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Adaptive Fuzzy Feedback Controller Design for Finite-Time Mittag-Leffler Synchronization of Fractional-Order Quaternion-Valued Reaction-Diffusion Fuzzy Molecular Modeling of Delayed Neural Networks

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ABSTRACT This paper addresses an adaptive fuzzy feedback controller design problem for finite-time Mittag-Leffler synchronization (FTMLS) of fractional-order quaternion-valued reaction-diffusion T-S fuzzy molecular modeling of delayed neural networks. A novel approach is proposed to effectively deal with the joint effects from fuzzy rules and reaction-diffusion terms for the class of T-S fuzzy fractional-order reaction-diffusion delayed quaternion-valued neural networks (FORDDQVNNs) under consideration. By employing Lyapunov stability theory, Caputo fractional derivative, several algebraic criteria are established to guarantee the FTMLS of T-S fuzzy FORDDQVNNs via designed fuzzy feedback controller. Moreover, the adaptive controller and parameter update laws are designed via adaptive control methods. Compared with existing results in the literature, we also show that our results are less conservative than existing ones with these illustrative T-S fuzzy FORDDQVNNs. A numerical example is presented to verify the analysis results and illustrate the effectiveness of the proposed FTMLS conditions.

INDEX TERMS Quaternion-valued neural networks (QVNNs), fractional derivatives, reaction-diffusion terms, Takagi-Sugeno fuzzy, adaptive control law.

I. INTRODUCTION

Based on these biological knowledge, the stability of molecular models of genetic regulatory networks, neural networks (NNs) and etc., has received more and more attention [1]–[3]. Note that these applications have important relationships with their dynamic behaviors, the internal dynamics of NNs, such as stability, multistability, synchronization, and so on, and have received increasing attention in past decades [4], [5]. In order to describe physical phenomena more accurately, fractional-order neural networks (FONNs) are recognized as a significant improvement over the integer-order NNs because of their long-term memory and hereditary properties [6], [7]. Subsequently, FONNs as a kind of important biological networks, have attracted increasing interests [8], [9]. In the study of FONNs, the discussion of dynamical behaviors is always a hot topic, such as Mittag-Leffler synchronization [10], stability [11] and so forth [12], [13]. Time delays, such as leakage delays, distributed delays, discrete delays, and neutral delays are widespread and inevitable in NNs [14]–[17]. It is a source of oscillation, divergence, instability, chaos and

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poor performance. Therefore, investigation of delayed NNs is not only of theoretical significance but also of practical significance.

As a typical collective behavior, synchronization has attracted considerable attention due to its theoretical importance and practical applications in various fields such as the modeling brain activity, cryptography, clock synchronization of sensor networks [18]–[20]. Until now, the problem of synchronization for fractional-order systems, particularly, dynamical networks [21] has been much analyzed, and widely control strategies, including the sliding mode control [22], impulsive control [23], the pinning control [24], and the adaptive control [10] have been concentrated on this topic. The above mentioned types of synchronization are shows that the trajectories of the response system can reach the trajectories of deriving system over the infinite horizon. In the application point of view, the synchronization should be realized in finite-time which is more and more important. Thus, it is necessary to analyze the finite-time synchronization of FONNs. Recently, many authors have paid their attention and interest for the analysis of finite-time synchronization of FONNs and some good results has been reported in [25]–[27]. For instance, the author [28] investigated the FTMLS of memristive BAM FONNs with time delays via state-feedback control. Chen *et al.* [29] studied FTMLS of memristor-based FONNs with parameters uncertainty by using Lyapunov-like method.

Reaction-diffusion NNs (RDNNs), in which the neuron states are dependent on both time and space, can perfectly describe the time and spatial evolutions. In comparison with the traditional NNs, RDNNs could realize better approximations of actual systems. It is thus reasonable and important to consider NNs with diffusion effects. Recently, many elegant achievements on qualitative analysis of dynamical behaviors for various RDNN models have been reported in [30]–[32]. In recent years, many efforts have been dedicated to investigating synchronization of RDNNs with time delays [33]–[35]. Also, relatively recently, reaction-diffusion terms have been incorporated into some fractional-order models [36], [37]. For example, Stamova and Stamov [38] developed impulsive control on Mittag-Leffler synchronization of FONNs with time-varying delays and reaction-diffusion terms. On the other hand, recent years have witnessed a rapid growing interest in adaptive control [39], [40] which is an important control technique and has been widely used to synchronization of NNs with reaction effects. These days, adaptive control has been applied to adjust control gains to achieve synchronization of FONNs with reaction effects. Based on the Caputo partial fractional derivative and adaptive control technique, some sufficient conditions for ensuring coupled networks synchronization of fractional-order reaction-diffusion systems were discussed in [41].

Takagi-Sugeno (T-S) fuzzy model [42] is widely recognized as an effective mathematical model, which supports various kinds of analyzes of which synchronization is a promising topic in the fuzzy control community, especially for nonlinear systems. Among various kinds of fuzzy methods, the T-S fuzzy systems are widely accepted as a useful tool for design and analysis of fuzzy control system [43]–[45]. Recently, the T-S fuzzy rules have been connected with the RDNNs and several accomplishments have been achieved. Based on the T-S fuzzy model, a fuzzy controller of state-feedback type was considered for fuzzy memristive-based RDNNs in [46]. In [47], the fuzzy adaptive stabilization problem was discussed for T-S fuzzy memristive RDNNs by employing the event-triggered sampled-data control. Authors in [48] analyzed the synchronization of RDNNs subject to partial couplings and T-S fuzzy nodes under pinning control. In [49], another fuzzy sampled-data controller was adopted to deal with the synchronization of T-S fuzzy RDNNs.

Above all, although the corresponding methods and techniques for studying real-valued NNs (RVNNs) or complex-valued NNs (CVNNs) cannot be directly used to investigate QVNNs, QVNNs can be converted into four real-valued systems by applying Hamilton rules to quaternion multiplication [50], [51]. Considering the simple representation of quaternion, which is easy to understand the geometrical meanings, QVNNs can be applied to various fields of science and engineering. Up to now, direct quaternion approach [52], plural decomposition approach [53], real decomposition approach [54], and have been proposed to investigate the dynamical analysis and synchronization for integer-order QVNNs. Recently, some researchers attempted to investigate the advantages of quaternions into FONNs. It is also necessary to point out that fractional-order QVNNs have many applications in engineering and science, such as wave propagation, electromagnetic waves, diffusion, and viscoelastic systems. So far, there have been some results on the dynamic properties of fractional-order QVNNs, but there are few results to propose Mittag-Leffler synchronization criteria for fractional-order QVNNs [55]–[57]. Furthermore, the FTMLS problem of fractional-order QVNNs by using linear feedback controllers have been investigated [58]. Since reaction-diffusion of CVNNs and QVNNs have storage capacity advantages in comparison to RVNNs, the synchronization issues of CVNNs and QVNNs with reaction-diffusion terms have received growing research interest in recent years [59]–[62]. However, to the best of our knowledge, these results are under the assumption that the reaction-diffusion QVNNs are of integer-order, and there are no results on the FTMLS of fractional-order systems via adaptive fuzzy feedback controller. Therefore, it is highly important and indeed imperative to study the FTMLS problem of FORDDQVNNs both in theoretical interest and practical applications.

Inspired by the above-mentioned arguments, in this paper aims to design an adaptive fuzzy feedback controller scheme for FTMLS problem of T-S fuzzy FORDDQVNNs. By virtue of the Green formula, Caputo fractional derivative and inequality technique, several algebraic criteria are established to guarantee the FTMLS problem of the proposed model. The main contributions of this paper are listed as below:

(i) First, the quaternion algebra is introduced in fractional-order reaction-diffusion system. To avoid the non-commutativity of quaternion multiplication, the QVNNs are decomposed into four RVNNs by using plural decomposition approach based on Hamilton rules: $i^2 = -1$, $j^2 = -1$, $k^2 = -1$, $ij = k$, $ji = -k$, $jk = i$, $kj = -i$, $ki = j$, and $ik = -j$.

(ii) This paper is one of the first paper that combines the fuzzy IF-THEN rules and the quaternion algebra with fractional-order RDNNs and attempts to achieve the FTMLS of T-S fuzzy FORDDQVNNs and the fuzzy-dependent adjustable matrix inequality technique is more flexible and helpful to reduce the conservatism.

(iii) By the construction of fuzzy feedback controller, Lyapunov functional, and some novel easily verifiable algebraic inequality conditions is established to achieve the FTMLS of T-S fuzzy FORDDQVNNs. It is worth noting that the effect of the reaction-diffusion on the FTMLS is considered in our results. Also the suitable adaptive controller is designed with adaptive law which guarantees the FTMLS of the proposed model.

(iv) Based on the previous papers on QVNNs, such as without reaction-diffusion terms [50]–[58], without fuzzy rules in [62], and without fractional-order [62], the effects of reaction-diffusion on the fractional-order system are additionally proposed in this paper, which means that our considered QVNNs are more general and may better meet practical requirements. Several corollaries are provided to show the advantages of the obtained results. It is noted that our results are comprehensive and include some existing ones [59]–[61] as special cases.

(v) To further illustrate the effectiveness of our theoretical result approach is demonstrated by numerical example and from the simulation results to comparing control scenarios are given.

Notation: Real numbers, complex numbers, and quaternion numbers are referred as $\mathbb{R}, \mathbb{C}$, and $\mathbb{Q}$ respectively. $\mathbb{R}^{n\times n}$, $\mathbb{C}^{n\times n}$, $\mathbb{Q}^{n\times n}$ represents the set of all $n\times n$ real-valued, complex-valued, quaternion-valued matrices, respectively. The Caputo fractional derivative operator ${}^{C}D_0^{\lambda}$ is chosen for fractional-order derivative with order λ. For $\wp = (\wp_1, \wp_2, \ldots, \wp_n)^T \in \mathbb{Q}^n$, let $|\wp| = (|\wp_1|, |\wp_2|, \ldots, |\wp_n|)^T$ be the modulus of $\wp$, and $\|\wp\| = \left(\sum_{\theta=1}^{n} |\wp_\theta|^2\right)^{\frac{1}{2}}$ be the norm of $\wp$.

II. MODEL DESCRIPTION AND PRELIMINARIES

A. QUATERNION ALGEBRA

Quaternions are an associative algebra defined over the real field $\mathbb{R}$. A real quaternion, simply called quaternion, can be written in the form

$$\hbar = \hbar^r + \hbar^i i + \hbar^j j + \hbar^k k \in \mathbb{Q}$$

with real coefficients $\hbar^r$, $\hbar^i$, $\hbar^j$ and $\hbar^k$ comprises a real part denoted by $\mathscr{R}(\hbar) = \hbar^r$, and a vector part with three imaginary components, denoted by $\mathscr{I}(\hbar) = \hbar^i i + \hbar^j j + \hbar^k k$.
The imaginary units, i, j, and k obey the following rules:

$$\begin{aligned} i^2 &= j^2 = k^2 = -1, \\ ij &= -ji = k, \\ jk &= -kj = i, \\ ki &= -ik = j, \end{aligned}$$

which implies immediately that the quaternion multiplication is not commutative.
For two quaternions $p = p^r + p^i i + p^j j + p^{(k)} k$ and $\hbar = \hbar^{(r)} + \hbar^{(i)} i + \hbar^{(j)} j + \hbar^{(k)} k$, the addition between them is defined by

$$p + \hbar = (p^r + \hbar^r) + (p^i + \hbar^i)i + (p^j + \hbar^j)j + (p^k + \hbar^k)k.$$

The product between them is defined as

$$\begin{aligned} p\hbar = &(p^r\hbar^r - p^i\hbar^i - p^j\hbar^j - p^k\hbar^k) \\ &+ (p^r\hbar^i + p^i\hbar^r) + p^j\hbar^k - p^k\hbar^j)i \\ &+ (p^r\hbar^j + p^j\hbar^r - p^i\hbar^k + p^k\hbar^i)j \\ &+ (p^r\hbar^k + p^k\hbar^r + p^i\hbar^j - p^j\hbar^i)k. \end{aligned}$$

For a quaternion $\hbar = \hbar^r + \hbar^i i + \hbar^j j + \hbar^k k$, the conjugate of $\hbar$, denoted by $\hbar^*$ or $\bar{\hbar}$, is defined as

$$\hbar^* = \bar{\hbar} = \hbar^r - \hbar^i i - \hbar^j j - \hbar^k k,$$

and the modulus of $\hbar$, denoted by $|\hbar|$, is defined as

$$|\hbar| = \sqrt{\hbar\hbar^*} = \sqrt{(\hbar^r)^2 + (\hbar^i)^2 + (\hbar^j)^2 + (\hbar^k)^2}.$$

Definition 1 [38]: For any $t > 0$, Caputo fractional derivative of order $\lambda (0 < \lambda < 1)$ for a function $\chi(t, z) \in \mathbb{C}^1[[0, b]\times\Omega, \mathbb{R}]$ is defined by

$$\frac{\partial^\lambda \chi(t, z)}{\partial t^\lambda} = \frac{1}{\Gamma(1-\lambda)} \int_0^t \frac{\partial \chi(s, z)}{\partial s} \frac{ds}{(t-s)^\lambda},$$

where $\Gamma(\upsilon) = \int_0^\infty e^{-t} t^{\upsilon-1} dt$. In the case when, $\frac{\partial^\lambda \chi(t,\cdot)}{\partial t^\lambda} = \frac{d^\lambda \chi(t)}{dt^\lambda} = {}^{C}D_0^\lambda \chi(t)$.

Definition 2 [10]: The one-parameter Mittag-Leffler function is defined as

$$\mathbb{E}_\lambda(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(n\lambda + 1)},$$

where $\lambda > 0$, and $x \in \mathbb{C}$.

In this paper, the fractional-order quaternion-valued reaction-diffusion molecular model of neural networks with time delay is considered as the following form:

$$\begin{aligned} \frac{\partial^\lambda \Im_\theta(t, z)}{\partial t^\lambda} = &\sum_{\alpha=1}^{m} \frac{\partial}{\partial z_\alpha}\left(q_{\theta\alpha} \frac{\partial \Im_\theta(t, z)}{\partial z_\alpha}\right) \\ &- a_\theta \Im_\theta(t, z) + \sum_{\varphi=1}^{n} b_{\theta\varphi} f_\varphi(\Im_\varphi(t, z)) \\ &+ \sum_{\varphi=1}^{n} d_{\theta\varphi} g_\varphi(\Im_\varphi(t - \sigma(t), z)) + \mathcal{I}_\theta, \end{aligned}$$

(or) the vector form

$$\frac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \Delta\Im(t,z) - \mathcal{A}\Im(t,z) + \mathcal{B}F(\Im(t,z)) + \mathcal{D}G(\Im(t-\sigma(t),z)) + \mathcal{I}, \quad (1)$$

where $\lambda \in (0,1)$, $\theta = 1,2,\ldots,n$; Ω is a bounded domain with smooth boundary $\partial\Omega$ in $\mathbb{R}^m$, and the space vector $z = (z_1, z_2, \ldots, z_m) \in \Omega$; $\Im(t,z) = \big(\Im_1(t,z), \Im_2(t,z), \ldots, \Im_n(t,z)\big)^T \in \mathbb{Q}^n$; $\Delta\Im(t,z) = \sum_{\alpha=1}^{m} \frac{\partial}{\partial z_\alpha}\Big(\mathcal{Q}\frac{\partial \Im_\theta(t,z)}{\partial z_\alpha}\Big)$; $\mathcal{Q} = diag(q_{1\alpha}, q_{2\alpha}, \ldots, q_{n\alpha}) \in \mathbb{R}^{n\times n}$ with $\mathcal{Q} > 0$ is transmission diffusion operator; $\mathcal{A} = diag(a_1, a_2, \ldots, a_n) \in \mathbb{R}^{n\times n}$ with $a_\theta > 0$; $\mathcal{B} = (b_{\theta\varphi})_{n\times n} \in \mathbb{Q}^{n\times n}$ and $\mathcal{D} = (d_{\theta\varphi})_{n\times n} \in \mathbb{Q}^{n\times n}$ are stands for the interconnection weight matrix; $F(\Im(t,z)) = \big(f_1(\Im_1(t,z)), f_2(\Im_2(t,z)), \ldots, f_n(\Im_n(t,z))\big)^T \in \mathbb{Q}^n$ and $G(\Im(t-\sigma(t),z)) = \big(g_1(\Im_1(t-\sigma(t),z)), g_2(\Im_2(t-\sigma(t),z)), \ldots, g_n(\Im_n(t-\sigma(t),z))\big)^T \in \mathbb{Q}^n$ define without and with time delay, the activation function respectively; $\mathcal{I} = (\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \in \mathbb{Q}^n$ is the external input; $\Im_\theta(t,z) \in \mathbb{Q}$ is the quaternion-valued state variable for the θth unit at time t and in space z and obesisously, $\Im_\theta = \Im_\theta^r + \Im_\theta^i i + \Im_\theta^j j + \Im_\theta^k k$; $b_{\theta\varphi} = b_{\theta\varphi}^r + b_{\theta\varphi}^i i + b_{\theta\varphi}^j j + b_{\theta\varphi}^k k$; $d_{\theta\varphi} = d_{\theta\varphi}^r + d_{\theta\varphi}^i i + d_{\theta\varphi}^j j + d_{\theta\varphi}^k k$; $f_\varphi(\Im_\varphi(t,z)) = f_\varphi^r(\Im_\varphi^r(t,z)) + f_\varphi^i(\Im_\varphi^i(t,z))i + f_\varphi^j(\Im_\varphi^j(t,z))j + f_\varphi^k(\Im_\varphi^k(t,z))k$; $g_\varphi(\Im_\varphi(t-\sigma(t),z)) = g_\varphi^r(\Im_\varphi^r(t-\sigma(t),z)) + g_\varphi^i(\Im_\varphi^i(t-\sigma(t),z))i + g_\varphi^j(\Im_\varphi^j(t-\sigma(t),z))j + g_\varphi^k(\Im_\varphi^k(t-\sigma(t),z))k$; $\mathcal{I}_\theta = \mathcal{I}_\theta^r + \mathcal{I}_\theta^i i + \mathcal{I}_\theta^j j + \mathcal{I}_\theta^k k$. The initial and boundary values of (1) are set as

$$\begin{cases} \Im(t,z) = 0, & (t,z) \in [-\sigma, +\infty)\times\partial\Omega, \\ \Im(s,z) = \psi^\iota(s,z), & (s,z) \in [-\sigma, 0]\times\Omega, \end{cases}$$

where $\psi^\iota(s,z)$ is bounded and continuous on $[-\sigma,0]\times\Omega$, $\psi^\iota(s,z) = (\psi_1^\iota(s,z), \psi_2^\iota(s,z)\ldots, \psi_n^\iota(s,z))^T$; $\psi^\iota(s,z) = \psi^{\iota r}(s,z) + \psi^{\iota i}(s,z)i + \psi^{\iota j}(s,z)j + \psi^{\iota k}(s,z)k$.

Viewing system (1) as the drive system, we introduce the response system as

$$\frac{\partial^\lambda \mathfrak{Z}_\theta(t,z)}{\partial t^\lambda} = \sum_{\alpha=1}^{m} \frac{\partial}{\partial z_\alpha}\Big(q_{\theta\alpha}\frac{\partial \mathfrak{Z}_\theta(t,z)}{\partial z_\alpha}\Big) - a_\theta \mathfrak{Z}_\theta(t,z) + \sum_{\varphi=1}^{n} b_{\theta\varphi} f_\varphi(\mathfrak{Z}_\varphi(t,z)) + \sum_{\varphi=1}^{n} d_{\theta\varphi} g_\varphi(\mathfrak{Z}_\varphi(t-\sigma(t),z)) + \mathcal{I}_\theta + u_\theta(t,z),$$

(or) the vector form

$$\frac{\partial^\lambda \mathfrak{Z}(t,z)}{\partial t^\lambda} = \Delta\mathfrak{Z}(t,z) - \mathcal{A}\mathfrak{Z}(t,z) + \mathcal{B}F(\mathfrak{Z}(t,z)) + \mathcal{D}G(\mathfrak{Z}(t-\sigma(t),z)) + \mathcal{I} + \hat{u}(t,z), \quad (2)$$

where $\lambda \in (0,1)$, $\Delta\mathfrak{Z}(t,z) = \sum_{\alpha=1}^{m}\Big(q_{\theta\alpha}\frac{\partial \mathfrak{Z}_\theta(t,z)}{\partial z_\alpha}\Big)$; $\mathfrak{Z}(t,z) = \big(\mathfrak{Z}_1(t,z), \ldots, \mathfrak{Z}_n(t,z)\big)^T \in \mathbb{Q}^n$; $\hat{u}(t,z) = (u_1(t,z), \ldots, u_n(t,z))^T \in \mathbb{Q}^n$ is the controller which will be designed.

Moreover,

$$\begin{cases} \mathfrak{Z}(t,z) = 0, & (t,z) \in [-\sigma, +\infty)\times\partial\Omega, \\ \mathfrak{Z}(s,z) = \psi^\varsigma(s,z), & (s,z) \in [-\sigma, 0]\times\Omega, \end{cases}$$

where $\psi^\varsigma(s,z)$ is bounded and continuous on $[-\sigma,0]\times\Omega$, $\psi^\varsigma(s,z) = (\psi_1^\varsigma(s,z), \psi_2^\varsigma(s,z)\ldots, \psi_n^\varsigma(s,z))^T$; $\psi^\varsigma(s,z) = \psi^{\varsigma r}(s,z) + \psi^{\varsigma i}(s,z)i + \psi^{\varsigma j}(s,z)j + \psi^{\varsigma k}(s,z)k$.

B. FUZZY LOGIC MOLECULAR MODELING

A fuzzy dynamic model has been proposed by Takagi and Sugeno [42] to represent different linear/nonlinear systems of different rules. Based on this, we shall construct T-S fuzzy system to describe molecular model of FOQVRDNNs structure. Similar to [47]–[49], we consider a T-S fuzzy molecular model, in which the ξth rule is formulated in the following form:

Plant rule ξ : If $\beta_1(t)$ is Ξ_1^ξ, $\beta_2(t)$ is Ξ_2^ξ, …, $\beta_r(t)$ is Ξ_r^ξ. Then

$$\begin{cases} \dfrac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \Delta\Im(t,z) - \mathcal{A}_\xi\Im(t,z) + \mathcal{B}_\xi F(\Im(t,z)) \\ \qquad + \mathcal{D}_\xi G(\Im(t-\sigma(t),z)) + \mathcal{I}, \\ \Im(t,z) = 0, \quad (t,z) \in [-\sigma, +\infty)\times\partial\Omega, \\ \Im(s,z) = \psi^\iota(s,z), \quad (s,z) \in [-\sigma, 0]\times\Omega, \end{cases} \quad (3)$$

where $\beta_\ell(t)$ ($\ell = 1,2,\ldots,r$) and Ξ_ℓ^ξ ($\xi = 1,2,\ldots,\zeta$) show the premise variable vectors and fuzzy sets, respectively; ζ is the number of fuzzy If-Then rules; $\mathcal{A}_\xi = diag(a_{\xi 1}, a_{\xi 2}, \ldots, a_{\xi n})$ with $\mathcal{A}_\xi > 0$; $\mathcal{B}_\xi = (b_{\xi\theta\varphi}) \in \mathbb{Q}^{n\times n}$; $\mathcal{D}_\xi = (d_{\xi\theta\varphi}) \in \mathbb{Q}^{n\times n}$.

By employing the weighted average fuzzy blending approach, the overall T-S fuzzy FORDDQVNNs (3) can be described as

$$\begin{cases} \dfrac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta} \Psi_\xi(\beta(t))\big\{\Delta\Im(t,z) - \mathcal{A}_\xi\Im(t,z) \\ + \mathcal{B}_\xi F(\Im(t,z)) + \mathcal{D}_\xi G(\Im(t-\sigma(t),z)) + \mathcal{I}\big\}, \\ \Im(t,z) = 0, \quad (t,z) \in [-\sigma, +\infty)\times\partial\Omega, \\ \Im(s,z) = \psi^\iota(s,z), \quad (s,z) \in [-\sigma, 0]\times\Omega, \end{cases} \quad (4)$$

where $\beta(t) = (\beta_1(t), \beta_2(t), \ldots, \beta_r(t))^T$, $\Psi_\xi(\beta(t)) = \frac{\prod_{\ell=1}^{r} \Xi_\ell^\zeta(\beta_\ell(t))}{\sum_{\xi=1}^{\zeta}\prod_{\ell=1}^{r} \Xi_\ell^\xi(\beta_\ell(t))}$, in which $\Xi_\ell^\zeta(\beta_\ell(t))$ is the grade of membership of $\beta_\ell(t)$ is Ξ_ℓ^ζ. According to the fuzzy theory it follows that $\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t)) = 1$ and $\Psi_\xi(\beta(t)) \geq 0$ for $(\xi = 1,2,\ldots,\zeta)$.

The considered T-S fuzzy response (2) is in the similar form (4),

$$\begin{cases} \dfrac{\partial^\lambda \mathfrak{Z}(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta} \Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}(t,z) - \mathcal{A}_\xi\mathfrak{Z}(t,z) \\ \qquad + \mathcal{B}_\xi F(\mathfrak{Z}(t,z)) + \mathcal{D}_\xi G(\mathfrak{Z}(t-\sigma(t),z)) \\ \qquad + \mathcal{I} + \hat{u}_\xi(t,z)\big\}, \\ \mathfrak{Z}(t,z) = 0, \quad (t,z) \in [-\sigma, +\infty)\times\partial\Omega, \\ \mathfrak{Z}(s,z) = \psi^\kappa(s,z), \quad (s,z) \in [-\sigma, 0]\times\Omega, \end{cases} \quad (5)$$

where $\hat{u}_\xi = (u_{\xi 1}, u_{\xi 2}, \ldots, u_{\xi n})^T$.

By applying the non-commutativity of quaternion multiplication with hamiltonian rules (4) and (5) be rewritten as the following four real-valued equations

$$\begin{aligned}\frac{\partial^\lambda \Im^r(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im^r(t,z)-\mathcal{A}_\xi\Im^r(t,z)+\mathcal{B}_\xi^r F^r(\Im^r(t,z))\\&-\mathcal{B}_\xi^i F^i(\Im^i(t,z))-\mathcal{B}_\xi^j F^j(\Im^j(t,z))-\mathcal{B}_\xi^k F^k(\Im^k(t,z))\\&+\mathcal{D}_\xi^r G^r(\Im^r(t-\sigma(t),z))-\mathcal{D}_\xi^i G^i(\Im^i(t-\sigma(t),z))\\&-\mathcal{D}_\xi^j G^j(\Im^j(t-\sigma(t),z))-\mathcal{D}_\xi^k G^k(\Im^k(t-\sigma(t),z))\\&+\mathcal{I}^r\big\},\end{aligned}\tag{6}$$

$$\begin{aligned}\frac{\partial^\lambda \Im^i(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im^i(t,z)-\mathcal{A}_\xi\Im^i(t,z)+\mathcal{B}_\xi^r F^i(\Im^i(t,z))\\&+\mathcal{B}_\xi^i F^r(\Im^r(t,z))+\mathcal{B}_\xi^j F^k(\Im^k(t,z))-\mathcal{B}_\xi^k F^k(\Im^j(t,z))\\&+\mathcal{D}_\xi^r G^i(\Im^i(t-\sigma(t),z))+\mathcal{D}_\xi^i G^r(\Im^r(t-\sigma(t),z))\\&+\mathcal{D}_\xi^j G^k(\Im^k(t-\sigma(t),z))-\mathcal{D}_\xi^k G^J(\Im^J(t-\sigma(t),z))\\&+\mathcal{I}^i\big\},\end{aligned}\tag{7}$$

$$\begin{aligned}\frac{\partial^\lambda \Im^j(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im^j(t,z)-\mathcal{A}_\xi\Im^j(t,z)+\mathcal{B}_\xi^r F^j(\Im^j(t,z))\\&-\mathcal{B}_\xi^i F^k(\Im^k(t,z))+\mathcal{B}_\xi^j F^r(\Im^r(t,z))+\mathcal{B}_\xi^k F^i(\Im^i(t,z))\\&+\mathcal{D}_\xi^r G^j(\Im^j(t-\sigma(t),z))-\mathcal{D}_\xi^i G^k(\Im^k(t-\sigma(t),z))\\&+\mathcal{D}_\xi^j G^r(\Im^r(t-\sigma(t),z))+\mathcal{D}_\xi^k G^i(\Im^i(t-\sigma(t),z))\\&+\mathcal{I}^j\big\},\end{aligned}\tag{8}$$

$$\begin{aligned}\frac{\partial^\lambda \Im^k(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im^k(t,z)-\mathcal{A}_\xi\Im^k(t,z)+\mathcal{B}_\xi^r F^k(\Im^k(t,z))\\&+\mathcal{B}_\xi^i F^j(\Im^j(t,z))-\mathcal{B}_\xi^j F^i(\Im^i(t,z))+\mathcal{B}_\xi^k F^r(\Im^r(t,z))\\&+\mathcal{D}_\xi^r G^k(\Im^k(t-\sigma(t),z))+\mathcal{D}_\xi^i G^j(\Im^j(t-\sigma(t),z))\\&-\mathcal{D}_\xi^j G^i(\Im^i(t-\sigma(t),z))+\mathcal{D}_\xi^k G^r(\Im^r(t-\sigma(t),z))\\&+\mathcal{I}^k\big\},\end{aligned}\tag{9}$$

(or) two complex-valued equations of drive system

$$\begin{aligned}\frac{\partial^\lambda \Im^R(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{&\Delta\Im^R(t,z)-\mathcal{A}_\xi\Im^R(t,z)\\&+\mathcal{B}_\xi^R F^R(\Im^R(t,z))-\mathcal{B}_\xi^I F^I(\Im^I(t,z))\\&+\mathcal{D}_\xi^R G^R(\Im^R(t-\sigma(t),z))\\&-\mathcal{D}_\xi^I G^I(\Im^I(t-\sigma(t),z))+\mathcal{I}^R\big\},\end{aligned}\tag{10}$$

$$\begin{aligned}\frac{\partial^\lambda \Im^I(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{&\Delta\Im^I(t,z)-\mathcal{A}_\xi\Im^I(t,z)\\&+\mathcal{B}_\xi^R F^I(\Im^I(t,z))+\mathcal{B}_\xi^I F^R(\Im^R(t,z))\\&+\mathcal{D}_\xi^R G^I(\Im^I(t-\sigma(t),z))\\&+\mathcal{D}_\xi^I G^R(\Im^R(t-\sigma(t),z))+\mathcal{I}^I\big\},\end{aligned}\tag{11}$$

and

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{Z}^r(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^r(t,z)-\mathcal{A}_\xi\mathfrak{Z}^r(t,z)\\&+\mathcal{B}_\xi^r F^r(\mathfrak{Z}^r(t,z))-\mathcal{B}_\xi^i F^i(\mathfrak{Z}^i(t,z))-\mathcal{B}_\xi^j F^j(\mathfrak{Z}^j(t,z))\\&-\mathcal{B}_\xi^k F^k(\mathfrak{Z}^k(t,z))+\mathcal{D}_\xi^r G^r(\mathfrak{Z}^r(t-\sigma(t),z))\\&-\mathcal{D}_\xi^i G^i(\mathfrak{Z}^i(t-\sigma(t),z))-\mathcal{D}_\xi^j G^j(\mathfrak{Z}^j(t-\sigma(t),z))\\&-\mathcal{D}_\xi^k G^k(\mathfrak{Z}^k(t-\sigma(t),z))+\mathcal{I}^r+\hat{u}_\xi^r(t,z)\big\},\end{aligned}\tag{12}$$

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{Z}^i(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^i(t,z)-\mathcal{A}_\xi\mathfrak{Z}^i(t,z)\\&+\mathcal{B}_\xi^r F^i(\mathfrak{Z}^i(t,z))+\mathcal{B}_\xi^i F^r(\mathfrak{Z}^r(t,z))+\mathcal{B}_\xi^j F^k(\mathfrak{Z}^k(t,z))\\&-\mathcal{B}_\xi^k F^k(\mathfrak{Z}^j(t,z))+\mathcal{D}_\xi^r G^i(\mathfrak{Z}^i(t-\sigma(t),z))\\&+\mathcal{D}_\xi^i G^r(\mathfrak{Z}^r(t-\sigma(t),z))+\mathcal{D}_\xi^j G^k(\mathfrak{Z}^k(t-\sigma(t),z))\\&-\mathcal{D}_\xi^k G^J(\mathfrak{Z}^J(t-\sigma(t),z))+\mathcal{I}^i+\hat{u}_\xi^i(t,z)\big\},\end{aligned}\tag{13}$$

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{Z}^j(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^j(t,z)-\mathcal{A}_\xi\mathfrak{Z}^j(t,z)\\&+\mathcal{B}_\xi^r F^j(\mathfrak{Z}^j(t,z))-\mathcal{B}_\xi^i F^k(\mathfrak{Z}^k(t,z))+\mathcal{B}_\xi^j F^r(\mathfrak{Z}^r(t,z))\\&+\mathcal{B}_\xi^k F^i(\mathfrak{Z}^i(t,z))+\mathcal{D}_\xi^r G^j(\mathfrak{Z}^j(t-\sigma(t),z))\\&-\mathcal{D}_\xi^i G^k(\mathfrak{Z}^k(t-\sigma(t),z))+\mathcal{D}_\xi^j G^r(\mathfrak{Z}^r(t-\sigma(t),z))\\&+\mathcal{D}_\xi^k G^i(\mathfrak{Z}^i(t-\sigma(t),z))+\mathcal{I}^j+\hat{u}_\xi^j(t,z)\big\},\end{aligned}\tag{14}$$

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{Z}^k(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^k(t,z)-\mathcal{A}_\xi\mathfrak{Z}^k(t,z)\\&+\mathcal{B}_\xi^r F^k(\mathfrak{Z}^k(t,z))+\mathcal{B}_\xi^i F^j(\mathfrak{Z}^j(t,z))-\mathcal{B}_\xi^j F^i(\mathfrak{Z}^i(t,z))\\&+\mathcal{B}_\xi^k F^r(\mathfrak{Z}^r(t,z))+\mathcal{D}_\xi^r G^k(\mathfrak{Z}^k(t-\sigma(t),z))\\&+\mathcal{D}_\xi^i G^j(\mathfrak{Z}^j(t-\sigma(t),z))-\mathcal{D}_\xi^j G^i(\mathfrak{Z}^i(t-\sigma(t),z))\\&+\mathcal{D}_\xi^k G^r(\mathfrak{Z}^r(t-\sigma(t),z))+\mathcal{I}^k+\hat{u}_\xi^k(t,z)\big\},\end{aligned}\tag{15}$$

(or) two complex-valued equations of response system

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{Z}^R(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^R(t,z)-\mathcal{A}_\xi\mathfrak{Z}^R(t,z)\\ &\quad+\mathcal{B}_\xi^R F^R(\mathfrak{Z}^R(t,z))-\mathcal{B}_\xi^I F^I(\mathfrak{Z}^I(t,z))\\ &\quad+\mathcal{D}_\xi^R G^R(\mathfrak{Z}^R(t-\sigma(t),z))\\ &\quad-\mathcal{D}_\xi^I G^I(\mathfrak{Z}^I(t-\sigma(t),z))+\mathcal{I}^R+\hat{u}_\xi^R(t,z)\big\}, \quad (16)\\ \frac{\partial^\lambda \mathfrak{Z}^I(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}^I(t,z)-\mathcal{A}_\xi\mathfrak{Z}^I(t,z)\\ &\quad+\mathcal{B}_\xi^R F^I(\mathfrak{Z}^I(t,z))+\mathcal{B}_\xi^I F^R(\mathfrak{Z}^R(t,z))\\ &\quad+\mathcal{D}_\xi^R G^I(\mathfrak{Z}^I(t-\sigma(t),z))\\ &\quad+\mathcal{D}_\xi^I G^R(\mathfrak{Z}^R(t-\sigma(t),z))+\mathcal{I}^I+\hat{u}_\xi^I(t,z)\big\}. \quad (17)\end{aligned}$$

Associated with system (6)-(9) and (12)-(15), initial and boundary value conditions are as follows

$$\begin{cases}\Im^\eta(t,z)=0, & (t,z)\in[-\sigma,+\infty)\times\partial\Omega,\\ \Im^\eta(s,z)=\psi^{\iota\eta}(s,z), & (s,z)\in[-\sigma,0]\times\Omega,\\ \mathfrak{Z}^\eta(t,z)=0, & (t,z)\in[-\sigma,+\infty)\times\partial\Omega,\\ \mathfrak{Z}^\eta(s,z)=\psi^{\varsigma\eta}(s,z), & (s,z)\in[-\sigma,0]\times\Omega.\end{cases} \quad (18)$$

Assumption 1: Throughout, this paper, we assume that $f_\theta^\eta(\cdot)$ and $g_\theta^\eta(\cdot)$ $(\theta = 1,2,\ldots,n, \eta = r,i,j,k)$ are respectively of function $f_\theta(\cdot)$ and $g_\theta(\cdot)$ satisfy the following inequalities for any $\vartheta_1,\vartheta_2\in\mathbb{R}$,

$$\begin{aligned}|f_\theta^\eta(\vartheta_1)-f_\theta^\eta(\vartheta_2)| &\le \mathcal{F}_\theta^\eta|\vartheta_1-\vartheta_2|,\\ |g_\theta^\eta(\vartheta_1)-g_\theta^\eta(\vartheta_2)| &\le \mathcal{G}_\theta^\eta|\vartheta_1-\vartheta_2|,\end{aligned}$$

where $\mathcal{F}_\theta^\eta, \mathcal{G}_\theta^\eta$ $(\eta = r,i,j,k)$ are positive constants.

Assumption 2: For any $\theta = 1,2,\ldots,n$, $\alpha = 1,2,\ldots,m$, the constants $q_{\theta\alpha}$ are such that $q_{\theta\alpha} > \hat{q}_{\theta\alpha} \ge 0$.

Now define the error $\wp(t,z) = \mathfrak{Z}(t,z)-\Im(t,z) \triangleq \wp^r(t,z)+i\wp^i(t,z)+j\wp^j(t,z)+k\wp^k(t,z)$, namely $\wp^r(t,z) = \mathfrak{Z}^r(t,z)-\Im^r(t,z)$, $\wp^i(t,z) = \mathfrak{Z}^i(t,z)-\Im^i(t,z)$, $\wp^j(t,z) = \mathfrak{Z}^j(t,z)-\Im^j(t,z)$, $\wp^k(t,z) = \mathfrak{Z}^k(t,z)-\Im^k(t,z)$. Simplicity we denote $t_\sigma = t-\sigma(t)$. Then the error dynamics system between (6)-(9) and (12)-(15), can be obtained with four parts as:

$$\begin{aligned}\frac{\partial^\lambda \wp^r(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\wp^r(t,z)-\mathcal{A}_\xi\wp^r(t,z)\\ &\quad+\mathcal{B}_\xi^r\big[F^r(\mathfrak{Z}^r(t,z))-F^r(\Im^r(t,z))\big]\\ &\quad-\mathcal{B}_\xi^i\big[F^i(\mathfrak{Z}^i(t,z))-F^i(\Im^i(t,z))\big]\\ &\quad-\mathcal{B}_\xi^j\big[F^j(\mathfrak{Z}^j(t,z))-F^j(\Im^j(t,z))\big]\\ &\quad-\mathcal{B}_\xi^k\big[F^k(\mathfrak{Z}^k(t,z))-F^k(\Im^k(t,z))\big]\\ &\quad+\mathcal{D}_\xi^r\big[G^r(\mathfrak{Z}^r(t_\sigma,z))-G^r(\Im^r(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^i\big[G^i(\mathfrak{Z}^i(t_\sigma,z))-G^i(\Im^i(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^j\big[G^j(\mathfrak{Z}^j(t_\sigma,z))-G^j(\Im^j(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^k\big[G^k(\mathfrak{Z}^k(t_\sigma,z))-G^k(\Im^k(t_\sigma,z))\big]\\ &\quad+\hat{u}_\xi^r(t,z)\big\}, \quad (19)\end{aligned}$$

$$\begin{aligned}\frac{\partial^\lambda \wp^i(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\wp^i(t,z)-\mathcal{A}_\xi\wp^i(t,z)\\ &\quad+\mathcal{B}_\xi^r\big[F^i(\mathfrak{Z}^i(t,z))-F^i(\Im^i(t,z))\big]\\ &\quad+\mathcal{B}_\xi^i\big[F^r(\mathfrak{Z}^r(t,z))-F^r(\Im^r(t,z))\big]\\ &\quad+\mathcal{B}_\xi^j\big[F^k(\mathfrak{Z}^k(t,z))-F^k(\Im^k(t,z))\big]\\ &\quad-\mathcal{B}_\xi^k\big[F^j(\mathfrak{Z}^j(t,z))-F^j(\Im^j(t,z))\big]\\ &\quad+\mathcal{D}_\xi^r\big[G^i(\mathfrak{Z}^i(t_\sigma,z))-G^i(\Im^i(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^i\big[G^r(\mathfrak{Z}^r(t_\sigma,z))-G^r(\Im^r(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^j\big[G^k(\mathfrak{Z}^k(t_\sigma,z))-G^k(\Im^k(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^k\big[G^j(\mathfrak{Z}^j(t_\sigma,z))-G^j(\Im^j(t_\sigma,z))\big]\\ &\quad+\hat{u}_\xi^i(t,z)\big\}, \quad (20)\end{aligned}$$

$$\begin{aligned}\frac{\partial^\lambda \wp^j(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\wp^j(t,z)-\mathcal{A}_\xi\wp^j(t,z)\\ &\quad+\mathcal{B}_\xi^r\big[F^j(\mathfrak{Z}^j(t,z))-F^j(\Im^j(t,z))\big]\\ &\quad-\mathcal{B}_\xi^i\big[F^k(\mathfrak{Z}^k(t,z))-F^k(\Im^k(t,z))\big]\\ &\quad+\mathcal{B}_\xi^j\big[F^r(\mathfrak{Z}^r(t,z))-F^r(\Im^r(t,z))\big]\\ &\quad+\mathcal{B}_\xi^k\big[F^i(\mathfrak{Z}^i(t,z))-F^i(\Im^i(t,z))\big]\\ &\quad+\mathcal{D}_\xi^r\big[G^j(\mathfrak{Z}^j(t_\sigma,z))-G^j(\Im^j(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^i\big[G^k(\mathfrak{Z}^k(t_\sigma,z))-G^k(\Im^k(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^j\big[G^r(\mathfrak{Z}^r(t_\sigma,z))-G^r(\Im^r(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^k\big[G^i(\mathfrak{Z}^i(t_\sigma,z))-G^i(\Im^i(t_\sigma,z))\big]\\ &\quad+\hat{u}_\xi^j(t,z)\big\}, \quad (21)\end{aligned}$$

$$\begin{aligned}\frac{\partial^\lambda \wp^k(t,z)}{\partial t^\lambda} &= \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\wp^k(t,z)-\mathcal{A}_\xi\wp^k(t,z)\\ &\quad+\mathcal{B}_\xi^r\big[F^k(\mathfrak{Z}^k(t,z))-F^k(\Im^k(t,z))\big]\\ &\quad+\mathcal{B}_\xi^i\big[F^j(\mathfrak{Z}^j(t,z))-F^j(\Im^j(t,z))\big]\\ &\quad-\mathcal{B}_\xi^j\big[F^i(\mathfrak{Z}^i(t,z))-F^i(\Im^i(t,z))\big]\\ &\quad+\mathcal{B}_\xi^k\big[F^r(\mathfrak{Z}^r(t,z))-F^r(\Im^r(t,z))\big]\\ &\quad+\mathcal{D}_\xi^r\big[G^k(\mathfrak{Z}^k(t_\sigma,z))-G^k(\Im^k(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^i\big[G^j(\mathfrak{Z}^j(t_\sigma,z))-G^j(\Im^j(t_\sigma,z))\big]\\ &\quad-\mathcal{D}_\xi^j\big[G^i(\mathfrak{Z}^i(t_\sigma,z))-G^i(\Im^i(t_\sigma,z))\big]\\ &\quad+\mathcal{D}_\xi^k\big[G^r(\mathfrak{Z}^r(t_\sigma,z))-G^r(\Im^r(t_\sigma,z))\big]\\ &\quad+\hat{u}_\xi^k(t,z)\big\}. \quad (22)\end{aligned}$$

The initial and boundary values of (19)-(22) are set as

$$\begin{cases}\wp^\eta(t,z)=0, & (t,z)\in[-\sigma,+\infty)\times\partial\Omega,\\ \wp^\eta(s,z)=\hat{\psi}^\eta(s,z), & (s,z)\in[-\sigma,0]\times\Omega,\end{cases}$$

where $\hat{\psi}^\eta(s,z) = \psi^{\varsigma\eta}(s,z)-\psi^{\iota\eta}(s,z)$ $(\eta = r,i,j,k)$.
We define, $\wp(t,z) = \big((\wp^r(t,z))^T, (\wp^i(t,z))^T, (\wp^j(t,z))^T, (\wp^k(t,z))^T\big)^T$, $\Delta\wp(t,z) = \big((\Delta\wp^r(t,z))^T, (\Delta\wp^i(t,z))^T, (\Delta\wp^j(t,z))^T, (\Delta\wp^k(t,z))^T\big)^T$, $\widehat{U}_\xi(t,z) = \big((\hat{u}_\xi^r(t,z))^T,$

$(\hat{u}^i_\xi(t,z))^T, (\hat{u}^j_\xi(t,z))^T, (\hat{u}^k_\xi(t,z))^T\big)^T$, $\widehat{F}(\hat{\wp}(t,z)) = \big((F^r(\mathfrak{Z}^r(t,z)) - F^r(\Im^r(t,z)))^T, (F^i(\mathfrak{Z}^i(t,z)) - F^i(\Im^i(t,z)))^T, (F^j(\mathfrak{Z}^j(t,z)) - F^j(\Im^j(t,z)))^T, (F^k(\mathfrak{Z}^k(t,z)) - F^k(\Im^k(t,z)))^T\big)^T$, $\widehat{G}(\hat{\wp}(t_\sigma,z)) = \big((G^r(\mathfrak{Z}^r(t_\sigma,z)) - G^r(\Im^r(t_\sigma,z)))^T, (G^i(\mathfrak{Z}^i(t_\sigma,z)) - G^i(\Im^i(t_\sigma,z)))^T, (G^j(\mathfrak{Z}^j(t_\sigma,z)) - G^j(\Im^j(t_\sigma,z)))^T, (G^k(\mathfrak{Z}^k(t_\sigma,z)) - G^k(\Im^k(t_\sigma,z)))^T\big)^T$,

$$\widehat{\mathcal{A}}_\xi = diag(\mathcal{A}_\xi, \mathcal{A}_\xi, \mathcal{A}_\xi, \mathcal{A}_\xi),$$

$$\widehat{\mathcal{B}}_\xi = \begin{bmatrix} \mathcal{B}^r_\xi & -\mathcal{B}^i_\xi & -\mathcal{B}^j_\xi & -\mathcal{B}^k_\xi \\ \mathcal{B}^i_\xi & \mathcal{B}^r_\xi & -\mathcal{B}^k_\xi & \mathcal{B}^j_\xi \\ \mathcal{B}^j_\xi & \mathcal{B}^k_\xi & \mathcal{B}^r_\xi & -\mathcal{B}^i_\xi \\ \mathcal{B}^k_\xi & -\mathcal{B}^j_\xi & \mathcal{B}^i_\xi & \mathcal{B}^r_\xi \end{bmatrix},$$

$$\widehat{\mathcal{C}}_\xi = \begin{bmatrix} \mathcal{C}^r_\xi & -\mathcal{C}^i_\xi & -\mathcal{C}^j_\xi & -\mathcal{C}^k_\xi \\ \mathcal{C}^i_\xi & \mathcal{C}^r_\xi & -\mathcal{C}^k_\xi & \mathcal{C}^j_\xi \\ \mathcal{C}^j_\xi & \mathcal{C}^k_\xi & \mathcal{C}^r_\xi & -\mathcal{C}^i_\xi \\ \mathcal{C}^k_\xi & -\mathcal{C}^j_\xi & \mathcal{C}^i_\xi & \mathcal{C}^r_\xi \end{bmatrix}.$$

Then the system (19)-(22) can be expressed as

$$\frac{\partial^\lambda \wp(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta} \Psi_\xi(\beta(t))\big\{\Delta\wp(t,z) - \widehat{\mathcal{A}}_\xi \wp(t,z) + \widehat{\mathcal{B}}_\xi \widehat{F}(\hat{\wp}(t,z)) + \widehat{\mathcal{C}}_\xi \widehat{G}(\hat{\wp}(t_\sigma,z)) + \widehat{U}_\xi(t,z)\big\}. \quad (23)$$

The initial and boundary values related to system (23) are of the form

$$\begin{cases} \wp(t,z) = 0, & (t,z) \in [-\sigma,+\infty)\times\partial\Omega, \\ \wp(s,z) = \hat{\psi}(s,z), & (s,z) \in [-\sigma,0]\times\Omega, \end{cases} \quad (24)$$

where $\hat{\psi}(s,z) = \big((\hat{\psi}^r(s,z))^T, (\hat{\psi}^i(s,z))^T, (\hat{\psi}^j(s,z))^T, (\hat{\psi}^k(s,z))^T\big)^T$.

Definition 3 [60]: The drive system (4) is said to be finite-time Mittag-Leffler synchronized with response system (5) for proposed controllers. That is, the state of error system (23) is said to be Mittag-Leffler stable in finite-time under initial condition (24), if there exist positive constants $\{\delta^\eta, \epsilon^\eta, \Theta, \omega, T\}$ $(\eta = r,i,j,k)$, $\delta = \sum_{\eta=r,i}^{j,k}\{\delta^\eta\}$, $\epsilon = \sum_{\eta=r,i}^{j,k}\{\epsilon^\eta\}$, $\epsilon > \delta$, $\|\hat{\psi}^\eta(s,z)\| \le \delta^\eta$, *and* $\|\hat{\psi}(s,z)\| \le \delta$, *such that* $\|\wp^\eta(t,z)\| \le \epsilon^\eta$ *and* $\|\wp(t,z)\| \le \|\hat{\psi}(s,z)\|\{\mathbb{E}_\lambda(-\Theta t^\lambda)\}^\omega < \epsilon$, *hold* ($t \ge 0, t \in F$ *and* F *is the interval* $[0,T)$).

Lemma 1 [36]: For a continuously differentiable with respect to its first argument function $\rho : [0,\nu]\times\Omega \to \mathbb{R}$, $\nu > 0$, *we have*

$$\frac{1}{2}\frac{\partial^\lambda \rho^2(t,z)}{\partial t^\lambda} \le \rho(t,z)\frac{\partial^\lambda \rho(t,z)}{\partial t^\lambda}, \quad t \ge 0, z \in \Omega,$$

where $0 < \lambda < 1$.

Lemma 2 [36]: Let Ω *be cube* $|z_k| < l_k$ $(k = 1,2,\ldots,m)$ *and* $\vartheta(z) = \vartheta(z_1,z_2,\ldots,z_m)$ *be a real-valued function which defines on* $\vartheta(z) \in \mathbb{C}^1(\Omega)$ *and it vanishes on the boundary* $\partial\Omega$ *of* Ω, *i.e.,* $\vartheta(z)|_{z\in\partial\Omega} = 0$. *Then*

$$\int_\Omega \vartheta^2(z)dz \le l_k^2 \int_\Omega \Big(\frac{\partial\vartheta(z)}{\partial z_k}\Big)^2 dz.$$

Lemma 3 [38]: Assume that the function $\mathbb{V} \in \mathbb{V}_0$ *is such that for* $t > 0$ *and the inequality* ${}^{\mathcal{C}}D_0^\lambda \mathbb{V}(t,\hat{\rho}(t,z)) \le -\hat{\pi}\mathbb{V}(t,\hat{\rho}(t,z))$, *where* $\lambda \in (0,1)$ *and* $\hat{\pi} > 0$. *Then*

$$\mathbb{V}(t,\hat{\rho}(t,z)) \le \sup_{-\sigma\le s\le 0} \mathbb{V}(0,\hat{\rho}(0,z))\mathbb{E}_\lambda(\hat{\pi}t^\lambda), \quad t > 0.$$

III. MAIN RESULTS

In this section, some sufficient conditions are derived by designing suitable controllers respectively to achieve FTMLS of the drive-response system (4) and (5). To achieve the FTMLS of drive-response system, the following controllers are designed

$$\hat{u}^\eta_\xi(t,z) = -\mu^\eta_{\xi\theta}\wp^\eta_\theta(t,z), \quad (25)$$

where $\xi = 1,2,\ldots,\zeta$; $\mu^\eta_{\xi\theta}$ $(\eta = r,i,j,k)$ are fuzzy feedback control gains to be determined later.

Remark 1: The control gains $\mu^\eta_{\xi\theta}$ *are considered to be not same in* $\hat{u}^r_\xi$, $\hat{u}^i_\xi$, $\hat{u}^j_\xi$ *and* $\hat{u}^k_\xi$, *so the conservatism can be improved.*

Theorem 1: Under Assumption 1 and 2, the system (4) and (5) achieve the FTMLS via controller (25) if the following conditions holds:

$$(i) \quad \Theta = (\mho - \mathfrak{L}) > 0, \quad (26)$$

$$(ii) \quad \mathbb{E}_\lambda(-\Theta t^\lambda) < \frac{\epsilon^2}{\delta^2}, \quad (27)$$

where,

$$\mho = \min_{1\le\theta\le n}\{\mho^r_\theta, \mho^i_\theta, \mho^j_\theta, \mho^k_\theta\}, \mathfrak{L} = \max_{1\le\theta\le n}\{\mathfrak{L}^r_\theta, \mathfrak{L}^i_\theta, \mathfrak{L}^j_\theta, \mathfrak{L}^k_\theta\},$$

$$\widehat{\mathcal{Q}} = \frac{\hat{q}_{\theta\alpha}}{l^2_\alpha},$$

$$\mho^r_\theta = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\Big[2(a_{\xi\theta} + \widehat{\mathcal{Q}} + \mu^r_{\xi\theta}) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^r_\varphi|b^r_{\xi\theta\varphi}| - \mathcal{F}^i_\varphi|b^i_{\xi\theta\varphi}| - \mathcal{F}^j_\varphi|b^j_{\xi\theta\varphi}| - \mathcal{F}^k_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi(|b^r_{\xi\varphi\theta}| + |b^i_{\xi\varphi\theta}| + |b^j_{\xi\varphi\theta}| + |b^k_{\xi\varphi\theta}|) + \mathcal{G}^r_\varphi|d^r_{\xi\theta\varphi}| - \mathcal{G}^i_\varphi|d^i_{\xi\theta\varphi}| - \mathcal{G}^j_\varphi|d^j_{\xi\theta\varphi}| - \mathcal{G}^k_\varphi|d^k_{\xi\theta\varphi}|\big)\Big]\Big\},$$

$$\mho^i_\theta = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\Big[2(a_{\xi\theta} + \widehat{\mathcal{Q}} + \mu^i_{\xi\theta}) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^i_\varphi|b^r_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi|b^i_{\xi\theta\varphi}| + \mathcal{F}^k_\varphi|b^j_{\xi\theta\varphi}| - \mathcal{F}^j_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^i_\theta(|b^r_{\xi\varphi\theta}| - |b^i_{\xi\varphi\theta}| + |b^k_{\xi\varphi\theta}| - |b^j_{\xi\varphi\theta}|) + \mathcal{G}^i_\varphi|d^r_{\xi\theta\varphi}| + \mathcal{G}^r_\varphi|d^i_{\xi\theta\varphi}| + \mathcal{G}^k_\varphi|d^j_{\xi\theta\varphi}| - \mathcal{G}^j_\varphi|d^k_{\xi\theta\varphi}|\big)\Big]\Big\},$$

$$\mho^j_\theta = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\Big[2(a_{\xi\theta} + \widehat{\mathcal{Q}} + \mu^j_{\xi\theta}) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^j_\varphi|b^r_{\xi\theta\varphi}| - \mathcal{F}^k_\varphi|b^i_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi|b^j_{\xi\theta\varphi}| + \mathcal{F}^i_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^j_\theta(|b^r_{\xi\varphi\theta}| - |b^j_{\xi\varphi\theta}| - |b^k_{\xi\varphi\theta}| + |b^i_{\xi\varphi\theta}|) + \mathcal{G}^j_\varphi|d^r_{\xi\theta\varphi}| - \mathcal{G}^k_\varphi|d^i_{\xi\theta\varphi}| + \mathcal{G}^r_\varphi|d^j_{\xi\theta\varphi}| + \mathcal{G}^i_\varphi|d^k_{\xi\theta\varphi}|\big)\Big]\Big\},$$

$$
\begin{aligned}
\mho_\theta^k &= \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^k)-\sum_{\varphi=}^{n}\big(\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^r| \\
&\quad +\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^i|-\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^k(|b_{\xi\varphi\theta}^r| \\
&\quad -|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^j|-|b_{\xi\varphi\theta}^i|)+\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^r|+\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^i| \\
&\quad -\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^j|+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^k|)\big)\big]\Big\},\\
\mathfrak{L}_\theta^r &= \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n \mathcal{G}_\theta^r\big(|d_{\xi\varphi\theta}^r|+|d_{\xi\varphi\theta}^i|+|d_{\xi\varphi\theta}^j|+|d_{\xi\varphi\theta}^k|\big)\Big\},\\
\mathfrak{L}_\theta^i &= \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n \mathcal{G}_\theta^i\big(|d_{\xi\varphi\theta}^r|-|d_{\xi\varphi\theta}^i|-|d_{\xi\varphi\theta}^j|+|d_{\xi\varphi\theta}^k|\big)\Big\},\\
\mathfrak{L}_\theta^j &= \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n \mathcal{G}_\theta^j\big(|d_{\xi\varphi\theta}^r|+|d_{\xi\varphi\theta}^i|-|d_{\xi\varphi\theta}^j|-|d_{\xi\varphi\theta}^k|\big)\Big\},\\
\mathfrak{L}_\theta^k &= \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n \mathcal{G}_\theta^k\big(|d_{\xi\varphi\theta}^r|-|d_{\xi\varphi\theta}^i|+|d_{\xi\varphi\theta}^j|-|d_{\xi\varphi\theta}^k|\big)\Big\}.
\end{aligned}
$$

Proof: Consider a Lyapunov function as follows

$$
\mathbb{V}(t,\wp(t,z)) = \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^n\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz. \tag{28}
$$

By calculating the Caputo fractional derivative of $\mathbb{V}(t,\wp(t,z))$ with $\lambda\in(0,1)$, along the trajectory of the error system, we can obtain

$$
\begin{aligned}
\frac{d^\lambda\mathbb{V}(t,\wp(t,z))}{dt^\lambda} &= \sum_{\eta=r,i}^{j,k}\Big\{\frac{1}{2}\frac{d^\lambda}{dt^\lambda}\Big(\int_\Omega\sum_{\theta=1}^n(\wp_\theta^\eta)^2(t,z)dz\Big)\Big\}\\
&= \sum_{\eta=r,i}^{j,k}\frac{1}{2}\sum_{\theta=1}^n\Big\{\frac{d^\lambda}{dt^\lambda}\Big(\int_\Omega(\wp_\theta^\eta)^2(t,z)dz\Big)\Big\}.
\end{aligned} \tag{29}
$$

In particular, we have

$$
\begin{aligned}
&\frac{d^\lambda}{dt^\lambda}\Big(\int_\Omega(\wp_\theta^\eta)^2(t,z)dz\Big)\\
&\quad=\frac{1}{\Gamma(1-\lambda)}\int_0^t\Big(\frac{d}{ds}\int_\Omega(\wp_\theta^\eta)^2(t,z)dz\Big)\frac{ds}{(t-s)^\lambda}\\
&\quad=\int_\Omega\frac{1}{\Gamma(1-\lambda)}\Big(\int_0^t\frac{\partial(\wp_\theta^\eta)^2(t,z)}{\partial s}\frac{ds}{(t-s)^\lambda}\Big)dz\\
&\quad=\int_\Omega\frac{\partial^\lambda(\wp_\theta^\eta)^2(t,z)}{\partial t^\lambda}dz.
\end{aligned} \tag{30}
$$

From (29) and (30), we get

$$
\frac{d^\lambda\mathbb{V}(t,\wp(t,z))}{dt^\lambda} = \sum_{\eta=r,i}^{j,k}\Big\{\frac{1}{2}\sum_{\theta=1}^n\int_\Omega\frac{\partial^\lambda(\wp_\theta^\eta)^2(t,z)}{\partial t^\lambda}dz\Big\}.
$$

It follows from Lemma 1 that

$$
\frac{d^\lambda\mathbb{V}(t,\wp(t,z))}{dt^\lambda} \le \sum_{\eta=r,i}^{j,k}\Big\{\sum_{\theta=1}^n\int_\Omega\wp_\theta^\eta(t,z)\frac{\partial^\lambda\wp_\theta^\eta(t,z)}{\partial t^\lambda}dz\Big\}.
$$

That is,

$$
\begin{aligned}
{}^{C}D_0^\lambda\mathbb{V}(t,\wp(t,z)) &\le \sum_{\theta=1}^n\int_\Omega\wp_\theta^r(t,z)\frac{\partial^\lambda\wp_\theta^r(t,z)}{\partial t^\lambda}dz\\
&\quad+\sum_{\theta=1}^n\int_\Omega\wp_\theta^i(t,z)\frac{\partial^\lambda\wp_\theta^i(t,z)}{\partial t^\lambda}dz\\
&\quad+\sum_{\theta=1}^n\int_\Omega\wp_\theta^j(t,z)\frac{\partial^\lambda\wp_\theta^j(t,z)}{\partial t^\lambda}dz\\
&\quad+\sum_{\theta=1}^n\int_\Omega\wp_\theta^k(t,z)\frac{\partial^\lambda\wp_\theta^k(t,z)}{\partial t^\lambda}dz.
\end{aligned}
$$

We denote,

$$
\begin{aligned}
\mathbb{W}_1 &= \sum_{\theta=1}^n\int_\Omega\wp_\theta^r(t,z)\frac{\partial^\lambda\wp_\theta^r(t,z)}{\partial t^\lambda}dz,\\
\mathbb{W}_2 &= \sum_{\theta=1}^n\int_\Omega\wp_\theta^i(t,z)\frac{\partial^\lambda\wp_\theta^i(t,z)}{\partial t^\lambda}dz,\\
\mathbb{W}_3 &= \sum_{\theta=1}^n\int_\Omega\wp_\theta^j(t,z)\frac{\partial^\lambda\wp_\theta^j(t,z)}{\partial t^\lambda}dz,\\
\mathbb{W}_4 &= \sum_{\theta=1}^n\int_\Omega\wp_\theta^k(t,z)\frac{\partial^\lambda\wp_\theta^k(t,z)}{\partial t^\lambda}dz.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\mathbb{W}_1 &\le \sum_{\xi=1}^\zeta\Psi_\xi(\beta(t))\sum_{\theta=1}^n\int_\Omega\wp_\theta^r(t,z)\Big\{\sum_{\alpha=1}^m\frac{\partial}{\partial z_\alpha}\Big(q_{\theta\alpha}\frac{\partial\wp_\theta^r(t,z)}{\partial z_\alpha}\Big)\\
&\quad-a_{\xi\theta}\wp_\theta^r(t,z)+\sum_{\varphi=1}^n b_{\xi\theta\varphi}^r\big[f_\varphi^r(\mathfrak{Z}_\varphi^r(t,z))-f_\varphi^r(\Im_\varphi^r(t,z))\big]\\
&\quad-\sum_{\varphi=1}^n b_{\xi\theta\varphi}^i\big[f_\varphi^i(\mathfrak{Z}_\varphi^i(t,z))-f_\varphi^i(\Im_\varphi^i(t,z))\big]\\
&\quad-\sum_{\varphi=1}^n b_{\xi\theta\varphi}^j\big[f_\varphi^j(\mathfrak{Z}_\varphi^j(t,z))-f_\varphi^j(\Im_\varphi^j(t,z))\big]\\
&\quad-\sum_{\varphi=1}^n b_{\xi\theta\varphi}^k\big[f_\varphi^k(\mathfrak{Z}_\varphi^k(t,z))-f_\varphi^k(\Im_\varphi^k(t,z))\big]\\
&\quad+\sum_{\varphi=1}^n d_{\xi\theta\varphi}^r\big[g_\varphi^r(\mathfrak{Z}_\varphi^r(t_\sigma,z))-g_\varphi^r(\Im_\varphi^r(t_\sigma,z))\big]\\
&\quad-\sum_{\varphi=1}^n d_{\xi\theta\varphi}^i\big[g_\varphi^i(\mathfrak{Z}_\varphi^i(t_\sigma,z))-g_\varphi^i(\Im_\varphi^i(t_\sigma,z))\big]\\
&\quad-\sum_{\varphi=1}^n d_{\xi\theta\varphi}^j\big[g_\varphi^j(\mathfrak{Z}_\varphi^j(t_\sigma,z))-g_\varphi^j(\Im_\varphi^j(t_\sigma,z))\big]\\
&\quad-\sum_{\varphi=1}^n d_{\xi\theta\varphi}^k\big[g_\varphi^k(\mathfrak{Z}_\varphi^k(t_\sigma,z))-g_\varphi^k(\Im_\varphi^k(t_\sigma,z))\big]\\
&\quad-\mu_{\xi\theta}^r\wp_\theta^r(t,z)\Big\}.
\end{aligned} \tag{31}
$$

By the boundary conditions and Green's formula, we have

$$\sum_{\alpha=1}^{m}\int_{\Omega}\wp_{\theta}^{r}(t,z)\frac{\partial}{\partial z_{\alpha}}\Big(q_{\theta\alpha}\frac{\partial\wp_{\theta}^{r}(t,z)}{\partial z_{\alpha}}\Big)dz = -\sum_{\alpha=1}^{m}\int_{\Omega}q_{\theta\alpha}\Big(\frac{\partial\wp_{\theta}^{r}(t,z)}{\partial z_{\alpha}}\Big)^{2}dz.$$

By using Assumption 2 and the light of Lemma 2, we have

$$\begin{aligned}\sum_{\alpha=1}^{m}\int_{\Omega}\wp_{\theta}^{r}(t,z)\frac{\partial}{\partial z_{\alpha}}\Big(q_{\theta\alpha}\frac{\partial\wp_{\theta}^{r}(t,z)}{\partial z_{\alpha}}\Big)dz &\leq -\sum_{\alpha=1}^{m}\int_{\Omega}\hat{q}_{\theta\alpha}\Big(\frac{\partial\wp_{\theta}^{r}(t,z)}{\partial z_{\alpha}}\Big)^{2}dz\\ &\leq -\sum_{\alpha=1}^{m}\int_{\Omega}\frac{\hat{q}_{\theta\alpha}}{l_{\alpha}^{2}}(\wp_{\theta}^{r})^{2}(t,z)dz\\ &\leq -\widehat{\mathcal{Q}}\int_{\Omega}(\wp_{\theta}^{r})^{2}(t,z)dz. \end{aligned}\tag{32}$$

According to Assumption 1 and the inequality $2|x||y| \leq x^2+y^2$, we have

$$\begin{aligned}&\sum_{\varphi=1}^{n}b_{\xi\theta\varphi}^{r}\int_{\Omega}\wp_{\theta}^{r}(t,z)[f_{\varphi}^{r}(\mathfrak{Z}_{\varphi}^{r}(t,z))-f_{\varphi}^{r}(\Im_{\varphi}^{r}(t,z))]dz\\ &\leq\sum_{\varphi=1}^{n}\mathcal{F}_{\varphi}^{r}|b_{\xi\theta\varphi}^{r}|\int_{\Omega}|\wp_{\theta}^{r}(t,z)||\wp_{\varphi}^{r}(t,z)|dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{F}_{\varphi}^{r}|b_{\xi\theta\varphi}^{r}|\int_{\Omega}\big((\wp_{\theta}^{r})^{2}(t,z)+(\wp_{\varphi}^{r})^{2}(t,z)\big)dz,\end{aligned}\tag{33}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}b_{\xi\theta\varphi}^{i}\int_{\Omega}\wp_{\theta}^{i}(t,z)[f_{\varphi}^{i}(\mathfrak{Z}_{\varphi}^{i}(t,z))-f_{\varphi}^{i}(\Im_{\varphi}^{i}(t,z))]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{F}_{\varphi}^{i}|b_{\xi\theta\varphi}^{i}|\int_{\Omega}\big((\wp_{\theta}^{i})^{2}(t,z)+(\wp_{\varphi}^{i})^{2}(t,z)\big)dz,\end{aligned}\tag{34}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}b_{\xi\theta\varphi}^{j}\int_{\Omega}\wp_{\theta}^{j}(t,z)[f_{\varphi}^{j}(\mathfrak{Z}_{\varphi}^{j}(t,z))-f_{\varphi}^{j}(\Im_{\varphi}^{j}(t,z))]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{F}_{\varphi}^{j}|b_{\xi\theta\varphi}^{j}|\int_{\Omega}\big((\wp_{\theta}^{j})^{2}(t,z)+(\wp_{\varphi}^{j})^{2}(t,z)\big)dz,\end{aligned}\tag{35}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}b_{\xi\theta\varphi}^{k}\int_{\Omega}\wp_{\theta}^{k}(t,z)[f_{\varphi}^{k}(\mathfrak{Z}_{\varphi}^{k}(t,z))-f_{\varphi}^{k}(\Im_{\varphi}^{k}(t,z))]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{F}_{\varphi}^{k}|b_{\xi\theta\varphi}^{k}|\int_{\Omega}\big((\wp_{\theta}^{k})^{2}(t,z)+(\wp_{\varphi}^{k})^{2}(t,z)\big)dz,\end{aligned}\tag{36}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}d_{\xi\theta\varphi}^{r}\int_{\Omega}\wp_{\theta}^{r}(t,z)\big[g_{\varphi}^{r}(\mathfrak{Z}_{\varphi}^{r}(t_{\sigma},z))-g_{\varphi}^{r}(\Im_{\varphi}^{r}(t_{\sigma},z))\big]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_{\varphi}^{r}|d_{\xi\theta\varphi}^{r}|\int_{\Omega}\big((\wp_{\theta}^{r})^{2}(t,z)+(\wp_{\varphi}^{r})^{2}(t_{\sigma},z)\big)dz,\end{aligned}\tag{37}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}d_{\xi\theta\varphi}^{i}\int_{\Omega}\wp_{\theta}^{i}(t_{\sigma},z)\big[g_{\varphi}^{i}(\mathfrak{Z}_{\varphi}^{i}(t_{\sigma},z))-g_{\varphi}^{i}(\Im_{\varphi}^{i}(t_{\sigma},z))\big]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_{\varphi}^{i}|d_{\xi\theta\varphi}^{i}|\int_{\Omega}\big((\wp_{\theta}^{i})^{2}(t,z)+(\wp_{\varphi}^{i})^{2}(t_{\sigma},z)\big)dz,\end{aligned}\tag{38}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}d_{\xi\theta\varphi}^{j}\int_{\Omega}\wp_{\theta}^{j}(t_{\sigma},z)\big[g_{\varphi}^{j}(\mathfrak{Z}_{\varphi}^{j}(t_{\sigma},z))-g_{\varphi}^{j}(\Im_{\varphi}^{j}(t_{\sigma},z))\big]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_{\varphi}^{j}|d_{\xi\theta\varphi}^{j}|\int_{\Omega}\big((\wp_{\theta}^{j})^{2}(t,z)+(\wp_{\varphi}^{j})^{2}(t_{\sigma},z)\big)dz,\end{aligned}\tag{39}$$

$$\begin{aligned}&\sum_{\varphi=1}^{n}d_{\xi\theta\varphi}^{k}\int_{\Omega}\wp_{\theta}^{k}(t_{\sigma},z)\big[g_{\varphi}^{k}(\mathfrak{Z}_{\varphi}^{k}(t_{\sigma},z))-g_{\varphi}^{k}(\Im_{\varphi}^{k}(t_{\sigma},z))\big]dz\\ &\leq\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_{\varphi}^{k}|d_{\xi\theta\varphi}^{k}|\int_{\Omega}\big((\wp_{\theta}^{k})^{2}(t,z)+(\wp_{\varphi}^{k})^{2}(t_{\sigma},z)\big)dz.\end{aligned}\tag{40}$$

Substituting (32)-(40) into (31), we obtain that

$$\begin{aligned}\mathbb{W}_{1}\leq\sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\Big\{&-\frac{1}{2}\sum_{\theta=1}^{n}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^{r})\\ &-\sum_{\varphi=1}^{n}\big(\mathcal{F}_{\varphi}^{r}|b_{\xi\theta\varphi}^{r}|-\mathcal{F}_{\varphi}^{i}|b_{\xi\theta\varphi}^{i}|-\mathcal{F}_{\varphi}^{j}|b_{\xi\theta\varphi}^{j}|-\mathcal{F}_{\varphi}^{k}|b_{\xi\theta\varphi}^{k}|\\ &+\mathcal{F}_{\theta}^{r}|b_{\xi\varphi\theta}^{r}|+\mathcal{G}_{\varphi}^{r}|d_{\xi\theta\varphi}^{r}|-\mathcal{G}_{\varphi}^{i}|d_{\xi\theta\varphi}^{i}|-\mathcal{G}_{\varphi}^{j}|d_{\xi\theta\varphi}^{j}|\\ &-\mathcal{G}_{\varphi}^{k}|d_{\xi\theta\varphi}^{k}|\big)\big]\int_{\Omega}(\wp_{\theta}^{r})^{2}(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}_{\theta}^{i}|b_{\xi\varphi\theta}^{i}|\int_{\Omega}(\wp_{\theta}^{i})^{2}(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}_{\theta}^{j}|b_{\xi\varphi\theta}^{j}|\int_{\Omega}(\wp_{\theta}^{j})^{2}(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}_{\theta}^{k}|b_{\xi\varphi\theta}^{k}|\int_{\Omega}(\wp_{\theta}^{k})^{2}(t,z)dz\\ &+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{r}|d_{\xi\varphi\theta}^{r}|\int_{\Omega}(\wp_{\theta}^{r})^{2}(t_{\sigma},z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{i}|d_{\xi\varphi\theta}^{i}|\int_{\Omega}(\wp_{\theta}^{i})^{2}(t_{\sigma},z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{j}|d_{\xi\varphi\theta}^{j}|\int_{\Omega}(\wp_{\theta}^{j})^{2}(t_{\sigma},z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{k}|d_{\xi\varphi\theta}^{k}|\int_{\Omega}(\wp_{\theta}^{k})^{2}(t_{\sigma},z)dz\Big\}.\end{aligned}\tag{41}$$

Similarly,

$$\begin{aligned}\mathbb{W}_{2}\leq\sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\Big\{&\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}_{\theta}^{r}|b_{\xi\varphi\theta}^{i}|\int_{\Omega}(\wp_{\theta}^{r})^{2}(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^{n}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^{i})-\sum_{\varphi=1}^{n}\big(\mathcal{F}_{\varphi}^{i}|b_{\xi\theta\varphi}^{r}|\\ &+\mathcal{F}_{\varphi}^{r}|b_{\xi\theta\varphi}^{i}|+\mathcal{F}_{\varphi}^{k}|b_{\xi\theta\varphi}^{j}|-\mathcal{F}_{\varphi}^{j}|b_{\xi\theta\varphi}^{k}|+\mathcal{F}_{\theta}^{i}|b_{\xi\varphi\theta}^{i}|\\ &+\mathcal{G}_{\varphi}^{i}|d_{\xi\theta\varphi}^{r}|+\mathcal{G}_{\varphi}^{r}|d_{\xi\theta\varphi}^{i}|+\mathcal{G}_{\varphi}^{k}|d_{\xi\theta\varphi}^{j}|\end{aligned}$$

$$
\begin{aligned}
&-\mathcal{G}^j_\varphi|d^k_{\xi\theta\varphi}|)\Big]\int_\Omega(\wp^i_\theta)^2(t,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^j_\theta|b^k_{\xi\varphi\theta}|\int_\Omega(\wp^j_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^k_\theta|b^j_{\xi\varphi\theta}|\int_\Omega(\wp^k_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^r_\theta|d^i_{\xi\varphi\theta}|\int_\Omega(\wp^r_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^i_\theta|d^r_{\xi\varphi\theta}|\int_\Omega(\wp^i_\theta)^2(t_\sigma,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^j_\theta|d^k_{\xi\varphi\theta}|\int_\Omega(\wp^j_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^k_\theta|d^j_{\xi\varphi\theta}|\int_\Omega(\wp^k_\theta)^2(t_\sigma,z)dz\Big\}, \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{W}_3 \leq{}& \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\Big\{\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^r_\theta|b^j_{\xi\varphi\theta}|\int_\Omega(\wp^r_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^i_\theta|b^k_{\xi\varphi\theta}|\int_\Omega(\wp^i_\theta)^2(t,z)dz-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}\\
&+\widehat{\mathcal{Q}}+\mu^j_{\xi\theta})-\sum_{\varphi=1}^{n}\big(\mathcal{F}^j_\varphi|b^r_{\xi\theta\varphi}|-\mathcal{F}^k_\varphi|b^i_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^j_{\xi\theta\varphi}|\\
&+\mathcal{F}^i_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^j_\theta|b^r_{\xi\theta\varphi}|+\mathcal{G}^j_\varphi|d^r_{\xi\theta\varphi}|-\mathcal{G}^k_\varphi|d^i_{\xi\theta\varphi}|\\
&+\mathcal{G}^r_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^i_\varphi|d^k_{\xi\theta\varphi}|)\Big]\int_\Omega(\wp^j_\theta)^2(t,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^k_\theta|b^i_{\xi\varphi\theta}|\int_\Omega(\wp^k_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^r_\theta|d^j_{\xi\varphi\theta}|\int_\Omega(\wp^r_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^i_\theta|d^k_{\xi\varphi\theta}|\int_\Omega(\wp^i_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^j_\theta|d^r_{\xi\varphi\theta}|\int_\Omega(\wp^j_\theta)^2(t_\sigma,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^k_\theta|d^i_{\xi\varphi\theta}|\int_\Omega(\wp^k_\theta)^2(t_\sigma,z)dz\Big\}, \qquad (43)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{W}_4 \leq{}& \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\Big\{\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^r_\theta|b^k_{\xi\varphi\theta}|\int_\Omega(\wp^r_\theta)^2(t,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^i_\theta|b^j_{\xi\varphi\theta}|\int_\Omega(\wp^i_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{F}^j_\theta|b^i_{\xi\varphi\theta}|\int_\Omega(\wp^j_\theta)^2(t,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}+\mathcal{Q}_\theta+\mu^k_{\xi\theta})-\sum_{\varphi=1}^{n}\big(\mathcal{F}^k_\varphi|b^k_{\xi\theta\varphi}|\\
&+\mathcal{F}^j_\varphi|b^i_{\xi\theta\varphi}|-\mathcal{F}^i_\varphi|b^j_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^k_\theta|b^r_{\xi\theta\varphi}|\\
&+\mathcal{G}^k_\theta|d^r_{\xi\theta\varphi}|+\mathcal{G}^j_\theta|d^i_{\xi\theta\varphi}|-\mathcal{G}^i_\theta|d^j_{\xi\theta\varphi}|\\
&+\mathcal{G}^r_\theta|d^k_{\xi\theta\varphi}|\big)\int_\Omega(\wp^k_\theta)^2(t,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^r_\theta|d^k_{\xi\varphi\theta}|\int_\Omega(\wp^r_\theta)^2(t_\sigma,z)dz\\
&-\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^i_\theta|d^j_{\xi\varphi\theta}|\int_\Omega(\wp^i_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^j_\theta|d^i_{\xi\varphi\theta}|\int_\Omega(\wp^j_\theta)^2(t_\sigma,z)dz\\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}^k_\theta|d^r_{\xi\varphi\theta}|\int_\Omega(\wp^k_\theta)^2(t_\sigma,z)dz\Big\}. \qquad (44)
\end{aligned}
$$

Combining from (41)-(44), we have

$$
\begin{aligned}
&{}^{C}D_0^\lambda\mathbb{V}(t,\wp(t,z))\\
&\leq \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\Big\{-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu^r_{\xi\theta})\\
&\quad-\sum_{\varphi=1}^{n}\big(\mathcal{F}^r_\varphi|b^r_{\xi\theta\varphi}|-\mathcal{F}^i_\varphi|b^i_{\xi\theta\varphi}|-\mathcal{F}^j_\varphi|b^j_{\xi\theta\varphi}|-\mathcal{F}^k_\varphi|b^k_{\xi\theta\varphi}|\\
&\quad+\mathcal{F}^r_\varphi(|b^r_{\xi\varphi\theta}|+|b^i_{\xi\varphi\theta}|+|b^j_{\xi\varphi\theta}|+|b^k_{\xi\varphi\theta}|)+\mathcal{G}^r_\varphi|d^r_{\xi\theta\varphi}|\\
&\quad-\mathcal{G}^i_\varphi|d^i_{\xi\theta\varphi}|-\mathcal{G}^j_\varphi|d^j_{\xi\theta\varphi}|-\mathcal{G}^k_\varphi|d^k_{\xi\theta\varphi}|)\Big]\int_\Omega(\wp^r_\theta)^2(t,z)dz\\
&\quad-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu^i_{\xi\theta})-\sum_{\varphi=1}^{n}\big(\mathcal{F}^i_\varphi|b^r_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^i_{\xi\theta\varphi}|\\
&\quad+\mathcal{F}^k_\varphi|b^j_{\xi\theta\varphi}|-\mathcal{F}^j_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^i_\theta(|b^r_{\xi\varphi\theta}|-|b^i_{\xi\varphi\theta}|+|b^k_{\xi\varphi\theta}|\\
&\quad-|b^j_{\xi\varphi\theta}|)+\mathcal{G}^i_\varphi|d^r_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^i_{\xi\theta\varphi}|+\mathcal{G}^k_\varphi|d^j_{\xi\theta\varphi}|-\mathcal{G}^j_\varphi|d^k_{\xi\theta\varphi}|)\Big]\\
&\quad\times\int_\Omega(\wp^i_\theta)^2(t,z)dz-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu^j_{\xi\theta})\\
&\quad-\sum_{\varphi=1}^{n}\big(\mathcal{F}^j_\varphi|b^r_{\xi\theta\varphi}|-\mathcal{F}^k_\varphi|b^i_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^j_{\xi\theta\varphi}|+\mathcal{F}^i_\varphi|b^k_{\xi\theta\varphi}|\\
&\quad+\mathcal{F}^j_\theta(|b^r_{\xi\varphi\theta}|-|b^j_{\xi\varphi\theta}-|b^k_{\xi\varphi\theta}|+|b^i_{\xi\varphi\theta}|)+\mathcal{G}^j_\varphi|d^r_{\xi\theta\varphi}|\\
&\quad-\mathcal{G}^k_\varphi|d^i_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^i_\varphi|d^k_{\xi\theta\varphi}|)\Big]\int_\Omega(\wp^j_\theta)^2(t,z)dz\\
&\quad-\frac{1}{2}\sum_{\theta=1}^{n}\Big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu^k_{\xi\theta})-\sum_{\varphi=1}^{n}\big(\mathcal{F}^k_\varphi|b^r_{\xi\theta\varphi}|+\mathcal{F}^j_\varphi|b^i_{\xi\theta\varphi}|\\
&\quad-\mathcal{F}^i_\varphi|b^j_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^k_\theta(|b^r_{\xi\varphi\theta}|-|b^k_{\xi\varphi\theta}+|b^j_{\xi\varphi\theta}|\\
&\quad-|b^i_{\xi\varphi\theta}|)+\mathcal{G}^k_\varphi|d^r_{\xi\theta\varphi}|+\mathcal{G}^j_\varphi|d^i_{\xi\theta\varphi}|-\mathcal{G}^i_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^k_{\xi\theta\varphi}|)\Big]
\end{aligned}
$$

$$\times\int_{\Omega}(\wp_{\theta}^{k})^{2}(t,z)dz+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{r}\big(|d_{\xi\varphi\theta}^{r}|+|d_{\xi\varphi\theta}^{i}|$$
$$+|d_{\xi\varphi\theta}^{j}|+|d_{\xi\varphi\theta}^{k}|\big)\int_{\Omega}(\wp_{\theta}^{r})^{2}(t_{\sigma},z)dz+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{i}$$
$$\times\big(|d_{\xi\varphi\theta}^{r}|-|d_{\xi\varphi\theta}^{i}|-|d_{\xi\varphi\theta}^{j}|+|d_{\xi\varphi\theta}^{k}|\big)\int_{\Omega}(\wp_{\theta}^{i})^{2}(t_{\sigma},z)dz$$
$$+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{j}\big(|d_{\xi\varphi\theta}^{r}|+|d_{\xi\varphi\theta}^{i}|-|d_{\xi\varphi\theta}^{j}|-|d_{\xi\varphi\theta}^{k}|\big)$$
$$\times\int_{\Omega}(\wp_{\theta}^{j})^{2}(t_{\sigma},z)dz+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_{\theta}^{k}\big(|d_{\xi\varphi\theta}^{r}|-|d_{\xi\varphi\theta}^{i}|$$
$$+|d_{\xi\varphi\theta}^{j}|-|d_{\xi\varphi\theta}^{k}|\big)\int_{\Omega}(\wp_{\theta}^{k})^{2}(t_{\sigma},z)dz\Big\}.$$

Thus,

$$\begin{aligned}
&{}^{\mathcal{C}}D_{0}^{\lambda}\mathbb{V}(t,\wp(t,z))\\
&\le\sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\Big\{-\frac{1}{2}\sum_{\theta=1}^{n}\mho_{\theta}^{r}\int_{\Omega}(\wp_{\theta}^{r})^{2}(t,z)dz\\
&\quad-\frac{1}{2}\sum_{\theta=1}^{n}\mho_{\theta}^{i}\int_{\Omega}(\wp_{\theta}^{i})^{2}(t,z)dz-\frac{1}{2}\sum_{\theta=1}^{n}\mho_{\theta}^{j}\int_{\Omega}(\wp_{\theta}^{j})^{2}(t,z)dz\\
&\quad-\frac{1}{2}\sum_{\theta=1}^{n}\mho_{\theta}^{k}\int_{\Omega}(\wp_{\theta}^{k})^{2}(t,z)dz+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{L}_{\theta}^{r}\int_{\Omega}(\wp_{\theta}^{r})^{2}(t_{\sigma},z)dz\\
&\quad+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{L}_{\theta}^{i}\int_{\Omega}(\wp_{\theta}^{i})^{2}(t_{\sigma},z)dz+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{L}_{\theta}^{j}\int_{\Omega}(\wp_{\theta}^{j})^{2}(t_{\sigma},z)dz\\
&\quad+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{L}_{\theta}^{k}\int_{\Omega}(\wp_{\theta}^{k})^{2}(t_{\sigma},z)dz\Big\}\\
&\le-\mho\sum_{\eta=r,i}^{j,k}\int_{\Omega}\sum_{\theta=1}^{n}\frac{1}{2}(\wp_{\theta}^{\eta})^{2}(t,z)dz\\
&\quad+\mathfrak{L}\sum_{\eta=r,i}^{j,k}\int_{\Omega}\sum_{\theta=1}^{n}\frac{1}{2}(\wp_{\theta}^{\eta})^{2}(t_{\sigma},z)dz\\
&\le-\mho\mathbb{V}(t,\wp(t,z))+\sup_{t_{\sigma}\le s\le t}\mathfrak{L}\mathbb{V}(t_{\sigma},\wp(t_{\sigma},z)).
\end{aligned}\tag{45}$$

As the above inequalities satisfies the Razumikhin condition [11], we have

$$\mathbb{V}(s,\wp(s,z))\le\mathbb{V}(t,\wp(t,z)),\quad t_{\sigma}\le s\le t,\quad t\ge 0.\tag{46}$$

From (45) and (46), we have

$${}^{\mathcal{C}}D_{0}^{\lambda}\mathbb{V}(t,\wp(t,z))\le-\Theta\mathbb{V}(t,\wp(t,z)).\tag{47}$$

Applying Lemma 3 in inequality (47), it follows that

$$\mathbb{V}(t,\wp(t,z))\le\sup_{\sigma\le s\le 0}\mathbb{V}(0,\hat{\psi}(s,\cdot))\mathbb{E}_{\lambda}(-\Theta t^{\lambda}),\quad t>0.$$

So, the equivalent inequality (28) can be derived as follows

$$\sum_{\eta=r,i}^{j,k}\int_{\Omega}\sum_{\theta=1}^{n}(\wp_{\theta}^{\eta})^{2}(t,z)dz\le\sum_{\eta=r,i}^{j,k}\Big(\sup_{\sigma\le s\le 0}\int_{\Omega}\sum_{\theta=1}^{n}(\hat{\psi}_{\theta}^{\eta})^{2}(s,z)dz\Big)\mathbb{E}_{\lambda}(-\Theta t^{\lambda}).$$

Denote $\|\hat{\psi}^{\eta}(s,z)\|=\Big(\sup_{\sigma\le s\le 0}\int_{\Omega}\sum_{\theta=1}^{n}(\hat{\psi}_{\theta}^{\eta})^{2}(s,z)dz\Big)^{\frac{1}{2}}$, then

$$\sum_{\eta=r,i}^{j,k}\int_{\Omega}\sum_{\theta=1}^{n}(\wp_{\theta}^{\eta})^{2}(t,z)dz\le\sum_{\eta=r,i}^{j,k}\|\hat{\psi}^{\eta}(s,z)\|^{2}\mathbb{E}_{\lambda}(-\Theta t^{\lambda}).$$

According to inequality (27) and Definition 3, it follows that

$$\sum_{\eta=r,i}^{j,k}\|\wp^{\eta}(t,z)\|^{2}\le\delta^{2}\frac{\epsilon^{2}}{\delta^{2}}.$$

Therefore,

$$\sum_{\eta=r,i}^{j,k}\|\wp^{\eta}(t,z)\|\le\epsilon.\tag{48}$$

Based on Definition 3, and the inequality (48), we can conclude that the system (4) is said to be FTMLS with the system (5) under controllers (25). □

Remark 2: Compared with the results in [62], the model in this paper has the fuzzy rules, fractional-order case and thus, our models are new and more general. Although Song et al. [62] studied finite-time anti-synchronization of memristive QVNNs with reaction-diffusion, and the controller was state feedback controller not adaptive fuzzy controller. Thus, to efficiently adjust the fuzzy control gains so that save control cost, we introduce adaptive control approach in (49). This is the first time to use an adaptive control method to study the FTMLS of T-S FORDDQVNNs.

The adaptive fuzzy controller is presented as follows:

$$\begin{cases}\hat{u}_{\xi}^{\eta}(t,z)=-\varpi_{\xi\theta}^{\eta}\wp_{\theta}^{\eta}(t,z),\\ {}^{\mathcal{C}}D_{0}^{\lambda}\varpi_{\xi\theta}^{\eta}=\gamma_{\xi\theta}^{\eta}(\wp_{\theta}^{\eta})^{2}(t,z)-\frac{\varrho^{\eta}}{2}(\varpi_{\xi\theta}^{\eta}(t)-\varpi^{\eta})^{2},\end{cases}\tag{49}$$

for $\theta=1,2,\ldots,n$; $\xi=1,2,\ldots,\zeta$, where $\varpi^{\eta}>0$ are tunable constants, $\varpi_{\xi\theta}^{\eta}(t)>0$ are tunable functions, $\varrho^{\eta}>0$ and $\gamma_{\xi\theta}^{\eta}>0$ are constants.

Theorem 2: Under Assumption 1 and 2, the system (4) and (5) achieve the FTMLS via adaptive controller (49) if the following conditions holds:

$$(i)\quad\Phi=\mathfrak{N}-\hat{\varrho}>0,\tag{50}$$

$$(ii)\quad\mathbb{E}_{\lambda}(-\Phi t^{\lambda})<\frac{\epsilon^{2}}{\mathcal{K}\delta^{2}}.\tag{51}$$

where,

$$\mathfrak{N}=\widehat{\mathfrak{K}}-\widehat{\mathfrak{P}},\ \hat{\varrho}=\min\{\varrho^{r},\varrho^{i},\varrho^{j},\varrho^{k}\},\ \widehat{\mathcal{Q}}=\frac{\hat{q}_{\theta\alpha}}{l_{\alpha}^{2}},$$

$$\widehat{\mathfrak{K}} = \min_{1\le\theta\le n}\{\mathfrak{K}_\theta^r, \mathfrak{K}_\theta^i, \mathfrak{K}_\theta^j, \mathfrak{K}_\theta^k\},$$
$$\widehat{\mathfrak{P}} = \max_{1\le\theta\le n}\{\mathfrak{P}_\theta^r, \mathfrak{P}_\theta^i, \mathfrak{P}_\theta^j, \mathfrak{P}_\theta^k\},$$
$$\begin{aligned}\mathfrak{K}_\theta^r = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^r)-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^r|\\ -\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^i|-\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^k|+\mathcal{F}_\varphi^r(|b_{\xi\varphi\theta}^r|\\ +|b_{\xi\varphi\theta}^i|+|b_{\xi\varphi\theta}^j|+|b_{\xi\varphi\theta}^k|)+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^r|-\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^i|\\ -\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^j|-\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^k|)\big]\Big\},\end{aligned}$$
$$\begin{aligned}\mathfrak{K}_\theta^i = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^i)-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^r|\\ +\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^i|+\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^i(|b_{\xi\varphi\theta}^r|\\ -|b_{\xi\varphi\theta}^i|+|b_{\xi\varphi\theta}^k|-|b_{\xi\varphi\theta}^j|)+\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^r|\\ +\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^i|+\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^j|-\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^k|)\big]\Big\},\end{aligned}$$
$$\begin{aligned}\mathfrak{K}_\theta^j = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^j)-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^r|\\ -\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^i|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^j(|b_{\xi\varphi\theta}^r|\\ -|b_{\xi\varphi\theta}^j|-|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^i|)+\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^r|-\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^i|\\ +\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^j|+\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^k|)\big]\Big\},\end{aligned}$$
$$\begin{aligned}\mathfrak{K}_\theta^k = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^k)-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^r|\\ +\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^i|-\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^k(|b_{\xi\varphi\theta}^r|\\ -|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^j|-|b_{\xi\varphi\theta}^i|)+\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^r|+\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^i|\\ -\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^j|+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^k|)\big]\Big\},\end{aligned}$$
$$\mathfrak{P}_\theta^r = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n\mathcal{G}_\theta^r\big(|d_{\xi\varphi\theta}^r|+|d_{\xi\varphi\theta}^i|+|d_{\xi\varphi\theta}^j|+|d_{\xi\varphi\theta}^k|\big)\Big\},$$
$$\mathfrak{P}_\theta^i = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n\mathcal{G}_\theta^i\big(|d_{\xi\varphi\theta}^r|-|d_{\xi\varphi\theta}^i|-|d_{\xi\varphi\theta}^j|+|d_{\xi\varphi\theta}^k|\big)\Big\},$$
$$\mathfrak{P}_\theta^j = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n\mathcal{G}_\theta^j\big(|d_{\xi\varphi\theta}^r|+|d_{\xi\varphi\theta}^i|-|d_{\xi\varphi\theta}^j|-|d_{\xi\varphi\theta}^k|\big)\Big\},$$
$$\mathfrak{P}_\theta^k = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^n\mathcal{G}_\theta^k\big(|d_{\xi\varphi\theta}^r|-|d_{\xi\varphi\theta}^i|+|d_{\xi\varphi\theta}^j|-|d_{\xi\varphi\theta}^k|\big)\Big\}.$$

Proof: Construct a Lyapunov function of the following form:

$$\mathbb{V}(t,\wp(t,z)) = \widehat{\mathbb{V}}(t,\wp(t,z))+\widehat{\mathbb{W}}(t,\wp(t,z)),$$

where,

$$\widehat{\mathbb{V}}(t,\wp(t,z)) = \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^n\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz,$$

$$\widehat{\mathbb{W}}(t,\wp(t,z)) = \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^n\frac{1}{2\gamma_{\xi\theta}^\eta}\Big(\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t)) \times(\varpi_{\xi\theta}^\eta(t)-\varpi^\eta)^2\Big)dz.$$

By computing the Caputo fractional derivative of $\mathbb{V}(t,\wp(t,z))$, one has

$$\begin{aligned}{}^{\mathcal{C}}D_0^\lambda\mathbb{V}(t,\wp(t,z)) \le& \sum_{\eta=r,i}^{j,k}\Big(\int_\Omega\sum_{\theta=1}^n\wp_\theta^\eta(t,z)\frac{\partial^\lambda\wp_\theta^\eta(t,z)}{\partial t^\lambda}dz\Big)\\ &+\sum_{\eta=r,i}^{j,k}\Big(\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma_{\xi\theta}^\eta}\Big\{\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\\ &\times(\varpi_{\xi\theta}^\eta(t)-\varpi^\eta)^{\mathcal{C}}D_0^\lambda\varpi_{\xi\theta}^\eta\Big\}dz\Big).\end{aligned}$$

Denote,

$$\widehat{W}_1 = \sum_{\eta=r,i}^{j,k}\Big(\int_\Omega\sum_{\theta=1}^n\wp_\theta^\eta(t,z)\frac{\partial^\lambda\wp_\theta^\eta(t,z)}{\partial t^\lambda}dz\Big),$$
$$\widehat{W}_2 = \sum_{\eta=r,i}^{j,k}\Big(\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma_{\xi\theta}^\eta}\{\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t)) \times(\varpi_{\xi\theta}^\eta(t)-\varpi^\eta)^{\mathcal{C}}D_0^\lambda\varpi_{\xi\theta}^\eta\}dz\Big),$$

and then

$$\begin{aligned}&\widehat{W}_1\\ &\le\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\Big\{-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}})-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^r|\\ &-\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^i|-\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^k|+\mathcal{F}_\varphi^r(|b_{\xi\varphi\theta}^r|\\ &+|b_{\xi\varphi\theta}^i|+|b_{\xi\varphi\theta}^j|+|b_{\xi\varphi\theta}^k|)+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^r|-\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^i|\\ &-\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^j|-\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^k|)\big]\int_\Omega(\wp_\theta^r)^2(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}})-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^r|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^i|\\ &+\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^i(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^i|\\ &+|b_{\xi\varphi\theta}^k|-|b_{\xi\varphi\theta}^j|)+\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^r|+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^i|\\ &+\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^j|-\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^k|)\big]\int_\Omega(\wp_\theta^i)^2(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}})-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^r|-\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^i|\\ &+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^j(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^j|\\ &-|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^i|)+\mathcal{G}_\varphi^j|d_{\xi\theta\varphi}^r|-\mathcal{G}_\varphi^k|d_{\xi\theta\varphi}^i|\\ &+\mathcal{G}_\varphi^r|d_{\xi\theta\varphi}^j|+\mathcal{G}_\varphi^i|d_{\xi\theta\varphi}^k|)\big]\int_\Omega(\wp_\theta^j)^2(t,z)dz\\ &-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}})-\sum_{\varphi=1}^n\big(\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^r|+\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^i|\\ &-\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^k(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^j|\end{aligned}$$

$$-|b^i_{\xi\varphi\theta}|)+\mathcal{G}^k_\varphi|d^r_{\xi\theta\varphi}|+\mathcal{G}^j_\varphi|d^i_{\xi\theta\varphi}|-\mathcal{G}^i_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^k_{\xi\theta\varphi}|)\big]$$
$$\times\int_\Omega(\wp^k_\theta)^2(t,z)dz-\int_\Omega\sum_{\theta=1}^n\varpi^r_{\xi\theta}(\wp^r_\theta)^2(t,z)dz$$
$$-\int_\Omega\sum_{\theta=1}^n\varpi^i_{\xi\theta}(\wp^i_\theta)^2(t,z)dz-\int_\Omega\sum_{\theta=1}^n\varpi^j_{\xi\theta}(\wp^j_\theta)^2(t,z)dz$$
$$-\int_\Omega\sum_{\theta=1}^n\varpi^k_{\xi\theta}(\wp^k_\theta)^2(t,z)dz+\frac{1}{2}\sum_{\theta=1}^n\sum_{\varphi=1}^n\mathcal{G}^r_\theta\big(|d^r_{\xi\varphi\theta}|$$
$$+|d^i_{\xi\varphi\theta}|+|d^j_{\xi\varphi\theta}|+|d^k_{\xi\varphi\theta}|\big)\int_\Omega(\wp^r_\theta)^2(t_\sigma,z)dz$$
$$+\frac{1}{2}\sum_{\theta=1}^n\sum_{\varphi=1}^n\mathcal{G}^i_\theta\big(|d^r_{\xi\varphi\theta}|-|d^i_{\xi\varphi\theta}|-|d^j_{\xi\varphi\theta}|+|d^k_{\xi\varphi\theta}|\big)$$
$$\times\int_\Omega(\wp^i_\theta)^2(t_\sigma,z)dz+\frac{1}{2}\sum_{\theta=1}^n\sum_{\varphi=1}^n\mathcal{G}^j_\theta\big(|d^r_{\xi\varphi\theta}|+|d^i_{\xi\varphi\theta}|$$
$$-|d^j_{\xi\varphi\theta}|-|d^k_{\xi\varphi\theta}|\big)\int_\Omega(\wp^j_\theta)^2(t_\sigma,z)dz+\frac{1}{2}\sum_{\theta=1}^n\sum_{\varphi=1}^n\mathcal{G}^k_\theta$$
$$\times\big(|d^r_{\xi\varphi\theta}|-|d^i_{\xi\varphi\theta}|+|d^j_{\xi\varphi\theta}|-|d^k_{\xi\varphi\theta}|\big)\int_\Omega(\wp^k_\theta)^2(t_\sigma,z)dz\Big\}.$$

Next,

$$\widehat{W}_2$$
$$\leq\sum_{\xi=1}^\zeta\Psi_\xi(\beta(t))\Big\{\int_\Omega\Big(\sum_{\theta=1}^n\frac{1}{\gamma^r_{\xi\theta}}(\varpi^r_{\xi\theta}(t)-\varpi^r)^{\mathcal{C}}D^\lambda_0\varpi^r_{\xi\theta}(t)\Big)dz$$
$$+\int_\Omega\Big(\sum_{\theta=1}^n\frac{1}{\gamma^i_{\xi\theta}}(\varpi^i_{\xi\theta}(t)-\varpi^i)^{\mathcal{C}}D^\lambda_0\varpi^i_{\xi\theta}(t)\Big)dz$$
$$+\int_\Omega\Big(\sum_{\theta=1}^n\frac{1}{\gamma^j_{\xi\theta}}(\varpi^j_{\xi\theta}(t)-\varpi^r)^{\mathcal{C}}D^\lambda_0\varpi^j_{\xi\theta}(t)\Big)dz$$
$$+\int_\Omega\Big(\sum_{\theta=1}^n\frac{1}{\gamma^k_{\xi\theta}}(\varpi^k_{\xi\theta}(t)-\varpi^k)^{\mathcal{C}}D^\lambda_0\varpi^k_{\xi\theta}(t)\Big)dz\Big\}$$
$$\leq\sum_{\xi=1}^\zeta\Psi_\xi(\beta(t))\Big\{\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma^r_{\xi\theta}}(\varpi^r_{\xi\theta}(t)-\varpi^r)$$
$$\times\{\gamma^r_{\xi\theta}(\wp_\theta)^2(t,z)-\frac{\varrho^r}{2}(\varpi^r_{\xi\theta}-\varpi^r)\}dz$$
$$+\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma^i_{\xi\theta}}(\varpi^i_{\xi\theta}(t)-\varpi^i)\{\gamma^i_{\xi\theta}(\wp_\theta)^2(t,z)$$
$$-\frac{\varrho^i}{2}(\varpi^i_{\xi\theta}-\varpi^i)\}dz+\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma^j_{\xi\theta}}(\varpi^j_{\xi\theta}(t)-\varpi^j)$$
$$\times\{\gamma^j_{\xi\theta}(\wp_\theta)^2(t,z)-\frac{\varrho^j}{2}(\varpi^j_{\xi\theta}-\varpi^j)\}dz$$
$$+\int_\Omega\sum_{\theta=1}^n\frac{1}{\gamma^k_{\xi\theta}}(\varpi^k_{\xi\theta}(t)-\varpi^k)\{\gamma^k_{\xi\theta}(\wp_\theta)^2(t,z)$$
$$-\frac{\varrho^k}{2}(\varpi^k_{\xi\theta}-\varpi^k)\}dz\Big\}$$
$$\leq\sum_{\xi=1}^\zeta\Psi_\xi(\beta(t))\Big\{\int_\Omega\sum_{\theta=1}^n(\varpi^r_{\xi\theta}(t)-\varpi^r)(\wp^r_\theta)^2(t,z)dz$$
$$+\int_\Omega\sum_{\theta=1}^n(\varpi^i_{\xi\theta}(t)-\varpi^i)(\wp^i_\theta)^2(t,z)dz$$
$$+\int_\Omega\sum_{\theta=1}^n(\varpi^j_{\xi\theta}(t)-\varpi^j)(\wp^j_\theta)^2(t,z)dz$$
$$+\int_\Omega\sum_{\theta=1}^n(\varpi^k_{\xi\theta}(t)-\varpi^k)(\wp^k_\theta)^2(t,z)dz$$
$$-\varrho^r\int_\Omega\sum_{\theta=1}^n\frac{1}{2\gamma^r_{\xi\theta}}(\varpi^r_{\xi\theta}(t)-\varpi^r)^2dz$$
$$-\varrho^i\int_\Omega\sum_{\theta=1}^n\frac{1}{2\gamma^i_{\xi\theta}}(\varpi^i_{\xi\theta}(t)-\varpi^i)^2dz$$
$$-\varrho^j\int_\Omega\sum_{\theta=1}^n\frac{1}{2\gamma^j_{\xi\theta}}(\varpi^j_{\xi\theta}(t)-\varpi^j)^2dz$$
$$-\varrho^k\int_\Omega\sum_{\theta=1}^n\frac{1}{2\gamma^k_{\xi\theta}}(\varpi^k_{\xi\theta}(t)-\varpi^k)^2dz\Big\}.$$

On the basis of above mentioned work, we can drive that

$$^{\mathcal{C}}D^\lambda_0\mathbb{V}(t,\wp(t,z))$$
$$\leq\sum_{\xi=1}^\zeta\Psi_\xi(\beta(t))\Big\{-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^r)$$
$$-\sum_{\varphi=1}^n\big(\mathcal{F}^r_\varphi|b^r_{\xi\theta\varphi}|-\mathcal{F}^i_\varphi|b^i_{\xi\theta\varphi}|-\mathcal{F}^j_\varphi|b^j_{\xi\theta\varphi}|-\mathcal{F}^k_\varphi|b^k_{\xi\theta\varphi}|$$
$$+\mathcal{F}^r_\varphi(|b^r_{\xi\varphi\theta}|+|b^i_{\xi\varphi\theta}|+|b^j_{\xi\varphi\theta}|+|b^k_{\xi\varphi\theta}|)+\mathcal{G}^r_\varphi|d^r_{\xi\theta\varphi}|$$
$$-\mathcal{G}^i_\varphi|d^i_{\xi\theta\varphi}|-\mathcal{G}^j_\varphi|d^j_{\xi\theta\varphi}|-\mathcal{G}^k_\varphi|d^k_{\xi\theta\varphi}|)\big]\int_\Omega(\wp^r_\theta)^2(t,z)dz$$
$$-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^i)-\sum_{\varphi=1}^n\big(\mathcal{F}^i_\varphi|b^r_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^i_{\xi\theta\varphi}|$$
$$+\mathcal{F}^k_\varphi|b^j_{\xi\theta\varphi}|-\mathcal{F}^j_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^i_\theta(|b^r_{\xi\varphi\theta}|-|b^i_{\xi\varphi\theta}|+|b^k_{\xi\varphi\theta}|$$
$$-|b^j_{\xi\varphi\theta}|)+\mathcal{G}^i_\varphi|d^r_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^i_{\xi\theta\varphi}|+\mathcal{G}^k_\varphi|d^j_{\xi\theta\varphi}|-\mathcal{G}^j_\varphi|d^k_{\xi\theta\varphi}|)\big]$$
$$\times\int_\Omega(\wp^i_\theta)^2(t,z)dz-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^j)$$
$$-\sum_{\varphi=1}^n\big(\mathcal{F}^j_\varphi|b^r_{\xi\theta\varphi}|-\mathcal{F}^k_\varphi|b^i_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^j_{\xi\theta\varphi}|+\mathcal{F}^i_\varphi|b^k_{\xi\theta\varphi}|$$
$$+\mathcal{F}^j_\theta(|b^r_{\xi\varphi\theta}|-|b^j_{\xi\varphi\theta}|-|b^k_{\xi\varphi\theta}|+|b^i_{\xi\varphi\theta}|)+\mathcal{G}^j_\varphi|d^r_{\xi\theta\varphi}|$$
$$-\mathcal{G}^k_\varphi|d^i_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^i_\varphi|d^k_{\xi\theta\varphi}|)\big]\int_\Omega(\wp^j_\theta)^2(t,z)dz$$
$$-\frac{1}{2}\sum_{\theta=1}^n\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^k)-\sum_{\varphi=}^n\big(\mathcal{F}^k_\varphi|b^r_{\xi\theta\varphi}|+\mathcal{F}^j_\varphi|b^i_{\xi\theta\varphi}|$$
$$-\mathcal{F}^i_\varphi|b^j_{\xi\theta\varphi}|+\mathcal{F}^r_\varphi|b^k_{\xi\theta\varphi}|+\mathcal{F}^k_\theta(|b^r_{\xi\varphi\theta}|-|b^k_{\xi\varphi\theta}|+|b^j_{\xi\varphi\theta}|$$
$$-|b^i_{\xi\varphi\theta}|)+\mathcal{G}^k_\varphi|d^r_{\xi\theta\varphi}|+\mathcal{G}^j_\varphi|d^i_{\xi\theta\varphi}|-\mathcal{G}^i_\varphi|d^j_{\xi\theta\varphi}|+\mathcal{G}^r_\varphi|d^k_{\xi\theta\varphi}|)\big]$$

$$
\begin{aligned}
&\times \int_\Omega (\wp_\theta^k)^2(t,z)dz + \frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^r\big(|d_{\xi\varphi\theta}^r| + |d_{\xi\varphi\theta}^i| + |d_{\xi\varphi\theta}^j| \\
&+|d_{\xi\varphi\theta}^k|\big)\int_\Omega (\wp_\theta^r)^2(t_\sigma,z)dz + \frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^i\big(|d_{\xi\varphi\theta}^r| \\
&-|d_{\xi\varphi\theta}^i| - |d_{\xi\varphi\theta}^j| + |d_{\xi\varphi\theta}^k|\big)\int_\Omega (\wp_\theta^i)^2(t_\sigma,z)dz \\
&+\frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^j\big(|d_{\xi\varphi\theta}^r| + |d_{\xi\varphi\theta}^i| - |d_{\xi\varphi\theta}^j| - |d_{\xi\varphi\theta}^k|\big) \\
&\times \int_\Omega (\wp_\theta^j)^2(t_\sigma,z)dz + \frac{1}{2}\sum_{\theta=1}^{n}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^k\big(|d_{\xi\varphi\theta}^r| - |d_{\xi\varphi\theta}^i| \\
&+|d_{\xi\varphi\theta}^j| - |d_{\xi\varphi\theta}^k|\big)\int_\Omega (\wp_\theta^k)^2(t_\sigma,z)dz \\
&-\varrho^r\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^r}(\varpi_{\xi\theta}^r(t)-\varpi^r)^2dz \\
&-\varrho^i\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^i}(\varpi_{\xi\theta}^i(t)-\varpi^i)^2dz \\
&-\varrho^j\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^j}(\varpi_{\xi\theta}^j(t)-\varpi^j)^2dz \\
&-\varrho^k\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^k}(\varpi_{\xi\theta}^k(t)-\varpi^k)^2dz\Big\} \\
\le &\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\Big\{-\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{K}_\theta^r\int_\Omega(\wp_\theta^r)^2(t,z)dz \\
&-\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{K}_\theta^i\int_\Omega(\wp_\theta^i)^2(t,z)dz - \frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{K}_\theta^j\int_\Omega(\wp_\theta^j)^2(t,z)dz \\
&-\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{K}_\theta^k\int_\Omega(\wp_\theta^k)^2(t,z)dz + \frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{P}_\theta^r\int_\Omega(\wp_\theta^r)^2(t_\sigma,z)dz \\
&+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{P}_\theta^i\int_\Omega(\wp_\theta^i)^2(t_\sigma,z)dz + \frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{P}_\theta^j\int_\Omega(\wp_\theta^j)^2(t_\sigma,z)dz \\
&+\frac{1}{2}\sum_{\theta=1}^{n}\mathfrak{P}_\theta^k\int_\Omega(\wp_\theta^k)^2(t_\sigma,z)dz \\
&-\varrho^r\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^r}(\varpi_{\xi\theta}^r(t)-\varpi^r)^2dz \\
&-\varrho^i\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^i}(\varpi_{\xi\theta}^i(t)-\varpi^i)^2dz \\
&-\varrho^j\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^j}(\varpi_{\xi\theta}^j(t)-\varpi^j)^2dz \\
&-\varrho^k\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^k}(\varpi_{\xi\theta}^k(t)-\varpi^k)^2dz\Big\} \\
\le &-\widehat{\mathfrak{K}}\sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz \\
&+\widehat{\mathfrak{P}}\sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2}(\wp_\theta^\eta)^2(t_\sigma,z)dz \\
&-\hat{\varrho}\sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^\eta}\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))(\varpi_{\xi\theta}^\eta(t)-\varpi^k)^2dz \\
\le &-\widehat{\mathfrak{K}}\widehat{\mathbb{V}}(t,\wp(t,z)) + \widehat{\mathfrak{P}}\sup_{t_\sigma\le s\le t}\widehat{\mathbb{V}}(t_\sigma,\wp(t_\sigma,z)) \\
&-\hat{\varrho}\widehat{\mathbb{W}}(t,\wp(t,z)).
\end{aligned}
$$

For $t_\sigma \le s \le t$, $t \ge 0$, based on the above inequality, the error state $\wp(t,z)$ satisfies the Razumikhin condition [11], which gives,

$$
\begin{aligned}
{}^{\mathcal{C}}D_0^\lambda\mathbb{V}(t,\wp(t,z)) &\le -(\widehat{\mathfrak{K}}-\widehat{\mathfrak{P}})\widehat{\mathbb{V}}(t,\wp(t,z)) - \hat{\varrho}\widehat{\mathbb{W}}(t,\wp(t,z)) \\
&\le -\mathfrak{N}\widehat{\mathbb{V}}(t,\wp(t,z)) - \hat{\varrho}\widehat{\mathbb{W}}(t,\wp(t,z)) \\
&\le -\Phi\mathbb{V}(t,\wp(t,z)).
\end{aligned}
$$

From Lemma 3, we have that

$$\mathbb{V}(t,\wp(t,z)) \le \sup_{\sigma\le s\le 0}\mathbb{V}(0,\hat{\psi}(s,z))\mathbb{E}_\lambda(-\Phi t^\lambda), \quad t>0.$$

It means that

$$
\begin{aligned}
&\sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz \\
&\le \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz + \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^\eta} \\
&\quad\times\Big(\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))(\varpi_{\xi\theta}^\eta(t)-\varpi^\eta)^2\Big)dz \\
&\le \Big\{\sum_{\eta=r,i}^{j,k}\Big(\sup_{\sigma\le s\le 0}\int_\Omega\sum_{\theta=1}^{n}(\hat{\psi}_\theta^\eta)^2(s,z)dz\Big) + \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^\eta} \\
&\quad\times\Big(\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))(\varpi_{\xi\theta}^\eta(0)-\varpi^\eta)^2\Big)dz\Big\}\mathbb{E}_\lambda(-\Phi t^\lambda).
\end{aligned}
$$

In view of $(\varpi_{\xi\theta}^\eta(0)-\varpi^\eta)$ is finite, it is obvious that there exists a positive constant $\mathcal{K}$ leading to

$$
\begin{aligned}
&\sum_{\eta=r,i}^{j,k}\Big(\sup_{\sigma\le s\le 0}\int_\Omega\sum_{\theta=1}^{n}(\hat{\psi}_\theta^\eta)^2(s,z)dz\Big) + \sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2\gamma_{\xi\theta}^\eta} \\
&\quad\times\Big(\sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))(\varpi_{\xi\theta}^\eta(0)-\varpi^\eta)^2\Big)dz \\
&\quad\le \mathcal{K}\sum_{\eta=r,i}^{j,k}\Big(\sup_{\sigma\le s\le 0}\int_\Omega\sum_{\theta=1}^{n}(\hat{\psi}_\theta^\eta)^2(s,z)dz\Big).
\end{aligned}
$$

Finally, we obtain that

$$
\begin{aligned}
&\sum_{\eta=r,i}^{j,k}\int_\Omega\sum_{\theta=1}^{n}\frac{1}{2}(\wp_\theta^\eta)^2(t,z)dz \\
&\qquad\le \mathcal{K}\sum_{\eta=r,i}^{j,k}\Big(\sup_{\sigma\le s\le 0}\int_\Omega\sum_{\theta=1}^{n}(\hat{\psi}_\theta^\eta)^2(s,z)dz\Big)\mathbb{E}_\lambda(-\Phi t^\lambda).
\end{aligned}
$$

According to inequality (51) and Definition 3, it follows that

$$\sum_{\eta=r,i}^{j,k} \|\wp^{\eta}(t,z)\|^2 \leq \mathcal{K}\delta^2 \frac{\epsilon^2}{\mathcal{K}\delta^2}.$$

That is

$$\sum_{\eta=r,i}^{j,k} \|\wp^{\eta}(t,z)\| \leq \epsilon.$$

According to Definition 3, we can conclude that the drive system (4) is said to be FTMLS with the response system (5) under adaptive controllers (49). □

Remark 3: Most industrial processes are spatiotemporal in nature, and the mathematical models of these nonlinear processes are generally expressed by nonlinear PDEs. In this paper, we designed adaptive fuzzy feedback controller scheme for FTMLS problem of T-S fuzzy FORDDQVNNs. Up to now, many interesting works concerning the property of fractional-order QVNNs without reaction-diffusion terms and fuzzy rules have been obtained, see [50]–[57]. Furthermore, the FTMLS problem of fractional-order QVNNs by using linear feedback controllers have been investigated [58]. In [62], finite/fixed-time synchronization problem was discussed for memristive QVNNs with integer-order case. To the best of our knowledge, the present study is the first attempt to analyze the FTMLS of T-S fuzzy FORDDQVNNs under adaptive fuzzy feedback controller scheme. Therefore, the theoretical results established in this paper are new and extend some previous ones.

Remark 4: In the implementation, due to the restrictions of equipments and influence of the environment, the reaction-diffusion phenomenon and fuzzy rules in CVNNs. Different from the existing reaction-diffusion CVNNs and without T-S fuzzy rules in [59]–[61], reaction-diffusion CVNNs without fractional-order case in [59]–[61], and the T-S fuzzy fractional-order reaction-diffusion CVNNs is newly built in (10), (11) and (16), (17), which not only considers the effect of the reaction-diffusion phenomenon but the fuzzy-dependent adjustable matrix inequality technique is more flexible and helpful to reduce the conservatism and compared with integer-order neurons, fractional-order neurons are helpful for effective signal detection and extraction. Thus, compared with the models in [59]–[61], the model in (10), (11) and (16), (17) is more applicable. T-S fuzzy FORDDQVNNs can be regarded as a generalization of fractional-order reaction-diffusion CVNNs, thus Theorem 1 and Theorem 2 can be used to estimate the FTMLS of T-S fuzzy fractional-order reaction-diffusion CVNNs.

Combining (10), (11) and (16), (17), can derive the following error system

$$\begin{aligned}\frac{\partial^{\lambda}\wp^{R}(t,z)}{\partial t^{\lambda}} &= \sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\big\{\Delta\wp^{R}(t,z)-\mathcal{A}_{\xi}\wp^{R}(t,z)\\ &+\mathcal{B}_{\xi}^{R}\big[F^{R}(\mathfrak{Z}^{R}(t,z))-F^{R}(\Im^{R}(t,z))\big]\\ &-\mathcal{B}_{\xi}^{I}\big[F^{I}(\mathfrak{Z}^{I}(t,z))-F^{I}(\Im^{I}(t,z))\big]\\ &+\mathcal{D}_{\xi}^{R}\big[G^{R}(\mathfrak{Z}^{R}(t_{\sigma},z))-G^{R}(\Im^{R}(t_{\sigma},z))\big]\\ &-\mathcal{D}_{\xi}^{I}\big[G^{I}(\mathfrak{Z}^{I}(t_{\sigma},z))-G^{I}(\Im^{I}(t_{\sigma},z))\big]\\ &+\hat{u}_{\xi}^{R}(t,z)\big\},\end{aligned} \tag{52}$$

$$\begin{aligned}\frac{\partial^{\lambda}\wp^{I}(t,z)}{\partial t^{\lambda}} &= \sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\big\{\Delta\wp^{I}(t,z)-\mathcal{A}_{\xi}\wp^{I}(t,z)\\ &+\mathcal{B}_{\xi}^{R}\big[F^{I}(\mathfrak{Z}^{I}(t,z))-F^{I}(\Im^{I}(t,z))\big]\\ &+\mathcal{B}_{\xi}^{I}\big[F^{R}(\mathfrak{Z}^{R}(t,z))-F^{R}(\Im^{R}(t,z))\big]\\ &+\mathcal{D}_{\xi}^{R}\big[G^{I}(\mathfrak{Z}^{I}(t_{\sigma},z))-G^{I}(\Im^{I}(t_{\sigma},z))\big]\\ &+\mathcal{D}_{\xi}^{I}\big[G^{R}(\mathfrak{Z}^{R}(t_{\sigma},z))-G^{R}(\Im^{R}(t_{\sigma},z))\big]\\ &+\hat{u}_{\xi}^{I}(t,z)\big\}.\end{aligned} \tag{53}$$

According to (52) and (53) can be rewritten as

$$\begin{aligned}\frac{\partial^{\lambda}\tilde{\wp}(t,z)}{\partial t^{\lambda}} &= \sum_{\xi=1}^{\zeta}\Psi_{\xi}(\beta(t))\big\{\Delta\tilde{\wp}(t,z)-\widetilde{\mathcal{A}}_{\xi}\tilde{\wp}(t,z)\\ &+\widetilde{\mathcal{B}}_{\xi}\widetilde{F}(\tilde{\wp}(t,z))+\widetilde{\mathcal{C}}_{\xi}\widetilde{G}(\tilde{\wp}(t_{\sigma},z))+\widetilde{U}_{\xi}(t,z)\big\},\end{aligned} \tag{54}$$

where, $\tilde{\wp}(t,z) = \big((\wp^{R}(t,z))^{T}, (\wp^{I}(t,z))^{T}\big)^{T}$, $\Delta\tilde{\wp}(t,z) = \big((\Delta\wp^{R}(t,z))^{T}, (\Delta\wp^{I}(t,z))^{T}\big)^{T}$, $\widetilde{F}(\tilde{\wp}(t,z)) = \big((F^{R}(\mathfrak{Z}^{R}(t,z))-F^{R}(\Im^{R}(t,z)))^{T}, (F^{I}(\mathfrak{Z}^{I}(t,z))-F^{I}(\Im^{I}(t,z)))^{T}\big)^{T}$, $\widetilde{G}(\tilde{\wp}(t_{\sigma},z)) = \big((G^{R}(\mathfrak{Z}^{R}(t_{\sigma},z))-G^{R}(\Im^{R}(t_{\sigma},z)))^{T}, (G^{I}(\mathfrak{Z}^{I}(t_{\sigma},z))-G^{I}(\Im^{I}(t_{\sigma},z)))^{T}\big)^{T}$, $\widetilde{U}_{\xi}(t,z) = \big((\hat{u}_{\xi}^{R}(t,z))^{T}, (\hat{u}_{\xi}^{I}(t,z))^{T}\big)^{T}$, $\widetilde{\mathcal{A}}_{\xi} = diag(\mathcal{A}_{\xi}, \mathcal{A}_{\xi})$, $\widetilde{\mathcal{B}}_{\xi} = \begin{bmatrix}\mathcal{B}_{\xi}^{R} & -\mathcal{B}_{\xi}^{I}\\ \mathcal{B}_{\xi}^{I} & \mathcal{B}_{\xi}^{R}\end{bmatrix}$, $\widetilde{\mathcal{C}}_{\xi} = \begin{bmatrix}\mathcal{C}_{\xi}^{R} & -\mathcal{C}_{\xi}^{I}\\ \mathcal{C}_{\xi}^{I} & \mathcal{C}_{\xi}^{R}\end{bmatrix}$.

In the following, we use the fuzzy feedback scheme to realize FTMLS between the system (54), then the controller can be designed as

$$\hat{u}_{\xi}^{R}(t,z) = -\hat{\mu}_{\xi\theta}^{R}\tilde{\wp}_{\theta}^{R}(t,z), \quad \hat{u}_{\xi}^{I}(t,z) = -\hat{\mu}_{\xi\theta}^{I}\tilde{\wp}_{\theta}^{I}(t,z), \tag{55}$$

where $\hat{\mu}_{\xi\theta}^{R}$ and $\hat{\mu}_{\xi\theta}^{I}$ represents the control gain.

Corollary 1: Under Assumption 1 and 2, the system (54) achieve the FTMLS via controller (55) if the following conditions holds:

$$(i) \quad \Lambda = (\mathfrak{B}-\mathfrak{E}) > 0,$$
$$(ii) \quad \mathbb{E}_{\lambda}(-\Lambda t^{\lambda}) < \frac{\epsilon^2}{\delta^2},$$

where,

$$\mathfrak{B} = \min_{1\leq\theta\leq n}\{\mathfrak{B}_{\theta}^{R}, \mathfrak{B}_{\theta}^{I}\},\ \mathfrak{E} = \max_{1\leq\theta\leq n}\{\mathfrak{E}_{\theta}^{R}, \mathfrak{E}_{\theta}^{I}\},$$

$$\begin{aligned}\mathfrak{B}_{\theta}^{R} = \min_{1\leq\theta\leq n}\Big\{&\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\tilde{\mu}_{\xi\theta}^{R})-\sum_{\varphi=1}^{n}\big(\mathcal{F}_{\varphi}^{R}|b_{\xi\theta\varphi}^{R}|\\ &-\mathcal{F}_{\varphi}^{I}|b_{\xi\theta\varphi}^{I}|+\mathcal{F}_{\varphi}^{R}(|b_{\xi\varphi\theta}^{R}|+|b_{\xi\varphi\theta}^{I}|)+\mathcal{G}_{\varphi}^{R}|d_{\xi\theta\varphi}^{R}|\\ &-\mathcal{G}_{\varphi}^{I}|d_{\xi\theta\varphi}^{I}|\big)\big]\Big\},\end{aligned}$$

$$\mathfrak{B}_\theta^I = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\tilde{\mu}_{\xi\theta}^I)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^I|b_{\xi\theta\varphi}^R| +\mathcal{F}_\varphi^R|b_{\xi\theta\varphi}^I|+\mathcal{F}_\theta^I(|b_{\xi\varphi\theta}^R|-|b_{\xi\varphi\theta}^I|)+\mathcal{G}_\varphi^I|d_{\xi\theta\varphi}^R| +\mathcal{G}_\varphi^R|d_{\xi\theta\varphi}^I|\big)\big]\Big\},$$

$$\mathfrak{E}_\theta^R = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^R\big(|d_{\xi\varphi\theta}^R|+|d_{\xi\varphi\theta}^I|\big)\Big\},$$

$$\mathfrak{E}_\theta^I = \max_{1\le\theta n}\Big\{\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^I\big(|d_{\xi\varphi\theta}^R|-|d_{\xi\varphi\theta}^I|\big)\Big\}.$$

Designing an adaptive controller $\hat{u}_\xi^R(t,z)$ and $\hat{u}_\xi^I(t,z)$ as follows:

$$\begin{cases}\hat{u}_\xi^R(t,z) = -\varpi_{\xi\theta}^R\wp_\theta^\eta(t,z),\\ \hat{u}_\xi^I(t,z) = -\varpi_{\xi\theta}^I\wp_\theta^\eta(t,z),\\ {}^CD_0^\lambda\varpi_{\xi\theta}^R = \gamma_{\xi\theta}^R(\wp_\theta^R)^2(t,z)-\frac{\varrho^R}{2}(\varpi_{\xi\theta}^R(t)-\varpi^R)^2,\\ {}^CD_0^\lambda\varpi_{\xi\theta}^I = \gamma_{\xi\theta}^I(\wp_\theta^I)^2(t,z)-\frac{\varrho^I}{2}(\varpi_{\xi\theta}^I(t)-\varpi^I)^2,\end{cases} \tag{56}$$

for $\theta = 1,2,\ldots,n$; $\xi = 1,2,\ldots,\zeta$, where $\varpi^R > 0$, $\varpi^I > 0$ are tunable constants; $\varpi_{\xi\theta}^R(t) > 0$, $\varpi_{\xi\theta}^I(t) > 0$ are tunable functions; $\varrho^R > 0$, $\varrho^I > 0$ and $\gamma_{\xi\theta}^R > 0$, $\gamma_{\xi\theta}^I > 0$ are constants.

Corollary 2: Under Assumption 1 and 2, the system (54) achieve the FTMLS via adaptive controller (56) if the following conditions holds:

$$(i)\quad \widehat{\Phi} = \mathfrak{T}-\bar{\varrho} > 0,$$
$$(ii)\quad \mathbb{E}_\lambda(-\widehat{\Phi}t^\lambda) < \frac{\epsilon^2}{\mathcal{K}\delta^2}.$$

where,

$$\mathfrak{T} = \widehat{\mathcal{S}}-\widehat{\mathcal{Z}},\ \widehat{\mathcal{S}} = \min_{1\le\theta\le n}\{\mathcal{S}_\theta^R,\mathcal{S}_\theta^I\},$$
$$\widehat{\mathcal{Z}} = \max_{1\le\theta\le n}\{\mathcal{Z}_\theta^R,\mathcal{Z}_\theta^I\},\ \bar{\varrho} = \min\{\varrho^R,\varrho^I\},$$

$$\mathcal{S}_\theta^R = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^R)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^R|b_{\xi\theta\varphi}^R| -\mathcal{F}_\varphi^I|b_{\xi\theta\varphi}^I|+\mathcal{F}_\varphi^R(|b_{\xi\varphi\theta}^R|+|b_{\xi\varphi\theta}^I|)+\mathcal{G}_\varphi^R|d_{\xi\theta\varphi}^R| -\mathcal{G}_\varphi^I|d_{\xi\theta\varphi}^I|\big)\big]\Big\},$$

$$\mathcal{S}_\theta^I = \min_{1\le\theta\le n}\Big\{\frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^I)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^I|b_{\xi\theta\varphi}^R| +\mathcal{F}_\varphi^R|b_{\xi\theta\varphi}^I|+\mathcal{F}_\theta^I(|b_{\xi\varphi\theta}^R|-|b_{\xi\varphi\theta}^I|)+\mathcal{G}_\varphi^I|d_{\xi\theta\varphi}^R| +\mathcal{G}_\varphi^R|d_{\xi\theta\varphi}^I|\big)\big]\Big\},$$

$$\mathcal{Z}_\theta^R = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^R\big(|d_{\xi\varphi\theta}^R|+|d_{\xi\varphi\theta}^I|\big)\Big\},$$

$$\mathcal{Z}_\theta^I = \max_{1\le\theta\le n}\Big\{\frac{1}{2}\sum_{\varphi=1}^{n}\mathcal{G}_\theta^I\big(|d_{\xi\varphi\theta}^R|-|d_{\xi\varphi\theta}^I|\big)\Big\}.$$

Remark 5: The result of Corollary 1 and 2 can also be applied to T-S fuzzy fractional-order reaction-diffusion CVNNs. Moreover, T-S fuzzy fractional-order reaction-diffusion RVNNs are also applicable to the results in this paper. This shows that the outcomes of this paper are more general.

If transmission delay term are not considered, T-S fuzzy FORDDQVNNs (4) is reduced to T-S fuzzy fractional-order reaction-diffusion QVNNs

$$\frac{\partial^\lambda\Im(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im(t,z)-\mathcal{A}_\xi\Im(t,z) +\mathcal{B}_\xi F(\Im(t,z))+\mathcal{I}\big\}. \tag{57}$$

and T-S FORDDQVNNs (5) is reduced to the controlled T-S fuzzy fractional-order reaction-diffusion QVNNs,

$$\frac{\partial^\lambda\mathfrak{Z}(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}(t,z)-\mathcal{A}_\xi\mathfrak{Z}(t,z) +\mathcal{B}_\xi F(\mathfrak{Z}(t,z))+\mathcal{I}+\hat{u}_\xi(t,z)\big\}. \tag{58}$$

In this case, we have the following corollary.

Corollary 3: Under Assumption 1 and 2, the system (57) and (58) achieve the FTMLS via controller (25) if the following conditions holds:

$$(i)\quad \Psi > 0,$$
$$(ii)\quad \mathbb{E}_\lambda(-\Psi t^\lambda) < \frac{\epsilon^2}{\delta^2},$$

where,

$$\Psi = \min_{1\le\theta\le n}\{\aleph_\theta^r,\aleph_\theta^i,\aleph_\theta^j,\aleph_\theta^k\},$$

$$\aleph_\theta^r = \frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^r)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^r-\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^i| -\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^k|+\mathcal{F}_\varphi^r(|b_{\xi\varphi\theta}^r|+|b_{\xi\varphi\theta}^i| +|b_{\xi\varphi\theta}^j|+|b_{\xi\varphi\theta}^k|)\big)\big],$$

$$\aleph_\theta^i = \frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^i)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^r|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^i| +\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^j|-\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^i(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^i| +|b_{\xi\varphi\theta}^k|-|b_{\xi\varphi\theta}^j|)\big)\big],$$

$$\aleph_\theta^j = \frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^j)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^r|-\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^i| +\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^j(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^j| -|b_{\xi\varphi\theta}^k|+|b_{\xi\varphi\theta}^i|)\big)\big],$$

$$\aleph_\theta^k = \frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\mu_{\xi\theta}^k)-\sum_{\varphi=1}^{n}\big(\mathcal{F}_\varphi^k|b_{\xi\theta\varphi}^r|+\mathcal{F}_\varphi^j|b_{\xi\theta\varphi}^i| -\mathcal{F}_\varphi^i|b_{\xi\theta\varphi}^j|+\mathcal{F}_\varphi^r|b_{\xi\theta\varphi}^k|+\mathcal{F}_\theta^k(|b_{\xi\varphi\theta}^r|-|b_{\xi\varphi\theta}^k| +|b_{\xi\varphi\theta}^j|-|b_{\xi\varphi\theta}^i|)\big)\big].$$

Corollary 4: Under Assumption 1 and 2, the system (57) and (58) achieve the FTMLS via adaptive controller (49) if the following conditions holds:

$$(i)\quad \Upsilon = \widehat{\Im} - \hat{\varrho} > 0,$$
$$(ii)\quad \mathbb{E}_\lambda(-\Upsilon t^\lambda) < \frac{\epsilon^2}{\mathcal{K}\delta^2}.$$

where,

$$\widehat{\Im} = \min_{1\le\theta\le n}\{\widehat{\Im}^r_\theta, \widehat{\Im}^i_\theta, \widehat{\Im}^j_\theta, \widehat{\Im}^k_\theta\}, \hat{\varrho} = \{\varrho^r, \varrho^i, \varrho^j, \varrho^k\},$$

$$\widehat{\Im}^r_\theta = \frac{1}{2}\big[2(a_{\xi\theta}+\mathcal{Q}_\theta+\varpi^r) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^r_\varphi|b^r_{\xi\theta\varphi}| - \mathcal{F}^i_\varphi|b^i_{\xi\theta\varphi}| - \mathcal{F}^j_\varphi|b^j_{\xi\theta\varphi}| - \mathcal{F}^k_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi(|b^r_{\xi\varphi\theta}| + |b^i_{\xi\varphi\theta}| + |b^j_{\xi\varphi\theta}| + |b^k_{\xi\varphi\theta}|)\big)\big],$$

$$\widehat{\Im}^i_\theta = \frac{1}{2}\big[2(a_{\xi\theta}+\mathcal{Q}_\theta+\varpi^i) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^i_\varphi|b^r_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi|b^i_{\xi\theta\varphi}| + \mathcal{F}^k_\varphi|b^j_{\xi\theta\varphi}| - \mathcal{F}^j_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^i_\theta(|b^r_{\xi\varphi\theta}| - |b^i_{\xi\varphi\theta}| + |b^k_{\xi\varphi\theta}| - |b^j_{\xi\varphi\theta}|)\big)\big],$$

$$\widehat{\Im}^j_\theta = \frac{1}{2}\big[2(a_{\xi\theta}+\mathcal{Q}_\theta+\varpi^j) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^j_\varphi|b^r_{\xi\theta\varphi}| - \mathcal{F}^k_\varphi|b^i_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi|b^j_{\xi\theta\varphi}| + \mathcal{F}^i_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^j_\theta(|b^r_{\xi\varphi\theta}| - |b^j_{\xi\varphi\theta}| - |b^k_{\xi\varphi\theta}| + |b^i_{\xi\varphi\theta}|)\big)\big],$$

$$\widehat{\Im}^k_\theta = \frac{1}{2}\big[2(a_{\xi\theta}+\widehat{\mathcal{Q}}+\varpi^k) - \sum_{\varphi=1}^{n}\big(\mathcal{F}^k_\varphi|b^r_{\xi\theta\varphi}| + \mathcal{F}^j_\varphi|b^i_{\xi\theta\varphi}| - \mathcal{F}^i_\varphi|b^j_{\xi\theta\varphi}| + \mathcal{F}^r_\varphi|b^k_{\xi\theta\varphi}| + \mathcal{F}^k_\theta(|b^r_{\xi\varphi\theta}| - |b^k_{\xi\varphi\theta}| + |b^j_{\xi\varphi\theta}| - |b^i_{\xi\varphi\theta}|)\big)\big].$$

IV. NUMERICAL EXAMPLE

In this section, we present a example to demonstrate our main results. In order to verify the effectiveness of the proposed control strategies for the synchronization between drive system (59) and response system (60).

Example: Consider the following 2D-dimensional drive system with two plant rules.

$$\frac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im(t,z) - \mathcal{A}_\xi\Im(t,z) + \mathcal{B}_\xi F(\Im(t,z)) + \mathcal{D}_\xi G(\Im(t-\sigma(t),z)) + \mathcal{I}\big\}. \tag{59}$$

Plant Rule 1: If $\beta_1(t)$ is Ξ^1_1, then,

$$\frac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im(t,z) - \mathcal{A}_1\Im(t,z) + \mathcal{B}_1 F(\Im(t,z)) + \mathcal{D}_1 G(\Im(t-\sigma(t),z)) + \mathcal{I}\big\}.$$

Plant Rule 2: If $\beta_2(t)$ is Ξ^2_2, then,

$$\frac{\partial^\lambda \Im(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\Im(t,z) - \mathcal{A}_2\Im(t,z) + \mathcal{B}_2 F(\Im(t,z)) + \mathcal{D}_2 G(\Im(t-\sigma(t),z)) + \mathcal{I}\big\}.$$

The parameters are as follows $\Delta\Im(t,z) = \sum_{\alpha=1}^{m}\left(q_{\theta\alpha}\frac{\partial\Im_\theta(t,z)}{\partial z_\alpha}\right)$, $m = n = 2$, $l_1 = 3$, $l_2 = 5$, $\Omega = \{z : z = (z_1,z_2)^T, |z_\alpha| < 1\}$, $\alpha = 1,2$, $\lambda = 0.93$, $\mathcal{A}_1 = diag(3.2, 3.9)$, $\mathcal{A}_2 = diag(4.7, 4.3)$, $\Im(t,z) = \Im^r(t,z) + \Im^i(t,z)i + \Im^j(t,z)j + \Im^k(t,z)k$, $F(\Im(t,z)) = ((f_1(\Im(t,z)))^T, (f_2(\Im(t,z)))^T)^T$, $G(\Im(t-\sigma(t),z)) = ((g_1(\Im(t-\sigma(t),z)))^T, (g_2(\Im(t-\sigma(t),z)))^T)^T$, $\sigma(t) = \frac{3e^t}{10+7e^t}$, $\Psi_1(\beta(t)) = cos^2(5tanh(\|\Im_1\| + \|\Im_2\|))$, $\Psi_2(\beta(t)) = sin^2(5tanh(\|\Im_1\| + \|\Im_2\|))$, $\mathcal{I}(t) = \begin{bmatrix} 2cos(t)-3sin(t)i-cos(t)j+2sin(t)k \\ 3sin(t)+cos(t)i+2cos(t)j-sin(t)k \end{bmatrix}$, $(q_{\theta\alpha})_{2\times2} = \begin{bmatrix} 1.9 & 0.2 \\ 0.2 & 2.9 \end{bmatrix}$, $\mathcal{B}_1 = \begin{bmatrix} b_{111} & b_{112} \\ b_{121} & b_{122} \end{bmatrix}$, $\mathcal{B}_2 = \begin{bmatrix} b_{211} & b_{212} \\ b_{221} & b_{222} \end{bmatrix}$, $\mathcal{D}_1 = \begin{bmatrix} d_{111} & d_{112} \\ d_{121} & d_{122} \end{bmatrix}$, $\mathcal{D}_2 = \begin{bmatrix} d_{211} & d_{212} \\ d_{221} & d_{222} \end{bmatrix}$, where,
$b_{111} = 1.3-0.9i-1.3j+0.9k$, $b_{112} = 2.1+1.2i-0.9j-2.1k$, $b_{121} = -1.9+2.3i+1.9j-2.3k$, $b_{122} = -1.4+0.9i+1.1j+0.9k$, $b_{211} = 2.3-1.9i+1.3j+0.7k$, $b_{212} = -1.3+0.9i-2.3j+1.9k$, $b_{221} = 2.1-1.7i-1.3j-0.9k$, $b_{222} = 0.9+1.9i+2.3j-1.5k$, $d_{111} = -3.7+2.9i-3.3j-2.9k$, $d_{112} = 3.1-3.3i+2.9j-3.1k$, $d_{121} = 1.9-2.7i+1.9j-2.7k$, $d_{122} = 2.4-1.9i-2.1j-1.9k$, $d_{211} = 3.3-2.9i+2.3j+2.7k$, $d_{212} = 2.9+2.3i-2.9j-2.3k$, $d_{221} = 2.1+1.7i+1.3j+2.9k$, $d_{222} = 2.9-3.9i+1.3j+1.7k$. With the original initial conditions set as $\Im_1(0,z) = -3.1\,cos(z) + 2.3\,cos(z)i + 1.8\,sin(z)j - 1.6cos(z)k$ and $\Im_2(0,z) = 2.4\,sin(z) - 3.1\,cos(z)i + 2.6sin(z)j + 0.6sin(z)k$, the drive system state trajectories can be described as in Fig. 1.

The controlled response system is depicted by

$$\frac{\partial^\lambda \mathfrak{Z}(t,z)}{\partial t^\lambda} = \sum_{\xi=1}^{\zeta}\Psi_\xi(\beta(t))\big\{\Delta\mathfrak{Z}(t,z) - \mathcal{A}_\xi\mathfrak{Z}(t,z) + \mathcal{B}_\xi F(\mathfrak{Z}(t,z)) + \mathcal{D}_\xi G(\mathfrak{Z}(t-\sigma(t),z)) + \mathcal{I} + \widehat{U}_\xi(t,z)\big\}, \tag{60}$$

where $\lambda = 0.93$, other parameters are the same as of as drive system (59).

Case 1: We choose $f^r_\varphi(\cdot) = g^r_\varphi(\cdot) = 2.17tanh(\cdot) + 0.03sign(\cdot)$, $f^i_\varphi(\cdot) = g^i_\varphi(\cdot) = 1.49tanh(\cdot) + 0.01sign(\cdot)$, $f^j_\varphi(\cdot) = g^j_\varphi(\cdot) = 2.17tanh(\cdot) + 0.03sign(\cdot)$, $f^k_\varphi(\cdot) = g^k_\varphi(\cdot) = 1.49tanh(\cdot) + 0.01sign(\cdot)$, as the activation function. Thus, Assumption $(\mathcal{H}_1)$ are $\mathcal{F}^r_\varphi = \frac{1}{4}$, $\mathcal{F}^i_\varphi = \frac{1}{2}$, $\mathcal{F}^j_\varphi = \frac{1}{4}$, $\mathcal{F}^k_\varphi = \frac{1}{2}$, $\mathcal{G}^r_\varphi = 0$, $\mathcal{G}^i_\varphi = \frac{1}{2}$, $\mathcal{G}^j_\varphi = 0$, $\mathcal{G}^k_\varphi = \frac{1}{2}$, $(\varphi = 1,2)$ and also $(\mathcal{H}_2)$ are $\hat{q}_{11} = 1.7$, $\hat{q}_{22} = 2.7$, $\hat{q}_{12} = \hat{q}_{21} = 0$.

Then, for controller (25), the parameters are designed as $\mu^r_{11} = 3.7$, $\mu^r_{12} = 3.1$, $\mu^r_{21} = 2.5$, $\mu^r_{22} = 2.9$, $\mu^i_{11} = 2.3$, $\mu^i_{12} = 3.5$, $\mu^i_{21} = 2.7$, $\mu^i_{22} = 3.7$, $\mu^j_{11} = 2.7$,

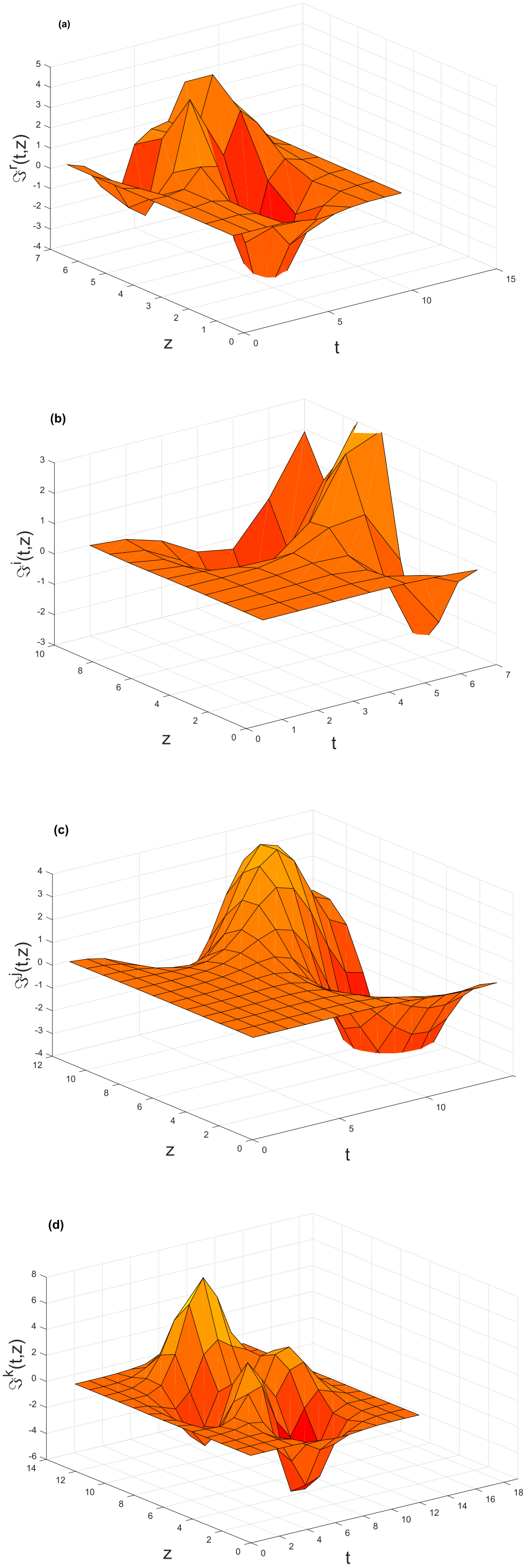

FIGURE 1. State trajectory of (*a*) $\Im^r(t,z)$, (*b*) $\Im^i(t,z)$, (*c*) $\Im^j(t,z)$, and (*d*) $\Im^k(t,z)$ of system (59) without control.

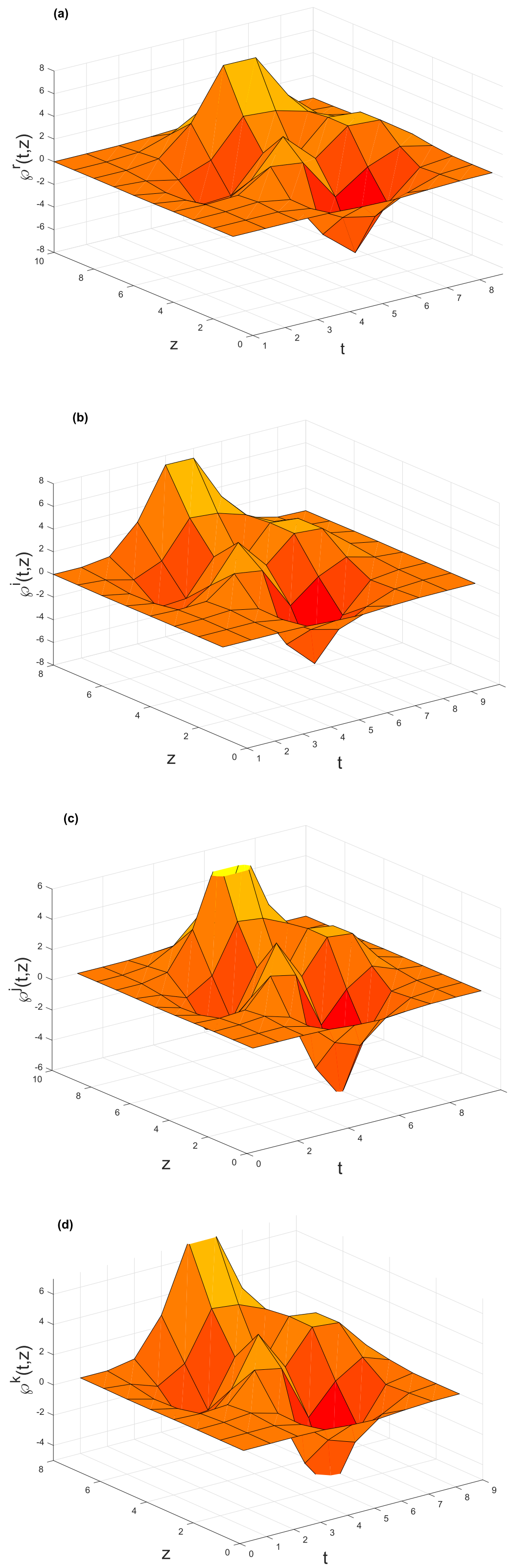

FIGURE 2. Synchronization error (*a*) $\wp^r(t,z)$, (*b*) $\wp^i(t,z)$, (*c*) $\wp^j(t,z)$, and (*d*) $\wp^k(t,z)$ of system (59) and (60) via fuzzy feedback controller (25) with fractional-order $\lambda = 0.93$..

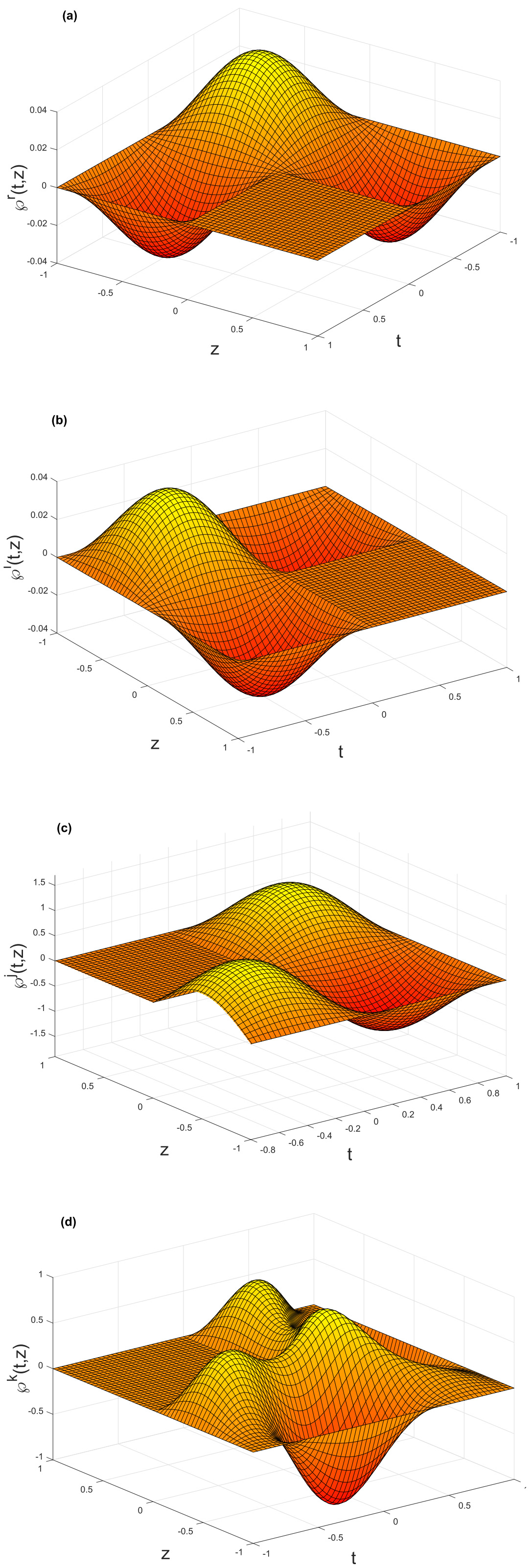

FIGURE 3. Synchronization error (a) $\wp^r(t,z)$, (b) $\wp^i(t,z)$, (c) $\wp^j(t,z)$, and (d) $\wp^k(t,z)$ of system (59) and (60) via adaptive controller (49) with fractional-order $\lambda = 0.93$..

$\mu^j_{12} = 4.1$, $\mu^j_{21} = 3.1$, $\mu^j_{22} = 2.8$, $\mu^k_{11} = 1.7$, $\mu^k_{12} = 2.7$, $\mu^k_{21} = 3.2$, and $\mu^k_{22} = 3.9$, combining with the proposed criteria in Theorem 1, one obtains the following results:

$$\mho^r_\theta = 9.4550,\ \mho^i_\theta = 1.1525,\ \mho^j_\theta = 4.8025,\ \mho^k_\theta = 4.0025,$$
$$\mathfrak{L}^r_\theta = 0,\ \mathfrak{L}^i_\theta = 1.4000,\ \mathfrak{L}^j_\theta = 0,\ \mathfrak{L}^k_\theta = 0.0025,$$

and, $\Theta = (\mho - \mathfrak{L}) = 0.2400 > 0$, holds and we can evaluate the finite time is about $t = 2.5$ according to the condition $\mathbb{E}_\lambda(-\Theta t^\lambda) < \frac{\epsilon^2}{\delta^2}$, when we assume $\epsilon = 6.70$, $\delta = 1.5$. Therefore, the drive system (59) can achieve FTMLS with response system (60) under the designed controller (25). Fig. 2 are synchronization errors $\wp^\eta(t,z)$ $(\eta = r, i, j, k)$ respectively, which have verified the feasibility and validness of the established theoretical results.

Case 2: We choose same as the activation function in case 1. Thus, Assumption $(\mathcal{H}_1)$ are $\mathcal{F}^r_\varphi = 1$, $\mathcal{F}^i_\varphi = 0$, $\mathcal{F}^j_\varphi = 1$, $\mathcal{F}^k_\varphi = 0$, $\mathcal{G}^r_\varphi = 0$, $\mathcal{G}^i_\varphi = \frac{2}{5}$, $\mathcal{G}^j_\varphi = 0$, $\mathcal{G}^k_\varphi = \frac{2}{5}$, $(\varphi = 1, 2)$ and $(\mathcal{H}_2)$ are $\hat{q}_{11} = 1.5$, $\hat{q}_{22} = 2$, $\hat{q}_{12} = \hat{q}_{21} = 0$.

Then, for adaptive controller (49), the parameters are designed as $\gamma^r_{11} = 4.09$, $\gamma^r_{12} = 2.70$, $\gamma^r_{21} = 2.08$, $\gamma^r_{22} = 3.10$, $\gamma^i_{11} = 2.53$, $\gamma^i_{12} = 3.70$, $\gamma^i_{21} = 3.07$, $\gamma^i_{22} = 4.07$, $\gamma^j_{11} = 3.11$, $\gamma^j_{12} = 3.97$, $\gamma^j_{21} = 4.13$, $\gamma^j_{22} = 2.97$, $\gamma^k_{11} = 3.63$, $\gamma^k_{12} = 3.07$, $\gamma^k_{21} = 3.80$, $\gamma^k_{22} = 3.36$, $\varpi^r = 2.71$, $\varpi^i = 0.93$, $\varpi^j = 1.57$, $\varpi^k = 0.89$, $\varrho^r = 0.73$, $\varrho^i = 0.21$, $\varrho^j = 0.97$, and $\varrho^k = 1.03$. By Theorem 2, we obtains the results:

$$\mathfrak{K}^r_\alpha = 1.5200,\ \mathfrak{K}^i_\alpha = 2.1700,\ \mathfrak{K}^j_\alpha = 1.7800,\ \mathfrak{K}^k_\alpha = 1.5900,$$
$$\mathfrak{P}^r_\alpha = 0,\ \mathfrak{P}^i_\alpha = 1.1200,\ \mathfrak{P}^j_\alpha = 0,\ \mathfrak{P}^k_\alpha = 0.0400,$$

$\mathfrak{N} = \widehat{\mathfrak{K}} - \widehat{\mathfrak{P}} = 0.4000$, and $\Phi = \mathfrak{N} - \hat{\varrho} = 0.1900 > 0$ holds, we can evaluate the finite-time is about $t = 0.6$ according to the condition $\mathbb{E}_\lambda(-\Phi t^\lambda) < \frac{\epsilon^2}{\mathcal{K}\delta^2}$, when we assume $\epsilon = 7.23$, $\delta = 1.9$, $\mathcal{K} = 1.05$. Therefore, the drive system (59) can achieve FTMLS with response system (60) under the designed controller (49). The simulation results are shown in Fig. 3, where the synchronization errors $\wp^\eta(t,z)(\eta = r, i, j, k)$, trend to be zero quickly with regard to time t.

V. CONCLUSION

In this paper, we introduces adaptive fuzzy feedback control schemes to investigate the FTMLS of T-S fuzzy FORDQVNNs. First, mainly by employing Hamilton rules, the studied multidimensional systems have been divided into the relevant real-valued ones. By designing new state-feedback controller and fuzzy adaptive controllers, then constructing a suitable Lyapunov functional and employing algebraic inequality methods, a novel FTMLS criterion of the proposed system can be obtained. Finally, a simulation example has been given to demonstrate the merits of the proposed approach. However, there are still two unsolved problems in this paper: (i) Stochastic factors are not considered in modeling, which is not consistent with some actual systems, and (ii) The impulsive controller would further reduce the contact consumption of the system compared to the continuous controllers. Therefore, inspired by [4], [32], [36], the FTMLS problem of stochastic FORDQVNNs via impulsive controller scheme will be studied in our future work.

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