

# Research Article

# Mixed *H*-Infinity and Passive Synchronization of Markovian Jumping Neutral-Type Complex Dynamical Networks with Randomly Occurring Distributed Coupling Time-Varying Delays and Actuator Faults

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Received 13 January 2021; Revised 28 March 2021; Accepted 17 April 2021; Published 4 May 2021

Academic Editor: Chong Fu

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This article examines mixed  $\mathscr{H}$ -infinity and passivity synchronization of Markovian jumping neutral-type complex dynamical network (MJNTCDN) models with randomly occurring coupling delays and actuator faults. The randomly occurring coupling delays are considered to design the complex dynamical networks in practice. These delays complied with certain Bernoulli distributed white noise sequences. The relevant data including limits of actuator faults, bounds of the nonlinear terms, and external disturbances are available for designing the controller structure. Novel Lyapunov–Krasovskii functional (LKF) is constructed to verify the stability of the error model and performance level. Jensen's inequality and a new integral inequality are applied to derive the outcomes. Sufficient conditions for the synchronization error system (SES) are given in terms of linear matrix inequalities (LMIs), which can be analyzed easily by utilizing general numerical programming. Numerical illustrations are given to exhibit the usefulness of the obtained results.

# 1. Introduction

Complex dynamical network (CDN) models are firstly investigated by Watts and Strogatz [1]. These models are sets of large-scale coupled nodes of interconnected systems, e.g., chemical substrates, microprocessors, and computers [2, 3]. Generally, CDN models are epitomized in practice, e.g., the World Wide Web, food webs, cellular and metabolic networks, neural systems, aviation systems, transportation systems, and electricity distribution systems [4, 5]. A number of these systems exhibit complexities in their topological and dynamical properties. The application of these networks can be found in streamlining the structure as well as reducing the control costs [6].

Particularly, synchronization remains normal along with the significant group of powerful performance as concerns CDNs, which has turned into an expanding concern furthermore, close inquire about the theme with its expressive ongoing years [7]. Synchronization is a regular and significant aggregate conduct of a complex network. As a key look into zone broad, endeavors are given to the comprehension of the synchronization system of such organization. As a central wonder in moving practices about systems, synchronization issue as concerns CDNs has appealed to an impressive consideration against different range, for example, environment, aerology, building, control matrices as well as banking sector [8, 9], because of its productive applications. Since inadequate data frequently show up in some practical systems, for example, just 5% of excitatory neurotransmitters can be received between two associated cortical regions of brain systems [10], not all the diverted data of clients in wireless systems are known to the transmitter [11]. Immediately, the synchronization issue of complex networks with partial coupling is tended to in [12]. For the model, a time-varying complex dynamical networks model and its synchronization criteria are investigated in [13]. Synchronization of CDNs with a weighted timevarying adjacency matrix was explored by means of distributed adaptive control in [14]. The coupled network models are regularly utilized as scientific instruments to demonstrate in break down systems. For asynchronous complex networks, it is specifically noteworthy to locate a lot of controllers to accomplish synchronization of dynamical practices of all nodes in systems.

Markovian jump networks (MJNs) are a class of stochastic hybrid models. They are dynamic networks whose structures are subjected to random parameter that changes abrupt condition disturbances, changing subsystem interconnections that occur nonlinearly [15–21]. When CDN models get unexpected changes in their structure, we can represent them using Markovian jumping complex networks, which have been studied by many researchers [22,23].

In practical systems, time delays are regularly experienced. Consequently, time delays in couplings and in dynamical nodes have received considerable attention [24, 25]. On the other hand, time delays are unavoidable as a result of constrained transmission speeds and traffic congestion. Many researchers have investigated the effects of time delays on stability and synchronization [26]. Indeed, the existence of time delays can be found in numerous systems, such as atomic reactors, population dynamic models, aircraft stabilization, natural frameworks, chemical engineering systems, and ship stabilization circuit theory [27, 28].

It is natural that actuator faults occur frequently, which can lead to unstable system operation [29]. It is imperative to design controllers with the capability of ensuring the stability of closed-loop systems in the event of faults. In this regard, various control methodologies are available to realize the control objectives subject to faults. Fault-tolerant control (FTC) systems can be divided into two types: passive and active approaches. Passive FTC controllers usually have basic structures with restricted fault-tolerant capabilities. In contrast, active FTC controllers have structures with better self-organization. Fault identification or estimation generally requires dynamic fault-tolerant controllers that can be changed in accordance with data limitations (see [30, 31]). Motivated by the above discussion, we study mixed  $\mathcal{H}$ -infinity and passivity-based synchronization of Markovian jumping neutral-type complex dynamical network (MJNTCDN) models with time-varying distributed coupling delays and actuator faults in this paper that are as follows:

- We analyze the mixed *H*-infinity and passivity synchronization of MJNTCDN models with distributed random coupling time-varying delays and actuator faults
- (2) The randomly occurring coupling delays satisfy the Bernoulli random binary procedure
- (3) Delay-dependent conditions are derived to guarantee the MJNTCDN models is mixed *ℋ*-infinity and passive performance at level γ
- (4) Improved Jensen's inequalities and integral inequalities are utilized to infer the sufficient conditions in terms of LMIs
- (5) Numerical results are provided to exhibit the effectiveness of the proposed method

Notations. The following notations are used throughout this paper.  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  denote the *n*-dimensional Euclidean space and the set of all  $m \times n$  real matrices, respectively, while  $\mathbf{P}_1 > 0$  means that matrix  $\mathbf{P}_1$  is real symmetric and positive definite. Superscript "T" stands for the transpose; diag{·} stands for a block diagonal matrix;  $\mathbf{A}^{-1}$  denotes the matrix inverse of  $\mathbf{A}$ ;  $\mathscr{C}\{\cdot\}$  stands for the mathematical expectation operator with respect to the given probability measure;  $\mathscr{L}_2[0,\infty)$  stands for the space of square-integrable vector functions over interval  $[0,\infty)$ ;  $\|\cdot\|$  denotes the Euclidean norm of a vector and its induced norm of a matrix. The symbol "\*" is used to represent the term of a symmetric matrix which can be inferred by symmetry; symbol "&" stands for the Kronecker product. If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. Given a complete probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t>0}, \mathcal{P})$  with a natural filtration  $\{\mathcal{F}_t\}_{t>0}$  fulfilling the typical conditions (i.e., the filtration contains all  $\mathscr{P}$ -invalid sets and is right continuous), where  $\Omega$  is the sample space,  $\mathcal{F}$  is the algebra of events, and  $\mathcal{P}$  is the probability measure defined on  $\mathcal{F}$ . The process { $\delta(t), t \leq 0$ } is a continuous homogeneous Markovian process with right continuous trajectories and takes values in the finite space  $S = \{1, 2, ..., N\}$ . In particular,  $\delta(t)$  is related to the change likelihood network  $\Theta = (\theta_{pq})_{\mathcal{N} \times \mathcal{N}} (p, q \in \mathcal{S})$  given by the transition rates:

$$P\{\delta(t+h) = q|\delta(t) = p\} = \begin{cases} \theta_{pq}h + o(h), & p \neq q, \\ 1 + \theta_{pp}h + o(h), & p = q, \end{cases}$$
(1)

where h > 0 is the visit time and  $\lim_{h \to 0} o(h)/h = 0$ . Here,  $\theta_{pq} \ge 0$  is the change rate from mode *p* at time *t* to mode *q* at time *t* + *h* if  $p \ne q$  and  $\theta_{pp} = -\sum_{q=1,q \ne p}^{\mathcal{N}} \theta_{ij}$ .

#### 2. Problem Statement and Preliminaries

We recognize a class of MJNTCDN models composed of N indistinguishable coupled nodes, in which each node has n-dimensional randomly occurring distributed coupling

delays and actuator faults and position of the elements of the *z*th hub is spoken to through the accompanying nonlinear dynamical subsystem:

$$\begin{aligned} \dot{\hat{r}}_{z}(t) &= \mathbf{A}_{1}(\delta(t))\hat{r}_{z}(t) + \mathbf{B}_{1}(\delta(t))f\left(t,\hat{r}_{z}(t)\right) + \mathbf{B}_{2}(\delta(t))f\left(t,\hat{r}_{z}(t-\vartheta(t))\right) + \mathbf{C}(\delta(t))\dot{\hat{r}}_{z}(t-\kappa(t)) \\ &+ (1-\beta(t))\sum_{j=1}^{N} \mathbf{g}_{zj}^{(1)}\Gamma_{1}(\delta(t))\hat{r}_{j}(t) + \beta(t)\sum_{j=1}^{N} \mathbf{g}_{zj}^{(2)}\Gamma_{2}(\delta(t))\int_{t-\rho(t)}^{t}\hat{r}_{j}(s)\mathrm{d}s + \mathbf{D}_{1}(\delta(t))u_{z}^{f}(t) + \mathbf{E}_{1}(\delta(t))v_{z}(t), \end{aligned}$$
(2)  
$$\hat{y}_{z}(t) &= \mathbf{A}_{2}(\delta(t))\hat{r}_{z}(t), \quad z = 1, 2, \dots, N, \\ \hat{r}_{z}(t) &= \Phi_{z}(t), \quad \forall t \in [-h, 0], \ h = \max\{\vartheta_{2}, \kappa_{2}\}, \end{aligned}$$

where  $\hat{r}_z(t) = (\hat{r}_{z1}(t), \hat{r}_{z2}(t), \dots, \hat{r}_{zn}(t))^T \in \mathbb{R}^n$  stands for the state vector *z*th node with respect to the model at time *t*,  $\hat{y}_z(t)$  is the measured output corresponding to the *z*th node;  $\Phi_{\tilde{x}}(t)$  is the continuous initial function of the *z*th node;  $u_z^f(t) \in \mathbb{R}^m$  denotes the fault control input vector of the *z*th hub;  $w_z(t) \in \mathbb{R}^p$  represents the constant external input vector which belongs to  $\mathcal{L}_2[0,\infty)$ ;  $f: \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$  is continuously differentiable smooth nonlinear vector-valued field, which satisfies sector-bounded conditions which will be characterized later;  $\mathbf{G}^{(q)} = (\mathbf{g}_{zj}^{(l)}) \in \mathbb{R}^{N \times N}$  (l = 1, 2) are the outer-coupling configuration matrices representing the topological structure of the model, in which  $\mathbf{g}_{zj}$  is defined as follows: if there is a connection from node *z* to node  $j(z \neq j)$ , then the coupling matrix  $\mathbf{g}_{zj}^{(1)} = \mathbf{g}_{jz}^{(1)} = 1$  and  $\mathbf{g}_{zj}^{(2)} = \mathbf{g}_{jz}^{(2)} = 1$ ; otherwise,  $\mathbf{g}_{zj}^{(1)} = \mathbf{g}_{jz}^{(1)} = 0$  and  $\mathbf{g}_{zj}^{(2)} = \mathbf{g}_{jz}^{(2)} = 0$ . The diagonal elements are defined by

$$\mathbf{g}_{zz}^{(1)} = -\sum_{j=1, j\neq z}^{N} \mathbf{g}_{zj}^{(1)}, 
\mathbf{g}_{zz}^{(2)} = -\sum_{j=1, j\neq z}^{N} \mathbf{g}_{zj}^{(2)}, \quad z = 1, 2, \dots, N.$$
(3)

 $\Gamma_a(\delta(t)) = \text{diag}\{u_{a1}(\delta(t)), u_{a2}(\delta(t)), \dots, u_{an}(\delta(t))\}\)$  (*a* = 1,2) are constant diagonal inner-coupling matrices. In addition,  $\mathbf{A}_1(\delta(t))$ ,  $\mathbf{A}_2(\delta(t))$ ,  $\mathbf{B}_1(\delta(t))$ ,  $\mathbf{B}_2(\delta(t))$ ,  $\mathbf{C}_1(\delta(t))$ ,  $\mathbf{D}_1(\delta(t))$ , and  $\mathbf{E}_1(\delta(t))$  are constant matrices with appropriate dimensions. For the time delay signals,  $\vartheta(t), \kappa(t)$ , and  $\rho(t)$  denote the retarded delays, distributed delays, and neutral time-varying delays, respectively, and are assumed to satisfy the following inequalities:

$$0 \le \vartheta_1 \le \vartheta(t) \le \vartheta_2,$$
  

$$\dot{\vartheta}(t) \le \varsigma_1 < 1,$$
  

$$0 \le \kappa_1 \le \kappa(t) \le \kappa_2,$$
  

$$\dot{\kappa}(t) \le \varsigma_2 < 1,$$
  

$$0 \le \rho(t) \le \overline{\rho},$$
  

$$\dot{\rho}(t) \le \mu_3,$$
  
(4)

where  $\vartheta_1, \vartheta_2, \kappa_1, \kappa_2, \varsigma_1, \varsigma_2, \mu_3$ , and  $\overline{\rho}$  are real constant scalars.

The randomly occurring coupling delay  $\beta(t) \in \mathbb{R}$  denotes a stochastic variable which is in the form of a Bernoulli distributed sequence defined by

$$\beta(t) = \begin{cases} 1, & \text{if delayed information exchanges happen,} \\ 0, & \text{if delayed information exchanges do not happen,} \end{cases}$$
(5)

The probability occurrence of stochastic variable  $\beta(t)$  is given by

$$Prob\{\beta(t) = 1\} = \overline{\beta},$$
  

$$Prob\{\beta(t) = 0\} = 1 - \overline{\beta},$$
(6)

where  $\overline{\beta} \in [0, 1]$  is a known constant. Then, we can obtain

$$\mathscr{C}\left\{\beta(t) - \overline{\beta}\right\} = 0,$$

$$\mathscr{C}\left\{(\beta(t) - \overline{\beta})^2\right\} = \overline{\beta}(1 - \overline{\beta}),$$

$$(7)$$

Assumption 1. For nonlinear function  $f(\cdot): \mathbb{R}^n \longrightarrow \mathbb{R}^n$ , there exist constant matrices  $\mathbf{G}_1$  and  $\mathbf{G}_2$  such that

$$\|f(t, \hat{r}_{z}(t)\| \leq \mathbf{G}_{1}\|\hat{r}_{z}(t)\|,$$

$$\|f(t, \hat{r}_{z}(t-\vartheta(t)))\| \leq \mathbf{G}_{2}\|\hat{r}_{z}(t-\tau(t))\|,$$

$$(8)$$

for any  $\hat{r}_z(t) \in \mathbb{R}^n$ .

To synchronize all the N indistinguishable nodes in model (2) to a common value, let us characterize the synchronization error vector  $\hat{e}_z(t) = \hat{r}_z(t) - \hat{s}(t)$  be the synchronization error, where  $\hat{s}(t) \in \mathbb{R}^n$  represents the state vector of the unforced isolate node and is chosen as

$$\begin{split} \widehat{s}(t) &= \mathbf{A}_{1}\left(\delta(t)\right)\widehat{s}(t) + \mathbf{B}_{1}\left(\delta(t)\right)f\left(t,\widehat{s}(t)\right) \\ &+ \mathbf{B}_{2}\left(\delta(t)\right)f\left(t,\widehat{s}\left(t-\vartheta(t)\right)\right) + \mathbf{C}_{1}\left(\delta(t)\right)\dot{\widehat{s}}\left(t-\kappa(t)\right), \\ \widehat{y}_{s}(t) &= \mathbf{A}_{2}\left(\delta(t)\right)\widehat{s}(t), \end{split}$$

$$\end{split}$$

$$(9)$$

where  $\hat{y}_s(t)$  is the unforced isolate output vector. We define the synchronization error dynamics as

$$\begin{aligned} \dot{\hat{e}}_{z}(t) &= \mathbf{A}_{1}(\delta(t))\hat{e}_{z}(t) + \mathbf{B}_{1}(\delta(t))h_{1}\left(t,\hat{e}_{z}(t)\right) + \mathbf{B}_{2}(\delta(t))h_{2}\left(t,\hat{e}_{z}(t-\vartheta(t))\right) + \mathbf{C}_{1}(\delta(t))\dot{\hat{e}}_{z}(t-\kappa(t)) \\ &+ (1-\beta(t))\sum_{j=1}^{N} \mathbf{g}_{zj}^{(1)}\Gamma_{1}(\delta(t))\hat{e}_{j}(t) + \beta(t)\sum_{j=1}^{N} \mathbf{g}_{zj}^{(2)}\Gamma_{2}(\delta(t))\int_{t-\rho(t)}^{t}\hat{e}_{j}(s)\mathrm{d}s + \mathbf{D}_{1}(\delta(t))u_{z}^{f}(t) + \mathbf{E}_{1}(\delta(t))v_{z}(t), \\ \tilde{y}_{z}(t) &= \mathbf{A}_{2}(\delta(t))\hat{e}_{z}(t), \quad z = 1, 2, \dots, N, \\ \hat{e}_{z}(t) &= \Phi_{z}(t), \quad \forall t \in [-h, 0], \ h = \max\{\vartheta_{2}, \kappa_{2}\}. \end{aligned}$$

where  $h_1(t, \hat{e}_z(t)) = f(t, \hat{r}_z(t)) - f(t, \hat{s}(t)), \quad h_2(t, \hat{e}_z(t-\vartheta(t))) = f(t, \hat{r}_z(t-\vartheta(t))) - f(t, \hat{s}(t-\vartheta(t))), \text{ and } \tilde{y}_z(t) = \hat{y}_z(t) - \hat{y}_s(t).$ 

The actuator fault model is used as [32, 33]

$$u_{z,j}^{f}(t) = \left(1 - \lambda_{z,j}(t)\right) u_{z,j}(t), \quad 0 \le \lambda_{z,j}(t) \le \lambda_{*j} < 1, \quad (11)$$

where z = (1, 2, ..., N) denotes the *z*th agent and j = (1, 2, ..., m) denotes the *j*th actuator,  $u_{z,j}$  is the input signal of the actuator,  $u_{z,j}^f$  is the output signal from the actuator,  $\lambda_{z,j}(t)$  is an unknown and piecewise continuous bounded actuator failure factor which shows the degree of effectiveness of the actuator, and  $\lambda_{*j}$  is a known constant which represents the upper limits of  $\lambda_{z,j}(t)$ ,  $\forall z$ . Note that, when  $\lambda_{z,j}(t) = 0$ , there is no deficiency of the actuator, i.e.,

the *j*th actuator of the *z*th agent is sound or typical; when  $0 < \lambda_{z,j}(t) < 1$ , the *j*th actuator is subject to fault.

We indicate

$$u_{z}^{f}(t) = \left[u_{z,1}^{f}(t), u_{z,2}^{f}(t), \dots, u_{z,m}^{f}(t)\right]^{I},$$
  

$$\lambda_{z}(t) = \operatorname{diag}\left\{\lambda_{z,1}(t), \lambda_{z,2}(t), \dots, \lambda_{z,m}(t)\right\}.$$
(12)

In this case, a uniform actuator fault model is represented as

$$u_z^f(t) = \left(\mathcal{F}_m - \lambda_z(t)\right) u_z(t). \tag{13}$$

Taking the actuator fault (13) into account, model (10) can be represented as follows:

$$\begin{cases} \dot{\hat{e}}_{z}(t) = \mathbf{A}_{1}(\delta(t))\hat{e}_{z}(t) + \mathbf{B}_{1}(\delta(t))h_{1}(t,\hat{e}_{z}(t)) + \mathbf{B}_{2}(\delta(t))h_{2}(t,\hat{e}_{z}(t-\vartheta(t))) + \mathbf{C}_{1}(\delta(t))\dot{\hat{e}}_{z}(t-\kappa(t)) \\ + (1-\beta(t))\sum_{j=1}^{N} \mathbf{g}_{zj}^{(1)}\Gamma_{1}(\delta(t))\hat{e}_{j}(t) + \beta(t)\sum_{j=1}^{N} \mathbf{g}_{zj}^{(2)}\Gamma_{2}(\delta(t)) \int_{t-\rho(t)}^{t} \hat{e}_{j}(s)ds + \mathbf{D}_{1}(\delta(t))(\mathcal{F}_{m} - \lambda_{z}(t))u_{z}(t) + \mathbf{E}_{1}(\delta(t))v_{z}(t), \\ \tilde{y}_{z}(t) = \mathbf{A}_{2}(\delta(t))\hat{e}_{z}(t), \quad z = 1, 2, \dots, N, \\ \hat{e}_{z}(t) = \Phi_{z}(t), \quad \forall t \in [-h, 0], \ h = \max\{\vartheta_{2}, \kappa_{2}\}, \end{cases}$$

$$(14)$$

where  $\overline{\lambda} = \text{diag}\{\overline{\lambda}_{*,1}, \overline{\lambda}_{*,2}, \dots, \overline{\lambda}_{*,m}\}.$ 

The following state feedback controller law is employed for the synchronization error model:

$$u_z(t) = \mathbf{K}_z(\delta(t))\hat{e}_z(t), \quad z = 1, 2, \dots, N,$$
(15)

where  $\mathbf{K}_{z}(\delta(t))$  is a real constant matrix representing the gain matrix of the feedback controller to be determined.

Therefore, the control input is substituted into the error model (14) leading to

$$\begin{cases} \dot{\hat{e}}_{z}(t) = \mathbf{A}_{1}(\delta(t))\hat{e}_{z}(t) + \mathbf{B}_{1}(\delta(t))h_{1}\left(t,\hat{e}_{z}(t)\right) + \mathbf{B}_{2}(\delta(t))h_{2}\left(t,\hat{e}_{z}\left(t-\vartheta(t)\right)\right) + \mathbf{C}_{1}\left(\delta(t)\right)\hat{\hat{e}}_{z}\left(t-\kappa(t)\right) \\ + (1-\beta(t))\sum_{j=1}^{N} \mathbf{g}_{zj}^{(1)}\Gamma_{1}\left(\delta(t)\right)\hat{e}_{j}\left(t\right) + \beta(t)\sum_{j=1}^{N} \mathbf{g}_{zj}^{(2)}\Gamma_{2}\left(\delta(t)\right) \int_{t-\rho(t)}^{t} \hat{e}_{j}\left(s\right)ds + \mathbf{D}_{1}\left(\delta(t)\right)\left(\mathcal{I}_{m}-\lambda_{z}\left(t\right)\right)K_{z}\left(\delta(t)\right)\hat{e}_{z}\left(t\right) + \mathbf{E}_{1}\left(\delta(t)\right)v_{z}\left(t\right), \\ \tilde{y}_{z}\left(t\right) = \mathbf{A}_{2}\left(\delta(t)\right)\hat{e}_{z}\left(t\right), \quad z = 1, 2, \dots, N, \\ \hat{e}_{z}\left(t\right) = \Phi_{z}\left(t\right), \quad \forall t \in [-h, 0], \ h = \max\{\vartheta_{2}, \kappa_{2}\}. \end{cases}$$

For convenience, every conceivable estimation of  $\delta(t)$  is meant by  $p, p \in S$  in the sequel. As such, we have (16)

$$\mathbf{A}_{1} (\delta(t)) = \mathbf{A}_{1p},$$

$$\mathbf{A}_{2} (\delta(t)) = \mathbf{A}_{2p},$$

$$\mathbf{B}_{1} (\delta(t)) = \mathbf{B}_{1p},$$

$$\mathbf{B}_{2} (\delta(t)) = \mathbf{B}_{2p},$$

$$\mathbf{C}_{1} (\delta(t)) = \mathbf{C}_{1p},$$

$$\mathbf{D}_{1} (\delta(t)) = \mathbf{D}_{1p},$$

$$\mathbf{E}_{1} (\delta(t)) = \mathbf{E}_{1p},$$

$$\Gamma_{1} (\delta(t)) = \Gamma_{1p},$$

$$\Gamma_{2} (\delta(t)) = \Gamma_{2p},$$
(17)

where  $\mathbf{A}_{1p}$ ,  $\mathbf{A}_{2p}$ ,  $\mathbf{B}_{1p}$ ,  $\mathbf{B}_{2p}$ ,  $\mathbf{C}_{1p}$ ,  $\mathbf{D}_{1p}$ , and  $\mathbf{E}_{1p}$  for any  $p \in S$  are known constant matrices of appropriate dimensions.

Then, error dynamical model (16) in virtue of the Kronecker product can be written in the following compact form:

$$\begin{aligned} \dot{\hat{e}}(t) &= \left( \left( \mathscr{F}_{N} \otimes \mathbf{A}_{1p} \right) + \left( \mathscr{F}_{N} \otimes \mathbf{D}_{1p} \right) \left( \mathscr{F}_{N} \otimes \mathbf{K}_{p} \right) - \left( \mathscr{F}_{N} \otimes \mathbf{D}_{1p} \right) \lambda(t) \left( \mathscr{F}_{N} \otimes \mathbf{K}_{p} \right) \right) \hat{e}(t) \\ &+ \left( \mathscr{F}_{N} \otimes \mathbf{B}_{1p} \right) \mathbb{H}_{1}(t, \hat{e}(t)) + \left( \mathscr{F}_{N} \otimes \mathbf{B}_{2p} \right) \mathbb{H}_{2}(t, \hat{e}(t - \vartheta(t))) + \left( \mathscr{F}_{N} \otimes \mathbf{C}_{1p} \right) \dot{\hat{e}}(t - \kappa(t)) \\ &+ (1 - \beta(t)) \left( \mathbf{G}^{(1)} \otimes \Gamma_{1p} \right) \hat{e}(t) + \beta(t) \left( \mathbf{G}^{(2)} \otimes \Gamma_{2p} \right) \int_{t - \rho(t)}^{t} \hat{e}(s) \mathrm{d}s + \left( \mathscr{F}_{N} \otimes \mathbf{E}_{1p} \right) \nu(t), \end{aligned}$$
(18)  
$$\tilde{\gamma}(t) = \left( \mathscr{F}_{N} \otimes \mathbf{A}_{2p} \right) \hat{e}(t), \end{aligned}$$

where

$$\widehat{e}(t) \coloneqq \left[\widehat{e}_{1}^{T}(t)\widehat{e}_{2}^{T}(t), \dots, \widehat{e}_{N}^{T}(t)\right]^{T}, \\
\mathbb{H}_{1}(t,\widehat{e}(t)) \coloneqq \left[h_{1}^{T}(t,\widehat{e}_{1}(t))h_{1}^{T}(t,\widehat{e}_{2}(t)), \dots, h_{1}^{T}(t,\widehat{e}_{N}(t))\right]^{T}, \\
\mathbb{H}_{2}(t,\widehat{e}(t-\vartheta(t))) \coloneqq \left[h_{2}^{T}(t,\widehat{e}_{1}(t-\vartheta(t)))h_{2}^{T}(t,\widehat{e}_{2}(t-\vartheta(t))), \dots, h_{2}^{T}(t,\widehat{e}_{N}(t-\vartheta(t)))\right]^{T}, \\
\lambda(t) \coloneqq di ag\{\lambda_{1}(t)\lambda_{2}(t), \dots, \lambda_{N}(t)\}, \\
\nu(t) \coloneqq \left[\nu_{1}^{T}(t)\nu_{2}^{T}(t), \dots, \nu_{N}^{T}(t)\right]^{T}, \\
\widetilde{y}(t) \coloneqq \left[\widetilde{y}_{1}^{T}(t)\widetilde{y}_{2}^{T}(t), \dots, \widetilde{y}_{N}^{T}(t)\right]^{T}, \\
\mathbf{K}_{p} \coloneqq diag\{\mathbf{K}_{1p}\mathbf{K}_{2p}, \dots, \mathbf{K}_{Np}\}.$$
(19)

In the following, we present the associated definitions and lemmas, which are required to derive the main results of this paper. Definition 1 (see [34]). Model (18) is said to have mixed  $\mathscr{H}$ -infinity and passive performance  $\gamma$ , if there exists a constant  $\gamma > 0$  such that

$$\int_{0}^{\mathcal{T}_{p}} \left[ -\sigma \tilde{y}^{T}(t) \tilde{y}(t) + 2(1-\sigma)\gamma \tilde{y}^{T}(t) v(t) \right] dt \ge -\gamma^{2} \int_{0}^{\mathcal{T}_{p}} \left[ v^{T}(t) v(t) \right] dt,$$

$$(20)$$

for any  $\mathcal{T}_p \ge 0$  and any nonzero  $v(t) \in \mathcal{L}_2[0,\infty)$ , where  $\sigma \in [0,1]$  represents the parameter that defines the trade-off between  $\mathcal{H}$ -infinity and passive performance.

*Definition 2.* The mixed  $\mathcal{H}$ -infinity and passive synchronization of model (18) can achieve the mixed  $\mathcal{H}$ -infinity and passive performance at level  $\gamma$ .

*Definition 3* (see [35]). For model (18), if there exists a scalar v > 0 such that the derivative of the Lyapunov function with respect to time *t* satisfies

$$\dot{V}(t) \le -v|e(t)|^2,$$
 (21)

then model (18) with v(t) = 0 is said to be quadratically stable.

*Remark 1.* This article considered the complex dynamic network system with the linear coupling strength on the mode at time t, while the rest of the coefficient is constant matrix, in the process of jump. This makes the complex dynamical network system more general.

*Remark* 2. Note that model (20) incorporates mixed  $\mathcal{H}$ -infinity and passivity performance level, which is a unique instance of dissipativity. For instance, if  $\sigma$  is estimated as 1, model (20) decreases to  $\mathcal{H}$ -infinity performance level; if  $\sigma$  is estimated as 0, model (20) decreases to the passivity performance; when  $\sigma$  takes (0, 1), model (20) represents the mixed  $\mathcal{H}$ -infinity and passivity performance level.

**Lemma 1** (see [36]). For any matrix  $\mathcal{N} \in \mathbb{R}^{n \times n}$  and  $\mathcal{N} = \mathcal{N}^T > 0$ , and given scalar  $\lambda > 0$ , the vector function is  $\Phi$ :  $[0, \lambda] \longrightarrow \mathbb{R}^n$  such that the following relation holds:

$$-\lambda \int_{0}^{\lambda} \Phi^{T}(s) \mathcal{N}\Phi(s) \mathrm{d}s \leq -\left(\int_{0}^{\lambda} \Phi(s) \mathrm{d}s\right)^{T} \mathcal{N}\left(\int_{0}^{\lambda} \Phi(s) \mathrm{d}s\right).$$
(22)

**Lemma 2** (see [36]). For any matrix  $\mathcal{N} \in \mathbb{R}^{n \times n}$  and  $\mathcal{N} = \mathcal{N}^T > 0$ , and given a scalar function  $\lambda := \lambda(t) > 0$ , the vector-valued function is  $\Phi: [-\lambda, 0] \longrightarrow \mathbb{R}^n$  such that the following relation holds:

$$-\lambda \int_{t-\lambda}^{t} \dot{\Phi}^{T}(s) \mathcal{N} \dot{\Phi}(s) ds \leq \begin{bmatrix} \Phi(t) \\ \Phi(t-\lambda) \end{bmatrix}^{T} \begin{bmatrix} -\mathcal{N} & \mathcal{N} \\ \star & -\mathcal{N} \end{bmatrix} \begin{bmatrix} \Phi(t) \\ \Phi(t-\lambda) \end{bmatrix}.$$
(23)

**Lemma 3** (see [37]). For a given matrices  $\mathcal{A}_{11}$ ,  $\mathcal{A}_{12}$ ,  $\mathcal{A}_{21}$ , and  $\mathcal{A}_{22}$  with appropriate dimensions,  $\begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21}^T & \mathcal{A}_{22} \end{bmatrix} < 0$  holds if and only if  $\mathcal{A}_{22} < 0$ ,  $\mathcal{A}_{12} - \mathcal{A}_{12}\mathcal{A}_{22}^{-1}\mathcal{A}_{12}^T < 0$ .

**Lemma 4** (see [38]). *The Kronecker product has the following properties:* 

- (1)  $(\alpha A) \otimes B = A \otimes (\alpha B)$
- $(2) (A+B) \otimes C = A \otimes C + B \otimes C$
- $(3) \ (A \otimes B)(C \otimes D) = (AC) \otimes (BD)$
- $(4) \ (A \otimes B)^T = A^T \otimes B^T$

#### 3. Main Results

In this section, we establish the LMI to determine a faulttolerant controller which is mixed  $\mathcal{H}$ -infinity and passive with synchronization error model (18).

**Theorem 1.** Given some constants such as  $\vartheta_1$ ,  $\vartheta_2$ ,  $\kappa_1$ ,  $\kappa_2$ ,  $\varsigma_1$ ,  $\varsigma_2$ ,  $\mu_3$ ,  $\overline{\rho}$ ,  $\beta_1$ ,  $\beta_2$ ,  $\sigma$ ,  $\overline{\beta} \in [0, 1]$ ,  $\gamma$ ,  $\overline{\lambda}^*$  and diagonal matrices  $\Gamma_{1p}$ ,  $\Gamma_{2p}$  ( $p \in S$ ),  $\mathbf{G}_1$ ,  $\mathbf{G}_2$ , and  $\mathbf{G}^{(1)}$ ,  $\mathbf{G}^{(2)}$  are known constant matrices, error model (18) satisfies a mixed  $\mathcal{H}$ -infinity and passivity performance  $\gamma > 0$  in the sense of Definitions 1 and 2, if there exist symmetric positive definite matrices  $\mathbf{X}_p > 0$  ( $p \in S$ ),  $\mathbf{P}_p > 0$ ,  $\mathbf{U}_m > 0$  ( $m = 2, \ldots, 11$ ),  $\mathbf{Z} > 0$ , and  $\mathbf{Y}_p$  and  $\mathbf{M}_p$  ( $p \in S$ ) are of appropriate dimension matrices such that the following successive LMIs hold:

where

$$\begin{split} \tilde{\Psi}_{p11} &= \theta_{pp} (\mathcal{F}_{N} \otimes \tilde{X}_{p}) + (\mathcal{F}_{N} \otimes \tilde{U}_{1}) + (\mathcal{F}_{N} \otimes \tilde{U}_{1}) + (\mathcal{F}_{N} \otimes \tilde{U}_{1}) + (\mathcal{F}_{N} \otimes \tilde{U}_{1}) - \frac{1}{\theta_{1}} (\mathcal{F}_{N} \otimes \tilde{U}_{1}) - \frac{1}{\theta_{1}} (\mathcal{F}_{N} \otimes \tilde{U}_{1}) \\ &- \frac{1}{\kappa_{1}} (\mathcal{F}_{N} \otimes \tilde{U}_{1}) - \frac{1}{\kappa_{2}} (\mathcal{F}_{N} \otimes \tilde{U}_{1}) + (\mathcal{F}_{N} \otimes \Lambda_{1}) (\mathcal{F}_{N} \otimes X_{p}) + (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes \Lambda_{1})^{T} + (\mathcal{F}_{N} \otimes D_{1}) \\ &\times (\mathcal{F}_{N} \otimes \Psi_{p}) + (\mathcal{F}_{N} \otimes \Psi_{p})^{T} (\mathcal{F}_{N} \otimes D_{1})^{T} - (\mathcal{F}_{N} \otimes D_{1})^{T} (\mathcal{F}_{N} \otimes \chi_{p}) - (\mathcal{F}_{N} \otimes \Psi_{p})^{T} \tilde{\lambda}_{1}^{T} (\mathcal{F}_{N} \otimes \tilde{\lambda}) \\ &+ (1 - \tilde{\rho}) (\mathcal{G}^{(1)} \otimes \Gamma_{1}) (\mathcal{F}_{N} \otimes X_{p}) + (1 - \tilde{\rho}) (\mathcal{F}_{N} \otimes X_{p})^{T} (\mathcal{F}_{N} \otimes \Psi_{p})^{T} (\mathcal{F}_{N} \otimes \tilde{\lambda}) \\ &- \rho_{1} (\mathcal{F}_{N} \otimes \Psi_{p})^{T} \tilde{\lambda}_{1}^{T} (\mathcal{F}_{N} \otimes L_{p}) + \rho_{1} (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{G}^{(1)} \otimes \Gamma_{1})^{T} + \rho_{1} (\mathcal{F}_{N} \otimes \Psi_{p})^{T} \tilde{\lambda}_{1}^{T} (\mathcal{F}_{N} \otimes \tilde{U}_{p}) \\ &- \rho_{1} (\mathcal{F}_{N} \otimes \Psi_{p})^{T} \tilde{\lambda}_{1}^{T} (\mathcal{F}_{N} \otimes \Lambda_{p}) + \rho_{1} (\mathcal{F}_{N} \otimes \Psi_{p})^{T} (\mathcal{F}_{N} \otimes \Psi_{p}) (\mathcal{G}^{(1)} \otimes \Gamma_{1})^{T} \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{G}^{(1)} \otimes \Gamma_{1})^{T} \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{G}^{(1)} \otimes \Gamma_{1})^{T} \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{G}^{(1)} \otimes \Gamma_{1})^{T} \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes \Lambda_{p})^{T} (\mathcal{F}_{N} \otimes \Psi_{p}) \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes \Lambda_{p})^{T} \\ &+ \rho_{1} (- \mathcal{F}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes X_{p}) \\ \\ &\bar{\Psi}_{p10} = (\mathcal{F}_{N} \otimes \mathcal{F}_{p}) + \rho_{1} (\mathcal{F}_{N} \otimes \mathcal{F}_{p}) \\ &\bar{\Psi}_{p11} = -\mathcal{F} (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes \Lambda_{p})^{T} \\ \\ &\bar{\Psi}_{p11} = -2 (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes \Lambda_{p}) \\ &+ \rho_{1} (\mathcal{F}_{N} \otimes V_{p}) \\ &\bar{\Psi}_{p11} = -\rho_{1} (\mathcal{F}_{N} \otimes X_{p}) (\mathcal{F}_{N} \otimes X_{p}) \\ \\ &\bar{\Psi}_{p11} = -\rho_{1} (\mathcal{F}_{N} \otimes X_{p}) \\ &\bar{\Psi}_{p11} = \rho_{1} (\mathcal{F}_{N} \otimes V_{p}) \\ &\bar{\Psi}_{p11} = -\rho_{1} (\mathcal{F}_{N} \otimes V_{p}) \\ &\bar{\Psi}_{p11} = -(\mathcal{F}_{N} \otimes V$$

Complexity

(26)

 $V(t,\widehat{e}(t),p) = \sum_{r=1}^{4} V_r(t,\widehat{e}(t),p),$ 

and the remaining parameters are zero. Then, the feedback controller gain is given by  $\mathbf{K}_p = \mathbf{Y}_p \mathbf{X}_p^{-1}$  which stabilizes the error dynamics.

*Proof.* We consider the following LKF with integral terms for the synchronization error model (18)

where

$$V_{1}(t,\hat{e}(t),p) = \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{P}_{p}) \hat{e}(t),$$

$$V_{2}(t,\hat{e}(t),p) = \int_{t-\vartheta_{1}}^{t} \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{U}_{2}) \hat{e}(s) ds + \int_{t-\vartheta(t)}^{t} \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{U}_{3}) \hat{e}(s) ds$$

$$+ \int_{t-\vartheta_{2}}^{t} \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{U}_{4}) \hat{e}(s) ds + \int_{t-\kappa_{1}}^{t} \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{U}_{5}) \hat{e}(s) ds$$

$$+ \int_{t-\kappa_{t}(t)}^{t} \hat{e}^{T}(s) (\mathscr{F}_{N} \otimes \mathbf{U}_{6}) \dot{e}(s) ds + \int_{t-\kappa_{2}}^{t} \hat{e}^{T}(t) (\mathscr{F}_{N} \otimes \mathbf{U}_{7}) \hat{e}(s) ds, \qquad (27)$$

$$V_{3}(t,\hat{e}(t),p) = \int_{t-\vartheta_{1}}^{t} \int_{u}^{t} \hat{e}^{T}(s) (\mathscr{F}_{N} \otimes \mathbf{U}_{8}) \dot{e}(s) ds du + \int_{t-\vartheta_{2}}^{t} \int_{u}^{t} \hat{e}^{T}(s) (\mathscr{F}_{N} \otimes \mathbf{U}_{9}) \dot{e}(s) ds du$$

$$+ \int_{t-\kappa_{1}}^{t} \int_{u}^{t} \hat{e}^{T}(s) (\mathscr{F}_{N} \otimes \mathbf{U}_{10}) \dot{e}(s) ds du + \int_{t-\kappa_{2}}^{t} \int_{u}^{t} \hat{e}^{T}(s) (\mathscr{F}_{N} \otimes \mathbf{U}_{11}) \dot{e}(s) ds du,$$

$$V_{4}(t,\hat{e}(t),p) = \overline{\rho} \int_{-\rho(t)}^{0} \int_{t+u}^{t} \hat{e}(s) (\mathscr{F}_{N} \otimes \mathbf{Z}) \hat{e}(s) ds du.$$

We use  $\mathscr{L}V(t, \hat{e}(t), p)$  to denote the weak infinitesimal operator of  $V(t, \hat{e}(t), p)$  [39], which is defined as

$$\mathscr{L}V(t,\hat{e}(t),p) = \lim_{\Delta \longrightarrow 0^+} \frac{1}{\Delta} \mathscr{E} \{ V((t+\Delta),\hat{e}(t+\Delta),\delta(t+\Delta)) | (t,\hat{e}(t),p) - V(t,\hat{e}(t),p) \},$$

$$\mathscr{L}V(t,\hat{e}(t),p) = \sum_{r=1}^{4} \mathscr{L}V_r(t,\hat{e}(t),p).$$
(28)

The derivative and mathematical expectation of (26) along the trajectories of the error model (18) is

$$\begin{split} \mathscr{C}\{\mathscr{Z}\mathcal{V}_{1}(t,\widehat{e}(t),p)\} &= \mathscr{C}\left\{2\widehat{e}^{T}(t)\left(\mathscr{F}_{N}\otimes\mathbf{P}_{p}\right)\widehat{e}(t) + \widehat{e}^{T}(t)\sum_{q=1}^{\mathscr{N}}\left(\mathscr{F}_{N}\otimes\mathbf{P}_{q}\right)\widehat{e}(t)\right\},\\ \mathscr{C}\{\mathscr{Z}\mathcal{V}_{2}(t,\widehat{e}(t),p)\} &= \mathscr{C}\left\{\widehat{e}^{T}(t)\left[\left(\mathscr{F}_{N}\otimes\mathbf{U}_{2}\right) + \left(\mathscr{F}_{N}\otimes\mathbf{U}_{3}\right) + \left(\mathscr{F}_{N}\otimes\mathbf{U}_{4}\right) + \left(\mathscr{F}_{N}\otimes\mathbf{U}_{5}\right) + \left(\mathscr{F}_{N}\otimes\mathbf{U}_{7}\right)\right]\widehat{e}(t) \\ &\quad + \widehat{e}^{T}(t)\left(\mathscr{F}_{N}\otimes\mathbf{U}_{6}\right)\widehat{e}(t) + \widehat{e}^{T}(t-\vartheta_{1})\left(\mathscr{F}_{N}\otimes\mathbf{U}_{2}\right)\widehat{e}(t-\vartheta_{1}) - (1-\varsigma_{1})\widehat{e}^{T}(t-\vartheta(t))\left(\mathscr{F}_{N}\otimes\mathbf{U}_{3}\right) \\ &\quad \times \widehat{e}(t-\vartheta(t)) - \widehat{e}^{T}(t-\vartheta_{2})\left(\mathscr{F}_{N}\otimes\mathbf{U}_{4}\right)\widehat{e}(t-\vartheta_{2}) - \widehat{e}^{T}(t-\kappa_{1})\left(\mathscr{F}_{N}\otimes\mathbf{U}_{5}\right)\widehat{e}(t-\kappa_{1}) \\ &\quad - (1-\varsigma_{2})\widehat{e}^{T}(t-\kappa(t))\left(\mathscr{F}_{N}\otimes\mathbf{U}_{6}\right)\widehat{e}(t-\kappa(t)) - \widehat{e}^{T}(t-\kappa_{2})\left(\mathscr{F}_{N}\otimes\mathbf{U}_{7}\right)\widehat{e}(t-\kappa_{2})\right\},\\ \mathscr{E}\{\mathscr{Z}\mathcal{V}_{3}(t,\widehat{e}(t),p)\} &= \mathscr{E}\left\{\widehat{e}^{T}(t)\left[\left(\mathscr{F}_{N}\otimes\mathbf{U}_{8}\right) + \vartheta_{2}\left(\mathscr{F}_{N}\otimes\mathbf{U}_{9}\right) + \kappa_{1}\left(\mathscr{F}_{N}\otimes\mathbf{U}_{10}\right) + \kappa_{2}\left(\mathscr{F}_{N}\otimes\mathbf{U}_{11}\right)\right]\widehat{e}(t)\end{split}$$

$$-\int_{t-\theta_{1}}^{t} \dot{\hat{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{8}\right) \dot{\hat{e}}(s) ds - \int_{t-\theta_{2}}^{t} \dot{\hat{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right) \dot{\hat{e}}(s) ds -\int_{t-\kappa_{1}}^{t} \dot{\hat{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \dot{\hat{e}}(s) ds - \int_{t-\kappa_{2}}^{t} \dot{\hat{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{11}\right) \dot{\hat{e}}(s) ds \bigg\},$$

$$\mathrm{E}\{\mathscr{L}V_{4}(t, \hat{e}(t), p)\} = \overline{\rho}^{2} \hat{e}^{T}(t) \left(\mathcal{F}_{N} \otimes \mathbf{Z}\right) \hat{e}(t) - (1-\mu_{3}) \overline{\rho} \int_{t-\rho(t)}^{t} \hat{e}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{Z}\right) \hat{e}(s) ds.$$

$$(29)$$

According to Lemmas 1 and 2, the integral terms in (29) can be rewritten as

$$-\int_{t-\theta_{1}}^{t} \dot{e}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{8}\right) \dot{\bar{e}}(s) ds \leq -\frac{1}{\theta_{1}} \left(\int_{t-\theta_{1}}^{t} \dot{\bar{e}}(s) ds\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{8}\right) \left(\int_{t-\theta_{1}}^{t} \dot{\bar{e}}(s) ds\right),$$

$$\leq -\frac{1}{\theta_{1}} \left[\hat{e}(t) - \hat{e}(t-\theta_{1})\right]^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{8}\right) \left[\hat{e}(t) - \hat{e}(t-\theta_{1})\right],$$

$$-\int_{t-\theta_{2}}^{t} \dot{\bar{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right) \dot{\bar{e}}(s) ds \leq -\frac{1}{\theta_{2}} \left(\int_{t-\theta_{2}}^{t} \dot{\bar{e}}(s) ds\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right) \left(\int_{t-\theta_{2}}^{t} \dot{\bar{e}}(s) ds\right),$$

$$\leq -\frac{1}{\theta_{2}} \left[\hat{e}(t) - \hat{e}(t-\theta_{2})\right]^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right) \left[\hat{e}(t) - \hat{e}(t-\theta_{2})\right],$$

$$-\int_{t-\kappa_{1}}^{t} \dot{\bar{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \dot{\bar{e}}(s) ds \leq -\frac{1}{\kappa_{1}} \left(\int_{t-\kappa_{1}}^{t} \dot{\bar{e}}(s) ds\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \left(\int_{t-\kappa_{1}}^{t} \dot{\bar{e}}(s) ds\right),$$

$$\leq -\frac{1}{\kappa_{1}} \left[\hat{e}(t) - \hat{e}(t-\kappa_{1})\right]^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \left(\hat{e}(t) - \hat{e}(t-\kappa_{1})\right],$$

$$-\int_{t-\kappa_{2}}^{t} \dot{\bar{e}}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{U}_{11}\right) \dot{\bar{e}}(s) ds \leq -\frac{1}{\kappa_{2}} \left(\int_{t-\kappa_{2}}^{t} \dot{\bar{e}}(s) ds\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \left(\hat{e}(t) - \hat{e}(t-\kappa_{1})\right],$$

$$(30)$$

$$\leq -\frac{1}{\kappa_{2}} \left[\hat{e}(t) - \hat{e}(t-\kappa_{2})\right]^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{10}\right) \left(\hat{e}(t) - \hat{e}(t-\kappa_{1})\right],$$

$$\leq -\frac{1}{\kappa_{2}} \left[\hat{e}(t) - \hat{e}(t-\kappa_{2})\right]^{T} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{11}\right) \left(\hat{e}(t) - \hat{e}(t-\kappa_{2})\right).$$

Applying Lemma 1 in (29), it follows that

$$-\overline{\rho} \int_{t-\rho(t)}^{t} \widehat{e}^{T}(s) \left(\mathcal{F}_{N} \otimes \mathbf{Z}\right) \widehat{e}(s) \mathrm{d}s \leq -\left(\int_{t-\rho(t)}^{t} \widehat{e}(s) \mathrm{d}s\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{Z}\right) \left(\int_{t-\rho(t)}^{t} \widehat{e}(s) \mathrm{d}s\right).$$
(31)

Besides, as per Assumption 1, we can obtain the following inequality:

$$\widehat{\boldsymbol{e}}^{T}(t)\left(\mathcal{F}_{N}\otimes\mathbf{G}_{1}\right)\left(\mathcal{F}_{N}\otimes\mathbf{G}_{1}\right)^{T}\widehat{\boldsymbol{e}}(t)-\mathbb{H}_{1}^{T}(t,\widehat{\boldsymbol{e}}(t))\mathbb{H}_{1}(t,\widehat{\boldsymbol{e}}(t))>0,$$

$$\widehat{\boldsymbol{e}}^{T}(t-\vartheta(t))\left(\mathcal{F}_{N}\otimes\mathbf{G}_{2}\right)\left(\mathcal{F}_{N}\otimes\mathbf{G}_{2}\right)^{T}\widehat{\boldsymbol{e}}(t-\vartheta(t))-\mathbb{H}_{1}^{T}(t,\widehat{\boldsymbol{e}}(t-\vartheta(t)))\mathbb{H}_{1}(t,\widehat{\boldsymbol{e}}(t-\vartheta(t)))>0.$$
(32)

Besides that, for any matrix  $\mathbf{M}_p$ ,  $p \in \mathcal{S}$  with appropriate dimension and scalars  $\beta_1$ ,  $\beta_2 > 0$ , and taking the mathematical expectation, the following condition holds:

$$0 = \mathscr{C}\left\{2\left[\widehat{e}^{T}\left(t\right) + \beta_{1}\dot{\widehat{e}}^{T}\left(t\right) + \beta_{2}\widehat{e}^{T}\left(t - \vartheta(t)\right)\right]\left(\mathscr{F}_{N}\otimes\mathbf{M}_{p}\right)\left[\left(\left(\mathscr{F}_{N}\otimes\mathbf{A}_{1p}\right) + \left(\mathscr{F}_{N}\otimes\mathbf{D}_{1p}\right)\left(\mathscr{F}_{N}\otimes\mathbf{K}_{p}\right) - \left(\mathscr{F}_{N}\otimes\mathbf{D}_{1p}\right)\lambda(t)\right)\right]\right\}$$

$$\times\left(\mathscr{F}_{N}\otimes\mathbf{K}_{p}\right)\widehat{e}\left(t\right) + \left(\mathscr{F}_{N}\otimes\mathbf{B}_{1p}\right)\mathbb{H}_{1}\left(t,\widehat{e}\left(t\right)\right) + \left(I\otimes\mathbf{B}_{2p}\right)\mathbb{H}_{2}\left(t,\widehat{e}\left(t - \vartheta(t)\right)\right) + \left(I\otimes\mathbf{C}_{1p}\right)\dot{\widehat{e}}\left(t - \kappa(t)\right)\right)\right]$$

$$+\left((1 - \beta(t))\left(\mathbf{G}^{(1)}\otimes\Gamma_{1p}\right) + \left(\overline{\beta} - \beta(t)\right)\left(\mathbf{G}^{(1)}\otimes\Gamma_{1p}\right)\widehat{e}\left(t\right) + \left(\overline{\beta}\left(\mathbf{G}^{(2)}\otimes\Gamma_{2p}\right) + \left(\beta(t) - \overline{\beta}\right)\left(\mathbf{G}^{(2)}\otimes\Gamma_{2p}\right)\right)\right)$$

$$\times \int_{t-\rho(t)}^{t}\widehat{e}\left(s\right)\mathrm{d}s + \left(\mathscr{F}_{N}\otimes\mathbf{E}_{1p}\right)\nu(t) - \dot{\widehat{e}}\left(t\right)\right]\right\}.$$

$$(33)$$

For the performance index, we can obtain  $\mathcal{J}_{p}(t) = \sigma \tilde{y}^{T}(t)\tilde{y}(t) - 2(1-\sigma)\gamma \tilde{y}^{T}(t)v(t) - \gamma^{2}v^{T}(t)v(t),$   $t \ge 0.$ (34) Substituting (32) and (33) into  $\mathscr{L}V(t, \hat{e}(t), p)$ , and from (29)–(31), it yields that

$$\mathscr{E}\{\mathscr{L}V(t,\widehat{e}(t),p)\} + \mathscr{J}_{p}(t) \leq \mathscr{E}\{\zeta^{T}(t)[\Psi_{p}]_{12\times 12}\zeta(t)\},$$
(35)

where

$$\begin{split} \Psi_{p19} &= \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{B}_{1p}\right) + \beta_{2} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right) \left(\mathcal{J}_{N} \otimes \mathbf{B}_{1p}\right), \Psi_{p110} = \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{B}_{2p}\right) \\ &+ \beta_{2} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right) \left(\mathcal{J}_{N} \otimes \mathbf{B}_{2p}\right), \Psi_{p111} = \overline{\beta} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathbf{G}^{(2)} \otimes \Gamma_{2p}\right), \Psi_{p112} = -2(1-\sigma)\gamma \\ &\times \left(\mathcal{J}_{N} \otimes \mathbf{A}_{2p}\right)^{T} + \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{E}_{1p}\right), \Psi_{p222} = \left(\mathcal{J}_{N} \otimes \mathbf{U}_{6}\right) + \vartheta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{8}\right) + \vartheta_{2} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{9}\right) \\ &+ \kappa_{1} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{10}\right) + \kappa_{2} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{11}\right) - \beta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} - \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)\beta_{1}, \Psi_{p24} = -\left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \beta_{2}, \\ \Psi_{p28} &= \beta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathbf{G}^{(2)} \otimes \Gamma_{2p}\right) + \beta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T}, \Psi_{p210} = \beta_{1} \left(I \otimes \mathbf{M}_{p}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{B}_{2p}\right), \\ \Psi_{p211} &= \beta_{1} \overline{\beta} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathbf{G}^{(2)} \otimes \Gamma_{2p}\right) + \beta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T}, \Psi_{p212} = \beta \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{E}_{1p}\right), \\ \Psi_{p33} &= -\left(\mathcal{J}_{N} \otimes \mathbf{U}_{2}\right) - \frac{1}{\vartheta_{1}} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{8}\right), \Psi_{p44} = -\left(1 - \varsigma_{1}\right) \left(\mathcal{J}_{N} \otimes \mathbf{U}_{3}\right) + \left(\mathcal{J}_{N} \otimes \mathbf{G}_{2}\right)^{T} \left(\mathcal{J}_{N} \otimes \mathbf{E}_{1p}\right), \\ \Psi_{p48} &= \beta_{1} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right) \left(\mathcal{J}_{N} \otimes \mathbf{C}_{1p}\right), \Psi_{p411} = \beta_{2} \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right) \overline{\beta} \left(\mathbf{G}^{(2)} \otimes \Gamma_{2p}\right), \Psi_{p412} = \beta_{2} \left(\mathcal{J}_{N} \otimes \mathbf{E}_{1p}\right) \\ \times \left(\mathcal{J}_{N} \otimes \mathbf{M}_{p}\right), \Psi_{p55} = -\left(\mathcal{J}_{N} \otimes \mathbf{U}_{4}\right) - \frac{1}{\vartheta_{2}} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{9}\right), \Psi_{p66} = -\left(\mathcal{J}_{N} \otimes \mathbf{U}_{5}\right) - \frac{1}{\vartheta_{2}} \left(\mathcal{J}_{N} \otimes \mathbf{U}_{10}\right), \\ \Psi_{p77} &= -\left(\mathcal{J}_{N} \otimes \mathcal{J}_{N}\right), \Psi_{p1111} = -\left(1 - \varsigma_{1}\right) \left(\mathcal{J}_{N} \otimes \mathbf{Z}\right), \Psi_{p1212} = -\gamma^{2}I, \end{split}$$

and the remaining elements of  $\Psi_p$  are zero. Let  $(\mathcal{F}_N \otimes \mathbf{X}_p) = (\mathcal{F}_N \otimes \mathbf{M}_p)^{-1}$ , and pre- and postmultiply matrix  $\Psi_p$  by diag{ $(\mathcal{F}_N \otimes \mathbf{X}_p) (\mathcal{F}_N \otimes \mathbf{X}_p) (\mathcal{F}_N \otimes \mathbf{X}_p) (\mathcal{F}_N \otimes \mathbf{X}_p) (\mathcal{F}_N \otimes \mathbf{X}_p)$ 

 $(\mathscr{I}_N \otimes \mathbf{X}_p)(\mathscr{I}_N \otimes \mathbf{X}_p)II(\mathscr{I}_N \otimes \mathbf{X}_p)I\}$ . By using the Schur complement (Lemma 3), we can obtain matrix  $\widehat{\Psi}_p$  in (24). Now we introduce the following new variables:

$$\begin{pmatrix} \mathscr{F}_{N} \otimes \mathbf{X}_{p} \end{pmatrix}^{T} (\mathscr{F}_{N} \otimes \mathbf{U}_{m}) (\mathscr{F}_{N} \otimes \mathbf{X}_{p}) = (\mathscr{F}_{N} \otimes \widehat{\mathbf{U}}_{m}), \quad (m = 2, ..., 11), \\ (\mathscr{F}_{N} \otimes \mathbf{X}_{p})^{T} (\mathscr{F}_{N} \otimes \mathbf{Z}) (\mathscr{F}_{N} \otimes \mathbf{X}_{p}) = (\mathscr{F}_{N} \otimes \widehat{\mathbf{Z}}), \\ \mathbf{Y}_{p} = \mathbf{K}_{p} \mathbf{X}_{p}.$$
(37)

These combinations are negative definite if the following conditions hold:

$$\mathscr{E}\left\{\mathscr{L}V(t,\widehat{e}(t),p)+\mathscr{J}_{p}(t)\right\} < \mathscr{E}\left\{\zeta^{T}(t)\widehat{\Psi}_{p}\zeta(t)\right\} < 0.$$
(38)

With zero initial condition, it can be inferred that for any  $\mathcal{T}_p$ 

$$\mathscr{C}\left\{\int_{0}^{\mathscr{T}_{p}}\left[\sigma\widetilde{y}^{T}(t)\widetilde{y}(t)-2(1-\sigma)\gamma\widetilde{y}^{T}(t)\nu(t)-\gamma^{2}\nu^{T}(t)\nu(t)\right]dt\right\} \leq \mathscr{C}\left\{\int_{0}^{\mathscr{T}_{p}}\mathscr{L}V(t,\widehat{e}(t),p)+\sigma\widetilde{y}^{T}(t)\widetilde{y}(t)-2(1-\sigma)\gamma\widetilde{y}^{T}(t)\nu(t)-\gamma^{2}\nu^{T}(t)\nu(t)\right]dt\right\} < 0,$$
(39)

which indicates that

$$\mathcal{J}_{p}(t) = \mathscr{C}\left\{\int_{0}^{\mathcal{T}_{p}} \left[\sigma \tilde{y}^{T}(t)\tilde{y}(t) - 2(1-\sigma)\gamma \tilde{y}^{T}(t)v(t)\right]dt\right\}$$
$$\leq \mathscr{C}\left\{\int_{0}^{\mathcal{T}_{p}} \gamma^{2} v^{T}(t)v(t)dt\right\}$$
(40)

holds if  $\overline{\Psi} < 0$ , and it can be obtained by integrating both sides of (40) that  $\mathcal{J}_p(t) \le 0$  holds, which means that the mixed  $\mathcal{H}$ -infinity and passive synchronization of the model is well achieved according to Definition 2. This completes the proof.

*Remark 3.* If there are no Markovian jumping parameters, then model (18) is simplified to

$$\begin{split} \dot{\hat{e}}(t) &= \left( \left( \mathcal{F}_{N} \otimes \mathbf{A}_{1} \right) + \left( \mathcal{F}_{N} \otimes \mathbf{D}_{1} \right) \left( \mathcal{F}_{N} \otimes \mathbf{K} \right) - \left( \mathcal{F}_{N} \otimes \mathbf{D}_{1} \right) \lambda(t) \left( \mathcal{F}_{N} \otimes \mathbf{K} \right) \right) \hat{e}(t) + \left( \mathcal{F}_{N} \otimes \mathbf{B}_{1} \right) \mathbb{H}_{1}(t, \hat{e}(t)) \\ &+ \left( \mathcal{F}_{N} \otimes \mathbf{B}_{2} \right) \mathbb{H}_{2}(t, \hat{e}(t - \vartheta(t))) + \left( \mathcal{F}_{N} \otimes \mathbf{C}_{1} \right) \dot{\hat{e}}(t - \kappa(t)) + (1 - \beta(t)) \left( \mathbf{G}^{(1)} \otimes \Gamma_{1} \right) \hat{e}(t) \\ &+ \beta(t) \left( \mathbf{G}^{(2)} \otimes \Gamma_{2} \right) \int_{t - \rho(t)}^{t} \hat{e}(s) \mathrm{d}s + \left( \mathcal{F}_{N} \otimes \mathbf{E}_{1} \right) \nu(t), \end{split}$$
(41)  
$$\tilde{y}(t) = \left( \mathcal{F}_{N} \otimes \mathbf{A}_{2} \right) \hat{e}(t), \end{split}$$

**Corollary 1.** Given some constants such as  $\vartheta_1$ ,  $\vartheta_2$ ,  $\kappa_1$ ,  $\kappa_2$ ,  $\zeta_1$ ,  $\zeta_2$ ,  $\mu_3$ ,  $\overline{\rho}$ ,  $\beta_1$ ,  $\beta_2$ ,  $\sigma$ ,  $\overline{\beta} \in [0, 1]$ ,  $\gamma$ ,  $\overline{\lambda}^*$  and diagonal matrices  $\Gamma_{1p}$ ,  $\Gamma_{2p}$  ( $p \in \mathcal{S}$ ),  $\mathbf{G}_1$ ,  $\mathbf{G}_2$ , and  $\mathbf{G}^{(1)}$ ,  $\mathbf{G}^{(2)}$  are known constant matrices, error model (41) satisfies a mixed  $\mathcal{H}$ -infinity and passivity performance  $\gamma > 0$  in the sense of Definitions 1 and 2, if there exist symmetric positive definite matrices  $\mathbf{X}_1 > 0$ ,  $\mathbf{P}_1 > 0$ ,  $\mathbf{U}_m > 0$  ( $m = 2, \ldots, 11$ ),  $\mathbf{Z} > 0$ , and  $\mathbf{Y}_1$  and  $\mathbf{M}_1$  are of

appropriate dimension matrices such that the following LMI holds:

$$\overline{\widehat{\Psi}} = \left[\overline{\widehat{\Psi}}_{14\times 14}\right] < 0, \tag{42}$$

where

$$\begin{split} \overline{\Psi}_{11} &= \left(\mathcal{F}_{N} \otimes \overline{U}_{2}\right) + \left(\mathcal{F}_{N} \otimes \overline{U}_{4}\right) + \left(\mathcal{F}_{N} \otimes \overline{U}_{5}\right) + \left(\mathcal{F}_{N} \otimes \overline{U}_{7}\right) - \frac{1}{\vartheta_{1}} \left(\mathcal{F}_{N} \otimes \overline{U}_{8}\right) - \frac{1}{\vartheta_{2}} \left(\mathcal{F}_{N} \otimes \overline{U}_{9}\right) - \frac{1}{\vartheta_{1}} \left(\mathcal{F}_{N} \otimes \overline{U}_{10}\right) \\ &- \frac{1}{\kappa_{2}} \left(\mathcal{F}_{N} \otimes \overline{U}_{11}\right) + \left(\mathcal{F}_{N} \otimes A_{1}\right) \left(\mathcal{F}_{N} \otimes X_{1}\right) + \left(\mathcal{F}_{N} \otimes X_{1}\right) \left(\mathcal{F}_{N} \otimes A_{1}\right)^{T} + \left(\mathcal{F}_{N} \otimes D_{1}\right) \left(\mathcal{F}_{N} \otimes Y_{1}\right) + \left(\mathcal{F}_{N} \otimes Y_{1}\right)^{T} \\ &\times \left(\mathcal{F}_{N} \otimes D_{1p}\right)^{T} - \left(\mathcal{F}_{N} \otimes D_{1}\right)^{\overline{\lambda}}_{4} \left(\mathcal{F}_{N} \otimes Y_{1}\right) - \left(\mathcal{F}_{N} \otimes Y_{1}\right)^{T} \overline{\lambda}_{*}^{T} \left(\mathcal{F}_{N} \otimes D_{1}\right) + \left(1 - \overline{\beta}\right) \left(\mathcal{F}_{N} \otimes X_{1}\right)^{T} \\ &+ \left(1 - \overline{\beta}\right) \left(\mathcal{F}_{N} \otimes X_{1}\right)^{T} \left(\mathcal{G}^{(1)} \otimes \Gamma_{1}\right)^{T} + \overline{\rho}^{2} \left(\mathcal{F}_{N} \otimes \overline{\lambda}\right), \overline{\Psi}_{12} = \left(\mathcal{F}_{N} \otimes \overline{\mu}_{1}\right) - \left(\mathcal{F}_{N} \otimes X_{1}\right) + \left(1 - \overline{\beta}\right) \left(\mathcal{F}_{N} \otimes X_{1}\right) \\ &\times \left(\mathcal{F}_{N} \otimes A_{1}\right)^{T} + \beta_{1} \left(\mathcal{F}_{N} \otimes Y_{1}\right)^{T} \left(\mathcal{F}_{N} \otimes D_{1}\right)^{T} - \beta_{1} \left(\mathcal{F}_{N} \otimes Y_{1}\right)^{T} \overline{\lambda}_{*}^{T} \left(\mathcal{F}_{N} \otimes D_{1}\right)^{T} + \left(1 - \overline{\beta}\right) \beta_{1} \left(\mathcal{F}_{N} \otimes X_{1}\right) \\ &\times \left(\mathcal{G}^{(1)} \otimes \Gamma_{1}\right)^{T}, \overline{\Psi}_{13} = \frac{1}{\vartheta_{1}} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{14} = \beta_{2} \left(\mathcal{F}_{N} \otimes X_{1}\right)^{T} \left(\mathcal{F}_{N} \otimes A_{1}\right) + \beta_{1} \left(\mathcal{F}_{N} \otimes \overline{Y}_{1}\right)^{T} \\ &- \beta_{2} \left(\mathcal{F}_{N} \otimes Y_{1}\right)^{T} \overline{\lambda}_{*}^{T} \left(\mathcal{F}_{N} \otimes D_{1}\right)^{T} + \beta_{2} \left(1 - \overline{\beta}\right) \left(\mathcal{F}_{N} \otimes X_{1}\right) \left(\mathcal{G}^{(1)} \otimes \Gamma_{1}\right)^{T}, \overline{\Psi}_{15} = \frac{1}{\vartheta_{2}} \left(\mathcal{F}_{N} \otimes \overline{U}_{9}\right), \\ &\overline{\Psi}_{16} = \frac{1}{\kappa_{1}} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{17} = \frac{1}{\kappa_{2}} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{18} = \left(\mathcal{F}_{N} \otimes X_{1}\right) \left(\mathcal{F}_{N} \otimes G_{1}\right), \overline{\Psi}_{115} = \left(\mathcal{F}_{N} \otimes X_{1}\right), \\ &+ \beta_{2} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{110} = \left(\mathcal{F}_{N} \otimes \overline{U}_{2}\right)^{T}, \overline{\Psi}_{1113} = \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right) \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{118} = \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \\ &\overline{\Psi}_{22} = \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right) \left(\mathcal{F}_{N} \otimes \overline{U}_{2}\right)^{T}, \overline{\Psi}_{113} = \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right) \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \overline{\Psi}_{22} = \beta_{1} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right), \\ &\overline{\Psi}_{22} = \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right)^{T}, \overline{\Psi}_{24} = -\beta_{2} \left(\mathcal{F}_{N} \otimes \overline{U}_{1}\right) \left(\mathcal{F}_{N} \otimes \overline{U}$$

Then, the feedback controller gain is given by  $\mathbf{K} = \mathbf{Y}_1 \mathbf{X}_1^{-1}$ .

*Proof.* The proof is similar to that of Theorem 1 by choosing  $\mathcal{S} = 1$ ,  $\theta_{pq} = 0$ . Hence, the proof is omitted.

*Remark* 4. For the case of  $\beta = 0$ , i.e., without Markovian jumping, random coupling matrices, distributed coupling time-varying delays, and external disturbance, model (2) becomes

 $\widehat{\Upsilon} = \left[\widehat{\Upsilon}_{11 \times 11}\right] < 0,$ 

$$\dot{\hat{e}}(t) = \left( \left( \mathscr{F}_N \otimes \mathbf{A}_1 \right) + \left( \mathscr{F}_N \otimes \mathbf{D}_1 \right) \left( \mathscr{F}_N \otimes \mathbf{K} \right) - \left( \mathscr{F}_N \otimes \mathbf{D}_1 \right) \lambda(t) \left( \mathscr{F}_N \otimes \mathbf{K} \right) \right) \hat{e}(t) + \left( \mathscr{F}_N \otimes \mathbf{B}_1 \right) \mathbb{H}_1(t, \hat{e}(t)) 
+ \left( \mathscr{F}_N \otimes \mathbf{B}_2 \right) \mathbb{H}_2(t, \hat{e}(t - \vartheta(t))) + \left( \mathscr{F}_N \otimes \mathbf{C}_1 \right) \dot{\hat{e}}(t - \kappa(t)).$$
(44)

**Corollary 2.** Given some constants 
$$\vartheta_1, \vartheta_2, \kappa_1, \kappa_2, \varsigma_1, \varsigma_2, \beta_1, \beta_2$$
,  $\overline{\lambda}^*$  and diagonal matrices  $\mathbf{G}_1$  and  $\mathbf{G}_2$ , error model (44) is quadratically stable, if there exist symmetric positive definite matrices  $\mathbf{X}_1 > 0$ ,  $\widehat{\mathbf{P}}_1 > 0$ ,  $\widehat{\mathbf{U}}_m > 0$  ( $m = 2, ..., 11$ ), and  $\mathbf{Y}_1$  and  $\mathbf{M}_1$  are of appropriate dimension matrices such that following LMI holds:

where

$$\begin{split} \hat{\mathbf{Y}}_{11} &= \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{2}\right) + \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{3}\right) + \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{4}\right) + \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{5}\right) + \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{7}\right) - \frac{1}{\vartheta_{1}} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{8}\right) - \frac{1}{\vartheta_{2}} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{9}\right) \\ &- \frac{1}{\kappa_{1}} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{10}\right) - \frac{1}{\kappa_{2}} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{11}\right) + \left(\mathcal{F}_{N} \otimes \mathbf{A}_{1}\right) \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right) + \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right) \left(\mathcal{F}_{N} \otimes \mathbf{Y}_{1}\right) + \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right)^{T} \\ &\times \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right)^{T} - \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right) \overline{\lambda}_{*} \left(\mathcal{F}_{N} \otimes \mathbf{Y}_{1}\right) - \left(\mathcal{F}_{N} \otimes \mathbf{Y}_{1}\right)^{T} \overline{\lambda}_{*}^{T} \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right)^{T}, \\ \hat{\mathbf{Y}}_{12} &= \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{8}\right), \\ \hat{\mathbf{Y}}_{14} &= \beta_{2} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{A}_{1}\right)^{T} + \beta_{2} \left(\mathcal{F}_{N} \otimes \mathbf{Y}_{1}\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right)^{T} - \beta_{2} \left(\mathcal{F}_{N} \otimes \mathbf{D}_{1}\right)^{T} \\ &\times \overline{\lambda}_{*}^{T} \left(\mathcal{F}_{N} \otimes \mathbf{Y}_{1}\right)^{T}, \\ \hat{\mathbf{Y}}_{15} &= \frac{1}{\vartheta_{2}} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right)^{T} \left(\mathcal{F}_{N} \otimes \mathbf{A}_{1}\right)^{T} + \beta_{2} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right), \\ \hat{\mathbf{Y}}_{18} &= \left(\mathcal{F}_{N} \otimes \mathbf{C}_{1}\right) \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{15} &= \frac{1}{\vartheta_{2}} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right), \\ \hat{\mathbf{Y}}_{16} &= \frac{1}{\kappa_{1}} \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{10}\right), \\ \hat{\mathbf{Y}}_{18} &= \left(\mathcal{F}_{N} \otimes \mathbf{C}_{1}\right) \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{17} &= \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right) - \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{U}_{9}\right), \\ \hat{\mathbf{Y}}_{14} &= -\left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right) - \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{22} &= \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right) - \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{210} &= \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{210} &= \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{210} &= \beta_{1} \left(\mathcal{F}_{N} \otimes \mathbf{X}_{1}\right), \\ \hat{\mathbf{Y}}_{41} &= \left(-1 - \varsigma_{1}\right) \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right), \\ \hat{\mathbf{Y}}_{41} &= \left(\mathcal{F}_{N} \otimes \hat{\mathbf{U}}_{1}\right), \\ \hat{\mathbf{Y}}$$

Then, the feedback controller gain is given by  $\mathbf{K} = \mathbf{Y}_1 \mathbf{X}_1^{-1}$ .

 $V(t,\hat{e}(t)) = \sum_{r_1=1} V_{r_1}(t,\hat{e}(t)),$ (47)

*Proof.* Consider the following Lyapunov–Krasovskii functional candidate:

and  $V_1(t, \hat{e}(t))$ ,  $V_2(t, \hat{e}(t))$ , and  $V_3(t, \hat{e}(t))$  are the same as defined in (26), and construct

(45)

$$\Pi^{T}(t) = \left[\widehat{e}^{T}(t)\widehat{e}^{T}(t)\widehat{e}^{T}(t-\vartheta_{1})\widehat{e}^{T}(t-\vartheta(t))\widehat{e}^{T}(t-\vartheta_{2})\widehat{e}^{T}(t-\kappa_{1})\widehat{e}^{T}(t-\kappa_{2})\widehat{e}^{T}(t-\kappa(t))\right]$$

$$\mathbb{H}_{1}^{T}(t,\widehat{e}(t))\mathbb{H}_{1}^{T}(t,\widehat{e}(t-\vartheta(t)))].$$
(48)

The proof of Corollary 2 is similar to that of Theorem 1. Therefore, it is omitted.  $\hfill \Box$ 

Remark 5. It is advantage specifying that a short time ago, many works regarding synchronization of CDNs have been reported in the literature, for instance, see [7, 40, 41]. Nevertheless, only very few papers have been focused on the issue of synchronization of NCDNs [42, 43]. It is noticed that all the abovementioned works on NCDNs have not considered the influence of sudden changes in parameters or environment, which can be represented using by Markovian jump parameters [44-48]. Moreover, so far in the literature, no work has been reported on *H*-infinity and passivity synchronization problem of Markovian jump NCDNs with distributed random coupling delay. Thus, the main contribution of this paper is to fill such a gap through employing a fault-tolerant control law based on *H*-infinity and passivity performance for achieving robust synchronization in Markovian jump NCDNs with random coupling delay against distributed time-varying actuator faults, which makes this work different from the existing works on Markovian jump NCDNs.

*Remark 6.* It should be mentioned that Theorem 1 provides a set of sufficient conditions for mean-square asymptotic synchronization of the Markov jump NCDNs subject to time-varying actuator faults and random coupling delay. It is noted that the proof of Theorem 1 is mainly based on the Lyapunov–Krasovskii stability theory, wherein the number of decision variables in the obtained LMI constraints plays an important role. It is obvious that the computational complexity increases when the number of decision variables becomes larger. Moreover, in this paper, we construct the Lyapunov–Krasovskii functional consisting of double integral terms. So, the estimation of those integral terms brings out some difficulties. In order to overcome this, we employ Jensen's integral inequality. It should be noted that the advantage of using Jensen's integral inequality is that it can significantly reduce the number of decision variables in the derivation of the main results. Besides, fortunately, all the computations in the main results are off-line, and with the aid of the existing standard convex optimization software, the proposed LMI conditions can be easily solved.

*Remark* 7. Theorem 1 develops mixed  $\mathcal{H}$ -infinity and passivity performance synchronization of MJNTCDNs with randomly occurring actuator faults. Theorem 1 makes full use of the information of the upper bounds of the discrete and distributed time-varying delays, which also brings us less conservativeness.

#### 4. Numerical Examples

In this section, two numerical examples are presented to illustrate the effectiveness of the proposed method to analyze synchronization with respect to MJNTCDN models.

*Example 1.* Consider the following MJNTCDN model along with distributed time-varying delays with 3 nodes, mode p = 1, 2 and N = 3.

$$\begin{cases} \dot{\hat{e}}(t) = \left( \left( \mathscr{F}_{N} \otimes \mathbf{A}_{1p} \right) + \left( \mathscr{F}_{N} \otimes \mathbf{D}_{1p} \right) \left( \mathscr{F}_{N} \otimes \mathbf{K}_{p} \right) - \left( \mathscr{F}_{N} \otimes \mathbf{D}_{1p} \right) \lambda(t) \left( \mathscr{F}_{N} \otimes \mathbf{K}_{p} \right) \right) \hat{e}(t) \\ + \left( \mathscr{F}_{N} \otimes \mathbf{B}_{1p} \right) \mathbb{H}_{1}(t, \hat{e}(t)) + \left( \mathscr{F}_{N} \otimes \mathbf{B}_{2p} \right) \mathbb{H}_{2}(t, \hat{e}(t - \vartheta(t))) + \left( \mathscr{F}_{N} \otimes \mathbf{C}_{1p} \right) \dot{\hat{e}}(t - \kappa(t)) \\ + (1 - \beta(t)) \left( \mathbf{G}^{(1)} \otimes \Gamma_{1p} \right) \hat{e}(t) + \beta(t) \left( \mathbf{G}^{(2)} \otimes \Gamma_{2p} \right) \int_{t - \rho(t)}^{t} \hat{e}(s) ds + \left( \mathscr{F}_{N} \otimes \mathbf{E}_{1p} \right) \nu(t), \\ \tilde{\gamma}(t) = \left( \mathscr{F}_{N} \otimes \mathbf{A}_{2p} \right) \hat{e}(t), \end{cases}$$

$$\tag{49}$$

#### Complexity

where  $\hat{e}(t) = (\hat{e}_1^T(t), \hat{e}_2^T(t))^T$  and the relevant parameters are given as follows:

Mode 1:

$$\mathbf{A}_{11} = \begin{bmatrix} -3.9 & 0.26 \\ 0.31 & -2.8 \end{bmatrix}, \\ \mathbf{A}_{21} = \begin{bmatrix} 0.13 & 0.14 \\ 0.16 & 0.13 \end{bmatrix}, \\ \mathbf{B}_{11} = \begin{bmatrix} 0.25 & 0.75 \\ 0.35 & 0.25 \end{bmatrix}, \\ \mathbf{B}_{21} = \begin{bmatrix} 0.14 & 0.15 \\ 0.45 & 0.23 \end{bmatrix}, \\ \mathbf{C}_{11} = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}, \\ \mathbf{D}_{11} = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix}, \\ \mathbf{E}_{11} = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$
(50)

$$\mathbf{A}_{12} = \begin{bmatrix} 2.5 & 0 \\ 0 & 2 \end{bmatrix},$$
  

$$\mathbf{A}_{22} = \begin{bmatrix} 0.15 & -1.08 \\ 0.75 & 2.019 \end{bmatrix},$$
  

$$\mathbf{B}_{12} = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & -0.2 \end{bmatrix},$$
  

$$\mathbf{B}_{22} = \begin{bmatrix} -0.23 & 1.02 \\ 0.56 & 1.021 \end{bmatrix},$$
  

$$\mathbf{C}_{12} = \begin{bmatrix} -0.025 & -1.01 \\ 2.3 & 1.75 \end{bmatrix},$$
  

$$\mathbf{D}_{12} = \begin{bmatrix} -0.025 & -1.01 \\ -1.3 & 1.75 \end{bmatrix},$$
  

$$\mathbf{E}_{12} = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix},$$
  
(51)

In addition, the inner-coupling matrices of nondelayed and delayed terms are taken as follows:

$$\Gamma_{11} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$\Gamma_{21} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$
(52)

The outer-coupling matrices are assumed to be  $\mathbf{g}_{zj}^{(1)} = \mathbf{G}^{(1)}, \, \mathbf{g}_{zj}^{(2)} = \mathbf{G}^{(2)}$  with

$$\mathbf{G}^{(1)} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 2 \end{bmatrix},$$

$$\mathbf{G}^{(2)} = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -2 & 2 \\ 2 & 0 & -2 \end{bmatrix}.$$
(53)

The nonlinear function  $f(t, \hat{r}_z(t))$  is chosen as

$$f(t, \hat{r}_{z}(t)) = \begin{bmatrix} 0.2\hat{r}_{z1}(t) - \tanh(0.1\hat{r}_{z1}(t)) \\ 0.1\hat{r}_{z2}(t) \end{bmatrix}.$$
 (54)

It can be observed that  $f(t, \hat{r}_z(t))$  fulfills Assumption 1 with the following:

$$\mathbf{G}_{1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\mathbf{G}_{2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$
(55)

The state and neutral, distributed time-varying delays satisfy  $\vartheta(t) = 0.5 + 0.5 \sin(t)$ , and  $\kappa(t) = 0.25 + 0.05 \sin(t)$ ,  $\rho(t) = 0.5 + 0.5 \cos(t - 1)$ , respectively.

Here we choose  $\vartheta_1 = 0.001$ ,  $\vartheta_2 = 1$ ,  $\gamma = 0.6$ ,  $\beta_1 = 0.05$ ,  $\beta_2 = 0.41$ ,  $\kappa_1 = 0.001$ ,  $\kappa_2 = 0.3$ ,  $\overline{p} = 1$ ,  $\zeta_1 = 0.01$ ,  $\zeta_2 = 0.1$ ,  $\mu_3 = 0.5$ ,  $\overline{\beta} = 0.5$ ,  $\sigma = 0.3$ , and  $\overline{\lambda}_* = 0.9$ . In addition, the Markov chain { $\delta(t) = p, t \ge 0$ } takes value in finite state space  $\mathcal{N} = \{1, 2\}$  and the transition probability matrix is given by

$$\delta(t) = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.2 \end{bmatrix}.$$
 (56)

By using MATLAB LMI Toolbox, we solved LMIs (24) and obtained the feasible solutions as follows:

 $\mathbf{P}_1 = \begin{bmatrix} 65.1265 & -34.7055 \\ -34.7055 & 62.4038 \end{bmatrix},$  $\mathbf{P}_2 = \begin{bmatrix} 22.9921 & -2.4665 \\ -2.4665 & 16.6365 \end{bmatrix},$  $\mathbf{U}_2 = \begin{bmatrix} 41.7827 & -4.6555 \\ -4.6555 & 35.9923 \end{bmatrix},$  $\mathbf{U}_3 = \begin{bmatrix} 387.9186 & 85.2083 \\ 85.2083 & 736.9597 \end{bmatrix},$  $\mathbf{U}_4 = \begin{bmatrix} 184.2395 & -7.9235 \\ -7.9235 & 166.1189 \end{bmatrix}$  $\mathbf{U}_5 = \begin{bmatrix} 41.7827 & -4.6555 \\ -4.6555 & 35.9923 \end{bmatrix},$  $\mathbf{U}_6 = \begin{bmatrix} 31.7143 & -2.0245 \\ -2.0245 & 28.9686 \end{bmatrix},$  $\mathbf{U}_7 = \begin{bmatrix} 48.5381 & -5.3557 \\ -5.3557 & 43.9116 \end{bmatrix},$  $\mathbf{U}_8 = \begin{bmatrix} 0.0918 & 0.0024 \\ 0.0024 & 0.0915 \end{bmatrix},$  $\mathbf{U}_9 = \begin{bmatrix} -69.7901 & 3.9091 \\ 3.9091 & -62.5765 \end{bmatrix},$  $\mathbf{U}_{10} = \begin{bmatrix} 0.0918 & 0.0024 \\ 0.0024 & 0.0915 \end{bmatrix},$  $\mathbf{U}_{11} = \begin{bmatrix} 18.4974 & 0.2475 \\ 0.2475 & 17.9481 \end{bmatrix},$  $\mathbf{X}_1 = \begin{bmatrix} 60.0845 & -35.8264 \\ -35.8264 & 55.3508 \end{bmatrix},$  $\mathbf{X}_2 = \begin{bmatrix} 5.0580 & 1.0693 \\ 1.0693 & 2.9108 \end{bmatrix},$  $\mathbf{Y}_1 = 10^3 \begin{bmatrix} -1.7000 & -0.5398\\ -0.5398 & -1.9253 \end{bmatrix},$  $\mathbf{Y}_2 = 10^3 \begin{bmatrix} 1.6331 & 3.3046 \\ 3.3046 & -0.4509 \end{bmatrix},$  $\mathbf{Z} = \begin{bmatrix} 96.6356 & -11.9607 \\ -11.9607 & 86.4221 \end{bmatrix}.$ 

(57)

We can obtain the following state feedback controller gains:

$$\mathbf{K}_{1} = \mathbf{Y}_{1}\mathbf{X}_{1}^{-1} = \begin{bmatrix} -55.5454 & -45.7051 \\ -48.4065 & -66.1147 \end{bmatrix},$$

$$\mathbf{K}_{2} = \mathbf{Y}_{2}\mathbf{X}_{2}^{-1} = 10^{3} \begin{bmatrix} 0.0899 & 1.1023 \\ 0.7439 & -0.4282 \end{bmatrix}.$$
(58)

Thus, it can be concluded that model (18) is mixed  $\mathscr{H}$ -infinity and passive at performance level  $\gamma$ . The maximum allowable upper bounds of  $\vartheta_2$  with different values  $\vartheta_1$  are given in Table 1.

*Example 2.* Consider a class of model in the form of (44) consisting of two-dimensional nodes with the following coefficient matrices:

$$\mathbf{A}_{1} = \begin{bmatrix} -0.9 & 0.26\\ 0.31 & -0.8 \end{bmatrix},$$
$$\mathbf{B}_{1} = \begin{bmatrix} 0.25 & 0.75\\ 0.35 & 0.25 \end{bmatrix},$$
$$\mathbf{B}_{2} = \begin{bmatrix} 0.14 & 0.15\\ 0.45 & 0.23 \end{bmatrix},$$
$$\mathbf{C}_{1} = \begin{bmatrix} -0.6 & 0.4\\ 0.5 & -0.4 \end{bmatrix},$$
$$\mathbf{D}_{1} = \begin{bmatrix} 1.2 & 1.4\\ 0.8 & 0.3 \end{bmatrix}.$$

It can be found that f satisfies Assumption 1 with

$$\mathbf{G}_{1} = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix},$$

$$\mathbf{G}_{2} = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix},$$
(60)

Based on the above parameters, we use the MATLAB LMI Toolbox, to solve the LMI in Corollary 2. We obtain the feasible solutions as follows. Therefore, the concerned model with time-varying delays is quadratically stable.

#### Complexity

TABLE 1: Maximum allowable bounds of  $\vartheta_2$  with different values of lower bound  $\vartheta_1$  for Example 1.

$\vartheta_1$	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$\vartheta_2$	2.1273	2.6124	2.6124	2.9345	3.0212	4.1213	5.2134

$$\begin{split} \mathbf{P}_{1} &= \begin{bmatrix} -4.4613 & -0.4878 \\ -0.4878 & -3.8748 \end{bmatrix}, \\ \mathbf{U}_{2} &= \begin{bmatrix} 19.5318 & 0.0775 \\ 0.0775 & 19.3311 \end{bmatrix}, \\ \mathbf{U}_{3} &= \begin{bmatrix} 22.8851 & 0.1539 \\ 0.1539 & 22.6357 \end{bmatrix}, \\ \mathbf{U}_{4} &= \begin{bmatrix} 73.0073 & 0.5419 \\ 0.5419 & 72.4158 \end{bmatrix}, \\ \mathbf{U}_{5} &= \begin{bmatrix} 19.5318 & 0.0775 \\ 0.0775 & 19.3311 \end{bmatrix}, \\ \mathbf{U}_{6} &= \begin{bmatrix} 10.2396 & 0.0042 \\ 0.0042 & 10.3032 \end{bmatrix}, \\ \mathbf{U}_{7} &= \begin{bmatrix} -99.7995 & -0.3808 \\ -0.3808 & -98.4768 \end{bmatrix}, \\ \mathbf{U}_{8} &= \begin{bmatrix} 0.0032 & -0.0000 \\ -0.0000 & 0.0033 \end{bmatrix}, \\ \mathbf{U}_{9} &= \begin{bmatrix} -32.4779 & -0.1621 \\ -0.1621 & -32.3669 \end{bmatrix}, \\ \mathbf{U}_{10} &= \begin{bmatrix} 0.0032 & -0.0000 \\ -0.0000 & 0.0033 \end{bmatrix}, \\ \mathbf{U}_{11} &= \begin{bmatrix} 36.7091 & 0.1316 \\ 0.1316 & 36.2684 \end{bmatrix}, \\ \mathbf{X}_{1} &= \begin{bmatrix} -2.1495 & -0.5201 \\ -0.5201 & -1.4914 \end{bmatrix}, \\ \mathbf{Y}_{1} &= \begin{bmatrix} 1.6994 & 1.3879 \\ 1.3879 & 0.4728 \end{bmatrix}. \end{split}$$

The corresponding control gain matrix is given as

$$\mathbf{K} = \begin{bmatrix} -0.6175 & -0.7152\\ -0.6214 & -0.1003 \end{bmatrix}.$$
 (62)

## 5. Conclusion

This article is concerned with mixed  $\mathscr{H}$ -infinity and passivity synchronization of MJNTCDNs models with randomly occurring distributed coupling time-varying delays and actuator faults. We have designed the fault-tolerant state feedback controller that is modeled by the Bernoulli random variable. By utilizing the Lyapunov–Krasovskii functional approach, the sufficient conditions are ensuring the mixed  $\mathscr{H}$ -infinity passive performance for the MJNCDN models which have been established in terms of LMIs. Two numerical examples are presented to illustrate the effectiveness of the proposed method. In future, we would investigate the occurrences of discrete sampled data control be described using stochastic variables and probability density functions [49]. Also, it is important to extend our results to analyze stochastic synchronization of MJNTCDN models with multiple time-varying delays via impulsive control and pinning control.

## **Data Availability**

No data were used to support this study.

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

# **Authors' Contributions**

N.B. was responsible for funding acquisition; G.R. conceptualized the study, performed formal analysis, and prepared the original draft and was responsible for methodology; G.R., M.U., and N.B. were responsible for software and reviewed and edited the manuscript; P.A., R.S., D.A., M.U., and C.P.L. supervised the study; G.R. and N.B. validated the data. All authors have read and agreed to the published version of the manuscript.

#### Acknowledgments

This research was supported by Rajamangala University of Technology Suvarnabhumi.

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