac{4}{h_1}U_4 - 4(h_{12})^2U_6, \\ \Phi^{(5,6)} &= -2(h_{12})^2U_6, \ \Phi^{(5,13)} = \frac{6}{h_1^2}U_4, \\ \Phi^{(5,15)} &= 6(h_{12})U_6, \ \Phi^{(6,6)} = -\frac{4}{h_2}U_5 - 4(h_{12})^2U_6, \\ \Phi^{(6,14)} &= \frac{6}{h_2^2}U_5, \ \Phi^{(6,15)} = 6(h_{12})U_6, \\ \Phi^{(7,7)} &= -\frac{4}{d_1}U_1, \ \Phi^{(7,8)} = \frac{6}{d_1^2}U_1, \\ \Phi^{(8,8)} &= -\frac{12}{d_1^3}U_1, \ \Phi^{(9,9)} = -\frac{4}{d_2}U_2, \ \Phi^{(9,10)} = \frac{6}{d_2^2}U_2, \\ \Phi^{(10,10)} &= -\frac{12}{d_2^3}U_2, \ \Phi^{(11,11)} = -\frac{4}{d_2 - d_1}U_3, \\ \Phi^{(11,12)} &= \frac{6}{(d_2 - d_1)^2}U_3, \ \Phi^{(12,12)} = -\frac{12}{(d_2 - d_1)^3}U_3, \\ \Phi^{(13,13)} &= -\frac{12}{h_1^3}U_4, \ \Phi^{(14,14)} = -\frac{12}{h_2^3}U_5, \ \Phi^{(15,15)} = -12U_6, \\ \Phi^{(16,16)} &= R_4 + R_5 + R_6 + h_1U_4 + h_2U_5 \\ &\quad + (h_{12})^4U_6 - 2F_1, \ \Phi^{(19,19)} = 2F_1A_2, \\ \Phi^{(17,17)} &= -R_4, \ \Phi^{(18,18)} = -R_5, \ \Phi^{(19,19)} = -(1 - \mu_2)R_6, \end{split}$$

Proof: Consider the following LKF candidate

$$\begin{split} V(t) &= \sum_{l=1}^{5} V_{l}(t), \\ V_{1}(t) &= \hat{x}^{T}(t)Q_{1}\hat{x}(t), \\ V_{2}(t) &= \int_{t-d_{1}}^{t} \hat{x}^{T}(s)R_{1}\hat{x}(s)ds + \int_{t-d_{2}}^{t} \hat{x}^{T}(s)R_{2}\hat{x}(s)ds \\ &+ \int_{t-d(t)}^{t} \hat{x}^{T}(s)R_{3}\hat{x}(s)ds, \\ V_{3}(t) &= \int_{t-h_{1}}^{t} \dot{x}^{T}(s)R_{4}\dot{\hat{x}}(s)ds + \int_{t-h_{2}}^{t} \dot{x}^{T}(s)R_{5}\dot{\hat{x}}(s)ds \\ &+ \int_{t-h(t)}^{t} \dot{x}^{T}(s)R_{6}\dot{\hat{x}}(s)ds, \\ V_{4}(t) &= \int_{-d_{1}}^{0} \int_{t+\theta}^{t} \hat{x}^{T}(s)U_{1}\hat{x}(s)dsd\theta + \int_{-d_{2}}^{0} \int_{t+\theta}^{t} \hat{x}^{T}(s) \\ &\times U_{2}\hat{x}(s)dsd\theta + \int_{-d_{2}}^{-d_{1}} \int_{t+\theta}^{t} \hat{x}^{T}(s)U_{3}\hat{x}(s)dsd\theta, \\ V_{5}(t) &= \int_{-h_{1}}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s)U_{4}\dot{\hat{x}}(s)dsd\theta + \int_{-h_{2}}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s) \\ &\times U_{5}\dot{\hat{x}}(s)dsd\theta + h_{12}^{3} \int_{-h_{2}}^{-h_{1}} \int_{t+\theta}^{t} \dot{x}^{T}(s)U_{6}\dot{\hat{x}}(s)dsd\theta. \end{split}$$

$$(56)$$

Then, using the same treatment theory as Theorem 1, we can easily get the Corollary 2.

Remark 3: Notice that Theorem 2 gives an adequate condition to co-plan the event-triggered filtering (8) and the event-triggered scheme in the communication (4). If the LMIs (47) and (48) is feasible, the desired filter parameters A_{fj} , B_{fj} and C_{fj} can be obtained.

Remark 4: In addition, computational complexity becomes a major issue, based on the size of the LMIs and the number of decision variables. However, larger LMIs improve performance. In recent years, an increasing number of researchers are working to improve the performance of time-delayed systems. The basic problem with time-delayed systems is to build better LKFs and estimate their derivatives using better integral inequality techniques. The proposed stability criteria in this study are derived from the constructions of various LKFs; the proposed LMI conditions are more complicated and have computational complexity. To meet these needs, some new research has been recently published using a different type of LKF that is more effective at reducing computational complexity while keeping less conservative results [44], [45]. In future work, we will apply Finsler's Lemma to reduce the number of decision variables and use this type of new LKF to reduce maintainability along with simple LMI conditions. Therefore, it is easy to use the provided method to reduce the maintainability and computational load of time-delayed systems with the new LKF in [44], [45].

IV. NUMERICAL EXAMPLES

In this part, numerical simulation results and application examples are provided to illustrate the designed filter's performance. Example 1 shows the feasibility of Theorem 2. By comparing the results in the previous literature, the advantages of the proposed result are presented in Example 2. Example 3 is given to benchmark the proposed filter design through studying a tunnel diode circuit.

Example 1: Consider the following T-S fuzzy neutral type system (57)

$$\dot{\xi}(t) = \overline{A}_0 \xi(t) + \overline{A}_1 \hat{x}(t - d(t)) + \overline{A}_2 \dot{\hat{x}}(t - h(t)) + \overline{B} w(t) + \overline{B}_1 e_k(t) + \overline{C} \hat{x}(t - \upsilon(t)),$$
$$e(t) = \overline{E} \xi(t), \tag{57}$$

with the following parameters:

$$A_{01} = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -2 & 0 \\ -0.2 & -1.1 \end{bmatrix}$$
$$A_{11} = \begin{bmatrix} -1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.9 & 0 \\ -1 & -0.8 \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} 0.3 & -0.15 \\ 0.5 & -0.2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.4 & -0.1 \\ 0.5 & -0.3 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 2 \\ -0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & -1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 2 & -0.5 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.5 & 0.2 \end{bmatrix}.$$

The membership functions are taken as $u_1(s(t)) = \begin{cases} \frac{3+\hat{x}(t)}{3}, & -3 \le \hat{x}(t) \le 0; \\ \frac{3-\hat{x}(t)}{3}, & 0 \le \hat{x}(t) \le 3; \end{cases}$ and $u_2(s(t)) = 1 - u_1(s(t)).$

0 otherwise, Subsequently, we will show the desired performance of the proposed ETCS (4) and the full order filter (7) for the

the proposed ETCS (4) and the full order filter (7) for the system (1) with $L_2 - L_{\infty}$ performance index γ according to Theorem 2. Before computation analysis, we assume $0.1 \leq d(t) \leq 0.2, \ 0.2 \leq h(t) \leq 0.4, \ \mu_1 = 0.2, \ \mu_2 = 0.3, \ \upsilon = 0.01$, the external disturbances $w(t) = \sin(0.1t)e^{(-0.1t)}$ and by applying the LMIs in Theorem 2, we can obtain the related trigger matrix $\phi = 2.0324$, and the following desired filter parameters:

$$A_{f1} = \begin{bmatrix} -15.0634 & -2.3412 \\ -4.1445 & -14.5469 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -0.5487 \\ -1.2421 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 0.5314 & 0.4478 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} -16.1543 & -3.5428 \\ -4.2432 & -15.3678 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} -0.6742 \\ -0.4587 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} 0.4812 & 0.3733 \end{bmatrix}.$$

Consider the initial state $\hat{x}(t) = [0.2 - 0.1]^T$ and $x_f(t) = [-1 \ 0.5]^T$. The related simulation results are displayed in Figure 2, Figure 2a present the state responses of $\xi(t)$, the corresponding curves of the state $\hat{x}_1(t), \hat{x}_2(t)$ and $x_{f1}(t), x_{f2}(t)$ are drawn in Figure 2b and Figure 2c, evolution of the error responses $e(t) = z(t) - z_f(t)$ are depicted in Figure 2d. Based on the above Figures 2a-2d, that the filtering error system is asymptotically stable in terms of $L_2 - L_\infty$ performance by applying the designed parameters. Furthermore, because of the event-triggered mechanism, the release instants and release intervals are shown in Figure 2e. Finally, figure 2f depicts the corresponding state responses of the considered neutral type fuzzy filtering system with different initial values. In view of the above simulation studies, the proposed event-triggered mechanism with modelled system (20) can significantly minimize the data communication burden. We can conclude that the event-triggered scheme-based filter designed here works well while minimizing unnecessary communication data.

TABLE 1.	Minimum $L_2 - L_0$	performance	level γ	for different d_2
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d_2	0.1	0.3	0.5	0.7	1.0
γ	0.0012	0.0254	0.0398	0.0478	0.1563

TABLE 2. Minimum $L_2 - L_\infty$ performance level γ for different μ_1 .

μ_1	0.01	0.1	0.3	0.5	0.7
γ	0.2682	0.3721	0.4168	0.4933	0.5267

Based on the ETCS and network environment, the network induced delay is fully investigated in the filter error system (20). In this part, the MAUB d_2 for guaranteeing the

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(c) Responses of $\hat{x}_2(t)$ and $x_{f2}(t)$ (d) Responses of z(t) and $z_f(t)$



(e) Response of the triggered in-(f) Response of $\xi(t)$ for various stants initial values

FIGURE 2. The panels (2a)-(2f) contain the simulation results of the state responses and release instants in Example 1.

stability of the error system (20) is essential. By this way solving Theorem 2, we can obtain the minimum $L_2 - L_{\infty}$ performance index for various d_2/μ_1 , and it is summarized in Table 1 and Table 2. From Table 1 and Table 2, we can see that the effects of the upper bound d_2/μ_1 on the $L_2 - L_{\infty}$ performance index γ .

Example 2: Consider the following T–S fuzzy system.

$$\dot{\hat{x}}(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[A_{0i} \hat{x}(t) + A_{1i} \hat{x}(t - d(t)) \\ + B_i w(t) \Big],$$
$$\hat{y}(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[C_i \hat{x}(t) \Big],$$
$$z(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[E_i \hat{x}(t) \Big],$$
(58)

where

$$A_{01} = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -2 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \\ A_{11} = \begin{bmatrix} -1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.8 \end{bmatrix},$$

$$B_{1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, B_{2} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}, C_{1} = \begin{bmatrix} 1 & 2 \end{bmatrix}, C_{2} = \begin{bmatrix} 1 & 0.5 \end{bmatrix}, E_{1} = \begin{bmatrix} 0.5 & -2 \end{bmatrix}, E_{2} = \begin{bmatrix} -0.3 & 0.3 \end{bmatrix}, u_{1}(s(t)) = \sin^{2} t, U_{2}(s(t)) = \cos^{2} t, w(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1, & 15 \le t \le 20 \\ 0, & else \end{cases}$$

According to the above parameters and let $0.4 \le d(t) \le 0.5$, $\upsilon = 0.3$. By using LMIs in Corollary 1 and MATLAB LMI toolbox, then the related trigger matrix $\phi = 8.3242$ and the subsequent matrices can be derived as follows.

$$A_{f1} = \begin{bmatrix} -8.1245 & -3.3252 \\ -2.3214 & -7.5463 \end{bmatrix}, B_{f1} = \begin{bmatrix} -2.1025 \\ -0.9425 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} 1.7458 & 0.8712 \end{bmatrix} A_{f2} = \begin{bmatrix} -10.4752 & -2.1024 \\ -4.1204 & -9.2867 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} -0.8412 \\ -0.7458 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.4785 & 0.1025 \end{bmatrix}.$$

Based on the zero initial condition, the state trajectories of $\xi(t)$ is depicted in Figure 3a, the corresponding simulation curves of the system states $\hat{x}_1(t), \hat{x}_2(t)$ and the state responses of the filters $x_{f1}(t)$, $x_{f2}(t)$ are shown in Figures 3b and 3c. Figure 3d demonstrates the responses of filtering error z(t) – $z_f(t)$ and the triggering instants are picturized in Figure 3e. Figure 3f depicts the corresponding state responses for the fuzzy filtering error system with different initial values. Furthermore, by solving LMIs in Corollary 1, we obtain the maximum allowable upper bounds of d_2 for various values of v, when $d_1 = 0.1$ (listed in Table 3), which clearly shows the effectiveness of our work. In addition, compared with the results of the proposed ETCS and the ETCS in [19], to show the performance of our proposed method. By designing $t \in (0, 35]$ Figure 3e presents the transmission instants and intervals under the proposed ETCS. Furthermore, the transmission trigger times during the interval (0, 35] of the proposed ETCS is 35 release instants. One can check that the proposed ETCS released less transmitted data than those in [19], which means the energy consumption is effectively reduced. In addition, Corollary 1 obtained here are not only less conservative results but has effectively saved the limited communication resources greatly under the release intervals in Figure 3e. In conclusion, all the simulation results have indicated our theoretical analysis for the proposed algorithmic filter.

TABLE 3.	Upper	bound	of d ₂	for	various	values	of v	and d ₁	= 0.1.
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v	0.1	0.3	0.5	0.55
[19]	-	-	0.4000	-
Corollary 1	0.5626	0.5421	0.4521	0.3914



(c) Responses of $\hat{x}_2(t)$ and $x_{f2}(t)$ (d) Responses of z(t) and $z_f(t)$



(e) Response of the triggered in-(f) Response of $\xi(t)$ for various stants initial values

FIGURE 3. The panels (3a)-(3f) contain the behavior of state trajectories and triggered instants in example 2.

V. APPLICATION

Example 3: Consider the tunnel diode circuit in [46] is considered to shown the performance of the proposed approach.

$$\begin{cases} C\dot{v}_{c}(t) = -0.002v_{c}(t) - 0.01v_{c}(t)^{3} + i_{L}(t) \\ y(t) = v_{c}(t) \\ L\dot{i}_{L}(t) = -v_{c}(t) - Ri_{L}(t) + w(t), \end{cases}$$
(59)

where $v_c(t)$ noted as voltage of the capacitor, $i_L(t)$ denotes the current of the inductor, w(t) indicates the disturbance, y(t) is the sampled-data measurement output. Furthermore, we initiate the parameters of the capacitor, inductor and resistance C = 20mF, L = 1000mH and $R = 10\Omega$, respectively.

For instance, $v_c(t) \in [-3, 3]$, the tunnel diode circuit can be described in the following fuzzy system with $\hat{x}(t) = [\hat{x}_1(t) \hat{x}_2(t)]^T = [v_c(t) i_L(t)]^T$ and $z(t) = \hat{x}_1(t)$.

$$\dot{\hat{x}}(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[A_{0i}(t) \hat{x}(t) + B_i(t) w(t) \Big]$$
$$\hat{y}(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[C_i(t) \hat{x}(t) \Big],$$



FIGURE 4. Schematic diagram of Tunnel diode circuit in [46].

$$z(t) = \sum_{i=1}^{2} u_i(s(t)) \Big[E_i(t) \hat{x}(t) \Big],$$
(60)

where

$$A_{01} = \begin{bmatrix} -0.1 & 50\\ -1 & -10 \end{bmatrix}, A_{02} = \begin{bmatrix} -4.6 & 50\\ -1 & -10 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0\\ 1 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$E_1 = E_2 \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Besides, we suppose the system membership functions are taken as $u_1(s(t)) = \begin{cases} \frac{3+\hat{x}(t)}{3}, & -3 \le \hat{x}(t) \le 0; \\ \frac{3-\hat{x}(t)}{3}, & 0 \le \hat{x}(t) \le 3; \\ 0 & otherwise, \end{cases}$

 $u_2(s(t)) = 1 - u_1(s(t))$. Let the bound of time delay v = 0.08 and event-triggered parameter $\delta = 0.9$. Utilizing LKF $V_1(t)$, $V_6(t)$ with Corollary 1 and the filtering gains are as follows:

$$A_{f1} = \begin{bmatrix} -3.2134 & 0.1574 \\ 0.2473 & -2.2154 \end{bmatrix}, B_{f1} = \begin{bmatrix} -2.0147 \\ 0.0112 \end{bmatrix},$$
$$C_{f1} = \begin{bmatrix} -2.0157 & 0.5240 \end{bmatrix}, A_{f2} = \begin{bmatrix} -10.0125 & 0.2584 \\ 8.0012 & -2.1291 \end{bmatrix},$$
$$B_{f2} = \begin{bmatrix} -6.3120 \\ 0.4102 \end{bmatrix}, C_{f2} = \begin{bmatrix} -1.0311 & 0.2471 \end{bmatrix}.$$

Moreover, to illustrate that the effectiveness of the fullorder event-triggered $L_2 - L_\infty$ filter design approach developed in Corollary 1, the responses of $\hat{x}(t)$ and $x_f(t)$ can be achieved by iterative estimations from initial value $\hat{x}(0) = x_f(0) = [0 \ 0]^T$, and the error can be easily obtained by $e(t) = z(t) - z_f(t)$ and the disturbance is considered as w(t) = exp(-0.1t)sin(0.1t). The evolution's of $\xi(t), \hat{x}(t), x_f(t)$ and e(t) are shown in Figures 5a, 5b, 5c and 5d. Figure 5f shows the related state trajectories for the fuzzy filtering error system with various initial conditions. With the above simulation results that the filtering error system (60) is asymptotically stable by applying the designed parameters. The transmission instants and intervals are presented in Figure 5e. In order to satisfied the $L_2 - L_\infty$ performance, the period is taken as $t \in (0, 25]$ in Figure 5e, which shows that the required transmission can save network resources. Moreover, from Figures 5a-5e, one can check ETCS cannot just mitigate the issue of resource constraints but also make the data in the transmission process faster and more stable. The triggering threshold can be chosen accordingly to the real system application requirements. Under the network communication parameters v = 0.08 and $\delta = 0.9$, therefore we get the minimum value of $L_2 - L_{\infty}$ performance $\gamma = 1.0254$. From the above simulation results and discussions, we can clearly illustrate that the filtering errors can achieve ultimate $L_2 - L_{\infty}$ performance under the designed event-triggered scheme in this paper, and it is consistent with our theoretical results.



(c) Responses of $\hat{x}_2(t)$ and $x_{f2}(t)$ (d) Responses of z(t) and $z_f(t)$



(e) Response of the triggered in-(f) Response of $\xi(t)$ for various stants initial values

FIGURE 5. Behavior of the responses for various states $\xi(t)$, $\hat{x}_1(t)$, $x_{f1}(t)$, $\hat{x}_2(t)$, $x_{f2}(t)$, e(t) and triggered instants in example 3.

TABLE 4.	Maximum	allowable	bound	$\boldsymbol{v} \text{ and }$	δ for	example	3.
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Methods	[41]	Example 3
v and δ	$v = 0.06$ and $\delta = 0.8$	$v = 0.08$ and $\delta = 0.9$

Example 4: As illustrated in [47], the partial element equivalent circuit (PEEC) model in Fig. 4(b) includes new circuit elements which involve retarded mutual coupling between the partial inductances of the form $Lp_{ij}i'_{j}(t - h_2)$ and retarded dependent current sources of the form $p_{ij}/p_{ii}i_{cj}(t - h_2)$. The general form of modeling PEEC can be modeled as

$$C_{0}\dot{\hat{x}}(t) + G_{0}\hat{x}(t) + C_{1}\dot{\hat{x}}(t - h_{2}) + G_{1}\hat{x}(t - h_{2}) = Bu(t, t - h_{2}), t \ge t_{0}; \hat{x}(t) = \hat{\phi}(t), t \le t_{0}.$$
(61)

To consider the asymptotic stability of the system with the mathematical deduction, the PEEC (61) can be rewritten as the following neutral system [17]:

$$\dot{\hat{x}}(t) = A_0 \hat{x}(t) + A_1 \hat{x}(t - h_2) + A_2 \dot{\hat{x}}(t - h_2),$$

$$\hat{x}(t) = \hat{\phi}(t), t \le t_0.$$
 (62)



FIGURE 6. Illustrate (i) shows the metal strip with two L_P cells (dashed means three capacitive cells) and (b) shows small PEEC model for metal strip.

If we take different time-varying delays into account, a more general form of PEEC (62) can be described by the following system [17]

$$\dot{\hat{x}}(t) = A_0 \hat{x}(t) + A_1 \hat{x}(t - d(t)) + A_2 \dot{\hat{x}}(t - h(t)),$$

$$\hat{x}(t) = \hat{\phi}(t), t \le t_0.$$
(63)

where the system parameters are given as [17]

$$A_{0} = 100 \begin{bmatrix} -2.105 & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix},$$

$$A_{1} = 100 \begin{bmatrix} 1 & 0 & -3 \\ -0.5 & -0.5 & -1 \\ -0.5 & -1.5 & 0 \end{bmatrix}, A_{2} = \frac{1}{72} \begin{bmatrix} -1 & 5 & 2 \\ 4 & 0 & 3 \\ -2 & 4 & 1 \end{bmatrix}.$$

Let us choose time varying delays $0.1 \le d(t) \le 0.8$, $0.1 \le h(t) \le 0.7$, $\mu_1 = 0.5$, $\mu_2 = 0.5$. Then by utilizing the above values and calling the MATLAB LMI toolbox, solving the LMI in Corollary 2, it is found that the system (54) is asymptotically stable and the state trajectories of the dynamical system converge to the zero equilibrium point with an initial state [1.5, -0.4, -0.3]^T shown in Figure 7.



FIGURE 7. State responses of the system (63) in example 4.

Remark 5: Consider the delayed neutral type system (54) with the input matrices as in [48] (Example 1). Solving the system (54), using the LMI in Corollary 2 and assume that $\mu_2 = 0$, we obtain the MAUB d_2 for different d_1 and μ_1 as shown in Table 5, which clearly shows the effectiveness of our work. The results obtained in this paper are significantly better than those in [48].

TABLE 5. MAUB of d_2 for various values of μ_1 and d_1 .

d_1	Methods	$\mu_1 = 0.5$	0.7	0.9
0.5	[48]	2.096	1.811	1.810
1.5	Corollary 2	2.121	1.922	1.910

VI. CONCLUSION

To conserve network bandwidth, we presented an eventtriggered $L_2 - L_\infty$ filtering for network-based neutral systems via T-S fuzzy approach. Using a new ETCS and the suitable LKF, Wirtinger's inequality technique, we established some sufficient conditions and constructed the LMIs to ensure the asymptotic stability of the filtering error system. The numerical examples demonstrate how the proposed communication scheme will greatly reduce network bandwidth consumption while maintaining the desired efficiency of the filtering error system. Moreover, the model proposed in this work can be also extended event-triggered mechanism to the discrete time domain with imperfect communication, such as packet dropouts and quantization. We will also target on the complex phenomena like the randomly occurring uncertainties, incomplete measurements, general switching system with repeated scalar nonlinearities, T-S fuzzy based piecewise Lyapunov function and event-triggered with asynchronous sampling, which makes the model more practical and will be investigated in our future work.

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