Chaos, Solitons and Fractals 150 (2021) 111212

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Strict dissipativity synchronization for delayed static neural networks: An event-triggered scheme

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ARTICLE INFO

Article history:
Received 19 January 2021
Revised 5 June 2021
Accepted 21 June 2021

Keywords:
Dissipativity
Event-triggered control
Lyapunov-Krasovskii functional
Static neural networks
Synchronization

ABSTRACT

This article addresses the investigation of strict dissipativity synchronization for a class of static neural networks under an event-triggered scheme. An event-triggered scheme is recommended, it can upgrade the exhibition of system dynamics and diminishes the network communication burden at the same time. Firstly, an appropriate Lyapunov-Krasovskii functional (LKF) with double and triple integral terms with the details on both lower and upper bounds of the delay is completely designed. Secondly, under the single and double Auxillary function-based integral inequalities (SAFBII and DAFBII, respectively) and generalized free weight matrix approach, a new class of delay-dependent adequate condition is proposed, so that the error system is $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ strict dissipative. A resilient distributed event-triggered control scheme is developed by this criterion in terms of linear matrix inequalities (LMIs). At last, simulation examples are provided to demonstrate the performance of the derived results.

1. Introduction

During the past few decades, neural networks (NNs) have been effectively applied in numerous areas, for example, pattern recognition, signal processing, solving optimization problems, static image processing, associative memories, target tracking, and automatic control, etc [1–4]. Time-delays are inescapable in executing the NNs because of the limited conduction speed and switching velocity of electronic components. It is notable, the appearance of time delays may cause the structured NNs to change the prescribed dynamical properties like instability of system and performance deterioration. Based on this aspect, various problems of NNs with respect to time-varying delays have been tackled [5–9]. It is worth pointing out that the dynamic behaviors of NNs are essential for each successful application, such as medicine and biology, economics, electronics, and telecommunications. According to the different positions of the weight matrices associated with the activation functions, NNs possibly divided into two kinds: local field neural networks (LFNNs) and static neural networks (SNNs) [10,11]. In the view of SNNs could be moved comparably to an LFNNs in the account of fulfilling definite hypothesis, maximum focus can be turned to LFNNs. As a tool for scientific computing and engineering application, an obvious characteristic of SNNs is its capability for implementing a nonlinear mapping from many neural inputs to many neural outputs [5]. On the other side, SNNs have been working a crucial act in the investigation based on stability and stabilization issues and countless remarkable results have been accounted in [12–14].

On the other hand, dissipative theory is the needed one in dynamical framework, which can instinctively give back the loss or dissipation of energy. Generally, dissipativity theory originates from electrical networks and by using an input-output description, presents a tool for analysis and synthesis of control systems, robotic system, electrical power system, engine system, combustion engines, circuit theory, damping, and electromechanical systems and so forth [15–17]. The dissipativity hypothesis additionally gives a crucial framework to examine control issues of SNNs. Furthermore, it fills in as a useful asset in symbolizing framework dynamics such as stability and passivity. Besides that, the theory of passivity assumes a significant part of circuits, networks and control systems. The principle thought of the passivity approach role is that, able to maintain the system internally stable [18–21]. Based

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https://doi.org/10.1016/j.chaos.2021.111212

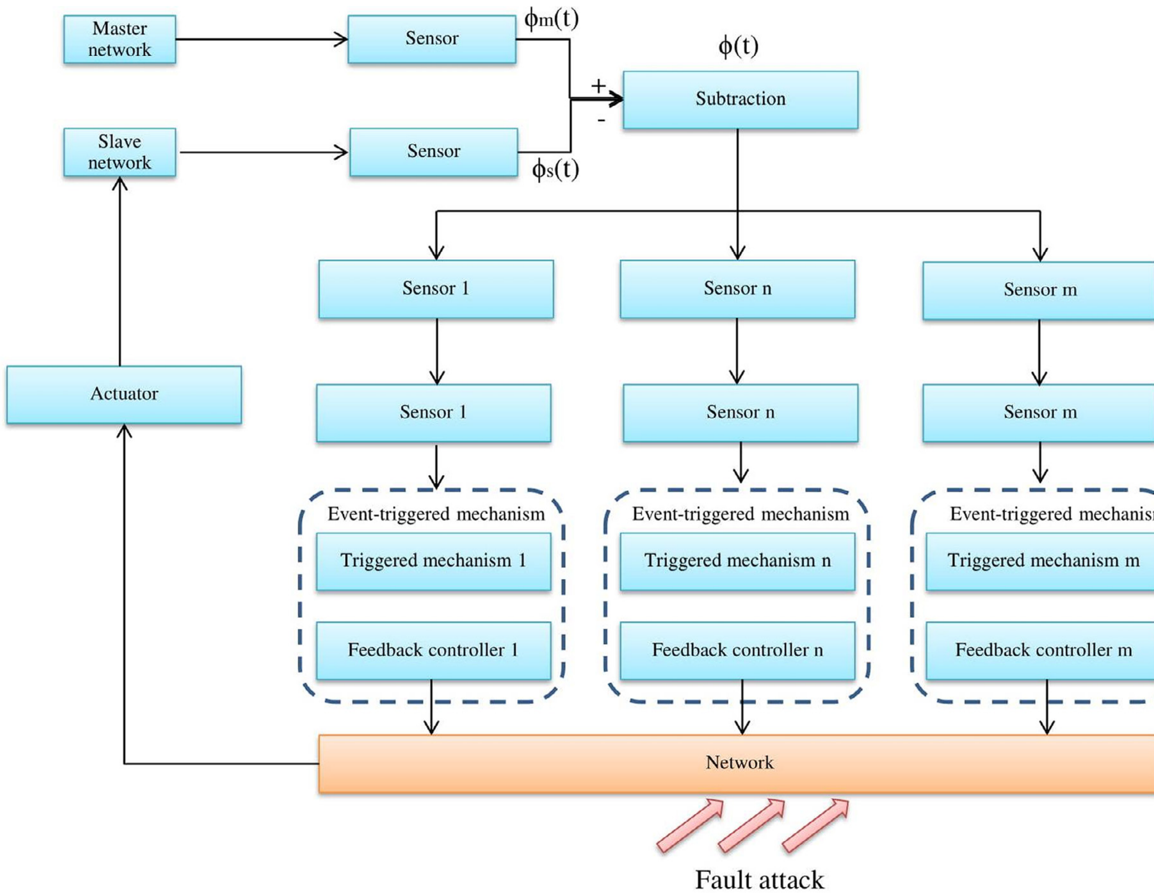

Fig. 1. The diagrammatic of master slave DETS.

on the framework of dissipativity, some effective results were investigated for continuous-time neural networks [22,23], discrete-time neural networks [24], and static neural networks [14] with stability and stabilization issues. For instance, in [22], authors proposed dissipative-based sampled-data synchronization control for complex dynamical networks with time-varying delay. Authors in [24], established a robust dissipative observer-based control design for discrete-time switched systems with time-varying delay. Event-triggered dissipative observer-based control for delay-dependent Takagi-Sugeno (T-S) fuzzy singular systems has been investigated in [23]. It is noted that the concept of dissipativity theory plays a vital part in analysis and control issues in the past years with respect to SNNs but only limited research has been done on the event-triggered control, which motivates our present research article. The diagrammatic of master-slave distributed event-triggered scheme (DETS) has been represented in Fig. 1.

The synchronization criteria is an active research area and it is a fascinating and remarkable one in numerous real systems. As reported, the interactions among a group of neurons may trigger synchronization, such as the synchronization of two neurons [25], the neural ensemble synchrony [26], and the partial synchronization of subsets of brain areas [27]. The synchronization has a large number of engineering background like biological model framework, combinational optimization, pattern recognition and harmonic oscillation generation, chemical systems, electrical circuits and systems, secure communications, image processing, parameter estimation, and neuroscience [27–30]. For example, the synchronization of two Hindmarsh-Rose neuronal models have been utilized for secure communications in the industrial internet of things in [29]. Recent studies on neural modeling demonstrated the influential role of synchrony between neuronal oscillators on constructing cognitive electronics in [31]. Recently, synchronization analysis for the static NNs get a lot of consideration, and some successful results for synchronization based control schemes with external disturbance (like dissipativity) have been proposed in the literatures. Accordingly, as of late, the dissipative synchronization concept has been successfully applied to various types of NNs in reports [28,30,32]. For example, authors in [32] studied event-triggered dissipative synchronization for Markovian jump neural networks with general transition probabilities. In [28], event-based synchronization control for memristive neural networks with time-varying delay has been proposed. Exponential synchronization of coupled stochastic memristor-based neural networks with time-varying probabilistic delay coupling and impulsive delay has been researched in [30]. Additionally, in pragmatic frameworks, resilient are typically inescapable in the controller performance. Because of this reality, the closed-loop system may be without a stable region. Subsequently, it is essential to plan a controller which need to permit the uncertain parameters. To defeat this trouble, resilient controller has been intended for different kinds of control systems (see [33–35]). It should be noted that, limited works have been expressed on resilient dissipative analysis in the research of synchronization techniques, which is the another direction of motivation in this work.

Note that the evolution of information technology and heavy transmission burdens in the efficiency of networks, an event-triggered control (ETC) has been actively engaged in research consideration. With advantages such as fast response, high reliability, easy implementation and robustness, ETS have been applied in military, manufacturing, transportation, NNs, network control systems, and social life [36–38]. Moreover, only when the event-triggering conditions are satisfied will the transmissions of the signals be triggered. In recent days, certain altered ETC has proved its usefulness to different system requirements. For example, decentralized ETC [39], adaptive ETC [36], distributed event-triggered scheme (DETS) [40–43], and deterministic ETC. So as to additionally diminish the release times, saving the network transmission resources and enhance the system dynamics behavior, nowadays researchers focusing on the distributed ETC. Many interesting results about event-triggered strategy have been obtained over the past few decades [40–43]. On another research frontier, controller failures has attracted more and more attention because of the difficulty of a dynamic system bigger, numerous components damaged by environment and machine errors could make a system having a failure [44–46]. Moreover, we will establish distributed event triggered scheme with the effect of controller failures is more effective to reduce the network transmission resources. To the best of author's knowledge, the dissipative synchronization for delayed static neural networks with distributed event-triggered control has not been studied so far, which motivates the present study.

Motivated by the above observations, this article focuses on the design of dissipative synchronization for delayed SNNs via DETS. The main contributions are summarized as below:

(i) An DETS is set up for the SNNs, which can successfully save network transmission resources and also enhance dynamic property of the system subject of controller failures.
(ii) A new dissipative error system model is proposed, which takes into account delayed SNNs, event-triggered schemes.
(iii) A suitable Lyapunov-Krasovski functional with single and double auxillary function-based integral inequalities (SAFBII, DAFBII), generalized free weighting matrix (GFM), Bessel-Legendre inequality approach is introduced to guarantee strict dissipative performance in the mean square of the error system, which provides the analysis framework of SNNs with time-varying delay.
(iv) The control gain matrices can be designed to achieve the desired performance by the use of the MATLAB LMIs Toolbox, and thus event-triggered parameters can be co-designed to guarantee the dissipative synchronization of the SNNs.
(v) In the end, the feasibility and advantage of the main results have been indicated by numerical examples section.

Notations: Let $\mathbb{R}^{n\times m}$ and $\mathbb{R}^n$ represent the set of all $n\times m$ real valued matrices and $n-$ dimensional Euclidean space. The expressions $X>0$ or $(X\geq 0)$ denote a positive definite or (semi-positive definite) matrix X, respectively; the superscripts T and -1 indicate that the transpose and inverse of a matrix. $*$ denotes the elements that are introduced due to corresponding symmetry. $\mathbb{N}$ represents the set of natural number. The space of square-integrable vector functions defined on $[0,\infty)$ is defined by $\mathcal{L}_2[0,\infty)$. I means the identity matrix of the appropriate dimensions, $Sym\{X\}=X+X^T$, $\otimes$ denotes the kronecker product, $diag\{\cdots\}$ means the block-diagonal matrix, and $\mathbb{E}\{\cdot\}$ is the expectation operator.

2. Preliminaries and problem formulation

Consider a class of SNNs consisting of interval-time varying delay:

$$\begin{cases}\dot{\phi}_m(t)=-A\phi_m(t)+f(H\phi_m(t-\rho(t))+I),\\ z_m(t)\ =D\phi_m(t),\end{cases}\tag{1}$$

where $\phi_m(t)=[\phi_{1m}^T(t),\phi_{2m}^T(t),\ldots,\phi_{nm}^T(t)]^T\in\mathbb{R}^n$ is the state of ith neuron with time t, $A=diag\{a_1,a_2,\ldots,a_n\}$ with $a_i>0,(i=1,2,\ldots,n)$ is the system known matrix, $f(H\phi_m(t))=[f_1^T(h_1\phi_{1m}(t)),f_2^T(h_2\phi_{2m}(t)),\ldots,f_n^T(h_n\phi_{nm}(t))]^T\in\mathbb{R}^n$ denotes the activation function of the neuron, $H\in\mathbb{R}^{n\times n}$ is the delayed connection weight matrix, D is the known matrix with compatible dimension, $I=[I_1,I_2,\ldots,I_n]^T\in\mathbb{R}^n$ is a external input vector, and $z_m(t)$ is the measurement output. $\rho(t)$ is a continuous and bounded differentiable function, which represent the time-varying delay and satisfies

$$\rho_1\leq\rho(t)\leq\rho_2,\ \ \dot{\rho}(t)\leq\rho_3,\tag{2}$$

where ρ_1,ρ_2, and ρ_3 are real constants. It is assumed that the neuron activation functions $f(\cdot)$ satisfy the following condition

$$(\mathbf{H_1})\quad \mathcal{G}_k^-\leq\frac{f_k(x_1)-f_k(x_2)}{x_1-x_2}\leq\mathcal{G}_k^+,\ \ k=1,2,\ldots,n.$$

For all $x_1,x_2\in\mathbb{R}$ and $x_1\neq x_2$. Throughout this paper, the master-slave scheme shall be adopted. We set system (1) to be a master system, and the relevant slave system is given as follows:

$$\begin{cases}\dot{\phi}_{\hat{s}}(t)=-A\phi_{\hat{s}}(t)+f(H\phi_{\hat{s}}(t-\rho(t))+I)+Bu(t)+C\omega(t),\\ z_{\hat{s}}(t)\ =D\phi_{\hat{s}}(t),\end{cases}\tag{3}$$

where $\phi_{\hat{s}}(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^n$ and $\omega(t)\in\mathbb{R}^p$ are the state vectors, control input and exogenous external disturbance, respectively, and $\omega(t)$ belongs to $\mathcal{L}_2[0,\infty)$. B and C are known matrices with appropriate dimensions.

Set $\phi(t)=\phi_m(t)-\phi_{\hat{s}}(t)$ and $z(t)=z_m(t)-z_{\hat{s}}(t)$, then we are able to write the synchronization-error system as follows:

$$\begin{cases}\dot{\phi}(t)=-A\phi(t)+g(H\phi(t-\rho(t)))+Bu(t)+C\omega(t)\\ z(t)\ =D\phi(t),\end{cases}\tag{4}$$

where $g(H\phi(t-\rho(t)))=f(H\phi_m(t-\rho(t))+I)-f(H\phi_{\hat{s}}(t-\rho(t))+I)$. Assume that the system (4) is controlled over a communication network by resorting to a novel event-triggering technique. Let $t_0\hbar,t_1\hbar,t_2\hbar,\ldots$ defined as the release times, which satisfy the event triggered condition and can be sent to the transmission channel. Therefore (4) can be explained as

$$\begin{cases}\dot{\phi}(t)=-A\phi(t)+g(H\phi(t-\rho(t)))+Bu(t_s\hbar)+C\omega(t),\\ z(t)\ =D\phi(t).\end{cases}$$

The goal of this work is to design an event-triggered controller as $u(t)=\mathcal{K}\phi(t)$

$$\begin{cases}\dot{\phi}(t)=-A\phi(t)+g(H\phi(t-\rho(t)))+B\mathcal{K}\phi(t_s\hbar)+C\omega(t),\\ z(t)\ =D\phi(t),\end{cases}\tag{5}$$

where $\phi(t_s\hbar)=[\phi_1^T(t_s^1\hbar)\ \phi_2^T(t_s^2\hbar)\ldots\phi_n^T(t_s^n\hbar)]^T$ and $\mathcal{K}$ denotes the control gain matrix and to be designed later. Assume that $\eta_s^j\in(0,\eta)$ denoted as the transmission delay, $\eta>0$ is a scalar, $j=1,2,\ldots,n$, $s\in\mathbb{N}$. The released state $\phi_j(t_s^j\hbar)$ occurs at the actuator at the time $t_s^j\hbar+\eta_s^j$. Subsequently, the designed system together with the following network transmission delay can be described as:

$$\Theta_s^j=min\{\delta|t_s^j+\eta_s^j+\delta\hbar\geq t_{s+1}^j+\eta_{s+1}^j,\ \ \delta=0,1,2,\ldots\}.\tag{6}$$

Let

$$\begin{aligned}&\mathbb{H}_\delta^j=[t_s^j\hbar+\eta_s^j+(\delta-1)\hbar,t_s^j\hbar+\eta_s^j+\delta\hbar),\\ &\mathbb{H}_{\Theta s}^j=[t_s^j\hbar+\eta_s^j+(\Theta_s^j-1)\hbar,t_{s+1}^j\hbar+\eta_{s+1}^j),\\ &\quad\delta=1,2,\ldots,\Theta_k^j-1.\end{aligned}\tag{7}$$

Moreover,

$$[t_s^j\hbar+\eta_s^j,t_{s+1}^j\hbar+\eta_{s+1}^j)=\bigcup_{\delta=1}^{\Theta_s^j}\mathbb{H}_\delta^j.\tag{8}$$

For $[t_s^j\hbar+\eta_s^j, t_{s+1}^j\hbar+\eta_{s+1}^j)$, define that

$$\eta^j(t)=\begin{cases} t-t_s^j\hbar, & t\in\mathbb{H}_1^j,\\ t-t_s^j\hbar-\hbar, & t\in\mathbb{H}_2^j,\\ \vdots & \vdots\\ t-t_s^j\hbar-(\Theta_s^j-1)\hbar, & t\in\mathbb{H}_{\Theta s}^j,\end{cases} \tag{9}$$

$$\nu_s(t)=\begin{cases} 0, & t\in\mathbb{H}_1^j,\\ \phi(t_s^j\hbar)-\phi(t_s^j\hbar+\hbar), & t\in\mathbb{H}_2^j,\\ \vdots & \vdots\\ \phi(t_s^j\hbar)-\phi(t_s^j\hbar+(\Theta_s^j-1)\hbar), & t\in\mathbb{H}_{\Theta s}^j.\end{cases} \tag{10}$$

Moreover, $0<\eta_s^j\le\eta^j(t)\le\eta$. In addition, we introduce the time-varying parameter as follows $\varrho_s(t)$, $\varrho_s(t)=diag\{\varrho_s^1(t),\varrho_s^2(t),\dots,\varrho_s^n(t)\}$. Here

$$\varrho_s^j(t)=\varrho_s^j(s\delta),\ t\in\mathbb{H}_\delta^j,\ j=1,2,\dots,n,$$

$$\varrho_{s(\delta+1)}^j=\bar{\varrho}+\frac{(\nu_{s\delta}^j)^T\nu_{s\delta}^j}{\epsilon+(\nu_{s\delta}^j)^T\nu_{s\delta}^j}(\nu_{s\delta}^j-\bar{\varrho}),$$

$$\nu_{s\delta}^j=\phi^j(t_s\hbar)-\phi^j(t_s\hbar+(\delta-1)\hbar),\ \delta=1,2,\dots,\Theta_s^j, \tag{11}$$

where $\epsilon>0$ denoted as constant, $\varrho_{s\delta}^j=diag\{\varrho_{s\delta}^1,\varrho_{s\delta}^2,\dots,\varrho_{s\delta}^n\}$, $\bar{\varrho}$ noted as upper bound of $\varrho_{s\delta}^1$, and $\bar{\varrho}\in[0,1)$, $0\le\varrho_{s1}^j<\bar{\varrho}$. Next, we consider the ETS together with time-varying term $\varrho_s(t)$, $t\in[t_s\hbar+\eta_s, t_{s+1}\hbar+\eta_{s+1})$.

$$\nu_s^T(t)\tilde{\chi}\nu_s(t)\le\bar{\varrho}\phi^T(t-\eta(t))\tilde{\chi}\phi(t-\eta(t)), \tag{12}$$

where $\phi(t-\eta(t))=[\phi_1^T(t-\eta^1(t)),\phi_2^T(t-\eta^2(t)),\dots,\phi_n^T(t-\eta^n(t))^T]$ and $\tilde{\chi}$ is the symmetric positive definite matrix.

Together with $\nu_s(t)$ and $\eta(t)$, for $t\in[t_s\hbar+\eta_s, t_{s+1}\hbar+\eta_{s+1})$, we can rewrite the control input as

$$u(t)=\mathcal{K}\phi(t-\eta(t))+\mathcal{K}\nu_s(t). \tag{13}$$

Moreover, the designed controller can be utilized with some faults and can be presented as follows:

$$u(t)=\beta(t)[\mathcal{K}\phi(t-\eta(t))+\mathcal{K}\nu_s(t)], \tag{14}$$

where $\beta(t)$ is a random variable can be described as follows:

$$\beta(t)=\begin{cases}1, & t\in[\hat{\kappa}(\mu_a+\mu_b),\hat{\kappa}(\mu_a+\mu_b)+\mu_b),\ \hat{\kappa}\in\mathbb{N}\\ 0, & t\in[\hat{\kappa}(\mu_a+\mu_b)+\mu_b,\hat{\kappa}+1(\mu_a+\mu_b)),\end{cases}$$

$$0<\eta(t)\le\eta, \tag{15}$$

where $\mu_a>0$ and $\mu_b>0$ noted as constants and termed as dwell times of $\beta(t)$ have unique values.

Therefore, (5) could be written as

$$\begin{cases}\dot{\phi}(t)=-A\phi(t)+g(H\phi(t-\rho(t))+\hat{u}(t)+C\omega(t),\\ \hat{u}(t)=\beta(t)[\mathcal{K}\phi(t-\eta(t))+\mathcal{K}\nu_s(t)],\\ z(t)=D\phi(t),\end{cases} \tag{16}$$

The aim of this paper is to propose new linear matrix inequality (LMI) based $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma-$ dissipative conditions for SNNs (16) with time-varying delays (2). To this end, the following Definiton and Lemmas will be useful.

Definition 2.1. [5] The system (16) is said to be strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma-$ dissipative for any $t_f\ge0$ and some scalar $\gamma>0$, the following inequality holds under zero initial conditions [21]:

$$\int_0^{t_f}u(\omega(t),z(t))dt\ge\gamma\int_0^{t_f}\omega^T(t)\omega(t)dt\ \forall t_f>0,$$

where $u(\omega(t),z(t))$ with $u(0,0)=0$ is a defined energy supply rate function for the system (16) and satisfies

$$u(\omega(t),z(t))=z^T(t)\mathcal{Q}z(t)+2z^T(t)\mathcal{S}\omega(t)+\omega^T(t)\mathcal{R}\omega(t),$$

where $\mathcal{Q}$, $\mathcal{S}$, and $\mathcal{R}$ are real matrices with $\mathcal{Q}^T=\mathcal{Q}$ and $\mathcal{R}^T=\mathcal{R}$, without loss of generality, it is assumed that $\mathcal{Q}\le0$.

Lemma 2.2. [6] *Let $Z>0$ and for given scalars α and β, the following relations are well defined for any differentiable function u in $[\alpha,\beta]\to\mathbb{R}^n$*

$$-\int_\alpha^\beta\dot{u}^T(s)Z\dot{u}(s)ds\le-\frac{1}{\beta-\alpha}\varphi_1^TZ\varphi_1-\frac{3}{\beta-\alpha}\varphi_2^TZ\varphi_2,$$

$$-\int_\alpha^\beta\int_\lambda^\beta\dot{u}^T(s)Z\dot{u}(s)dsd\lambda\le-2\varphi_3^TZ\varphi_3-4\varphi_4^TZ\varphi_4,$$

$$-\int_\alpha^\beta\int_\alpha^\lambda\dot{u}^T(s)Z\dot{u}(s)dsd\lambda\le-2\varphi_5^TZ\varphi_5-4\varphi_6^TZ\varphi_6,$$

where

$$\varphi_1=u(\beta)-u(\alpha),\ \varphi_2=u(\beta)+u(\alpha)-\frac{2}{\beta-\alpha}\int_\alpha^\beta u(s)ds,$$

$$\varphi_3=u(\beta)-\frac{1}{\beta-\alpha}\int_\alpha^\beta u(s)ds,\ \varphi_5=u(\alpha)-\frac{1}{\beta-\alpha}\int_\alpha^\beta u(s)ds,$$

$$\varphi_4=u(\beta)+\frac{2}{\beta-\alpha}\int_\alpha^\beta u(s)ds-\frac{6}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\lambda^\beta u(s)dsd\lambda,$$

$$\varphi_6=u(\alpha)-\frac{4}{\beta-\alpha}\int_\alpha^\beta u(s)ds+\frac{6}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\lambda^\beta u(s)dsd\lambda.$$

Lemma 2.3. [4] *For symmetric positive definite matrix $Q\in\mathbb{R}^{n\times n}$, any matrices X,Y, and vector $\phi:[\alpha,\beta]\to\mathbb{R}^n$ such that the integration concerned is well defined and the following inequality holds*

$$\int_\alpha^\beta\phi^T(s)Q\phi(s)ds\ge Sym\{\eta_a^TX\eta_b+\eta_a^TY\eta_c\}-(\beta-\alpha)\eta_a^T\left(\frac{3XQ^{-1}X^T+YQ^{-1}Y^T}{3}\right)\eta_a,$$

where η_a is any vector and η_b, η_c are defined as

$$\eta_b=\int_\alpha^\beta\phi(s)ds\ \text{and}\ \eta_c=-\eta_b+\frac{2}{\beta-\alpha}\int_\alpha^\beta\int_s^\beta\phi(u)duds.$$

Lemma 2.4. [40] *For any $u\in[0,1]$, $s,r\in\mathbb{N}$, the shifted Lengendre polynomial is*

$$y_s(u)=(-1)^s\sum_{r=0}^s\hat{\rho}_r^su^r,$$

where $\hat{\rho}_r^s=(-1)^r\binom{s}{r}\binom{s+r}{r}$ and the binomial co-efficient $\binom{s}{r}=\frac{s!}{((s-r)!r!)}$. Correspondingly, the polynomial matrix

$$\mathbb{T}_p(u)=[y_0(u)I_n,y_1(u)I_n,\dots,y_p(u)I_n]^T,$$

where $n\in\mathbb{N}$, $p\in\mathbb{N}$. Moreover, $\mathbb{T}_p(0)$ and $\mathbb{T}_p(1)$ can be defined as

$$\mathbb{T}_p(0)=\begin{bmatrix}I_n\\-I_n\\\vdots\\(-1)^pI_n\end{bmatrix}=\hat{\mho}_a,\ \mathbb{T}_p(1)=\begin{bmatrix}I_n\\I_n\\\vdots\\I_n\end{bmatrix}=\mho_a. \tag{17}$$

Lemma 2.5. [4] *For any constant matrix $M\in\mathbb{R}^{n\times n}$, $M=M^T>0$, scalar $\eta>0$, vector function $w:[0,\eta]\to\mathbb{R}^n$ such that the following relation holds*

$$\eta\int_0^\eta w^T(s)Mw(s)ds\le\left[\int_0^\eta w(s)ds\right]^TM\left[\int_0^\eta w(s)ds\right].$$

Definition 2.6. [19] The system (16) is stated that passive, for any $t_f \geq 0$ and all solutions of (16) with $\phi(0) = 0$, there exist a scalar $\gamma > 0$, ensure that the following inequality

$$2\int_0^{t_f} z^T(s)w(s)ds \geq -\gamma \int_0^{t_f} w^T(s)w(s)ds$$

is satisfied under the zero initial condition.

Remark 2.7. The derivative of Lengendre polynomial matrix in Lemma 2.4, can be defined as follows:

$$\frac{d}{du}\mathbb{T}_p(u) = \bar{\Lambda}_p\mathbb{T}_p(u) = \Lambda_p\mathbb{T}_{p-1}(u),$$

$$\frac{d}{du}(u\mathbb{T}_p(u)) = \mathbb{T}_p(u) + \Psi_p\mathbb{T}_p(u),$$

where $\bar{\Lambda}_p = [\Lambda_p \ \ 0_{n(p+1),n}]$, $\Lambda_p = \nu_p \otimes I_n$, and $\Psi_p = \hat{e}_p \otimes I_n$. The matrices $\nu_p \in \mathbb{R}^{p+1\times p}$ and $\hat{e}_p \in \mathbb{R}^{p+1\times p+1}$ are defined as follows

$$\nu_p(s,r) = \begin{cases} 0, & if\ s \geq r, \\ (2s-1)(1-(-1)^{s+r}), & if\ s < r, \end{cases}$$

$$\hat{e}_p(s,r) = \begin{cases} 0, & if\ s > r, \\ s, & if\ s = r, \\ 2s-1, & if\ s < r. \end{cases}$$

3. Main results

3.1. Strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ dissipative synchronization analysis

In this part, we will discuss dissipativity criteria for SNNs (16) with event-triggered controller. Based on the framework of Lyapunov-Krasovskii functional (LKF) and the integral inequality approaches, we will give the $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ dissipative conditions in the following Theorems 3.1 and 3.3. Moreover, we represent the block entry matrices.

$e_j = [0_{n\times(j-1)n}\ I_{n\times n}\ 0_{n\times(35-j)n}]^T \in \mathbb{R}^{35n\times n}$, $j = 1, 2, \ldots, 35$, for example

$$e_5 = [0\ 0\ 0\ 0\ I\ \underbrace{0\ 0\ 0}_{30\ times}],\ \eta_1(t) = [e_1^T\ e_{28}^T]^T = \mathbb{E}_1^T,$$

$$\hat{\phi}(t) = [e_1^T\ e_5^T]^T,\ \ \eta_2(t) = [e_1^T\ \ e_8^T]^T,\ \hat{\mathbb{S}} = g^T(H\phi(s)),$$

$$\eta_3(t) = [e_1^T\ \ e_7^T]^T,\ \ \rho_{1t} = \rho(t) - \rho_1,\ \ \rho_{2t} = \rho_2 - \rho(t),$$

$$\zeta_1^T(t) = [\phi^T(t)\ \phi^T(t-\rho(t))\ \phi^T(t-\rho_1)\ \phi^T(t-\rho_2)\ \dot{\phi}^T(t)\ \ \frac{1}{\rho(t)}\int_{t-\rho(t)}^t \phi^T(s)ds\ \ \frac{1}{\rho_{2t}}\int_{t-\rho_2}^{t-\rho(t)} \phi^T(s)ds],$$

$$\zeta_2^T(t) = [\frac{1}{\rho_{1t}}\int_{t-\rho(t)}^{t-\rho_1} \phi^T(s)ds\ \ \frac{1}{\rho^2(t)}\int_{t-\rho(t)}^t\int_\theta^t \phi^T(s)dsd\theta\ \ \aleph_1\ \aleph_2\ \int_{t-\lambda\rho_1}^t \phi^T(s)ds\ \int_{t-\lambda\rho_1}^t \hat{\mathbb{S}}ds\ \int_{t-\rho_2}^{t-\lambda\rho_2} \phi^T(s)ds],$$

$$\zeta_3^T(t) = [\int_{t-\rho_2}^{t-\lambda\rho_2} \hat{\mathbb{S}}ds\ \int_{t-\rho(t)}^{t-\rho_1} \hat{\mathbb{S}}ds\ \int_{t-\rho_2}^{t-\rho(t)} \hat{\mathbb{S}}ds\ \ \frac{1}{\rho_1}\int_{t-\rho_1}^t \phi^T(s)ds\ \ \frac{1}{\rho_{21}}\int_{t-\rho_2}^{t-\rho_1} \phi^T(s)ds\ \aleph_3\ \aleph_4\ \phi^T(t-\eta(t))],$$

$$\zeta_4^T(t) = [\phi^T(t-\eta)\ \ \frac{1}{\eta(t)}\int_{t-\eta(t)}^t \phi^T(s)ds\ \ \frac{1}{\eta-\eta(t)}\int_{t-\eta_1}^{t-\eta(t)} \phi^T(s)ds\ \aleph_5\ \aleph_6],$$

$$\zeta_5^T(t) = [g^T(H\phi(t))\ g^T(H(\phi(t-\rho_1)))\ g^T(H(\phi(t-\rho(t))))\ \ g^T(H(\phi(t-\rho_2)))\ \nu_s^T(t)\ \tilde{\Omega}_{11}\ \tilde{\Omega}_{12}\ w(t)]$$

$$\zeta^T(t) = [\zeta_1^T(t)\ \zeta_2^T(t)\ \zeta_3^T(t)\ \zeta_4^T(t)\ \zeta_5^T(t)],\ \ \rho_{21} = \rho_2 - \rho_1$$

$$\aleph_1 = \frac{1}{(\rho(t)-\rho_1)^2}\int_{t-\rho(t)}^{t-\rho_1}\int_\theta^{t-\rho_1} \phi^T(s)dsd\theta,$$

$$\aleph_2 = \frac{1}{(\rho_2-\rho(t))^2}\int_{t-\rho_2}^{t-\rho(t)}\int_\theta^{t-\rho(t)} \phi^T(s)dsd\theta,$$

$$\aleph_3 = \frac{1}{\rho_1^2}\int_{-\rho_1}^0\int_{t+\theta}^t \phi^T(s)dsd\theta,$$

$$\aleph_4 = \frac{1}{(\rho_2-\rho_1)^2}\int_{-\rho_2}^{-\rho_1}\int_{t+\theta}^t \phi^T(s)dsd\theta,$$

$$\aleph_5 = \frac{1}{(\eta(t))^2}\int_{t-\eta(t)}^t\int_u^t \phi^T(s)dsdu,$$

$$\aleph_6 = \frac{1}{(\eta-\eta(t))^2}\int_{t-\eta}^{t-\eta(t)}\int_u^{t-\eta(t)} \phi^T(s)dsdu,$$

$$\lambda_1(t) = \Big[\phi^T(t)\ \int_{t-\rho_1}^t \phi^T(s)ds\ \int_{t-\rho_2}^{t-\rho_1} \phi^T(s)ds\ \frac{1}{\rho_1}\int_{-\rho_1}^0\int_{t+\theta}^t \times \phi^T(s)dsd\theta\ \ \frac{1}{\rho_2-\rho_1}\int_{t-\rho_2}^{t-\rho_1}\int_{t+\theta}^t \phi^T(s)dsd\theta\Big]^T,$$

$$\mathbb{P} = [\mathbb{P}_{ij}], \hat{U}_2 = \begin{bmatrix} U_2^{11} & U_2^{12} \\ & U_2^{22} \end{bmatrix}, \hat{U}_3 = \begin{bmatrix} U_3^{11} & U_3^{12} \\ & U_3^{22} \end{bmatrix},$$

$(i, j = 1, 2, \ldots, 15)$.

Theorem 3.1. *Under Assumption* ($\mathbf{H_1}$), *for given matrix* $\mathcal{K}$ *and scalars* ρ_1, ρ_2, ρ_3, η, $\bar{\varrho}$ *and* $0 < \lambda < 1$, *if there exist positive symmetric matrices* $\mathbb{P} \in \mathbb{R}^{5n\times 5n}, R_1, R_2, R_3, Q_1, Q_2, \mathcal{V} \in \mathbb{R}^{2n\times 2n}, \hat{U}_i \in \mathbb{R}^{2n\times 2n}, T_1, T_i, i = 2, 3, 4, \mathcal{W}_1, \mathcal{W}_2, S, S_1, S_2, S_3, W_3, W_4, \mathcal{W}, \hat{\mathcal{V}}_a, \hat{\mathcal{V}}_b \in \mathbb{R}^{n\times n}$, *positive diagonal matrices* $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$, *and any compatible matrices* $\mathcal{F}_i$, $\mathcal{X}_i, \mathcal{Y}_i \in \mathbb{R}^{9n\times 2n}$, $\tilde{\chi} > 0, i = 1, 2,$ *such that the follow matrix inequalities are hold*

$$\begin{bmatrix} \hat{U}_4 & \mathcal{W} \\ & \hat{U}_4 \end{bmatrix} \geq 0,$$

$$\Sigma = \begin{bmatrix} \Pi & \Gamma_{12} \\ * & -(\mathcal{R} - \gamma I) \end{bmatrix} < 0, \tag{18}$$

where $\Gamma_{12} = [\mathcal{F}_1 C^T\ \ 0\ \ 0\ \ 0\ \ \mathcal{F}_2 C^T\ \ \overbrace{0\ 0\ 0}^{30\ times}]^T$,

and the elements of the matrix $\Pi = \sum_{i=0}^{10}\Pi_i$ *are represented in Appendix A then, for any initial condition, the error system (16) is dissipative. That is, the slave system (3) and master system (1) are* $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ *dissipative synchronous via DETS, such that for* $\rho(t) \in [\rho_1, \rho_2]$ *and* $\beta(t) = \{0, 1\}$.

Proof. Choose a LKF candidate for system (16) as follows:

$$V(t) = \sum_{i=1}^7 V_i(t) + \tilde{V}(t), \tag{19}$$

where

$$V_1(t) = \lambda_1^T(t)\mathbb{P}\lambda_1(t),$$

$$V_2(t) = \int_{t-\rho_1}^t \phi^T(s)R_1\phi(s)ds + \int_{t-\rho(t)}^{t-\rho_1} \phi^T(s)R_2\phi(s)ds + \int_{t-\rho_2}^t \phi^T(s)R_3\phi(s)ds,$$

$$V_3(t) = \rho_2\int_{t-\rho_2}^t\int_{t+\beta}^t \dot{\phi}^T(s)Q_1\dot{\phi}(s)dsd\beta + (\rho_2-\rho_1)\int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^t \dot{\phi}^T(s)Q_2\dot{\phi}(s)dsd\beta + \int_{t-\rho_1}^{t-\rho_2}\int_\beta^{t-\rho_1} \hat{\phi}^T(s)\mathcal{V}\hat{\phi}(s)dsd\beta,$$

$$V_4(t) = \lambda\rho_1 \int_{t-\lambda\rho_1}^{t}\int_{\beta}^{t} \eta_1^T(s)\hat{U}_2\eta_1(s)dsd\beta + (\rho_2-\lambda\rho_2)\int_{t-\rho_2}^{t-\lambda\rho_2}\int_{\beta}^{t}\eta_1^T(s)\hat{U}_3\eta_1(s)dsd\beta + (\rho_2-\rho_1)\int_{-\rho_2}^{\rho_1}\int_{\beta}^{t}\eta_1^T(s)\hat{U}_4\eta_1(s)dsd\beta,$$

$$V_5(t) = \int_{-\rho_1}^{0}\int_{-\rho_1}^{\theta}\int_{t+\beta}^{t}\dot{\phi}^T(s)T_1\dot{\phi}(s)dsd\beta d\theta + \int_{-\rho_1}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{\phi}^T(s)T_2\dot{\phi}(s)dsd\beta d\theta + \int_{-\rho_2}^{-\rho_1}\int_{-\rho_2}^{\theta}\int_{t+\beta}^{t}\dot{\phi}^T(s)T_3\dot{\phi}(s)dsd\beta d\theta + \int_{-\rho_2}^{-\rho_1}\int_{\theta}^{-\rho_1}\int_{t+\beta}^{t}\dot{\phi}^T(s)T_4\dot{\phi}(s)dsd\beta d\theta,$$

$$V_6(t) = (\rho(t)-\rho_1)\eta_2^T(t)\mathcal{W}_1\eta_2(t) + (\rho_2-\rho(t))\eta_3^T(t)\mathcal{W}_2\eta_3(t),$$

$$V_7(t) = \eta_4^T(t)S\eta_4(t) + \int_{t-\eta(t)}^{t}\phi^T(s)S_1\phi(s)ds + \int_{t-\eta}^{t-\eta(t)}\phi^T(s)S_2\phi(s)ds + \eta\int_{t-\eta}^{t}\int_{\beta}^{t}\dot{\phi}^T(s)S_3\dot{\phi}(s)dsd\beta,$$

$$\tilde{V}(t) = \rho_2\int_{t-\rho_2}^{t}\phi^T(s)W_3\phi(s)ds - \int_{t-\rho_2}^{t}\phi^T(s)dsW_3 \times \int_{t-\rho_2}^{t}\phi(s)ds + \rho_2\int_{t-\rho_2}^{t}g^T(H\phi(s))W_4g(H\phi(s))ds - \int_{t-\rho_2}^{t}g^T(H\phi(s))dsW_4\int_{t-\rho_2}^{t}g(H\phi(s))ds.$$

The derivatives of $V_i(t)$ and $\tilde{V}$ in terms of t together with the trajectories of system (16), where $i = 1, 2, \ldots, 7$ yield

$$\dot{V}(t) = \sum_{i=1}^{7}\dot{V}_i(t) + \tilde{V} < 0, \tag{20}$$

where

$$\dot{V}_1(t) = Sym(\lambda_1^T(t)\mathbb{P}\dot{\lambda}_1(t)) = \zeta^T(t)\Pi_0\zeta(t),$$

$$\dot{V}_2(t) \le \phi^T(t)R_1\phi(t) + \phi^T(t-\rho_1)(R_2-R_1)\phi(t-\rho_1) - (1-\rho_3)\phi^T(t-\rho(t))R_2\phi(t-\rho(t)) + \phi^T(t)R_3\phi(t) - \phi^T(t-\rho_2)R_3\phi(t-\rho_2), \le \zeta^T(t)\Pi_1\zeta(t),$$

$$\dot{V}_3(t) \le \rho_2^2\dot{\phi}^T(t)Q_1\dot{\phi}(t) + \rho_{21}^2\dot{\phi}^T(t)Q_2\dot{\phi}(t) - \rho_2\int_{t-\rho_2}^{t}\dot{\phi}^T(s)Q_1\dot{\phi}(s)ds - \rho_{21}\int_{t-\rho_1}^{t-\rho_2}\dot{\phi}^T(s) \times Q_2\dot{\phi}(s)ds + \rho_{21}\hat{\phi}^T(t)\mathcal{V}\hat{\phi}(t) - \int_{t-\rho_2}^{t-\rho_1}\hat{\phi}^T(s)\mathcal{V}\hat{\phi}(s)ds,$$

Utilizing Lemma 2.2, we get

$$-\rho_2\int_{t-\rho(t)}^{t}\dot{\phi}^T(s)Q_1\dot{\phi}(s)ds - \rho_2\int_{t-\rho_2}^{t-\rho(t)}\dot{\phi}^T(s)Q_1\dot{\phi}(s)ds$$
$$\le -\zeta^T(t)[(e_1-e_2)Q_1(e_1-e_2)^T + 3(e_1+e_2-2e_6) \times Q_1(e_1+e_2-2e_6)^T + 5(e_1-e_2-6e_6+12e_9) \times Q_1(e_1-e_2-6e_6+12e_9)^T + (e_2-e_4)Q_1 \times (e_2-e_4)^T + 3(e_2+e_4-2e_7)Q_1(e_2+e_4-2e_7)^T + 5(e_2-e_4-6e_7+12e_{11})Q_1(e_2-e_4-6e_7+12e_{11})^T]\zeta(t)$$

Similar to the above case, we get

$$\rho_{21}\int_{t-\rho_1}^{t-\rho_2}\dot{\phi}^T(s)Q_2\dot{\phi}(s)ds$$
$$\le -\zeta^T(t)[(e_3-e_2)Q_2(e_3-e_2)^T + 3(e_3+e_2-2e_8) \times Q_2(e_3+e_2-2e_8)^T + 5(e_3-e_2-6e_8+12e_{10}) \times Q_2(e_3-e_2-6e_8+12e_{10})^T + (e_2-e_4)Q_2 \times (e_2-e_4)^T + 3(e_2+e_4-2e_7)Q_2(e_2+e_4-2e_7)^T + 5(e_2-e_4-6e_7+12e_{11})Q_2(e_2-e_4-6e_7+12e_{11})^T]\zeta(t).$$

We can establish the subsequent zero-value term to further reduce the conservatism of the system (16)

$$\zeta^T(t)[e_2\hat{\mathcal{V}}_a e_2^T - e_4\hat{\mathcal{V}}_a e_4^T]\zeta(t) - 2\int_{t-\rho_2}^{t-\rho(t)}\hat{\phi}^T(s)\hat{\mathcal{V}}_a\dot{\hat{\phi}}(s)ds = 0,$$
$$\zeta^T(t)[e_3\hat{\mathcal{V}}_b e_3^T - e_2\hat{\mathcal{V}}_b e_2^T]\zeta(t) - 2\int_{t-\rho(t)}^{t-\rho_1}\hat{\phi}^T(s)\hat{\mathcal{V}}_b\dot{\hat{\phi}}(s)ds = 0, \tag{21}$$

where $\hat{\mathcal{V}}_a = \hat{\mathcal{V}}_a^T$ and $\hat{\mathcal{V}}_b = \hat{\mathcal{V}}_b^T$ are symmetric matrices, combining (21), we get

$$= \zeta^T(t)\hat{\mho}\zeta^T(t) - \int_{t-\rho_2}^{t-\rho(t)}\hat{\phi}^T(s)\bar{\mathcal{V}}_a\hat{\phi}(s)ds - \int_{t-\rho(t)}^{t-\rho_1}\hat{\phi}^T(s)\bar{\mathcal{V}}_b\hat{\phi}(s)ds,$$

$\hat{\mho} = \rho_{21}\begin{bmatrix} e_1 \\ e_5 \end{bmatrix}\mathcal{V}\begin{bmatrix} e_1 \\ e_5 \end{bmatrix}^T - e_2[\bar{\mathcal{V}}_a + \bar{\mathcal{V}}_b]e_2^T + e_3\bar{\mathcal{V}}_b e_3^T - e_4\bar{\mathcal{V}}_a e_4^T$. Next, for any matrices $\mathcal{X}_i, \mathcal{Y}_i \in \mathbb{R}^{9n\times 2n}$, i = 1,2, letting $\mathcal{X}_0$ be

$$\mathcal{X}_0 = [\phi(t)\ g(H\phi(t))\ \phi(t-\rho(t))\ g(H\phi(t-\rho(t)))\ e_7\ e_8\ e_9\ e_{10}\ \omega(t)] = \hat{\Pi}_a\zeta(t),$$

By Lemma 2.3 to calculate the single integral term $\dot{V}_3(t)$, we get

$$-\int_{t-\rho_2}^{t-\rho(t)}\hat{\phi}^T(s)\bar{\mathcal{V}}_a\hat{\phi}(s)ds - \int_{t-\rho(t)}^{t-\rho_1}\hat{\phi}^T(s)\bar{\mathcal{V}}_b\hat{\phi}(s)ds \le \zeta^T(t)[sym\{\hat{\Pi}_1\} + \gamma_a + \gamma_b]\zeta(t),$$

where

$$\hat{\Pi}_1 = \hat{\Pi}_a^T\mathcal{X}_1\gamma_1 + \hat{\Pi}_a^T\mathcal{Y}_1\gamma_2 + \hat{\Pi}_a^T\mathcal{X}_2\gamma_3 + \hat{\Pi}_a^T\mathcal{Y}_2\gamma_4,$$
$$\gamma_1 = [(\rho(t)-\rho_1)e_8^T \quad e_3^T - e_2^T]^T,$$
$$\gamma_2 = [(\rho(t)-\rho_1)^2e_8^T + 2e_{10}^T \quad e_3^T + e_2^T - 2e_8^T]^T,$$
$$\gamma_3 = [(\rho_2-\rho(t))e_7^T \quad e_2^T - e_4^T]^T,$$
$$\gamma_4 = [(\rho_2-\rho(t))^2e_7^T + 2e_{11}^T \quad e_2^T + e_4^T - 2e_{11}^T]^T,$$
$$\gamma_a = (\rho(t)-\rho_1)[\hat{\Pi}_a^T\mathcal{X}_1\bar{\mathcal{V}}_a^{-1}\mathcal{X}_1^T\hat{\Pi}_a + \frac{1}{3}\hat{\Pi}_a^T\mathcal{Y}_1\bar{\mathcal{V}}_a^{-1}\mathcal{Y}_1^T\hat{\Pi}_a],$$
$$\gamma_b = (\rho_2-\rho(t))[\hat{\Pi}_a^T\mathcal{X}_2\bar{\mathcal{V}}_b^{-1}\mathcal{X}_2^T\hat{\Pi}_a + \frac{1}{3}\hat{\Pi}_a^T\mathcal{Y}_2\bar{\mathcal{V}}_b^{-1}\mathcal{Y}_2^T\hat{\Pi}_a],$$
$$\hat{\Pi}_a = [e_1^T \quad e_2^T \quad e_{28}^T \quad e_{29}^T \quad e_7^T \quad e_8^T \quad e_{11}^T \quad e_{10}^T \quad e_{35}^T]^T.$$

$$\dot{V}_3(t) \le \zeta^T(t)\Pi_2\zeta(t).$$

$$\dot{V}_4(t) = \lambda^2\rho_1^2\eta_1^T(t)\hat{U}_2\eta_1(t) - \lambda\rho_1\int_{t-\lambda\rho_1}^{t}\eta_1^T(s)\hat{U}_2\eta_1(s)ds + (\rho_2-\lambda\rho_2)^2\eta_1^T(t)\hat{U}_3\eta_1(t) - \rho_{21}\int_{t-\rho_2}^{t-\rho_1}\eta_1^T(s)\hat{U}_4\eta_1(s)ds + \rho_{21}^2\eta_1^T(t)\hat{U}_4\eta_1(t) - (\rho_2-\lambda\rho_2)\int_{t-\rho_2}^{t-\lambda\rho_2}\eta_1^T(s)\hat{U}_3\eta_1(s)ds.$$

By utilizing Lemma 2.5, it obtains

$$-\lambda\rho_1\int_{t-\lambda\rho_1}^{t}\eta_1^T(s)\hat{U}_2\eta_1(s)ds - (\rho_2-\lambda\rho_2)\int_{t-\rho_2}^{t-\lambda\rho_2}\eta_1^T(s)\hat{U}_3\eta_1(s)ds$$

$$\leq -\zeta^T(t)\left\{\begin{bmatrix} e_{12} \\ e_{13}\end{bmatrix}\begin{bmatrix} U_2^{11} & U_2^{12} \\ & U_2^{22}\end{bmatrix}\begin{bmatrix} e_{12} \\ e_{13}\end{bmatrix}^T - \begin{bmatrix} e_{14} \\ e_{15}\end{bmatrix}\begin{bmatrix} U_3^{11} & U_3^{12} \\ & U_3^{22}\end{bmatrix}\begin{bmatrix} e_{14} \\ e_{15}\end{bmatrix}^T\right\}\zeta(t).$$

By Lemma 2.5 and reciprocal convex technique utilized in [11], we get

$$\begin{aligned}-\rho_{21}\int_{t-\rho_2}^{t-\rho_1}\eta_1^T(s)\hat{U}_4\eta_1(s)ds &\leq -\rho_{21}\left[\int_{t-\rho(t)}^{t-\rho_1}\eta_1^T(s)\hat{U}_4\eta_1(s)ds + \int_{t-\rho_2}^{t-\rho(t)}\eta_1^T(s)\hat{U}_4\eta_1(s)ds\right],\\ &\leq -\int_{t-\rho(t)}^{t-\rho_1}\eta_1^T(s)ds\hat{U}_4\int_{t-\rho(t)}^{t-\rho_1}\eta_1(s)ds - \int_{t-\rho_2}^{t-\rho(t)}\eta_1^T(s)ds\hat{U}_4\int_{t-\rho_2}^{t-\rho(t)}\eta_1(s)ds,\\ &\leq -\zeta^T(t)\left\{\gamma_5\begin{bmatrix}\hat{U}_4 & \mathcal{W} \\ & \hat{U}_4\end{bmatrix}\gamma_5^T\right\}\zeta(t),\\ &\leq \zeta^T(t)\Pi_3\zeta(t),\end{aligned}$$

where $\gamma_5 = \begin{bmatrix} e_8\delta_1 & e_{16} \\ e_7\delta_2 & e_{17}\end{bmatrix}$, $\delta_1 = \rho_{1t}$, $\delta_2 = \rho_{2t}$.

$$\begin{aligned}\dot{V}_5(t) &= \dot{\phi}^T(t)\left[\frac{(\rho_2-\rho_1)^2}{2}(T_1+T_2) + \frac{\rho_1^2}{2}(T_1+T_2)\right]\dot{\phi}(t)\\ &- \int_{-\rho_1}^{0}\left[\int_{t-\rho_1}^{t+\theta}\dot{\phi}^T(s)T_1\dot{\phi}(s) + \int_{t+\theta}^{t}\dot{\phi}^T(s)T_2\dot{\phi}(s)\right]dsd\theta\\ &- \int_{-\rho_2}^{-\rho_1}\int_{t-\rho_2}^{t+\theta}\dot{\phi}^T(s)T_3\dot{\phi}(s)dsd\theta\\ &- \int_{-\rho_2}^{-\rho_1}\int_{t+\theta}^{t-\rho_1}\dot{\phi}^T(s)T_4\dot{\phi}(s)dsd\theta.\end{aligned}$$

Utilizing Lemma 2.2, the following inequality can be rewritten as

$$\begin{aligned}&-\left[\int_{-\rho_1}^{0}\int_{t-\rho_1}^{t+\theta}\dot{\phi}^T(s)T_1\dot{\phi}(s) + \int_{-\rho_1}^{0}\int_{t+\theta}^{t}\dot{\phi}^T(s)T_2\dot{\phi}(s)\right]dsd\theta\\ &\leq -2\zeta^T(t)[(e_3-e_{18})T_1(e_3-e_{18})^T + 2(e_3-4e_{18}+6e_{20})\\ &T_1(e_3-4e_{18}+6e_{20})^T + (e_1-e_{18})T_2(e_1-e_{18})^T\\ &+2(e_1+2e_{18}-6e_{20})T_2(e_1+2e_{18}-6e_{20})^T]\zeta(t).\end{aligned}$$

Similarly

$$\begin{aligned}&-\left[\int_{-\rho_2}^{-\rho_1}\int_{t-\rho_2}^{t+\theta}\dot{\phi}^T(s)T_3\dot{\phi}(s) + \int_{-\rho_2}^{-\rho_1}\int_{t+\theta}^{t-\rho_1}\dot{\phi}^T(s)T_4\dot{\phi}(s)\right]dsd\theta\\ &\leq -2\zeta^T(t)[(e_4-e_{19})T_3(e_4-e_{19})^T + 2(e_4-4e_{19}+6e_{21})\\ &T_3(e_4-4e_{19}+6e_{21})^T + (e_3-e_{19})T_4(e_3-e_{19})^T\\ &+2(e_3+2e_{19}-6e_{21})T_4(e_3+2e_{19}-6e_{21})^T]\zeta(t).\end{aligned}$$

$$\dot{V}_5(t) \leq \zeta^T(t)\Pi_4\zeta(t). \quad (22)$$

Now, let us focus on $V_6(t)$ in (19), we have

$$\begin{aligned}\dot{V}_6(t) &= \dot{\rho}(t)\eta_2^T(t)\mathcal{W}_1\eta_2(t) + 2(\rho(t)-\rho_1)\eta_2^T(t)\mathcal{W}_1\dot{\eta}_2(t)\\ &- \dot{\rho}(t)\eta_3^T(t)\mathcal{W}_2\eta_3(t) + 2(\rho_2-\rho(t))\eta_3^T(t)\mathcal{W}_2\dot{\eta}_3(t)\\ &= \dot{\rho}(t)\begin{bmatrix}\phi(t) \\ \gamma_1(t)\end{bmatrix}\mathcal{W}_1\begin{bmatrix}\phi(t) \\ \gamma_1(t)\end{bmatrix}^T - \dot{\rho}(t)\begin{bmatrix}\phi(t) \\ \gamma_2(t)\end{bmatrix}\mathcal{W}_2\\ &\times\begin{bmatrix}\phi(t) \\ \gamma_2(t)\end{bmatrix}^T + Sym\left\{\begin{bmatrix}\phi(t) \\ \gamma_1(t)\end{bmatrix}^T\mathcal{W}_1\begin{bmatrix}(\rho(t)-\rho_1)\dot{\phi}(t) \\ \mathbb{G}^a\end{bmatrix}\right.\\ &\left.+\begin{bmatrix}\phi(t) \\ \gamma_2(t)\end{bmatrix}^T\mathcal{W}_2\begin{bmatrix}(\rho_2-\rho(t))\dot{\phi}(t) \\ \mathbb{G}^b\end{bmatrix}\right\},\end{aligned}$$

$$\dot{V}_6(t) \leq \zeta^T(t)\Pi_5\zeta(t), \quad (23)$$

where

$$\mathbb{G}^a = \phi(t-\rho_1) - (t-\dot{\rho}(t))\phi(t-\rho(t)) - \dot{\rho}(t)\gamma_1(t)$$
$$\mathbb{G}^b = (t-\dot{\rho}(t))\phi(t-\rho(t)) - \phi(t-\rho_2) - \dot{\rho}(t)\gamma_2(t)$$
$$\gamma_1(t) = e_8^T\zeta(t),\ \gamma_2(t) = e_7^T\zeta(t).$$

Before obtaining the derivative of $V_7(t)$, define that

$$\begin{aligned}\eta_4^T(t) &= [\eta(t)\Omega_1^T(t)\ \ (\eta-\eta(t))\Omega_2^T(t)],\\ \Omega_1(t) &= \frac{1}{\eta(t)}\int_{t-\eta(t)}^{t}\tilde{\Omega}_{11}\phi(s)ds,\\ \Omega_2(t) &= \frac{1}{\eta-\eta(t)}\int_{-\eta}^{-\eta(t)}\tilde{\Omega}_{12}\phi(s)ds\\ \tilde{\Omega}_{11} &= \mathbb{T}_P\left(\frac{s-t+\eta(t)}{\eta(t)}\right),\ \tilde{\Omega}_{12} = \mathbb{T}_P\left(\frac{s-t+\eta}{\eta-\eta(t)}\right).\end{aligned} \quad (24)$$

Furthermore, we get

$$\begin{aligned}\frac{d}{dt}[\eta(t)\Omega_1(t)] &= \dot{\eta}(t)\Omega_1(t) + \eta(t)\\ &\times\int_0^1\mathbb{T}_P(\delta)\dot{\phi}(s(\delta))[\delta\dot{\eta}(t)+1-\dot{\eta}(t)]d\delta\\ &= \dot{\eta}(t)\Omega_1(t) + \dot{\eta}(t)\eta(t)\int_0^1\delta\mathbb{T}_P(\delta)\dot{\phi}(s(\delta))d\delta\\ &+ (1-\dot{\eta}(t))\eta(t)\int_0^1\mathbb{T}_P(\delta)\dot{\phi}(s(\delta))d\delta,\end{aligned}$$

where $\delta = \frac{s-t+\eta(t)}{\eta(t)}$ then $s = \delta\eta(t) + t - \eta(t)$. Utilizing the novel techniques with (17) and Remark 2.7, we obtain

$$\begin{aligned}\eta(t)\int_0^1\delta\mathbb{T}_P(\delta)\dot{\phi}(s(\delta))d\delta &= \mathbb{T}_P(1)\phi(t) - \int_0^1\phi(s(\delta))\frac{d}{d\delta}(\delta\mathbb{T}_P(\delta))d\delta,\\ &= \mho_a\phi(t) - \Omega_1(t) - \Psi_p\Omega_1(t).\end{aligned}$$

Similarly,

$$\begin{aligned}&\eta(t)\int_0^1\mathbb{T}_P(\delta)\dot{\phi}(s(\delta))d\delta\\ &= \mathbb{T}_P(1)\phi(t) - \mathbb{T}_P(0)\phi(t-\eta(t)) - \int_0^1\phi(s(\delta))\frac{d}{d\delta}(\mathbb{T}_P(\delta))d\delta\\ &= \mho_a\phi(t) - \hat{\mho}_a\phi(t-\eta(t)) - \bar{\Lambda}_P\Omega_1(t).\end{aligned}$$

Correspondingly, the derivative of $(\eta-\eta(t))\Omega_2(t)$ gives,

$$\begin{aligned}\frac{d}{dt}[(\eta-\eta(t))\Omega_2(t)] &= -\dot{\eta}(t)\Omega_2(t) - \dot{\eta}(t)[\mho_a\phi(t-\eta(t))\\ &- \Omega_2(t) - \bar{\Phi}_T\Omega_2(t)] + \mho_a\phi(t-\eta(t))\\ &- \hat{\mho}_a\phi(t-\eta) - \bar{\Lambda}_P\Omega_2(t).\end{aligned} \quad (25)$$

Combining (24)-(25), we get

$$\frac{d}{dt}\{\eta_4^T(t)S\eta_4(t)\} = Sym\{(\tilde{\gamma}_{11}+\dot{\eta}(t)\tilde{\gamma}_{12})^TS\hat{\varphi}_s\}. \quad (26)$$

Therefore, the derivative of $V_7(t)$ becomes

$$\begin{aligned}\dot{V}_7(t) &\leq Sym\{(\tilde{\gamma}_{11}+\tilde{\gamma}_{12})^TS\hat{\varphi}_s\} + \phi^T(t)S_1\phi(t)\\ &- \phi^T(t-\eta)S_2\phi(t-\eta) + \eta^2\dot{\phi}^T(t)S_3\dot{\phi}(t)\\ &- \eta\int_{t-\eta}^{t}\dot{\phi}^T(s)S_3\dot{\phi}(s)ds.\end{aligned}$$

From Lemma 2.2, the above inequality can be written as

$$-\eta\int_{t-\eta}^{t}\dot{\phi}^T(s)S_3\dot{\phi}(s)ds = -\zeta^T(t)[(e_1-e_{22})S_3$$
$$\times(e_1-e_{22})^T + 3(e_1+e_{22}-2e_{24})S_3(e_1+e_{22}-2e_{24})^T$$
$$+5(e_1-e_{22}-6e_{24}+12e_{26})S_3(e_1-e_{22}-6e_{24}+12e_{26})^T$$
$$+(e_{22}-e_{23})S_3(e_{22}-e_{23})^T + 3(e_{22}+e_{23}-2e_{25})S_3$$
$$\times(e_{22}+e_{23}-2e_{25})^T + 5(e_{22}-e_{23}-6e_{25}+12e_{27})S_3$$
$$\times(e_{22}-e_{23}-6e_{25}+12e_{27})^T]\zeta(t)$$
$$\dot{V}_7(t)\le\zeta^T(t)\Pi_6\zeta(t). \quad (27)$$

$$\dot{V}(t)\le\zeta^T(t)\Big[\rho_2(e_1W_3e_1^T - e_4W_3e_4^T) - 2(e_1-e_4^T)$$
$$W_3(\rho(t)e_6 + (\rho_2-\rho(t))e_7) + \rho_2(e_{28}W_4e_{28}^T - e_{31}W_4e_{31}^T)$$
$$-2(e_{28}-e_{31})W_4(e_{16}^T+e_{17}^T)\Big]\zeta(t)$$
$$\dot{V}_8(t)\le\zeta^T(t)\Pi_7\zeta(t). \quad (28)$$

In addition, for any compatible matrices $\mathcal{F}_j(j=1,2)$, one can get the following equation based on system (16)

$$0 = Sym(\phi^T(t)\mathcal{F}_1^T + \dot{\phi}^T(t)\mathcal{F}_2^T)[-\dot{\phi}^T(t) - A\phi^T(t)$$
$$+g(H\phi(t-\rho(t))) + B[\beta(t)[\mathcal{K}\phi(t-\eta(t)) + \mathcal{K}\nu_s(t)]] + C\omega(t)]$$
$$=\zeta^T(t)\Pi_8\zeta(t). \quad (29)$$

From the neuron activation function in $(\mathbf{H_1})$, the subsequent relations hold for any $\mathcal{X},\mathcal{Y},\mathcal{Z}\ge 0$

$$\neg_z^T(t)\,\triangle_a\,\neg_z(t)\ge 0,\ \ \neg_z^T(t-\rho(t))\,\triangle_b\,\neg_z(t-\rho(t))\ge 0,$$
$$\neg_V^T(t)\,\triangle_c\,\neg_V(t)\ge 0, \quad (30)$$

where

$$\neg_z(t) = [\phi(t)g^T(H\phi(t))],\ \ \neg_V(t) = [\neg_z^T(t)\neg_z(t-\rho(t))]$$
$$\triangle_a = \begin{bmatrix}-\mathcal{G}_1H\mathcal{X} & \mathcal{G}_2H\mathcal{X}\\ * & -\mathcal{X}\end{bmatrix},\ \triangle_b = \begin{bmatrix}-\mathcal{G}_1H\mathcal{Y} & \mathcal{G}_2H\mathcal{Y}\\ * & -\mathcal{Y}\end{bmatrix},$$
$$\triangle_c = \begin{bmatrix}-\mathcal{G}_1H\mathcal{Z} & \mathcal{G}_2H\mathcal{Z} & \mathcal{G}_1H\mathcal{Z} & -\mathcal{G}_2H\mathcal{Z}\\ * & -\mathcal{Z} & -\mathcal{G}_2H\mathcal{Z} & \mathcal{Z}\\ * & * & -\mathcal{G}_1H\mathcal{Z} & \mathcal{G}_2H\mathcal{Z}\\ * & * & * & -\mathcal{Z}\end{bmatrix},$$

$\mathcal{X} = diag\{x_{11},\ x_{12},\dots,x_{1n}\}$, $\mathcal{Y} = diag\{y_{21},\ y_{22},\dots,y_{2n}\}$, and $\mathcal{Z} = diag\{z_{31},\ z_{32},\dots,z_{3n}\}$.

Therefore from (30), we get $\zeta^T(t)\Pi_9\zeta(t)$ and also combining (20)-(30) with (12), it can be obtained that

$$\dot{V}(t) - u(w(t),z(t)) + \gamma w^T(t)w(t)\le\zeta^T(t)\Pi\zeta(t),$$

where $\Pi = \sum_{i=0}^{10}\Pi_i$ is defined in Theorem 3.1. Suppose $\Pi<0$, we can obtain

$$\dot{V}(t) - u(w(t),z(t)) + \gamma w^T(t)w(t)\le 0. \quad (31)$$

Integrating (31) from 0 to t_f, the following inequality holds:

$$\gamma\int_0^{t_f}w^T(t)w(t)dt - \int_0^{t_f}u(w(t),z(t))dt\le -V(t_f)+V(0).$$

Under the zero initial condition $V(0)=0, \forall t_f>0$, it is clear that

$$\gamma\int_0^{t_f}w^T(t)w(t)dt - \int_0^{t_f}u(w(t),z(t))dt\le -V(t_f)\le 0.$$

Thus, the error system (16) is $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma-$dissipative in the sense of Definition 2.1, which shows that the master system (1) and the slave system (3) are dissipative synchronous. Moreover, if $\omega(t)=0$ and following the similar procedures used above, we can obtain the error system (16) is asymptotic stable. The proof is completed. □

Remark 3.2. The synchronization of sampled-data control system has reported in an extraordinary volume of literatures see [28,30] and the references therein. It is worth mentioning that the sampled-data control with synchronization analysis is adopted in the above literature. In contrast, the resilent distributed event-triggered control (RDETC) with fault approach is developed in this paper. Generally, the RDETC presents better dynamic performance of the closed-loop system and reducing the amount of controller updates and lessing the network communication than the other controller as explained in the introduction section. The result in Theorem 3.3 fills the gap on the resilient event-triggered control of SNNs.

3.2. Resilient dissipative event-triggered controller design

It is observed that arguments in the previous literature works the modelled controller could be very sensitive or fragile in terms of various changes in the feedback gain. The following resilient control idea being implemented to frame a feedback control gain. In the following Theorem 3.3, we will discuss resilient event-triggered control framework for the SNNs. By the way, the control gain can be modelled as $\mathcal{K}+\bar{\Upsilon}(t)\Delta\mathcal{K}(t)$, where

$$\Delta\mathcal{K}(t) = \mathbb{G}\mathcal{H}(t)\mathbb{F}, \quad (32)$$

where $\mathbb{G}$ and $\mathbb{F}$ denoted as known matrices and $\mathcal{H}(t)$ defined as unknown matrix which satisfies $\mathcal{H}^T(t)\mathcal{H}(t)=I$. Moreover, $\bar{\Upsilon}(t)$ taken as random variable and can be utilized to model the randomly occurring controller gain fluctuations. $\bar{\Upsilon}(t)$ is termed as Bernoulli-distributed white sequences take off the values of 0 or 1 with $P_r\{\bar{\Upsilon}(t)=1\} = \mathbb{E}\{\bar{\Upsilon}(t)\} = \bar{\Upsilon}$. Now consider the following system with resilient event-triggered controller

$$\begin{cases}\dot{\phi}(t) &= -A\phi(t) + g(H\phi(t-\rho(t))) + Bu(t) + C\omega(t),\\ u(t) &= \beta(t)[(\mathcal{K}+\bar{\Upsilon}(t)\Delta\mathcal{K}(t))\phi(t-\eta(t))\\ &\quad +(\mathcal{K}+\bar{\Upsilon}(t)\Delta\mathcal{K}(t))\nu_s(t)],\\ z(t) &= D\phi(t).\end{cases} \quad (33)$$

Theorem 3.3. *Under Assumption* $(\mathbf{H_1})$, *for given scalars* $\rho_1,\ \rho_2,\ \rho_3,\ \eta,\ \lambda,\ \tilde{\lambda},\ \tilde{\alpha},\ \bar{\varrho}$, *and matrices* $\mathcal{Q}=\mathcal{Q}^T, \mathcal{S}, \mathcal{R}=\mathcal{R}^T$, *the neural network (33) is* $(\mathcal{Q},\mathcal{S},\mathcal{R})-\gamma-$ *dissipative and asymptotically stable in mean square sense and using the event-triggered controller (14), if there exist positive symmetric matrices* $\hat{\mathbb{P}}\in\mathbb{R}^{5n\times 5n}, \hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{Q}_1, \hat{Q}_2, \hat{\mathbb{V}}\in\mathbb{R}^{2n\times 2n}, \hat{U}_i\in\mathbb{R}^{2n\times 2n}, \hat{T}_1, \hat{T}_i, i=2,3,4, \hat{\mathbb{W}}_1, \hat{\mathbb{W}}_2, \hat{S}, \hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{W}_3, \hat{W}_4, \hat{\mathbb{W}}, \hat{\mathbf{K}}$, $\bar{V}_a, \bar{V}_b\in\mathbb{R}^{n\times n}$, *positive diagonal matrices* $\mathcal{X},\mathcal{Y},\mathcal{Z}$, *and any compatible matrices* $\mathcal{F}$, $\mathcal{X}_i, \mathcal{Y}_i\in\mathbb{R}^{9n\times 2n}, \tilde{\chi}>0, i=1,2,\ \beta(t)=\{0,1\}$ *and* $\rho(t)\in[\rho_1,\rho_2]$ *in such a way the subsequent inequalities hold*

$$\begin{bmatrix}\hat{U}_4 & \hat{\mathbb{W}}\\ & \hat{U}_4\end{bmatrix}\ge 0,$$

$$\begin{bmatrix}\tilde{\hat{\Pi}} & \tilde{\mathbf{\Gamma}}_{12} & \Lambda_1 & \Lambda_2^T\\ * & -(\mathcal{R}-\gamma I) & 0 & 0\\ & * & -\tilde{\lambda}I & 0\\ & * & * & -\tilde{\lambda}I\end{bmatrix}<0, \quad (34)$$

where $\tilde{\hat{\Pi}} = \sum_{i=0}^{10}\tilde{\hat{\Pi}}_i$ *are defined in Appendix B and*

$$\tilde{\mathbf{\Gamma}}_{12} = [\tilde{\alpha}C^T\ \ 0\ \ 0\ \ 0\ \ C^T\ \ \overbrace{0\ 0\ 0}^{29\ times}]^T,$$

$$\Lambda_1 = [\mathbb{G}\tilde{\alpha}B^T\beta(t)\ 0\ 0\ 0\ \mathbb{G}B^T\beta(t)\ \underbrace{0\ 0\ 0}_{30\ times}]^T,$$

$$\Lambda_2 = [\overbrace{0\ 0\ 0}^{21\ times}\ \mathcal{F}^T\mathbb{F}^T\ \underbrace{0\ 0\ 0}_{9\ times}\ \mathcal{F}^T\mathbb{F}^T\ 0\ 0\ 0].$$

Then, the error system (33) is dissipative synchronous. Moreover, the desired resilient event-triggered control gain can be achieved by $\mathcal{K} = \hat{\mathbf{K}}\mathcal{F}^{-1}$, *such that for* $\rho(t) \in [\rho_1, \rho_2]$ *and* $\beta(t) = \{0, 1\}$.

Proof. Utilizing (32) and the condition in (18), we obtain

$$\Sigma + \Re\mathcal{H}(t)\Im + \Im^T\mathcal{H}^T(t)\Re^T,$$

where

$$\Re = [(\mathbb{G}\mathcal{F}_1B\beta(t))^T\ 0\ 0\ 0\ (\mathbb{G}\mathcal{F}_2B\beta(t))^T\ \overbrace{0\ 0\ 0}^{30\ times}]^T,$$

$$\Im = [\overbrace{0\ 0\ 0}^{21\ times}\ \mathbb{F}\ \underbrace{0\ 0\ 0}_{9\ times}\ \mathbb{F}\ 0\ 0\ 0].$$

Furthermore, it follows from Lemma 3.5 of [39] that there exists scalar $\tilde{\lambda} > 0$, such that $\Sigma + \tilde{\lambda}^{-1}\Re\Re^T + \tilde{\lambda}\Im^T\Im$. Then, by using schur complement, it is easy to obtain

$$\tilde{\Lambda} = \begin{bmatrix} \Sigma & \Re & \lambda\Im^T \\ & -\tilde{\lambda}I & 0 \\ & * & -\tilde{\lambda}I \end{bmatrix}. \tag{35}$$

The proof follows the similar procedure as in Theorem 3.1. Define

$$\begin{cases} \hat{\mathbb{P}} = \mathcal{F}^T \otimes \mathbb{P} \otimes \mathcal{F},\ \hat{R}_1 = \mathcal{F}^TR_1\mathcal{F},\ \hat{R}_2 = \mathcal{F}^TR_2\mathcal{F}, \\ \hat{R}_3 = \mathcal{F}^TR_3\mathcal{F},\ \hat{Q}_1 = \mathcal{F}^TQ_1\mathcal{F},\ \hat{Q}_2 = \mathcal{F}^TQ_2\mathcal{F}, \\ \hat{\mathcal{W}}_1 = \mathcal{F}^T\mathcal{W}_1\mathcal{F},\ \hat{\mathcal{W}}_2 = \mathcal{F}^T\mathcal{W}_2\mathcal{F},\ \hat{\mathcal{V}} = \mathcal{F}^T\mathcal{V}\mathcal{F}, \\ \bar{\hat{\mathcal{V}}}_a = \mathcal{F}^T\bar{\mathcal{V}}_a\mathcal{F},\ \bar{\hat{\mathcal{V}}}_b = \mathcal{F}^T\bar{\mathcal{V}}_b\mathcal{F},\ \hat{U}_j^{1k} = \mathcal{F}^TU_j^{1k}\mathcal{F}, \\ \hat{U}_j^{22} = \mathcal{F}^TU_j^{22}\mathcal{F},\ \bar{\hat{U}}_4 = \mathcal{F}^T\hat{U}_4\mathcal{F},\ \hat{T}_1 = \mathcal{F}^TT_1\mathcal{F}, \\ \hat{T}_2 = \mathcal{F}^TT_2\mathcal{F},\ \hat{T}_3 = \mathcal{F}^TT_3\mathcal{F},\ \hat{T}_4 = \mathcal{F}^TT_4\mathcal{F}, \\ \hat{S} = \mathcal{F}^TS\mathcal{F},\ \hat{S}_1 = \mathcal{F}^TS_1\mathcal{F},\ \hat{S}_2 = \mathcal{F}^TS_2\mathcal{F}, \\ \hat{S}_3 = \mathcal{F}^TS_3\mathcal{F},\ \hat{W}_3 = \mathcal{F}^TW_3\mathcal{F},\ \hat{W}_4 = \mathcal{F}^TW_4\mathcal{F}, \\ \hat{\mathcal{W}} = \mathcal{F}^T\mathcal{W}\mathcal{F},\ \hat{\tilde{\chi}} = \mathcal{F}^T\tilde{\chi}\mathcal{F}, \\ j = 2, 3,\ k = 1, 2. \end{cases} \tag{36}$$

Moreover, $\mathcal{F}_2 = \mathcal{F}^{-1}, \mathcal{F}_1 = \tilde{\alpha}\mathcal{F}^{-1}, \hat{\mathbf{K}} = \mathcal{K}\mathcal{F}$. Then performing congruence transformation to (35) with $diag\{\underbrace{\mathcal{F}, \mathcal{F}, \mathcal{F}}_{32\ times}, \underbrace{I, I, I}_{5\ times}\}$, we obtain the condition (34). □

4. Passivity analysis

Willems [21] built up an efficient structure for dissipative systems, including passive modelled systems, by presenting the entry of a storage function and a supply rate. Inspired by the above facts, in this following segment, we discuss the synchronization problem for passivity analysis of SNNs with event-triggered scheme.

Remark 4.1. Theorem 3.1 established a strict dissipativity synchronization analysis of SNNs (16). In the view of Theorem 3.1, we obtain the passivity analysis of error system (16) with respect to $\mathcal{Q} = 0, \mathcal{S} = I$, and $\mathcal{R} = 2\gamma I$ in the following Theorem 4.2.

Theorem 4.2. *Under Assumption* $(\mathbf{H_1})$, *for given scalars* $\rho_1, \rho_2, \rho_3, \eta, \tilde{\lambda}, \tilde{\alpha}, \bar{\varrho}$, *and matrices* $\mathcal{Q} = \mathcal{Q}^T, \mathcal{S}, \mathcal{R} = \mathcal{R}^T$, *the error system (33) is passive with the event-triggered controller (14), if there exist positive symmetric matrices* $\hat{\mathbb{P}}_l \in \mathbb{R}^{5n\times 5n}, \hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{Q}_1, \hat{Q}_2, \hat{\mathcal{W}}_1, \hat{\mathcal{W}}_2, \hat{\mathcal{V}} \in \mathbb{R}^{2n\times 2n}, \bar{\hat{U}}_i \in \mathbb{R}^{2n\times 2n}, \hat{T}_1, \hat{T}_i, i = 2, 3, 4, \hat{S}, \hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{W}_3, \hat{W}_4, \hat{\mathcal{W}}, \hat{\mathbf{K}}, \bar{\hat{\mathcal{V}}}_a, \bar{\hat{\mathcal{V}}}_b \in \mathbb{R}^{n\times n}$, *positive diagonal matrices* $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$, *and any compatible matrices* $\mathcal{F}$, $\mathcal{X}_i, \mathcal{Y}_i \in \mathbb{R}^{9n\times 2n}, \hat{\tilde{\chi}} > 0, i = 1, 2,\ \beta(t) = \{0, 1\}$ *and* $\rho(t) \in [\rho_1, \rho_2]$ *in such a way the subsequent inequalities hold*

$$\begin{bmatrix} \bar{\hat{U}}_4 & \hat{\mathcal{W}} \\ & \bar{\hat{U}}_4 \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \tilde{\hat{\Pi}} & \tilde{\mathbf{\Gamma}}_{12} \\ * & -\gamma \end{bmatrix} < 0, \tag{37}$$

where $\tilde{\hat{\Pi}} = \sum_{i=0}^{10}\tilde{\hat{\Pi}}_i$ *and* $\sum_{i=0}^{9}\tilde{\hat{\Pi}}_i$ *are defined in Appendix B,* $\tilde{\hat{\Pi}}_{10} = -e_1\mathcal{F}De_{33} + \bar{\varrho}e_{22}\hat{\tilde{\chi}}e_{22}^T - e_{32}\hat{\tilde{\chi}}e_{32}^T$ *and the remaining terms are defined in Theorem 3.3.*

Then for any initial condition, the error system (33) is passive. For this case, the slave system (3) and master system (1) are passive synchronous. Furthermore, the desired resilient event-triggered controller gain can be achieved by $\mathcal{K} = \hat{\mathbf{K}}\mathcal{F}^{-1}$, *such that for* $\rho(t) \in [\rho_1, \rho_2]$ *and* $\beta(t) = \{0, 1\}$.

Proof. Considering the same LKF as in Theorem 3.1 and following the similar proof of Theorem 3.1, we can see that the subsequent inequality is true.

Therefore,

$$\dot{V}(t) - 2z^T(t)w(t) - w^T(t)\gamma w(t) \leq \zeta^T(t)\Pi\zeta(t),$$

where $\zeta(t)$ and Π is defined in Theorem 3.1. Hence if $\Pi < 0$ holds, then the above inequality implies that

$$\dot{V}(t) - 2z^T(t)w(t) - w^T(t)\gamma w(t) \leq 0. \tag{38}$$

Integrating (38) from 0 to t_f, under zero initial condition, we get

$$2\int_0^{t_f} z^T(s)w(s)ds \geq V(t_f) - V(0) - \gamma\int_0^{t_f} w^T(s)w(s)ds$$
$$\geq -\gamma\int_0^{t_f} w^T(s)w(s)ds, \tag{39}$$

for all $t_f \geq 0$. Therefore, the error system (33) is passive with respect to Definition 2.6. □

Remark 4.3. If taking $w(t) = 0$ and $z(t) = 0$, then the error system (5) turns into the subsequent system (40).

$$\begin{cases} \dot{\phi}(t) &= -A\phi(t) + g(H\phi(t - \rho(t)) + Bu(t), \\ u(t) &= \beta(t)[\mathcal{K}\phi(t - \eta(t)) + \mathcal{K}\nu_s(t)]. \end{cases} \tag{40}$$

A stabilization criterion for synchronization network (40) with time-varying delay can be discussed in the following Corollary 4.4. The other procedures of Corollary 4.4 for synchronization network (40) are similar to the proof of Theorem 3.1.

Corollary 4.4. *Under Assumption* $(\mathbf{H_1})$, *for given scalars* $\rho_1, \rho_2, \rho_3, \eta, \lambda, \tilde{\lambda}, \tilde{\alpha}, \bar{\varrho}$, *and the error system (40) is asymptotically stable in the sense of event-triggered controller (14), if there exist positive symmetric matrices* $\hat{\mathbb{P}} \in \mathbb{R}^{5n\times 5n}, \hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{Q}_1, \hat{Q}_2, \hat{\mathcal{W}}_1, \hat{\mathcal{W}}_2, \hat{\mathcal{V}} \in \mathbb{R}^{2n\times 2n}, \bar{\hat{U}}_i \in \mathbb{R}^{2n\times 2n}, \hat{T}_1, \hat{T}_i, i = 2, 3, 4, \hat{S}, \hat{S}_1, \hat{S}_2, \hat{S}_3, \hat{W}_3, \hat{W}_4, \hat{\mathcal{W}}, \hat{\mathbf{K}}$, *symmetric matrices* $\bar{\hat{\mathcal{V}}}_a, \bar{\hat{\mathcal{V}}}_b \in \mathbb{R}^{n\times n}$, *diagonal matrices* $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$, *and any compatible matrices* $\mathcal{F}$, $\mathcal{X}_i, \mathcal{Y}_i \in \mathbb{R}^{9n\times 2n}, \tilde{\chi} > 0, i = 1, 2,\ \beta(t) = \{0, 1\}$ *and* $\rho(t) \in [\rho_1, \rho_2]$ *in such a way the subsequent inequalities hold*

$$\begin{bmatrix} \bar{\hat{U}}_4 & \mathcal{W} \\ & \bar{\hat{U}}_4 \end{bmatrix} > 0,$$

$$\tilde{\hat{\Pi}} < 0, \tag{41}$$

where $\tilde{\hat{\Pi}} = \sum_{i=0}^{9}\tilde{\hat{\Pi}}_i$ *are same as defined in Theorem 3.3.*

Then, the error system without resilient are asymptotically synchronous. Moreover, the control gain matrix is given by $\mathcal{K} = \hat{\mathbf{K}}\mathcal{F}^{-1}$.

Proof. Setting $\omega(t) = 0$ and construct

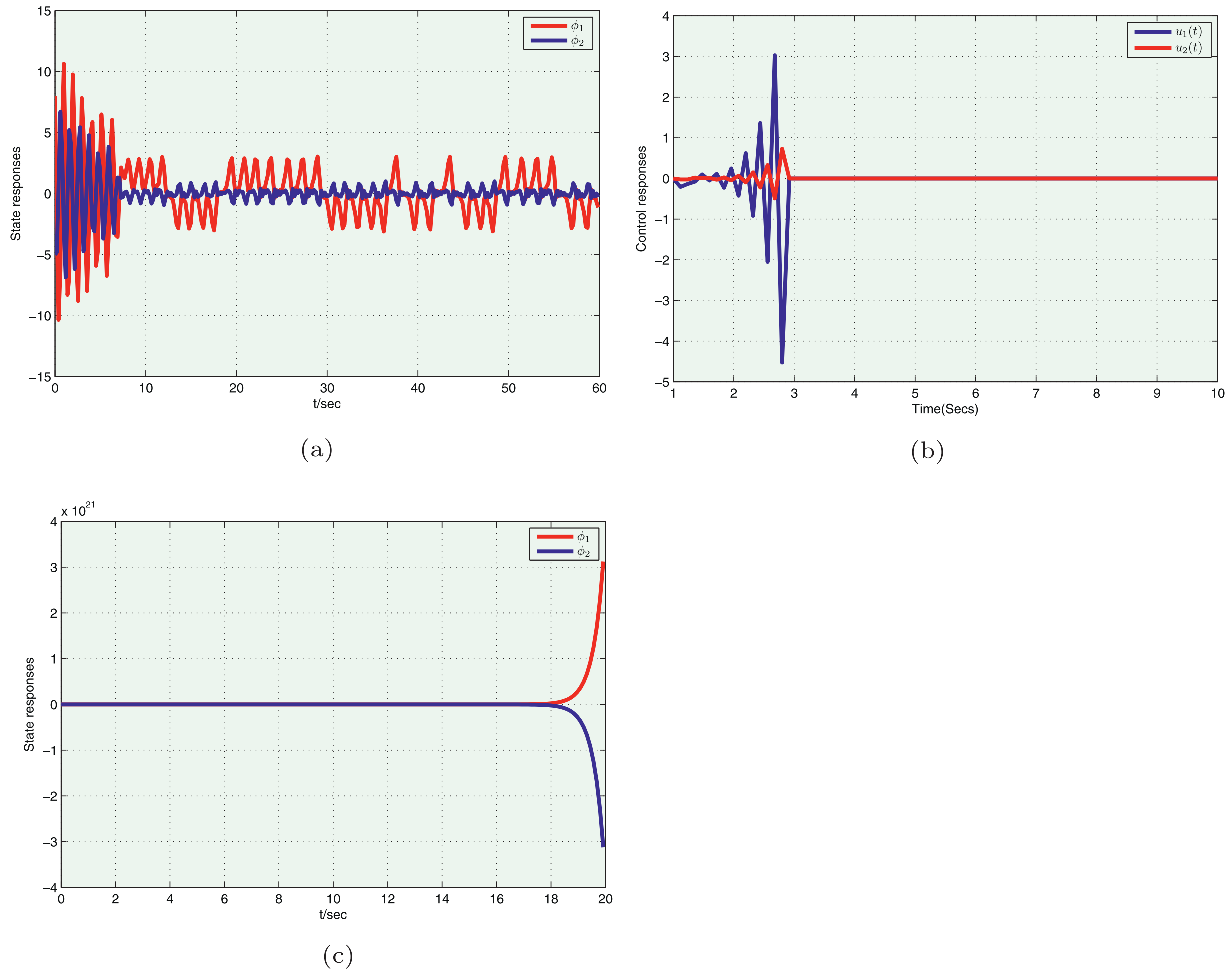

Fig. 2. The panels (a)-(c) contain the evolution of curves for case I in Example 5.1.

$\zeta^T(t) = [\zeta_1^T(t) \;\; \zeta_2^T(t) \;\; \zeta_3^T(t) \;\; \zeta_4^T(t) \;\; \zeta_5^T(t)]$ $\zeta_5^T(t) = [g^T(H(\phi(t-\rho_1))) \;\; g^T(H(\phi(t-\rho(t)))) \;\; g^T(H(\phi(t-\rho_2))) \;\; \nu_s^T(t) \;\; \tilde{\Omega}_{11}^T(t) \;\; \tilde{\Omega}_{12}^T(t)]$. The remaining vector elements are defined in Theorem 3.1. The proof is the same as that of Theorem 3.3. □

5. Numerical examples

In this section, the following numerical examples are discussed to prove the usefulness of the derived results in this paper

Example 5.1. Consider the following system with the parameters as follows:

$$\begin{cases} \dot{\phi}(t) = -A\phi(t) + g(H\phi(t-\rho(t)) + Bu(t) + C\omega(t), \\ u(t) = \beta(t)[\mathcal{K}\phi(t-\eta(t)) + \mathcal{K}\nu_s(t)], \\ z(t) = D\phi(t), \end{cases} \tag{42}$$

$$A = \begin{bmatrix} 7.0214 & 0 \\ 0 & 7.4367 \end{bmatrix}, \; H = \begin{bmatrix} -6.49 & -12.02 \\ -0.68 & 5.66 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.5 & -0.1 \\ -0.4 & 0.2 \end{bmatrix}, \; C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -1 & 0.4 \\ 0 & 1 \end{bmatrix}, \; \mathbb{G} = diag\{0.5, 0.5\}, \; \mathbb{F} = diag\{0.1, 0.1\}.$$

The objective of this example is to find the maximum permissible time delay ρ_2, such that the system is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ dissipative. For this, we choose

$$\mathcal{Q} = diag\{-1, -1\}, \; \mathcal{R} = diag\{3, 3\}, \; \mathcal{S} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.5 \end{bmatrix}.$$

In this example, the activation functions are assumed to be $g(\phi_j) = 0.5(|\phi_j + 1| - |\phi_j - 1|)$. It can be verified that Assumption $(\mathbf{H_1})$ is hold with $G_j^- = 0, G_j^+ = 1, j = 1, 2$. Thus, $\mathcal{G}_1 = diag\{0, 0\}$, $\mathcal{G}_2 = diag\{0.5, 0.5\}$. By using MATLAB LMI toolbox and by taking $\eta = 0.5$, $\rho(t) = 1 + 0.5sint$, which means $\rho_2 = 1.5$ and the time-derivative of the delay satisfies $\rho_3 = 0.5$, $\rho_1 = 0.5$. Based on the above parameter value and solve the LMI conditions in Theorem 3.3 for two cases. In this simulation, we introduce two cases to validate the design control performance.

Case:I (with fault):

In this case $\beta(t)$ is assume to be fixed with respect to the below parameters. In addition, some of the efficient results have been produced with respect to fault approach based on the traditional approaches, for instance [47] and [48]. Such a controller with fault have not conventionally, (i.e) switching between two different approaches are very fast. In the sense of practical applications, it may be difficult or impossible for implementation. Furthermore, the fast switching will cause multiple damages to equipments. With reference to these facts, it is appropriate to design a controller with fault belongs to the intervals. According to the key idea in [47] and [48], one could design fault approach. However, the corresponding variable $\beta(t)$ with respect to traditional Bernoulli variable and let $\mathbb{E}[\beta(t)] = 0.889$.

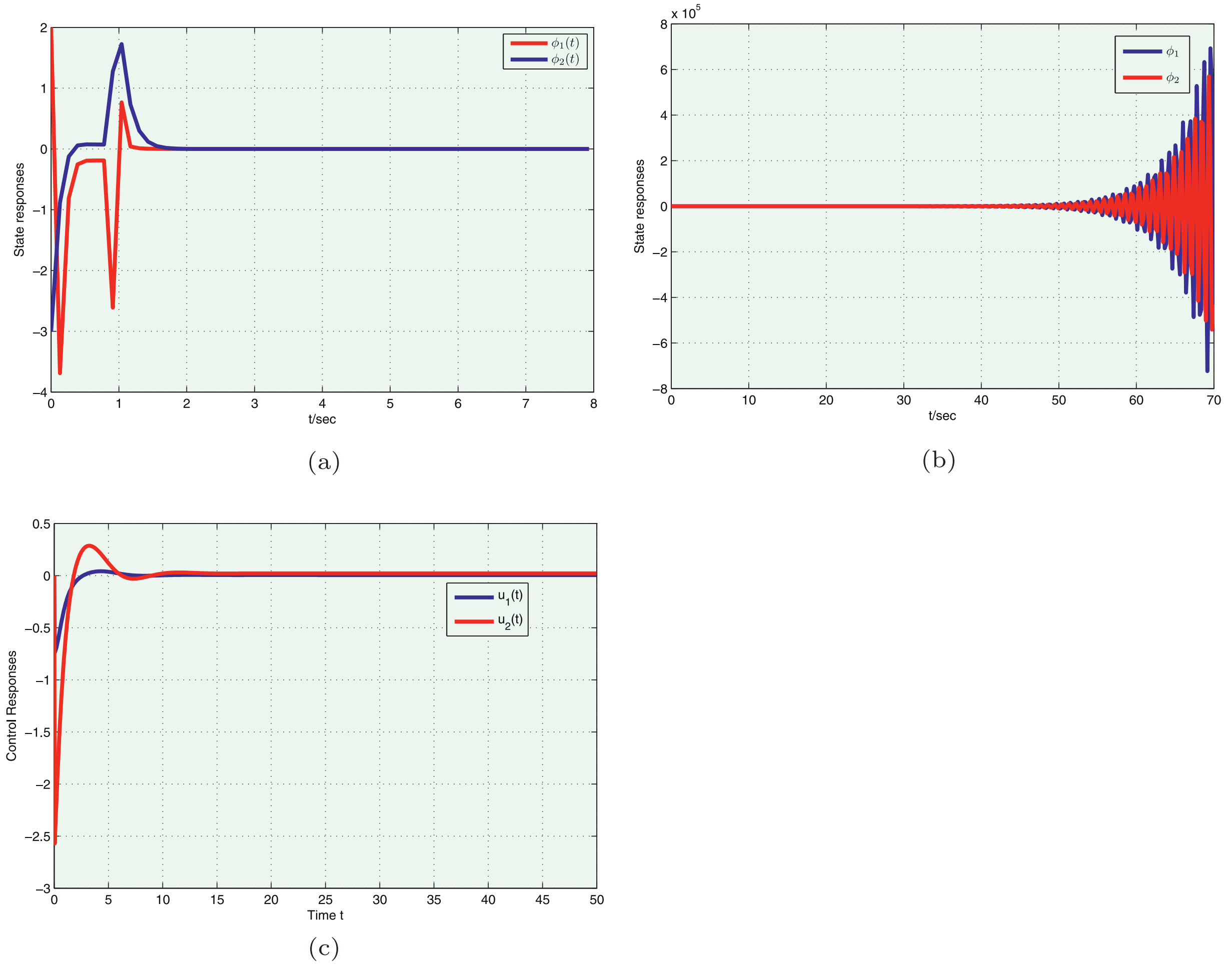

Fig. 3. The panels (a)-(c) contain the evolution of curves for case II in Example 5.1.

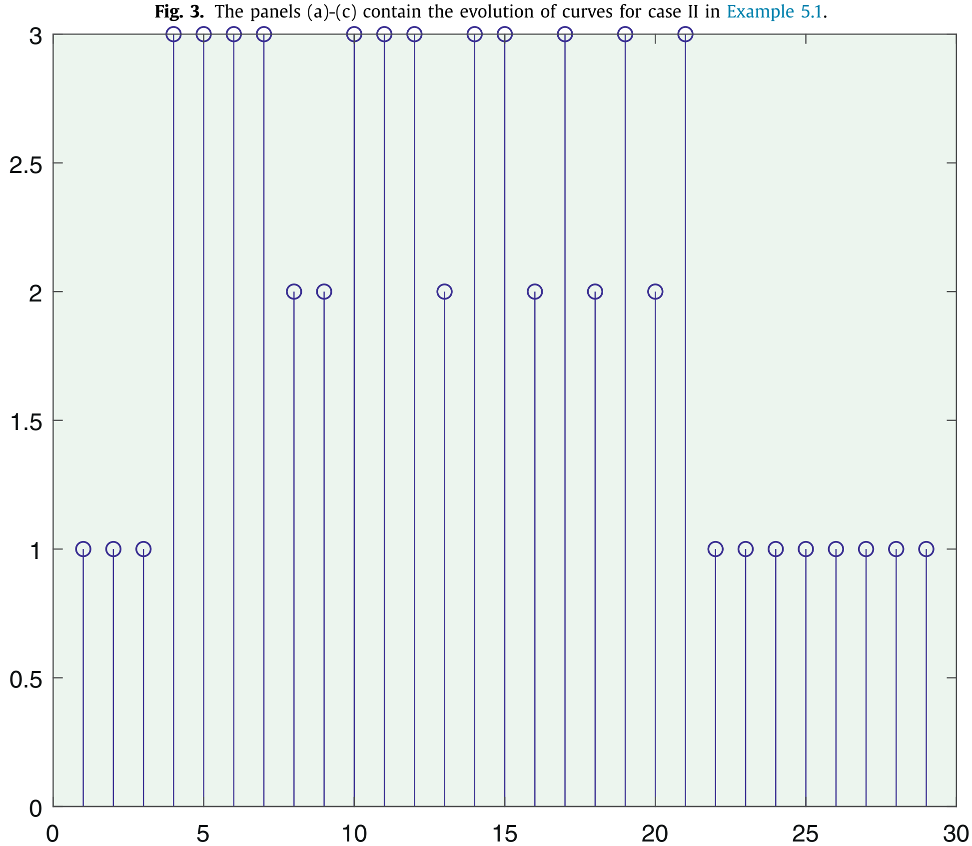

Fig. 4. Event-triggered release interval in Example 5.1.

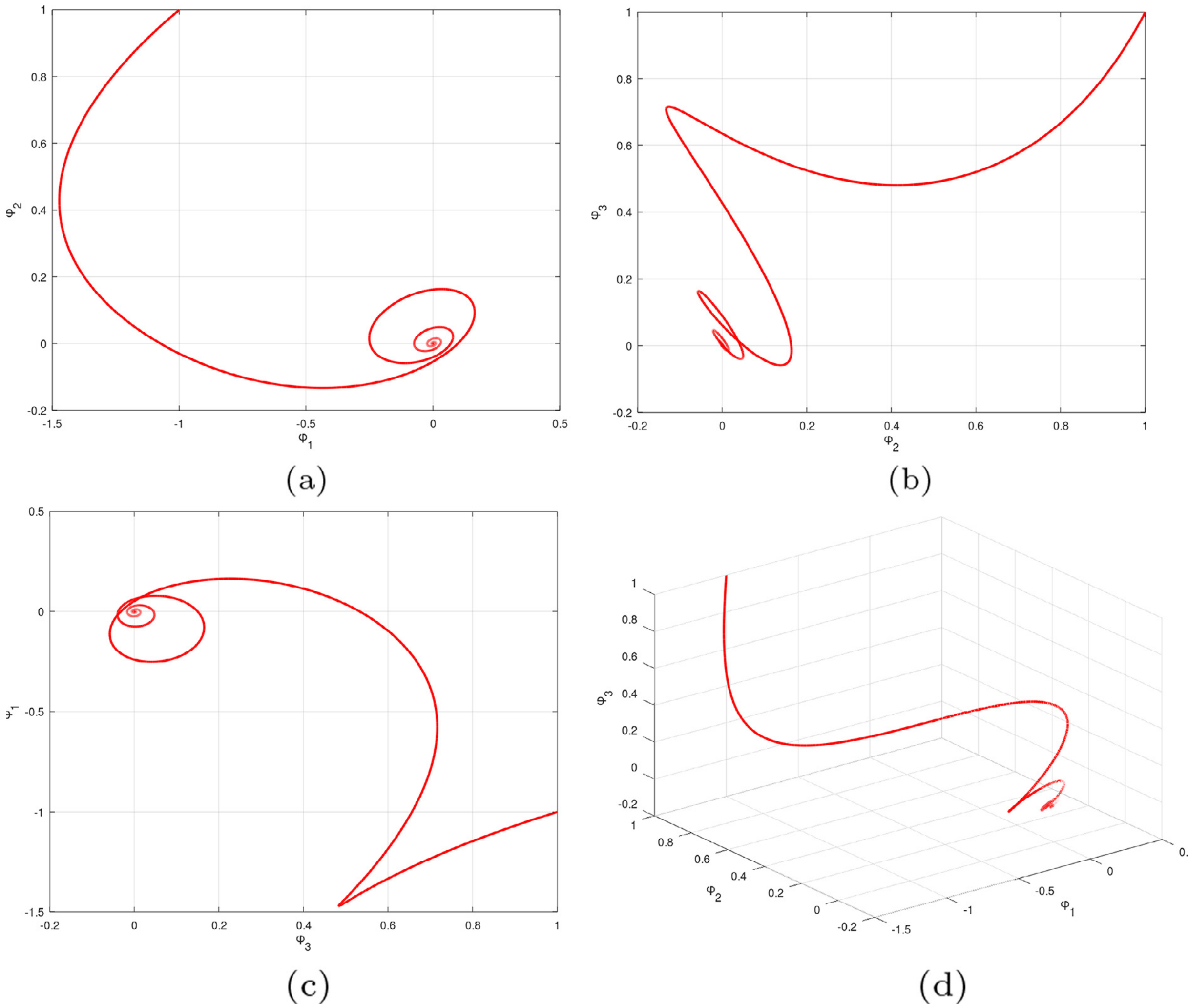

Fig. 5. Behavior of various solutions trajectories ϕ_1, ϕ_2, ϕ_3 in Example 5.2.

We can solve the LMIs in Theorem 3.3 and applying the above parameters, fault value, the corresponding control gain matrix and triggered parameters are obtained as follows:

$$\tilde{\chi} = 10^2 \times \begin{bmatrix} 2.2748 & 0.0001 \\ 0.0001 & 0.1215 \end{bmatrix},$$
$$\mathcal{K} = \begin{bmatrix} -0.0263 & 0.4221 \\ 0.1411 & -0.2874 \end{bmatrix}.$$

For the simulation purpose, we consider the exogenous disturbance input $w(t) = 0.5 + \sin(0.13\exp(0.5t))$ and the initial condition $\phi_0 = [-3, 3]^T$, the error simulation of open-loop system is given Fig. 2c in which the trajectories diverge. The corresponding closed-loop error response curves are depicted in Fig. 2a, which shows that the trajectories converges quickly to zero. In addition, the event-triggered scheme is applied to dissipative problem for SNNs in this paper, which greatly reduce the communication burden. The corresponding triggered events for dissipative case is given in Fig. 4, which indicates that the proposed event-triggered controller scheme can reduce the number of controller executions effectively and save resources. The corresponding control input has been shown in Fig. 2b. Concluded, from Fig. 2 it is revealed that the proposed controller effectively stabilize the considered system even in the presence of fault approach.

Case:II (without fault approach):

For this case, no fault approach. Furthermore, solving the LMIs presented in Theorem 3.3 with the same system parameters as mentioned in the previous case, the controller gain matrices and event-triggered parameters are solved out as follows

$$\lambda = 10^2 \times \begin{bmatrix} 5.6335 & 4.8231 \\ 4.8231 & 3.5412 \end{bmatrix},$$
$$\mathcal{K} = \begin{bmatrix} -1.5342 & 0.5672 \\ 0.3118 & -2.5922 \end{bmatrix}.$$

Using the above gain matrices and with the randomized initial condition, the error responses of both closed and open-loop system, control variation of the system (42) with respect to Case II are depicted in Fig. 3(a–c).

Example 5.2. Consider a general form of the SNNs can be expressed as [1]

$$\frac{d\phi_{mi}(t)}{dt} = -\hat{\alpha}_i\phi_{mi}(t) + \sum_{j=1}^{n} g_j(H_{ij}\phi_{mj}(t\rho(t))), \tag{43}$$

where $\hat{\alpha}_i$ is the inverse of the time constant governing the rate of change of the ith neuron. Moreover, a vector form of the system given in (43) can be expressed as

$$\frac{d\phi_m(t)}{dt} = -A\phi_m(t) + g(H\phi_m(t - \rho(t))). \tag{44}$$

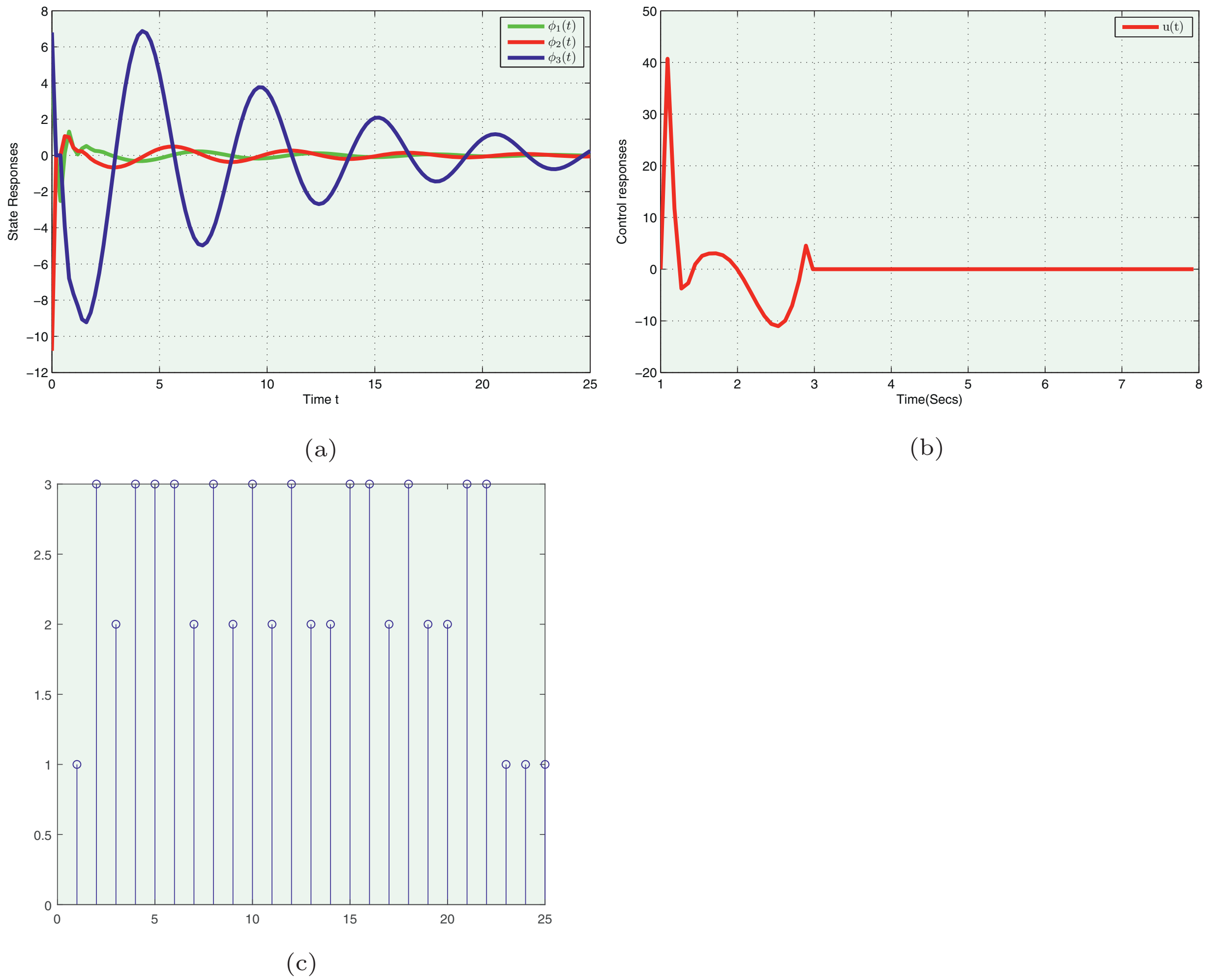

Fig. 6. (a) error responses; (b) control responses; (c) release time and release interval of the error system (44).

Now, we consider the event-triggered synchronization with delayed SNNs. Based on the system (44), let us take into account the master and slave system as follows:

$$\begin{cases} master: \frac{d\phi_m(t)}{dt} = -A\phi_m(t) + g(H\phi_m(t-\rho(t))), \\ slave: \frac{d\phi_s(t)}{dt} = -A\phi_s(t) + g(H\phi_s(t-\rho(t))) + Bu(t), \end{cases}$$

$error = slave - master:$

$$\left\{ error: \frac{d\phi(t)}{dt} = -A\phi(t) + g(H\phi(t-\rho(t))) + Bu(t), \right. \tag{45}$$

where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \; H = \begin{bmatrix} 0.2 & -0.1 & 0 \\ 0.1 & 0.3 & -0.2 \\ -0.2 & 0.1 & 0.2 \end{bmatrix},$$

$B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T.$

In order to get a synchronization for the above SNNs (45), let us consider the activation function as $g(\phi(t)) = tanh(\phi(t))$ with $\mathcal{G}_1 = diag\{0, 0, 0, 0\}$, $\mathcal{G}_2 = diag\{1, 1, 1, 1\}$. Also, $\rho_2 = 0.3$, $\rho_1 = 0.2$, $\eta = 0.2$, and using the Matlab LMI Control Toolbox to solve the LMIs of Corollary 4.4, we can obtain the corresponding control gain matrix and triggered parameters as follows:

$$\mathcal{K} = \begin{bmatrix} 0.9836 & 0.3125 & 0.4086 \end{bmatrix}.$$

$$\lambda = \begin{bmatrix} 73.3163 & -0.0150 & -0.0105 \\ -0.0150 & 73.2579 & -0.0138 \\ -0.0105 & -0.0138 & 73.3012 \end{bmatrix}.$$

The behavior of the uncontrolled system (45) is depicted in Fig. 5. Moreover, under the randomized initial conditions corresponding dynamic behaviors of error system (45) is displayed in Fig. 6a and the control input of the considered system is shown in Fig. 6b and transmission intervals are presented in Fig. 6c; it is obvious that the error system (45) is asymptotically stable under the event-triggered controller (13). In other words, the considered SNNs designed in this paper is feasible, which can synchronize the considered system effectively.

6. Conclusion and future directions

In this paper, the distributed event-triggered control for SNNs was studied with dissipative synchronization and time-varying delay. In view of the time-varying delay techniques and adequate conditions have been acquired to guarantee the SNNs is strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma -$ dissipative and passivity subject to synchronization criteria. Then, by taking the influence of the controller failures into account, a novel error model with the DETS has been established. The desired controller gain and event-triggered parameters have been obtained by solving a set of linear matrix inequalities. Finally, simulation examples are given to verify the effectiveness of the proposed method. The presented results and approaches in this article can be extended to many complex dynamic systems, such as multiagent systems, stochastic delayed NNs, and the semi-Markovian jump-delayed NNs.

Funding

Not applicable

Availability of data and materials

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. The elements of the matrix $\sum_{i=0}^{10} \Pi_i$

$$
\begin{aligned}
\Pi_0 &= Sym(\upsilon_1^T \mathbb{P} \upsilon_2)),\\
\upsilon_1 &= [e_1 \quad \rho_1 e_{18} \quad (\rho_2-\rho_1)e_{19} \quad \rho_1 e_{20} \quad (\rho_2-\rho_1)e_{21}]^T,\\
\upsilon_2 &= [e_5 \quad e_1-e_3 \quad e_3-e_4 \quad e_1-e_{18} \quad e_3-e_{20}]^T,\\
\Pi_1 &= e_1^T R_1 e_1 + e_3(R_2-R_1)e_3 - (1-\rho_3)e_2^T R_1 e_2\\
&\quad + e_1^T R_3 e_1 - e_4^T R_3 e_4,\\
\Pi_2 &= -(e_1-e_2)Q_1(e_1-e_2)^T - 3(e_1+e_2-2e_6)\\
&\quad \times Q_1(e_1+e_2-2e_6)^T - 5(e_1-e_2-6e_6+12e_9)\\
&\quad \times Q_1(e_1-e_2-6e_6+12e_9)^T - (e_2-e_4)Q_1\\
&\quad \times (e_2-e_4)^T - 3(e_2+e_4-2e_7)Q_1(e_2+e_4-2e_7)^T\\
&\quad - 5(e_2-e_4-6e_7+12e_{11})Q_1(e_2-e_4-6e_7+12e_{11})^T\\
&\quad + (e_3-e_2)Q_2(e_3-e_2)^T + 3(e_3+e_2-2e_8)\\
&\quad \times Q_2(e_3+e_2-2e_8)^T + 5(e_3-e_2-6e_8+12e_{10})\\
&\quad \times Q_2(e_3-e_2-6e_8+12e_{10})^T + (e_2-e_4)Q_2\\
&\quad \times (e_2-e_4)^T + 3(e_2+e_4-2e_7)Q_2(e_2+e_4-2e_7)^T\\
&\quad + 5(e_2-e_4-6e_7+12e_{11})Q_2(e_2-e_4-6e_7+12e_{11})^T\\
&\quad + Sym\{\hat{\Pi}_1\} + \gamma_a + \gamma_b,\\
\Pi_3 &= \begin{bmatrix} e_1 \\ e_{28} \end{bmatrix} \left[\lambda^2 \rho_1^2 \hat{U}_2 + (\rho 2 - \lambda\rho_2)^2 \hat{U}_3 + \rho_{21}^2 \hat{U}_4\right] \begin{bmatrix} e_1 \\ e_{28} \end{bmatrix}^T\\
&\quad + \begin{bmatrix} e_{12} \\ e_{13} \end{bmatrix} \begin{bmatrix} U_2^{11} & U_2^{12} \\ & U_2^{22} \end{bmatrix} \begin{bmatrix} e_{12} \\ e_{13} \end{bmatrix}^T - \begin{bmatrix} e_{14} \\ e_{15} \end{bmatrix}\\
&\quad \times \begin{bmatrix} U_3^{11} & U_3^{12} \\ & U_3^{22} \end{bmatrix} \begin{bmatrix} e_{14} \\ e_{15} \end{bmatrix}^T - \gamma_5 \begin{bmatrix} \hat{U}_4 & \mathcal{W} \\ & \hat{U}_4 \end{bmatrix} \gamma_5^T,\\
\Pi_4 &= e_5\Big[\frac{(\rho_2-\rho_1)^2}{2}(T_1+T_2) + \frac{\rho_1^2}{2}(T_1+T_2)\Big]e_5\\
&\quad - 2[(e_3-e_{18})T_1(e_3-e_{18})^T + 2(e_3-4e_{18}+6e_{20})\\
&\quad T_1(e_3-4e_{18}+6e_{20})^T + (e_1-e_{18})T_2(e_1-e_{18})^T\\
&\quad + 2(e_1+2e_{18}-6e_{20})T_2(e_1+2e_{18}-6e_{20})^T]\\
&\quad - 2[(e_4-e_{19})T_1(e_4-e_{19})^T + 2(e_4-4e_{19}+6e_{21})\\
&\quad T_3(e_4-4e_{19}+6e_{21})^T + (e_3-e_{19})T_2(e_3-e_{19})^T\\
&\quad + 2(e_3+2e_{19}-6e_{21})T_4(e_3+2e_{19}-6e_{21})^T,\\
\Pi_5 &= \dot{\rho}(t)\mathbb{E}_5^T \mathcal{W}_1 \mathbb{E}_5 - \dot{\rho}(t)\mathbb{E}_6^T \mathcal{W}_2 \mathbb{E}_6\\
&\quad + Sym\{\mathbb{E}_5^T \mathcal{W}_1 \mathbb{E}_7 + \mathbb{E}_5^T \mathcal{W}_2 \mathbb{E}_8\},\\
\Pi_6 &= Sym\{(\tilde{\gamma}_{11} + \tilde{\gamma}_{12})^T S \hat{\varphi}_s\} + e_5^T S_1 e_1\\
&\quad - e_3^T s_2 e_3 - [(e_1-e_{22})S_3(e_1-e_{22})^T\\
&\quad + 3(e_1+e_{22}-2e_{24})S_3(e_1+e_{22}-2e_{24})^T\\
&\quad + 5(e_1-e_{22}-6e_{24}+12e_6)S_3(e_1-e_{22}-6e_{24}+12e_6)^T\\
&\quad + (e_{22}-e_{23})S_3(e_{22}-e_{23})^T + 3(e_{22}+e_{23}-2e_{25})S_3\\
&\quad \times (e_{22}+e_{23}-2e_{25})^T + 5(e_{22}-e_{23}-6e_{25}+12e_{27})S_3\\
&\quad \times (e_{22}-e_{23}-6e_{25}+12e_{27})^T],\\
\Pi_7 &= \rho_2(e_1 W_3 e_1^T - e_4 W_3 e_4^T) - 2(e_1 - e_4^T)\\
&\quad W_3 \rho_2 e_6 + (\rho_2 - \rho(t))e_7) + \rho_2(e_{27} w_4 e_{27}^T - e_{29} w_4 e_{29}^T)\\
&\quad - 2(e_{27}-e_{29})W_4(e_{27}^T + e_{29}^T),\\
\Pi_8 &= 2[e_1^T \mathcal{F}_1^T + e_5 \mathcal{F}_2^T][-e_5^T - Ae_1^T + e_{30}^T + B\beta(t)\mathcal{K}e_{22}^T\\
&\quad + B\beta(t)\mathcal{K}e_{32}^T],\\
\Pi_9 &= \begin{bmatrix} e_1 \\ e_{28} \end{bmatrix} \triangle_a \begin{bmatrix} e_1 \\ e_{28} \end{bmatrix}^T + \begin{bmatrix} e_2 \\ e_{30} \end{bmatrix}\\
&\quad \triangle_b \begin{bmatrix} e_2 \\ e_{30} \end{bmatrix}^T + \begin{bmatrix} e_1 \\ e_{28} \\ e_2 \\ e_{30} \end{bmatrix} \triangle_c \begin{bmatrix} e_1 \\ e_{28} \\ e_2 \\ e_{30} \end{bmatrix}^T,\\
\Pi_{10} &= -e_1 D \mathcal{Q} e_1^T - e_1 D \mathcal{S} e_{33}^T - \bar{\varrho} e_{22} \tilde{\chi} e_{22}^T + e_{32} \tilde{\chi} e_{32}^T,\\
\bar{\mathcal{V}}_a &= \mathcal{V} + \begin{bmatrix} 0 & \hat{\mathcal{V}}_a \\ \hat{\mathcal{V}}_a & 0 \end{bmatrix}, \quad \bar{\mathcal{V}}_b = \mathcal{V} + \begin{bmatrix} 0 & \hat{\mathcal{V}}_b \\ \hat{\mathcal{V}}_b & 0 \end{bmatrix},\\
\tilde{\gamma}_{11} &= [\mathfrak{B}_{11}^T \ \mathfrak{B}_{12}^T \ \mathfrak{B}_{13}^T]^T,\\
\mathfrak{B}_{11} &= [-A \ \underbrace{0\ 0\ 0}_{20\ times} \ B\beta(t)\mathcal{K} \ \underbrace{0\ 0\ 0}_{9\ times} \ B\beta(t)\mathcal{K} \ 0\ 0\ 0],\\
\mathfrak{B}_{12} &= [\underbrace{0\ 0\ 0}_{21\ times} \ -\hat{\mho}_a \ \underbrace{0\ 0\ 0}_{11\ times} \ -\Lambda_p \ 0],\\
\mathfrak{B}_{13} &= [\underbrace{0\ 0\ 0}_{21\ times} \ \mho_a \ -\hat{\mho}_a \ \underbrace{0\ 0\ 0}_{11\ times} \ -\bar{\Lambda}_p],\\
\tilde{\gamma}_{12} &= \begin{bmatrix} \underbrace{0\ 0\ 0}_{21\ times} \ 0 \ \underbrace{0\ 0\ 0}_{12\ times} \ 0 \\ \underbrace{0\ 0\ 0}_{21\ times} \ \hat{\mho}_a \ \underbrace{0\ 0\ 0}_{12\ times} \ -\Psi_p + \bar{\Lambda}_p \\ \underbrace{0\ 0\ 0}_{21\ times} \ -\hat{\mho}_a \ \underbrace{0\ 0\ 0}_{12\ times} \ \bar{\Lambda}_p \end{bmatrix}, \mathbb{E}_5 = \begin{bmatrix} e_1 \\ e_8 \end{bmatrix},\\
\hat{\varphi}_s &= \begin{bmatrix} \underbrace{0\ 0\ 0}_{33\ times} \ 0\ 0 \\ \underbrace{0\ 0\ 0}_{33\ times} \ \eta \mathbb{I}_{nP} \ 0 \\ \underbrace{0\ 0\ 0}_{33\ times} \ \eta \mathbb{I}_{nP} \ 0 \end{bmatrix}, \mathbb{E}_6 = \begin{bmatrix} e_1 \\ e_7 \end{bmatrix}.\\
\mathbb{E}_7 &= \begin{bmatrix} \rho_1 e_5 \\ e_3 - (1-\rho_3)e_2 - \rho_3 e_8 \end{bmatrix},\\
\mathbb{E}_8 &= \begin{bmatrix} \rho_2 e_5 \\ (1-\rho_3)e_2 - e_4 - \dot{\rho}(t)e_7 \end{bmatrix}.
\end{aligned}
$$

Appendix B. The elements of the matrix $\sum_{i=0}^{10} \bar{\hat{\Pi}}_i$

Replace $R_i,\ Q_i,\ \mathcal{V},\ \hat{U}_j,\ T_j,\ S,\ S_i,\ W_3,\ W_4, i = 1,2,3,\ j = 1,2,3,4$ by (36) and remaining matrix elements $\sum_{i=0}^{7} \bar{\hat{\Pi}}_i$, $\bar{\hat{\Pi}}_9$ are same as in Theorem 3.1, and

$$
\begin{aligned}
\bar{\hat{\Pi}}_8 &= Sym[\tilde{\alpha} e_1^T + e_5][-\mathcal{F} e_5^T - A\mathcal{F} e_1^T + \mathcal{F} e_{29}^T + B\beta(t)\hat{\mathbf{K}} e_{22}^T\\
&\quad + B\beta(t)\hat{\mathbf{K}} e_{32}^T],\\
\bar{\hat{\Pi}}_{10} &= -e_1 \mathcal{F} D \mathcal{Q} e_1^T - e_1 \mathcal{F} D \mathcal{S} e_{33}^T - \bar{\varrho} e_{22} \hat{\tilde{\chi}} e_{22}^T + e_{32} \hat{\tilde{\chi}} e_{32}^T,
\end{aligned}
$$

$$\tilde{\Gamma}_{12} = [\tilde{\alpha}C^T \quad 0 \quad 0 \quad 0 \quad C^T \quad \overbrace{0\;0\;0}^{28\ times}]^T,$$

$$\tilde{\gamma}_{11} = [\mathfrak{B}_{11}^T\ \mathfrak{B}_{12}^T\ \mathfrak{B}_{13}^T]^T,$$

$$\mathfrak{B}_{11} = [-A\ \ \underbrace{0\;0\;0}_{20\ times}\ B\beta(t)\hat{\mathbf{K}}\ \underbrace{0\;0\;0}_{9\ times}\ B\beta(t)\hat{\mathbf{K}}\ 0\ 0\ 0],$$

$$\mathfrak{B}_{12} = [\underbrace{0\;0\;0}_{21\ times}\ -\hat{\mho}_a\mathcal{F}\ \underbrace{0\;0\;0}_{11\ times}\ -\bar{\Lambda}_p\mathcal{F}\ 0],$$

$$\mathfrak{B}_{13} = [\underbrace{0\;0\;0}_{21\ times}\ \mho_a\mathcal{F}\ -\hat{\mho}_a\mathcal{F}\ \underbrace{0\;0\;0}_{11\ times}\ -\bar{\Lambda}_p\mathcal{F}],$$

$$\tilde{\gamma}_{12} = \begin{bmatrix} \underbrace{0\;0\;0}_{21\ times}\ 0\ \underbrace{0\;0\;0}_{12\ times}\ 0 \\ \underbrace{0\;0\;0}_{21\ times}\ \hat{\mho}_a\mathcal{F}\ \underbrace{0\;0\;0}_{12\ times}\ (-\Psi_p + \bar{\Lambda}_p)\mathcal{F} \\ \underbrace{0\;0\;0}_{21\ times}\ -\hat{\mho}_a\mathcal{F}\ \underbrace{0\;0\;0}_{12\ times}\ \bar{\Lambda}_p\mathcal{F} \end{bmatrix}.$$

CRediT authorship contribution statement

R. Vadivel: Conceptualization, Methodology, Writing - original draft. **P. Hammachukiattikul:** Formal analysis, Validation, Writing - review & editing. **Nallappan Gunasekaran:** Software, Visualization. **R. Saravanakumar:** Methodology, Formal analysis. **Hemen Dutta:** Software, Supervision.

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