IET Control Theory & Applications

Research Article

Robust event-triggered T–S fuzzy system with successive time-delay signals and its application

ISSN 1751-8644
Received on 23rd June 2020
Revised 14th November 2020
Accepted on 7th December 2020
E-First on 18th March 2021
doi: 10.1049/iet-cta.2020.0750
www.ietdl.org

Rajarathinam Vadivel[1,2], Young Hoon Joo[1] ✉

[1]*School of IT Information and Control Engineering, Kunsan National University, 558, Daehak-ro, Gunsan-si, Jeollabuk-do, 54150, Republic of Korea*

[2]*Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Phuket 83000, Thailand*

✉ *E-mail: yhjoo@kunsan.ac.kr*

Abstract: This study is relevant to the topic of a robust event-triggered mechanism for the Takagi–Sugeno (T–S) fuzzy system with successive time-delay (STD) signals and its application, where the uncertainties satisfy the randomly occurring form. Firstly, an event-triggered communication scheme is introduced, which can adaptively adjust the communication threshold to save limited communication resource. The primary aim of this study is to model an event-triggered mechanism with STD, which ensures that the suggested T–S fuzzy system achieves extended dissipative with permissible uncertainties. Secondly, by using the relaxed integral inequality technique, single and double auxillary function-based integral inequalities to evaluate the derivative of the designed Lyapunov–Krasovskii functional, quadratically stable condition is established for the delayed fuzzy system in terms of linear matrix inequalities and analyse the H_∞, $\mathscr{L}_2-\mathscr{L}_\infty$, passivity, mixed H_∞ and passivity, $(\mathscr{Q},\mathscr{S},\mathscr{R})$-dissipativity execution by choosing the weighting matrices can be solved simultaneously in a standard framework based on the idea of extended dissipative. Finally, simulation studies are given to verify the effectiveness of the derived results, among them one example was supported by the real-life application of the benchmark problem in the sense of STD signals.

$\hat{\theta}$	frequency bias factor
R	speed drop
$\Delta\hat{P}_v$	generator valve position
$\Delta\hat{P}_m$	turbine generator mechanical output
Δf	deviation of frequency
K_p	proportional gain of the local PI controller
K_I	integral gain of the local PI controller
$\tilde{T}_{ch}$	turbine time constant
$\hat{T}_g$	governor time constant
$\bar{M}$	moment of inertia of the generator
$\bar{D}$	generator damping coefficient
ΔPc	set point

1 Introduction

In recent years, time-delay has received extensive attention due to its reasonable applications in numerous practical systems, such as economic systems, neural networks, networked control systems, engineering systems, and so on [1–3]. It is acknowledged to be a significant purpose for the instability and poor execution of the system. To beat stability analysis issues, different methodologies are suggested for the time-delay systems in the reference section, for example the free matrix variables, convex polyhedron method, and successive time-delay (STD) approach. Meanwhile, it merits specifying that the STD system was first proposed in [4]. Such a system might be experienced in numerous real applications, for example, the networked control system and power system models [5, 6]. For instance, in organised controlled systems, signals transmitted starting with one point then onto the next may encounter two segments of networks, which can probably induce successive delays, one from the sensor to the controller, and the other from the controller to the actuator, having distinctive properties because of variable system transmission conditions. Next, with the advance of renewable energy approach, the integration of these techniques and power system is increasing the interest of research. Owing to the expansive application of an open communication network, researchers pay close attention to the stability analysis of STD load frequency control (LFC) system. Compared with the single time-varying delay approach, event-triggered communication scheme with this model is under a stronger background of applications, particularly the LFC system. Therefore, taking the model with STD components into consideration is meaningful. In the process of our research on this subject, we find that there is still much room for further development of the existing results [7–10]. Recently, the relaxed and auxillary function based integral inequalities plays an important role in the stability criteria for time-delay systems [11, 12]. For the parameter uncertainties: as we know, either the external environment changes or internal disturbance can cause changes in the system parameters. In the past analysis of the closed-loop system based on event-triggered mechanism, the uncertain factors in the system were often ignored, and a simple deterministic network model was discussed. Although uncertainties are very small in many cases, it has a great impact on the stability and performance index of the system. Therefore, it is necessary to investigation for a non-linear system with uncertainty has been turned into an incredible issue in STD and many results have been examined (see [13–15]).

On the other hand, the fuzzy logic theory has been generally chosen for modelling complex non-linear systems. Among the different fuzzy logic model, the Takagi–Sugeno (T–S) fuzzy pattern is famous among researchers because of its amazing capacity in taking care of non-linear systems, which can be depicted as a group of linear subsystems with the assistance of IF-THEN fuzzy rules and fuzzy membership functions [16–18]. Besides, this strategy has turned quite specific and well known for handling the issue of some difficult non-linear systems. Thus, from the sense of both theoretical point of view and real-life applications an enormous amount of results related for T–S fuzzy systems have been stated in the previous years [19, 20]. Researchers in [21], improved delay-dependent stability criteria has been studied for quadratic T–S fuzzy system. In [22], extended dissipative analysis has been discussed for uncertain T–S fuzzy system with time-varying delay and randomly occurring gain variations. Furthermore, in [23], the authors proposed event-triggered reliable control for T–S fuzzy uncertain systems with weighted based inequality. Quite recently in [24], event-triggered control (ETC) of T–S fuzzy networked system has been studied for distributed delay method. In many complicated control systems, communication networks are utilised to exchange information and signals between components in a distributed system. By combining T–S fuzzy systems and event-triggered communication scheme, a new networked T–S fuzzy system will be proposed in this paper, which is not a simple combination of closed-loop and fuzzy systems. In this paper, on the basis of previous work in the literature, the modelled system under a robust event-triggered communication scheme that determines when and which data should be transmitted, is described as the T–S fuzzy system with an successive time delay method.

In any case, not quite the same as the conventional control systems, networked systems definitely bring new issues, and difficulties inferable from the inserted communication network [25, 26], and fault approaches [27, 28]. It is amazingly evident to look at this the fundamental advantages of the event-triggered mechanism can be finished up as low broadcast frequency via networks and decrease network utilisation. On the other hand, the threshold limit of the conventional event-triggered mechanism is likely ahead of time and has difficulty in adjustment for a variety of the thought about the system. To beat inadequacy exhibited over, the considered adaptive event-triggered mechanism in networked systems is a current research subject (see [29, 30]). Be that as its way, the considered adaptive law has been introduced through a simple structure. With this result that adaptive event-triggered mechanism can be more efficient in decreasing the communication burden resources. Currently, a variety of issues have been focused under the adaptive ETC scheme. For example, the problem of synchronisation control for T–S fuzzy neural networked systems has been studied in [31]. Based upon the adaptive event-triggered mechanism, the study of network-based H_∞ control for T–S fuzzy systems have been investigated in [32, 33]. In [34], event-triggered stabilisation has been addressed for T–S fuzzy systems via asynchronous premise constraints. In any case, how to design successive time-varying delay never completely examined in the previously stated works, particularly in response to event-triggered T–S fuzzy approach.

Especially, in the physical systems, dissipativity issue is profoundly connected with the idea of energy dissipation. This implies that hypothesis has given a basic structure to the control-based issues on planning in the investigation of linear and non-linear system models by means of an information yield description through the energy-related design [35]. Willem's introduced this idea in 1972 (see [36]), it has been accepted as extraordinary research from the researchers due to its extensive variety of practical recognition. For this reason, in the present day, the thought of dissipativity is approved to be essential and useful tool for control engineering realisations, for example, mechanical technology, burning motors, and electromechanical system modellings (see [37, 38]). Additionally, the dissipativity theory recommends another outlining execution list through the adaptable parameters $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$ distinguished with other performance indexes, for example, H_∞ control and passivity. Therefore, various specialists have taken interest in the linear matrix inequality (LMI) structure analysis with the extended-dissipativity based examination of time-delay systems in the past years. In addition, designing a system to be extended-dissipativity will enable not only closed-loop stability but also effective in control strategy via external noise attenuation. Many previous studies have been considered on extended-dissipativity issues based on event-triggered strategy. Researchers in [39], non-linear multi-agent systems have been studied for a distributed asynchronous event-triggered mechanism with disturbances. Event-triggered mechanism and extended dissipative analysis for network control systems have been discussed in [40]. In [41], the problem of event-triggered extended dissipative control for switched systems have been considered under the finite-time stability analysis. Furthermore, in [42], authors proposed event-triggered dissipative state estimation for Markov jump neural networks. In any case, no related work has been a breakthrough in bringing together T–S fuzzy system investigation for event-triggered mechanism with STD. In this manner, to meet this request, a noteworthy commitment of this paper is to fill such a gap by making the main endeavour to discuss about the extended dissipativity analysis for T–S fuzzy system, which covers at the same time $\mathscr{L}_2 - \mathscr{L}_\infty, H_\infty$, passivity, and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity analysis. Consequently, this incompletion motivates our present examination on this issue.

Motivated by the above discussions, in this paper, the extended dissipativity for T–S fuzzy system via STD signal is discussed in terms of the event-triggered mechanism. The existing methods in [23, 24], are dealt with only event-triggered approach for T–S fuzzy system. In this paper, we projected STD approach with one benchmark problem, which further shows the significance of our research. The main contributions of this paper as follows:

(i) By using suitable Lyapunov–Krasovskii functional (LKF), the sufficient condition is derived for T–S fuzzy system with STD through ETC scheme.
(ii) Merge with a more tightly estimation of the LKF derivative, single auxillary function-based integral inequality (SAFBII), double auxillary function-based integral inequality (DAFBII), and relaxed integral inequality (RII) techniques, extended dissipativity criterion with less conservativeness is determined in terms of LMIs.
(iii) The ETC is intended to stabilise the considered T–S fuzzy system. Furthermore, the proposed stability criteria build up the connection between the STD in system and communication delay in the controller and the acquired conditions can be converted over into LMIs, which can be checked by MATLAB LMI toolbox.
(iv) Simulation examples are given to demonstrate the viability and less conservatism of the developed approaches. In the application perspective, the obtained theoretical result is validated with the single-area LFC system.

This paper is organised as follows. Section 2 describes the T–S fuzzy STD model description and gives a theoretical background. Extended dissipative criteria for robust event-triggered mechanism and the constructed controller are summarised in Section 3. Simulation results, comparison and applications are conducted and explained in Sections 4–6, respectively to demonstrate the adequacy and less conservatism of the proposed approaches. Lastly, conclusion and future directions are given in Section 7.

Notations: A set of fairly standard notations is used in this paper. $\mathbb{N}$ and $\mathbb{R}^n$ means the positive integers and n-dimensional Euclidean space, respectively. $\mathbb{R}^{n \times m}$ is the arrangement of $n \times m$ real matrices. $X > 0$ $(X \geq 0)$ denotes positive definite (semi-positive definite) matrix X; the superscripts T and -1 means that the transpose and inverse of a matrix, respectively. $*$ denotes the elements that are introduced due to corresponding symmetry. I means the identity matrix of the appropriate dimensions and diag$\{\ldots\}$ means the block-diagonal matrix. MAUBs denotes the maximum allowable upper bounds. Prob$\{s\}$ defines the probability of the occurrence s.

2 Preliminaries and problem formulation

Consider the non-linear system as follows:

$$\dot{m}(t) = f(m(t), m(t - \kappa_1(t) - \kappa_2(t)), u(t), v(t)), \tag{1}$$

where $m(t) \in \mathbb{R}^n$ represents the state vector, $u(t) \in \mathbb{R}^l$ is the control input, $v(t) \in \mathbb{R}^q$ is the disturbance input which belongs to $L_2[0, \infty)$. $f(\cdot)$ denote the non-linear function, $\kappa_1(t)$ and $\kappa_2(t)$ denote two time-varying delay satisfying

$$\begin{cases} 0 \leq \kappa_1(t) \leq \kappa_1,\ 0 \leq \kappa_2(t) \leq \kappa_2,\ \kappa(t) = \kappa_1(t) + \kappa_2(t) \\ \dot{\kappa}_1(t) \leq \mu_1,\ \dot{\kappa}_2(t) \leq \mu_2,\ \tilde{h} = \kappa_1 + \kappa_2,\ \mu = \mu_1 + \mu_2, \end{cases} \tag{2}$$

where $\kappa_1, \kappa_2, \mu_1$ and μ_2 are non-negative constants. Then, the non-linearities in system (1) can be expressed in terms of linear subsystems based on T–S fuzzy IF-THEN rules is as follows:

Plant rule i:

IF $h_1(t)$ is M_{i1} and $h_2(t)$ is $M_{i2}\ldots h_p(t)$ is M_{ip}.
THEN

$$\dot{m}(t) = \bar{A}_i m(t) + \bar{A}_{di} m(t-\kappa(t)) + \bar{B}_i u(t) + \bar{D}_i v(t),$$

where $i = 1, 2, \ldots, r$, r denotes the number of IF-THEN rules, $h(t) = [h_1(t)\, h_2(t) \ldots h_p(t)]^T$ is premise variable, $M_{ik}(i = 1, 2, \ldots, r, k = 1, 2, \ldots, p)$ is the fuzzy set. $\bar{A}_i = A_i + \alpha(t)\Delta A_i(t), \bar{A}_{di} = A_{di} + \varrho(t)\Delta A_{di}(t)$, $\bar{B}_i = B_i + \gamma(t)\Delta B_i(t)$, $\bar{D}_i = D_i + \sigma(t)\Delta D_i(t)$
and the appropriate dimension real constant matrices are denoted as A_i, A_{di}, B_i, and D_i. $\alpha(t), \varrho(t), \gamma(t)$, and $\sigma(t)$ are commonly stochastic variables which are mutually independent Bernoulli-distributed sequences. The unknown matrices $\Delta A_i(t), \Delta A_{di}(t), \Delta B_i(t)$, and $\Delta D_i(t)$ are noted as norm-bounded parametric uncertainties to be of the form:

$$[\Delta A_i(t)\; \Delta A_{di}(t)\; \Delta B_i(t)\; \Delta D_i(t)] = H_{1i} F_i(t) [E_{1i}\; E_{2i}\; E_{3i}\; E_{4i}] \tag{3}$$

where $H_{1i}, E_{1i}, E_{2i}, E_{3i}$, and E_{4i} are the constant matrices with proper dimensions, $F_i(t)$ is unknown real and possibly time-varying matrices satisfying $F_i(t)^T F_i(t) \le I$. Supposed that all elements $F_i(t)$ is Lebesgue measurable, $\Delta A_i(t), \Delta A_{di}(t), \Delta B_i(t)$, and $\Delta D_i(t)$ are said to admissible if (3) hold. With the combination of product inference, centre-average defuzzifier and singleton fuzzifier the T–S fuzzy system (1) is expressed as follows:

$$\begin{aligned}\dot{m}(t) = \sum_{i=1}^{r} w_i(h(t)) \big[\bar{A}_i m(t) + \bar{A}_{di} m(t-\kappa(t)) \\ + \bar{B}_i u(t) + \bar{D}_i v(t)\big],\end{aligned} \tag{4}$$

where

$$w_i(h(t)) = \frac{\beta_i(h(t))}{\sum_{i=1}^{r} \beta_i(h(t))},\; \beta_i(h(t)) = \prod_{j=1}^{p} M_{ij}(h_j(t)),$$

and $M_{ij}(h_j(t))$ is the grade membership of $h_j(t)$ in M_{ij}. It is clear that $\beta_i(h(t)) \ge 0$ and $\sum_{i=1}^{r} \beta_i(h(t)) \ge 0$. Therefore

$$w_i(h(t)) \ge 0, \quad i = 1, \ldots, r, \quad \sum_{i=1}^{r} w_i(h(t)) = 1. \tag{5}$$

Networked control systems contain various parts, for example, sensors, controllers, and actuators associated by means of communication networks. It is notable that the periodic time-triggered scheme was broadly utilised in various framework of the systems, since it is anything but difficult to outline and keep up. Be that as it may, the time-triggered scheme could build the amount of the network, and lose the restricted resource of the network for the logic that it transfer various pointless information over the system. Along these lines, the consider event-triggered mechanism was produced to conquer these inadequacies without degrading the coveted system execution. The structure of the event-triggered mechanism is that a choice generator is developed to decide, either to spread the information over the network previously the data are discharged into the network. All over this paper, the system (1) is controlled through a network. We expect that the sections (n) and the estimation error $m(t)$ are gathered into v nodes, so the signals relating to node $l \in \{1, 2, \ldots, v\}$ can be meant as $m_l(t) \in \mathbb{R}^{n_l}$ with $n_1 + n_2 + \cdots + n_v = n$. Now, we indicate the lth event triggering discharged instant by $[t^l_{k_l} h]^{\infty}_{k_l}$ and coming immediately $t^l_{k_l+1} h$ of lth event generator is described by

$$\begin{aligned}t^l_{k_l+1} h = t^l_{k_l} h + \min_{\bar{l} \in Z} \Big\{ \bar{l} h \,\Big|\, s_l^T(\hat{i}^l_{k_l} h) \Phi_{1l} s_l(\hat{i}^l_{k_l} h) > \tilde{\beta}_l(t) m_l^T \\ \times (t^l_{k_l} h) \Phi_{2l} m_l(t^l_{k_l} h) + \delta_l(t) m_l^T(\hat{i}^l_{k_l} h) \Phi_{2l} m_l(t^l_{k_l} h) \Big\},\end{aligned} \tag{6}$$

where $\hat{i}^l_{k_l} = t^l_{K_1} + \tilde{l}$, $s_l(\hat{i}^l_{K_1} h) = m_l(\hat{i}^l_{K_1} h) - m_l(t^l_{K_1} h)$, $t^l_{K_1} h$ denotes $k_{l\text{th}}$ communication instant of the lth event, and $\Phi_{\tilde{i}l} > 0 (\tilde{i} = 1, 2)$ are the generated parameters remain to be evaluated. The terms $\tilde{\beta}_l(t), \delta_l(t)$ with $\tilde{\beta}_l(0) > 0$, $\delta_l(0) > 0$ are the activity controlled by the following adaptive laws:

$$\begin{aligned}\dot{\tilde{\beta}}_l(t) = \frac{1}{\tilde{\beta}_l(t)} \Big[\frac{1}{\tilde{\beta}_l(t)} - \rho_{1l}\Big] \Big[s_l^T(\hat{i}^l_{kl} h) \Phi_{1l} s_l(\hat{i}^l_{kl} h) - \lambda_{1l} m_l^T \\ \times (\hat{i}^l_{k_l} h) \Phi_{2l} m_l(\hat{i}^l_{kl} h) + \lambda_{1l} m_l^T(t^l_{k_l} h) \Phi_{2l} m_l(t^l_{kl} h)\Big],\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{\delta}_l(t) = \frac{1}{\delta_l(t)} \Big[\frac{1}{\delta_l(t)} - \rho_{2l}\Big] \Big[s_l^T(\hat{i}^l_{kl} h) \Phi_{1l} s_l(\hat{i}^l_{kl} h) - \lambda_{2l} m_l^T \\ \times (t^l_{k_l} h) \Phi_{2l} m_l(t^l_{kl} h) + \lambda_{2l} m_l^T(\hat{i}^l_{k_l} h) \Phi_{2l} m_l(\hat{i}^l_{kl} h)\Big],\end{aligned} \tag{8}$$

where ρ_{il}, $\lambda_{\tilde{i}l}(\tilde{i} = 1, 2; l = 1, 2, \ldots, v)$ are likely positive constants such that $1/\rho_{1l} < \tilde{\beta}_l(t) \le \lambda_{2l}, 1/\rho_{2l} < \delta_l(t) \le \lambda_{1l}$. In addition, we will exhibit a few indication as follows:

$$\begin{aligned}&\bar{\Phi}_{\tilde{i}} = \text{diag}(\Phi_{\tilde{i}1}, \ldots, \Phi_{\tilde{i}v}) > 0 (\tilde{i} = 1, 2),\; \tilde{\beta}(t) = \text{diag}(\tilde{\beta}_1(t), \ldots, \tilde{\beta}_v(t)),\\ &\delta(t) = \text{diag}(\delta_1(t), \ldots, \delta_v(t)),\; \chi_{\tilde{i}} = \text{diag}(\lambda_{\tilde{i}1} I_{n1}, \ldots, \lambda_{\tilde{i}v} I_{nv}),\\ &\Lambda_{\tilde{i}} = \text{diag}(\rho_{\tilde{i}1} I_{n1}, \ldots, \rho_{\tilde{i}v} I_{nv}) (\tilde{i} = 1, 2).\end{aligned} \tag{9}$$

In this paper, we utilise the event-triggered plan in (6) to minimise useless data transmission. Therefore, we plan to model the controller

$$u(t) = K[m(t^1_{k_1} h)\, m(t^2_{k_2} h) \ldots m(t^n_{k_n} h)]^T, \quad t \in [t_k h,\, t_{k+1} h], \tag{10}$$

where K is the matrix of controller gain and to be resolved later,

$$t_k h = \max_{l = 1, 2, \ldots, v} \{t^l_{k_l}\}, \quad t_{k+1} h = \max_{l = 1, 2, \ldots, v} \{t^l_{k_l+1}\}.$$

Let $l_k = t_{k+1} - t_k$, then the following period $[t_k h, t_{k+1} h)$ can be defined as

$$[t_k h, t_{k+1} h) = \bigcup_{i=0}^{l_k - 1} \Phi_i,$$

where $\Phi_{\tilde{i}} = [t_k h + \tilde{i} h,\, t_k h + \tilde{i} h + h)$. Define $\eta_2(t) = t - t_k h - \tilde{i} h$ for $t \in \Phi_{\tilde{i}}$. Therefore, $\eta_2(t)$ can be expressed as

$$\begin{cases} 0 \le \eta_2(t) \le h, t \in \Phi_{\tilde{i}}, \\ \dot{\eta}_2(t) = 1. \end{cases} \tag{11}$$

Therefore, the threshold error $s_l(t_k h + \tilde{i} h)$ can be composed as follows:

$$m_l(t^l_{k_l} h) = m_l(t - \eta_2(t)) - s_l(t - \eta_2(t)), t \in \Phi_{\tilde{i}}.$$

Denoting $s^T(\cdot) = [s_1^T(\cdot), \ldots, s_v^T(\cdot)]$, as $s(t - \eta_2(t)) = col\{s_1(t - \eta_2(t)), s_2(t - \eta_2(t)), \ldots, s_v(t - \eta_2(t))\}$. At that point, the design controller can be presented as

$$u(t) = K(m(t - \eta_2(t)) - s(t - \eta_2(t))),\; t \in \Phi_{\tilde{i}}.$$

Controller Rule **z**: IF $h_1(t)$ is M_{z1} and $h_2(t)$ is M_{z2} and … and $h_p(t)$ is M_{zp},

$$u(t) = \sum_{z=1}^{r} w_z(h(t)) [K_z(m(t - \eta_2(t)) - s(t - \eta_2(t)))]. \tag{12}$$

Substituting the above controller into the system (4) generating the closed-loop system

$$\dot{m}(t) = \sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\Big[\bar{A}_i m(t) + \bar{A}_{di} m(t-\kappa(t)) + \bar{B}_i[K_z(m(t-\eta_2(t)) - s(t-\eta_2(t)))] + \bar{D}_i v(t)\Big]. \tag{13}$$

Besides, the output arrangement of the system (13) is explained as follows:

$$y(t) = m(t) + m(t-\kappa(t)). \tag{14}$$

Before presenting our main results, the following assumption, definitions and instrumental lemmas are introduced to verify the stability criteria for the closed-loop system.

Assumption 1: Matrices $\Theta_1, \Theta_2, \Theta_3$, and Θ_4 fulfil the accompanying conditions:

(1) $\Theta_1 = \Theta_1^{\mathrm{T}} \le 0, \Theta_3 = \Theta_3^{\mathrm{T}} > 0, \Theta_4 = \Theta_4^{\mathrm{T}} \ge 0$,
(2) $(\|\Theta_1\| + \|\Theta_2\|)\cdot\|\Theta_4\| = 0$.

Definition 1: [38] The T–S fuzzy system (13) with (14) is said to be extended dissipative, there exists a scalar $\omega > 0$, and the given matrices $\Theta_1, \Theta_2, \Theta_3$, and Θ_4 satisfying Assumption 1 such that for any $t_f \ge 0$ and all $v(t) \in L_2[0,\infty)$ the following inequality holds:

$$\int_0^{t_f} J(t)\,\mathrm{d}t \ge \sup_{0\le t\le t_f} y^{\mathrm{T}}(t)\Theta_4 y(t) + \omega,$$

where $J(t) = y^{\mathrm{T}}(t)\Theta_1 y(t) + 2y^{\mathrm{T}}(t)\Theta_2 v(t) + v^{\mathrm{T}}(t)\Theta_3 v(t)$.

Definition 2: [38] For system (13), if there exists a scalar $\nu > 0$, such that the Lyapunov function derivative corresponding with time t states

$$\dot{V}(t) \le -\nu\left|m(t)\right|^2,$$

therefore, system (13) with $v(t) = 0$ is said to be quadratically stable.

Lemma 1: [15] Given matrices H, E, and F with $F^{\mathrm{T}}F = I$ and a scalar $\epsilon > 0$, the succeeding inequality holds:

$$HFE + (HFE)^{\mathrm{T}} \le \epsilon HH^{\mathrm{T}} + \epsilon^{-1}E^{\mathrm{T}}E.$$

Lemma 2: [11] For block symmetric matrices $\hat{Q}_6 = \mathrm{diag}\{Q_6, 3Q_6, 5Q_6\}$ with $Q_6 > 0$, any matrix S, the following inequality holds:

$$\Upsilon(t) \le -\frac{1}{h}\tilde{\chi}^{\mathrm{T}}(t)\begin{bmatrix}F_1\\F_2\end{bmatrix}^{\mathrm{T}}\left(\begin{bmatrix}\hat{Q}_6 & S\\ * & \hat{Q}_6\end{bmatrix} + \begin{bmatrix}\frac{h-\eta_2(t)}{h}U_1 & 0\\ 0 & \frac{\eta_2(t)}{h}U_2\end{bmatrix}\right)\begin{bmatrix}F_1\\F_2\end{bmatrix}\tilde{\chi}(t),$$

$$\tilde{\chi}(t) = -\int_{t-\eta_2(t))}^{t}\dot{u}^{\mathrm{T}}(s)Q_6\dot{u}(s)\,\mathrm{d}s - \int_{t-h}^{t-\eta_2(t)}\dot{u}^{\mathrm{T}}(s)Q_6\dot{u}(s)\,\mathrm{d}s,$$

$$\xi(t) = [u^{\mathrm{T}}(t)\; u^{\mathrm{T}}(t-\eta_2(t))\; u^{\mathrm{T}}(t-h)\; v_1^{\mathrm{T}}(t)\; v_2^{\mathrm{T}}(t)\; v_3^{\mathrm{T}}(t)\; v_4^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$F_1 = \Lambda_1,\; F_2 = \Lambda_2,$$

$$\Lambda_1 = \mathrm{col}[\hat{s}_1 - s_2\,\hat{s}_1 + \hat{s}_2 - 2\hat{s}_4\,\hat{s}_1 - \hat{s}_2 + 6\hat{s}_4 - 12\hat{s}_6],$$

$$\Lambda_2 = \mathrm{col}[\hat{s}_2 - \hat{s}_3\,\hat{s}_2 + \hat{s}_3 - 2\hat{s}_5\,\hat{s}_2 - \hat{s}_3 + 6\hat{s}_5 - 12\hat{s}_7],$$

$$U_1 = \hat{Q}_6 - S\hat{Q}_6^{-1}S^{\mathrm{T}},\; U_2 = \hat{Q}_6 - S^{\mathrm{T}}\hat{Q}_6^{-1}S,$$

$$v_1(t) = \frac{1}{\eta_2(t)}\int_{t-\eta_2(t)}^{t}u^{\mathrm{T}}(s)\,\mathrm{d}s,\; v_2(t) = \frac{1}{h-\eta_2(t)}\int_{t-h}^{t-\eta_2(t)}u^{\mathrm{T}}(s)\,\mathrm{d}s,$$

$$v_3(t) = \frac{1}{\eta_2^2(t)}\int_{t-\eta_2(t)}^{t}\int_s^t u^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\,\mathrm{d}s,$$

$$v_4(t) = \frac{1}{(h-\eta_2(t))^2}\int_{t-h}^{t-\eta_2(t)}\int_s^{t-\eta_2(t)} u^{\mathrm{T}}(\theta)\,\mathrm{d}\theta\,\mathrm{d}s,$$

$$\hat{s}_j = [0_{n\times(j-1)_n}\quad I\quad 0_{n\times(7-j)n}],\; j = 1,2,\ldots,7.$$

Lemma 3: [12] Let $Z > 0$ and for given scalars α and $\hat{\beta}$, the following relation is well defined for any differentiable function u in $[\alpha, \hat{\beta}] \to \mathbb{R}^n$.

$$-\int_\alpha^{\hat{\beta}}\dot{u}^{\mathrm{T}}(s)Z\dot{u}(s)\,\mathrm{d}s \le -\frac{1}{\hat{\beta}-\alpha}\varphi_1^{\mathrm{T}}Z\varphi_1 - \frac{3}{\hat{\beta}-\alpha}\varphi_2^{\mathrm{T}}Z\varphi_2,$$

$$-\int_\alpha^{\hat{\beta}}\dot{u}^{\mathrm{T}}(s)Z\dot{u}(s)\,\mathrm{d}s \le -\frac{1}{\hat{\beta}-\alpha}\varphi_1^{\mathrm{T}}Z\varphi_1 - \frac{3}{\hat{\beta}-\alpha}\varphi_2^{\mathrm{T}}Z\varphi_2 - \frac{5}{\hat{\beta}-\alpha}\varphi_7^{\mathrm{T}}Z\varphi_7,$$

$$-\int_\alpha^{\hat{\beta}}\int_\lambda^{\hat{\beta}}\dot{u}^{\mathrm{T}}(s)Z\dot{u}(s)\,\mathrm{d}s\,\mathrm{d}\lambda \le -2\varphi_3^{\mathrm{T}}Z\varphi_3 - 4\varphi_4^{\mathrm{T}}Z\varphi_4,$$

$$-\int_\alpha^{\hat{\beta}}\int_\alpha^{\lambda}\dot{u}^{\mathrm{T}}(s)Z\dot{u}(s)\,\mathrm{d}s\,\mathrm{d}\lambda \le -2\varphi_5^{\mathrm{T}}Z\varphi_5 - 4\varphi_6^{\mathrm{T}}Z\varphi_6,$$

where

$$\varphi_1 = u(\hat{\beta}) - u(\alpha),\; \varphi_2 = u(\hat{\beta}) + u(\alpha) - \frac{2}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s,$$

$$\varphi_3 = u(\hat{\beta}) - \frac{1}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s,\; \varphi_5 = u(\alpha) - \frac{1}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s,$$

$$\varphi_4 = u(\hat{\beta}) + \frac{2}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s - \frac{6}{(\hat{\beta}-\alpha)^2}\int_\alpha^{\hat{\beta}}\int_\lambda^{\hat{\beta}}u(s)\,\mathrm{d}s\,\mathrm{d}\lambda,$$

$$\varphi_6 = u(\alpha) - \frac{4}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s + \frac{6}{(\hat{\beta}-\alpha)^2}\int_\alpha^{\hat{\beta}}\int_\lambda^{\hat{\beta}}u(s)\,\mathrm{d}s\,\mathrm{d}\lambda,$$

$$\varphi_7 = u(\hat{\beta}) - u(\alpha) - \frac{2}{\hat{\beta}-\alpha}\int_\alpha^{\hat{\beta}}u(s)\,\mathrm{d}s + \frac{12}{(\hat{\beta}-\alpha)^2}\int_\alpha^{\hat{\beta}}\int_\theta^{\hat{\beta}}u(s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

The main objective of this paper is summarised in the following problem 1:

Problem 1: For the given T–S fuzzy system (13), to achieve quadratically stable under the adaptive event-triggered mechanism through the following objectives:

(1) New LKF with communication delay signals is proposed with $v(t) = 0$ to derive the quadratically stable condition of (13).

(2) The suitable event-triggered strategy, a norm-bounded parametric uncertainties, and control gain matrix K_z are designed from the suitable LMIs to ensure the closed-loop system (13) is extended dissipative, there exists a scalar $\omega > 0$.

The following section provides the solution to Problem 1 for the extended dissipativity condition.

3 Main results

In this section, we will give the new quadratic stability sufficient conditions, and develop event-triggered controller to ensure extended dissipativity criteria for the given T–S fuzzy system (15) with successive time-varying delay and communication delay signal.

A. Event-triggered quadratically stable and extended dissipative analysis for the following nominal system by considering successive time-varying delays:

$$\begin{aligned}\dot{m}(t) = \sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\big[A_i m(t) + A_{di} m(t-\kappa(t))\\ +B_i[K_z(m(t-\eta_2(t)) - s(t-\eta_2(t)))] + D_i v(t)\big].\end{aligned} \tag{15}$$

For the sake of simplicity of matrix and vector representation, $e_l \in \mathbb{R}^{21\mathbb{N}\times\mathbb{N}} (l = 1, 2, \ldots, 21)$ are defined as block entry matrices (such as $e_3 = [0\,0\,I\,\underbrace{0\,0\,0}_{18\text{ times}}]^{\mathrm{T}}$). The other notations are defined as:

$$\begin{aligned}\hat{\xi}^{\mathrm{T}}(t) &= \big[m^{\mathrm{T}}(t)\, m^{\mathrm{T}}(t-\kappa(t))\, m^{\mathrm{T}}(t-\kappa_1(t))\, m^{\mathrm{T}}(t-\tilde{h})\\ &\times m^{\mathrm{T}}(t-\kappa_1)\, m^{\mathrm{T}}(t-\eta_2(t))\, m^{\mathrm{T}}(t-h)\, \upsilon_1^{\mathrm{T}}(t)\, \upsilon_2^{\mathrm{T}}(t)\, \upsilon_3^{\mathrm{T}}(t)\\ &\times \upsilon_4^{\mathrm{T}}(t)\, s^{\mathrm{T}}(t-\eta_2(t))\, v^{\mathrm{T}}(t)\, m^{\mathrm{T}}(t-\kappa_2(t))\, m^{\mathrm{T}}(t-\kappa_2)\\ &\times \hat{\xi}_1^{\mathrm{T}}(t)\, \hat{\xi}_2^{\mathrm{T}}(t)\big],\\ \hat{\xi}_1^{\mathrm{T}}(t) &= \Big[\frac{1}{\kappa_2(t)}\int_{t-\kappa_2(t)}^{t} m^{\mathrm{T}}(s)\,\mathrm{d}s\, \frac{1}{\kappa_2-\kappa_2(t)}\int_{t-\kappa_2}^{t-\kappa_2(t)} m^{\mathrm{T}}(s)\,\mathrm{d}s\\ &\times \frac{1}{\kappa_2^2(t)}\int_{t-\kappa_2(t)}^{t}\int_{u}^{t} m^{\mathrm{T}}(s)\,\mathrm{d}s\mathrm{d}u\Big],\\ \hat{\xi}_2^{\mathrm{T}}(t) &= \Big[\frac{1}{(\kappa_2-\kappa_2(t))^2}\int_{t-\kappa_2}^{t-\kappa_2(t)}\int_{u}^{t-\kappa_2(t)} m^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}u\\ &\times \frac{1}{\kappa_1}\int_{t-\kappa_1}^{t} m^{\mathrm{T}}(s)\,\mathrm{d}s\, \frac{1}{\kappa_1^2}\int_{t-\kappa_1}^{t}\int_{t+\hat{\beta}}^{t} m^{\mathrm{T}}(s)\,\mathrm{d}s\,\mathrm{d}\hat{\beta}\Big],\\ \hbar_{13} &= [I\,0\,-I\,\underbrace{0\,0\,0}_{18\text{ times}}]^{\mathrm{T}},\ \hbar_{23}, \text{ and } \hbar_{24} \text{ follow similarly}.\end{aligned} \tag{16}$$

Theorem 1: For a given scalars $\kappa_1, \kappa_2, \mu_1, \mu_2, h$ and $0 < \epsilon < 1$, matrices Θ_1, Θ_2, Θ_3, and Θ_4 satisfying Assumption 2.1, the T–S fuzzy system (15) with (14) is extended dissipative and quadratically stable, if there exist matrices $P > 0, Q_l > 0, l = 1, 2, 3, 4, \hat{R}_1 > 0, \hat{R}_2 > 0, S_j > 0, j = 1, 2, 3\ldots, 7$, $Q_6 > 0$, $\bar{\Phi}_1 > 0$, $\bar{\Phi}_2 > 0$, $\bar{K}_z$ and $\bar{P}$, such that the following conditions are satisfied

$$\begin{aligned}\Pi &= \begin{bmatrix}\epsilon\bar{P} - \Theta_4 & -\Theta_4\\ \star & (1-\epsilon)\bar{P} - \Theta_4\end{bmatrix} > 0,\\ \tilde{\Omega}_{iz} &= \begin{bmatrix}\Xi_\ell & \Sigma_{iz}\\ \Sigma_{iz}^{\mathrm{T}} & -\hat{\Psi}\end{bmatrix} < 0,\ \ell = 1, 2, 3, 4,\end{aligned} \tag{17}$$

where

$$\begin{aligned}\Xi_1 &= \Xi_{iz} - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{23}\kappa_1^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}} < 0,\\ \Xi_2 &= \Xi_{iz} - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2\tilde{h}^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}} < 0,\\ \Xi_3 &= \Xi_{iz} - \hbar_{24}\kappa_1 h^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}} < 0,\\ \Xi_4 &= \Xi_{iz} - \hbar_{24}h^{-1}\bar{R}_2\hbar_{24}^{\mathrm{T}} < 0,\ \Xi_{iz} = \sum_{j=1}^{5}\hat{\Theta}_j,\\ \hat{\Psi} &= \mathrm{diag}\{\bar{R}_1 - 2\bar{P},\ \bar{R}_2 - 2\bar{P},\ \bar{Q}_6 - 2\bar{P},\ \bar{S}_4 - 2\bar{P},\ \bar{S}_5 - 2\bar{P},\\ &\quad \bar{S}_3 - 2\bar{P},\ \bar{S}_6 - 2\bar{P}\},\\ \Sigma_{iz} &= \Big[\sqrt{\kappa_1}\Lambda_{iz}^{\mathrm{T}}\sqrt{\tilde{h}}\Lambda_{iz}^{\mathrm{T}}\, h\Lambda_{iz}^{\mathrm{T}}\frac{\kappa_1}{\sqrt{2}}\Lambda_{iz}^{\mathrm{T}}\frac{\kappa_1}{\sqrt{2}}\Lambda_{iz}^{\mathrm{T}}\kappa_1\Lambda_{iz}^{\mathrm{T}}\frac{\tilde{h}}{\sqrt{2}}\Lambda_{iz}^{\mathrm{T}}\Big],\\ \Lambda_{iz}^{\mathrm{T}} &= [\bar{P}A_i\,\bar{P}A_{di}\,0\,0\,0\,B_i\bar{K}_z\,\overbrace{0\,0\,0}^{5\text{ times}} - B_i\bar{K}_z\,\bar{P}D_i\,\overbrace{0\,0\,0}^{8\text{ times}}]^{\mathrm{T}},\\ \hat{\Theta}_{1_{iz}} &= \mathrm{sym}\{e_1\bar{P}(A_i e_1^{\mathrm{T}} + A_{di}e_2^{\mathrm{T}} + D_i e_{13}^{\mathrm{T}}) + e_1 B_i[\bar{K}_z(e_6 - e_{12})^{\mathrm{T}}]\}\\ &\quad -(1-\mu_1)e_3 Q_1 e_3^{\mathrm{T}} + e_1\Big(\sum_{i=1}^{4} Q_i + S_1 + S_2\Big)e_1^{\mathrm{T}} - e_4 Q_2 e_4^{\mathrm{T}}\\ &\quad -(1-\mu)e_2 Q_3 e_2^{\mathrm{T}} - e_5 Q_4 e_5^{\mathrm{T}} - (1-\mu_2)e_{14}S_1 e_{14}^{\mathrm{T}} - e_{15}S_2 e_{15}^{\mathrm{T}},\\ \hat{\Theta}_2 &= -(e_1 - e_3)\kappa_1^{-1}\hat{R}_2(e_1 - e_3)^{\mathrm{T}} - (e_2 - e_4)\tilde{h}^{-1}\hat{R}_2(e_2 - e_4)^{\mathrm{T}}\\ &\quad -(e_2 - e_3)\kappa_2^{-1}\hat{R}_2(e_2 - e_3)^{\mathrm{T}} - (e_1 - e_3)\kappa_1^{-1}\hat{R}_1(e1 - e_3)^{\mathrm{T}}\\ &\quad -(e_3 - e_4)\kappa_1^{-1}\hat{R}_1(e_3 - e_4)^{\mathrm{T}},\\ \hat{\Theta}_3 &= (e_{12} - e_6)(I_n + \chi_2\Lambda_2)\bar{\Phi}_2(e_{12} - e_6)^{\mathrm{T}} + e_6(I_n + \Lambda_1\chi_1)\bar{\Phi}_2 e_6^{\mathrm{T}}\\ &\quad -e_{12}(\Lambda_1 + \Lambda_2)\bar{\Phi}_1 e_{12}^{\mathrm{T}} + e_1 S_7 e_1^{\mathrm{T}} + \Upsilon_1^{\mathrm{T}}\mathbb{Z}_1\Upsilon_1,\\ \Upsilon_1 &= [\Upsilon_{11}^{\mathrm{T}}\,\Upsilon_{12}^{\mathrm{T}}]^{\mathrm{T}},\ \Upsilon_{11} = [\Upsilon_{11}^1\, e_1 - e_6 + 6e_8 - 12e_{10}],\\ \Upsilon_{12} &= [\Upsilon_{12}^1\, e_6 - e_7 + 6e_9 - 12e_{11}],\ \Upsilon_{11}^1 = [e_1 - e_6\, e_1 + e_6 - 2e_8],\\ \Upsilon_{12}^1 &= [e_6 - e_7\, e_6 + e_7 - 2e_9],\\ \mathbb{Z}_1 &= \begin{bmatrix}\frac{2h - \eta_2(t)}{h}\mathbb{Z}_2 & S\\ * & \frac{h + \eta_2(t)}{h}\mathbb{Z}_2\end{bmatrix},\ \mathbb{Z}_2 = \mathrm{diag}\{Q_6,\ 3Q_6,\ 5Q_6\}\\ \hat{\Theta}_4 &= -2(e_5 - e_{20})S_4(e_5 - e_{20})^{\mathrm{T}} - 2(e_1 - e_{20})S_5(e_1 - e_{20})^{\mathrm{T}}\\ &\quad -4(e_5 - 4e_{20} + 6e_{21})S_4(e_5 - 4e_{20} + 6e_{21})^{\mathrm{T}}\\ &\quad -4(e_1 + 2e_{20} - 6e_{21})S_4(e_1 + 2e_{20} - 6e_{21})^{\mathrm{T}} - 2(e_1 - e_{16})\\ &\quad \times S_6(e_1 - e_{16})^{\mathrm{T}} - 4(e_1 + 2e_{16} - 6e_{18})S_6(e_1 + 2e_{16} - 6e_{18})^{\mathrm{T}}\\ &\quad -2(e_{14} - e_{17})S_6(e_{14} - e_{17})^{\mathrm{T}} - 4(e_{14} + 2e_{17} - 6e_{19})\\ &\quad \times S_6(e_{14} + 2e_{17} - 6e_{19})^{\mathrm{T}} - (e_1 - e_{14})S_6(e_1 - e_{14})^{\mathrm{T}}\\ &\quad -3(e_1 + e_{14} - 2e_{16})S_6(e_1 + e_{14} - 2e_{16})^{\mathrm{T}}\\ &\quad -5(e_1 - e_{14} - 6e_{16} + 12e_{18})S_6(e_1 - e_{14} - 6e_{16} + 12e_{18})^{\mathrm{T}},\\ \hat{\Theta}_5 &= -e_1\Theta_1 e_1^{\mathrm{T}} + e_1\Theta_1 e_2^{\mathrm{T}} - e_1\Theta_2 e_{13}^{\mathrm{T}} - e_2\Theta_1 e_2^{\mathrm{T}}\\ &\quad +e_2\Theta_2 e_{13}^{\mathrm{T}} - e_{13}\Theta_3 e_{13}^{\mathrm{T}}.\end{aligned}$$

Moreover, if the foregoing condition holds, the desired controller gain matrix is given by $K_z = \bar{K}_z\bar{P}^{-1}$, and the scalar ω is defined in Definition 1 is chosen as $\omega = -V(0) - \|\bar{P}\|\ \sup_{-r \le s \le 0}|\phi(s)|^2$.

Proof: Define an LKF as follows:

$$V(t) = \sum_{\ell=1}^{4} V_\ell(t), \tag{18}$$

where

$$
\begin{aligned}
V_1(t) &= m^{\mathrm{T}}(t)Pm(t) + \int_{t-\kappa_1(t)}^{t} m^{\mathrm{T}}(\alpha)Q_1 m(\alpha)\,\mathrm{d}\alpha \\
&\quad + \int_{t-\tilde{h}}^{t} m^{\mathrm{T}}(\alpha)Q_2 m(\alpha)\,\mathrm{d}\alpha + \int_{t-\kappa(t)}^{t} m^{\mathrm{T}}(\alpha)Q_3 m(\alpha)\,\mathrm{d}\alpha \\
&\quad + \int_{t-\kappa_1}^{t} m^{\mathrm{T}}(\alpha)Q_4 m(\alpha)\,\mathrm{d}\alpha + \int_{t-\kappa_2(t)}^{t} m^{\mathrm{T}}(\alpha)S_1 m(\alpha)\,\mathrm{d}\alpha \\
&\quad + \int_{t-\kappa_2}^{t} m^{\mathrm{T}}(\alpha)S_2 m(\alpha)\,\mathrm{d}\alpha, \\
V_2(t) &= \int_{t-\kappa_1}^{t}\int_{t+\theta}^{t} \dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}s \\
&\quad + \int_{t-\tilde{h}}^{t}\int_{t+\theta}^{t} \dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}s \\
&\quad +\kappa_2\int_{t-\kappa_2}^{t}\int_{t+\theta}^{t} \dot{m}^{\mathrm{T}}(\alpha)S_3\dot{m}(\alpha)\,\mathrm{d}\alpha\,\mathrm{d}\theta, \\
V_3(t) &= h\int_{h}^{0}\int_{t+\theta}^{t} \dot{m}^{\mathrm{T}}(s)Q_6\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\
&\quad + \int_{t-\eta(t)}^{t} m^{\mathrm{T}}(s)S_7 m(s)\,\mathrm{d}s + \frac{1}{2}\tilde{\beta}^2(t) + \frac{1}{2}\delta^2(t), \\
V_4(t) &= \int_{-\kappa_1}^{0}\int_{-\kappa_1}^{\lambda}\int_{t+\hat{\beta}}^{t} \dot{m}^{\mathrm{T}}(\alpha)S_4\dot{m}(\alpha)\,\mathrm{d}s\,\mathrm{d}\hat{\beta}d\lambda \\
&\quad + \int_{-\kappa_1}^{0}\int_{\lambda}^{0}\int_{t+\hat{\beta}}^{t} \dot{m}^{\mathrm{T}}(\alpha)S_5\dot{m}(\alpha)\,\mathrm{d}s\,\mathrm{d}\hat{\beta}d\lambda \\
&\quad + \int_{-\tilde{h}}^{0}\int_{-\theta}^{0}\int_{t+u}^{t} \dot{m}^{\mathrm{T}}(\alpha)S_6\dot{m}(\alpha)\,\mathrm{d}s\,\mathrm{d}u\,\mathrm{d}\theta\,.
\end{aligned}
$$

Then, calculating the time derivative of the LKF along the trajectory of (15) yields

$$
\begin{aligned}
\dot{V}_1(t) &\le 2m^{\mathrm{T}}(t)P\big[A_i m(t) + A_{di}m(t-\kappa(t)) + B_i[K_z(m(t-\eta_2(t)) \\
&\quad -s(t-\eta_2(t)))] + D_i v(t)\big] + m^{\mathrm{T}}(t)Q_1 m(t) - (1-\mu_1) \\
&\quad \times m(t-\kappa_1(t))^{\mathrm{T}}Q_1 m(t-\kappa_1(t)) + m^{\mathrm{T}}(t)Q_2 m(t) \\
&\quad -m(t-\tilde{h})^{\mathrm{T}}Q_2 m(t-\tilde{h}) + m^{\mathrm{T}}(t)Q_3 m(t) - (1-\mu) \\
&\quad \times m(t-\kappa(t))^{\mathrm{T}}Q_3 m(t-\kappa(t)) + m^{\mathrm{T}}(t)Q_4 m(t) \\
&\quad -m(t-\kappa_1)^{\mathrm{T}}Q_4 m(t-\kappa_1) + m^{\mathrm{T}}(t)S_1 m(t) \\
&\quad -(1-\mu_2(t))m(t-\kappa_2(t))^{\mathrm{T}}S_1 m(t-\kappa_2(t)) \\
&\quad +m^{\mathrm{T}}(t)S_2 m(t) - m(t-\kappa_2)^{\mathrm{T}}S_2 m(t-\kappa_2), \qquad (19) \\
&\le \sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\hat{\xi}^{\mathrm{T}}(t)\hat{\Theta}_{1_{iz}}\hat{\xi}(t), \\
\dot{V}_2(t) &\le \kappa_1\dot{m}^{\mathrm{T}}(t)\hat{R}_1\dot{m}(t) - \int_{t-\kappa_1}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\quad +h\dot{m}^{\mathrm{T}}(t)\hat{R}_2\dot{m}(t) - \int_{t-h}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\quad +\kappa_2^2\dot{m}^{\mathrm{T}}(t)S_3\dot{m}(t) - \kappa_2\int_{t-\kappa_2}^{t}\dot{m}^{\mathrm{T}}(\alpha)S_3\dot{m}(\alpha)\,\mathrm{d}\alpha\,.
\end{aligned}
$$

Note that

$$
\begin{aligned}
&-\int_{t-\kappa_1}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha - \int_{t-\tilde{h}}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&= -\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha - \int_{t-\kappa_1}^{t-\kappa_1(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\quad -\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha - \int_{t-\kappa(t)}^{t-\kappa_1(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \qquad (20) \\
&\quad -\int_{t-\tilde{h}}^{t-\kappa(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha\,.
\end{aligned}
$$

Let $\hat{\omega} = \kappa_1(t)/\kappa_1$ and $\check{\omega} = \kappa_2(t)/\kappa_2$. Then

$$
\begin{aligned}
-\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha &= -\kappa_1^{-1}\int_{t-\kappa_1(t)}^{t}\kappa_1\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&= -\kappa_1^{-1}\int_{t-\kappa_1(t)}^{t}\kappa_1(t)\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \qquad (21) \\
&\quad -\kappa_1^{-1}\int_{t-\kappa_1(t)}^{t}[\kappa_1-\kappa_1(t)]\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha\,.
\end{aligned}
$$

From (21), we have

$$
\begin{aligned}
&-\kappa_1^{-1}\int_{t-\kappa_1(t)}^{t}[\kappa_1-\kappa_1(t)]\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&= -(1-\hat{\omega})\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \qquad (22) \\
&\le -(1-\hat{\omega})\kappa_1^{-1}\int_{t-\kappa_1(t)}^{t}\kappa_1(t)\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha\,.
\end{aligned}
$$

By Jensen's inequality, together with (21) and (22), we get the following:

$$
\begin{aligned}
&-\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha - \int_{t-\kappa(t)}^{t-\kappa_1(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\le \hat{\xi}^{\mathrm{T}}(t)\big[-(e_1-e_3)[\kappa_1^{-1}\hat{R}_2 + (1-\hat{\omega})\kappa_1^{-1}\hat{R}_2](e_1-e_3)^{\mathrm{T}} \\
&\quad -(e_2-e_3)[\kappa_2^{-1}\hat{R}_2 + (1-\check{\omega})\kappa_2^{-1}\hat{R}_2](e_2-e_3)^{\mathrm{T}}\big]\hat{\xi}(t),
\end{aligned}
$$

similarly, the following inequality holds

$$
\begin{aligned}
&-\int_{t-\tilde{h}}^{t-\kappa(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_2\dot{m}(\alpha)\,\mathrm{d}\alpha \le \hat{\xi}^{\mathrm{T}}(t)\big[-(e_2-e_4)[\tilde{h}^{-1}\hat{R}_2 \\
&\quad +\hat{\omega}\kappa_1\tilde{h}^{-2}\times\hat{R}_2 + \check{\omega}\kappa_2\tilde{h}^{-2}\hat{R}_2](e_2-e_4)^{\mathrm{T}}\big]\hat{\xi}(t), \\
&-\int_{t-\kappa_1(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha - \int_{t-\kappa_1}^{t-\kappa_1(t)}\dot{m}^{\mathrm{T}}(\alpha)\hat{R}_1\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\le \hat{\xi}^{\mathrm{T}}(t)\big[-(e_1-e_3)[\kappa_1^{-1}\hat{R}_1](e_1-e_3)^{\mathrm{T}} \\
&\quad -(e_3-e_4)[\kappa_1^{-1}\hat{R}_1](e_3-e_4)^{\mathrm{T}}\big]\hat{\xi}(t)\,.
\end{aligned}
$$

By using the statement (ii) of Lemma 3, we have

$$
\begin{aligned}
&-\kappa_2\int_{t-\kappa_2}^{t}\dot{m}^{\mathrm{T}}(\alpha)S_3\dot{m}(\alpha)\,\mathrm{d}\alpha = -\kappa_2\int_{t-\kappa_2(t)}^{t}\dot{m}^{\mathrm{T}}(\alpha)S_3\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&\quad -\kappa_2\int_{t-\kappa_2}^{t-\kappa_2(t)}\dot{m}^{\mathrm{T}}(\alpha)S_3\dot{m}(\alpha)\,\mathrm{d}\alpha \\
&= -\hat{\xi}^{\mathrm{T}}(t)\big[(e_1-e_{14})S_3(e_1-e_{14})^{\mathrm{T}} + 3(e_1+e_{14}-2e_{16})S_3 \\
&\quad \times(e_1+e_{14}-2e_{16})^{\mathrm{T}} + 5(e_1-e_{14}-6e_{16}+12e_{18})S_3 \qquad (23) \\
&\quad \times(e_1-e_{14}-6e_{16}+12e_{18})^{\mathrm{T}}\big]\hat{\xi}(t) - \hat{\xi}^{\mathrm{T}}(t)\big[(e_{14}-e_{15})S_3 \\
&\quad \times(e_{14}-e_{15})^{\mathrm{T}} + 3(e_{14}+e_{15}-2e_{17})S_3(e_{14}+e_{15}-2e_{17})^{\mathrm{T}} \\
&\quad +5(e_{14}-e_{15}-6e_{17}+12e_{19}) \\
&\quad S_3(e_{14}-e_{15}-6e_{17}+12e_{19})^{\mathrm{T}}\big]\hat{\xi}(t)\,.
\end{aligned}
$$

Combining (20)–(23), one can obtain

$$
\begin{aligned}
\dot{V}_2(t) &\le \hat{\xi}^{\mathrm{T}}(t)\hat{\Theta}_2\hat{\xi}(t) + \hat{\xi}^{\mathrm{T}}(t)\big[-(1-\hat{\omega})[(e_1-e_3)\kappa_1^{-1}\hat{R}_2 \\
&\quad \times(e_1-e_3)^{\mathrm{T}} - \hat{\omega}(e_2-e_4)\kappa_1\tilde{h}^{-2}\hat{R}_2(e_2-e_4)^{\mathrm{T}} \\
&\quad \times -\check{\omega}(e_2-e_4)\times\kappa_2\tilde{h}^{-2}\hat{R}_2(e_2-e_4)^{\mathrm{T}} \qquad (24) \\
&\quad -(1-\check{\omega})(e_2-e_3)\times\kappa_2^{-1}\hat{R}_2(e_2-e_3)^{\mathrm{T}}\big]\hat{\xi}(t),
\end{aligned}
$$

$$\dot{V}_3(t) \le \sum_{l=1}^{\upsilon}\Big\{m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{2l}m_l(t_{k_l}^l h) + \frac{\delta_l(t)-\lambda_{1l}}{\tilde{\beta}_l(t)}m_l^{\mathrm{T}}(i_{k_l}^l h)$$
$$\Phi_{1l}m_l(i_{k_l}^l h) + \Big[\frac{1}{\tilde{\beta}_l(t)-\rho_{1l}}\Big]\lambda_{1l}m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{2l}m_l(t_{k_l}^l h)$$
$$\times m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{3l}m_l(t_{k_l}^l h) + \frac{\delta_l(t)-\lambda_{2l}}{\tilde{\beta}_l(t)}m_l^{\mathrm{T}}(i_{k_l}^l h)$$
$$\Phi_{1l}m_l(i_{k_l}^l h) + \Big[\frac{1}{\delta_l(t)-\rho_{2l}}\Big]\lambda_{2l}m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{2l}m_l(t_{k_l}^l h)$$
$$-\rho_{1l}[s_l^{\mathrm{T}}(i_{k_l}^l h)\Phi_{1l}s_l(i_{k_l}^l h) - \lambda_{1l}m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{3l}m_l(t_{k_l}^l h)]$$
$$-\rho_{2l}[s_l^{\mathrm{T}}(i_{k_l}^l h)\Phi_{1l}s_l(i_{k_l}^l h)\lambda_{2l}m_l^{\mathrm{T}}(t_{k_l}^l h)\Phi_{2l}m_l(t_{k_l}^l h)]\Big\} \quad (25)$$
$$+h^2\dot{m}^{\mathrm{T}}(t)Q_6\dot{m}(t) - h\int_{t-h}^{t}\dot{m}^{\mathrm{T}}(s)Q_6\dot{m}(s)\,\mathrm{d}s$$
$$+m^{\mathrm{T}}(t)S_3m(t),$$
$$\le [s(t-\eta_2(t)) - m(t-\eta_2(t))]^{\mathrm{T}}(I_n + \Lambda_2\chi_2)\bar{\Phi}_2$$
$$\times[s(t-\eta_2(t)) - m(t-\eta_2(t))] + m^{\mathrm{T}}(t-\eta_2(t))$$
$$(I_n + \Lambda_1\chi_1)\bar{\Phi}_2 m(t-\eta_2(t)) - s^{\mathrm{T}}(t-\eta_2(t))(\Lambda_1+\Lambda_2)$$
$$\times\bar{\Phi}_1 s(t-\eta_2(t)) + h^2\dot{m}^{\mathrm{T}}(t)Q_6\dot{m}(t)$$
$$-h\int_{t-h}^{t}\dot{m}^{\mathrm{T}}(s)Q_6\dot{m}(s)\,\mathrm{d}s + m^{\mathrm{T}}(t)S_7m(t).$$

Utilizing Lemma 2 in (25), we get

$$-h\int_{t-h}^{t}\dot{m}^{\mathrm{T}}(s)Q_6\dot{m}(s)\,\mathrm{d}s$$
$$= -h\Big\{\int_{t-\eta_2(t)}^{t}Q_{6\dot{m}}(s)\,\mathrm{d}s + \int_{t-h}^{t-\eta_2(t)}Q_{6\dot{m}}(s)\,\mathrm{d}s\Big\},$$
$$\le \hat{\xi}^{\mathrm{T}}(t)[\Upsilon_1^{\mathrm{T}}\mathbb{Z}_1\Upsilon_1 + \Pi]\hat{\xi}(t),$$

where

$$Q_{6\dot{m}}(s) = \dot{m}^{\mathrm{T}}(s)Q_6\dot{m}(s),\ \Pi_1 = \Upsilon_{11}^{\mathrm{T}}S\mathbb{Z}_2^{-1}S^{\mathrm{T}}\Upsilon_{11},$$
$$\Pi_1 = \Upsilon_{12}^{\mathrm{T}}S^{\mathrm{T}}\mathbb{Z}_2^{-1}S\Upsilon_{12},\ \Pi = \frac{h-\eta_2(t)}{h}\Pi_1 + \frac{\eta_2(t)}{h}\Pi_2.$$

Leading to:

$$\dot{V}_3(t) \le \hat{\xi}^{\mathrm{T}}(t)\hat{\Theta}_3\hat{\xi}(t), \quad (26)$$

$$\dot{V}_4(t) = \dot{m}^{\mathrm{T}}(t)\Big[\frac{\kappa_1^2}{2}(S_4+S_5) + \frac{\tilde{h}^2}{2}S_6\Big]\dot{m}(t)$$
$$-\int_{-\kappa_1}^{0}\int_{t-\kappa_1}^{t+\hat{\beta}}\dot{m}^{\mathrm{T}}(s)S_4\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\hat{\beta}$$
$$-\int_{-\kappa_1}^{0}\int_{t+\hat{\beta}}^{t}\dot{m}^{\mathrm{T}}(s)S_5\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\hat{\beta} \quad (27)$$
$$-\int_{-\tilde{h}}^{0}\int_{t+\theta}^{t}\dot{m}^{\mathrm{T}}(s)S_6\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Using Lemma 3 in (27), one can obtain

$$-\int_{-\kappa_1}^{0}\int_{t-\kappa_1}^{t+\hat{\beta}}\dot{m}^{\mathrm{T}}(s)S_4\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\hat{\beta} - \int_{-\kappa_1}^{0}\int_{t+\hat{\beta}}^{t}\dot{m}^{\mathrm{T}}(s)S_5\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\hat{\beta}$$
$$\le -\hat{\xi}^{\mathrm{T}}(t)\big[2(e_5-e_{20})S_4(e_5-e_{20})^{\mathrm{T}} + 2(e_1-e_{20})$$
$$S_5(e_1-e_{20})^{\mathrm{T}} + 4(e_5 - 4e_{20} + 6e_{21})$$
$$S_4(e_5 - 4e_{20} + 6e_{21})^{\mathrm{T}} + 4(e_1 + 2e_{20} - 6e_{21})$$
$$S_4(e_1 + 2e_{20} - 6e_{21})^{\mathrm{T}}\big]\hat{\xi}(t),$$
$$-\int_{-\tilde{h}}^{0}\int_{t+\theta}^{t}\dot{m}^{\mathrm{T}}(s)S_6\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\theta = -\int_{t-\kappa(t)}^{t}\int_{\theta}^{t}\dot{m}^{\mathrm{T}}(s)S_6\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\theta$$
$$-\int_{t-\tilde{h}}^{t-\kappa(t)}\int_{\theta}^{t-\kappa(t)}\dot{m}^{\mathrm{T}}(s)S_6\dot{m}(s)\,\mathrm{d}s\,\mathrm{d}\theta$$
$$-(\tilde{h}-\kappa(t))\int_{t-\kappa(t)}^{t}\dot{m}^{\mathrm{T}}(s)S_6\dot{m}(s)\,\mathrm{d}s,$$
$$\le -\hat{\xi}^{\mathrm{T}}(t)\big[2(e_1-e_{16})S_6(e_1-e_{16})^{\mathrm{T}} + 4(e_1 + 2e_{16} - 6e_{18})$$
$$\times S_6(e_1 + 2e_{16} - 6e_{18})^{\mathrm{T}} + 2(e_{14}-e_{17})S_6(e_{14}-e_{17})^{\mathrm{T}}$$
$$+4(e_{14} + 2e_{17} - 6e_{19})S_6(e_{14} + 2e_{17} - 6e_{19})^{\mathrm{T}}$$
$$+(e_1-e_{14})S_6(e_1-e_{14})^{\mathrm{T}} + 3(e_1 + e_{14} - 2e_{16})S_6$$
$$\times(e_1 + e_{14} - 2e_{16})^{\mathrm{T}} + 5(e_1 - e_{14} - 6e_{16} + 12e_{18})S_6$$
$$\times(e_1 - e_{14} - 6e_{16} + 12e_{18})^{\mathrm{T}}\big]\hat{\xi}(t).$$

From (27), one can get

$$\dot{V}_4(t) \le \hat{\xi}^{\mathrm{T}}(t)\hat{\Theta}_4\hat{\xi}(t). \quad (28)$$

Combining (19)–(28), we can obtain

$$\dot{V}(t) - J(t) \le \sum_{i=1}^{r}\sum_{z=1}^{r}w_i(h(t))w_z(h(t))\hat{\xi}^{\mathrm{T}}(t)\big[\bar{\Phi}_{c_{iz}} - (1-\hat{\omega})(e_1-e_3)$$
$$\times\kappa_1^{-1}\hat{R}_2(e_1-e_3)^{\mathrm{T}} - \hat{\omega}(e_2-e_4)\kappa_1\tilde{h}^{-2}\hat{R}_2(e_2-e_4)^{\mathrm{T}}$$
$$-\check{\omega}(e_2-e_4)\kappa_2\tilde{h}^{-2}\hat{R}_2(e_2-e_4)^{\mathrm{T}}$$
$$-(1-\check{\omega})(e_2-e_3)\kappa_2^{-1}\hat{R}_2(e_2-e_3)^{\mathrm{T}}\big]\hat{\xi}(t),$$
$$\le \sum_{i=1}^{r}\sum_{z=1}^{r}w_i(h(t))w_z(h(t))\hat{\xi}^{\mathrm{T}}(t)M(\hat{\omega},\check{\omega})_{iz}\hat{\xi}(t), \quad (29)$$

where

$$M(\hat{\omega},\check{\omega})_{iz} = \bar{\Phi}_{c_{iz}} - \hat{\omega}\hbar_{24}\kappa_1h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}} - (1-\hat{\omega})\hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}}$$
$$-\check{\omega}\hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}} - (1-\check{\omega})\hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}$$
$$= \hat{\omega}[\bar{\Phi}_{c_{iz}} - \hbar_{24}\kappa_1h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}}] + (1-\hat{\omega})[\bar{\Phi}_{c_{iz}}$$
$$-\hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}}] - \check{\omega}\hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}}$$
$$-(1-\check{\omega})\hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}$$
$$= \hat{\omega}[\bar{\Phi}_{c_{iz}} - \hbar_{24}\kappa_1h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}} - \check{\omega}\hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}}$$
$$-(1-\check{\omega})\hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}] + (1-\hat{\omega})[\bar{\Phi}_{c_{iz}}$$
$$-\hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}} - \check{\omega}\hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}}$$
$$-(1-\check{\omega})\hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}]$$
$$= \hat{\omega}[\check{\omega}(\bar{\Phi}_{c_{iz}} - \hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{T}) + (1-\check{\omega})(\bar{\Phi}_{c_{iz}}$$
$$-\hbar_{24}\kappa_2h^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}})]$$
$$+(1-\hat{\omega})[\check{\omega}(\bar{\Phi}_{c_{iz}} - \hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2h^{-2}$$
$$\hat{R}_2\hbar_{24}^{\mathrm{T}}) + (1-\check{\omega})(\bar{\Phi}_{c_{iz}} - \hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}}$$
$$-\hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}})].$$

Then, we obtain

$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\bar{\Phi}_{c_{iz}} - \hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{23}\kappa_1^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}\} < 0,$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\bar{\Phi}_{c_{iz}} - \hbar_{13}\kappa_1^{-1}\hat{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2\tilde{h}^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}}\} < 0, \quad (30)$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\bar{\Phi}_{c_{iz}} - \hbar_{24}\kappa_1\tilde{h}^{-2}\hat{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\hat{R}_2\hbar_{23}^{\mathrm{T}}\} < 0,$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\bar{\Phi}_{c_{iz}} - \hbar_{24}\tilde{h}^{-1}\hat{R}_2\hbar_{24}^{\mathrm{T}}\} < 0,$$

where

$$\bar{\Phi}_{c_{iz}} = \hat{\Theta}_{1_{iz}} + \sum_{j=1}^{4}\hat{\Theta}_j + \Phi_{d_{iz}}^{\mathrm{T}}[\kappa_1\hat{R}_1 + \tilde{h}\hat{R}_2 + h^2Q_6 + \frac{\kappa_1^2}{2}(S_4 + S_5) + \kappa_2^2 S_3 + \frac{\tilde{h}^2}{2}S_6]\Phi_{d_{iz}}. \quad (31)$$

Let $J = \mathrm{diag}\{\underbrace{P^{-1},\ldots,P^{-1}}_{12\,\text{times}}, I, \underbrace{P^{-1},\ldots,P^{-1}}_{8\,\text{times}}\}$, $\bar{P} = P^{-1}$, $\bar{R}_v = P^{-1}\hat{R}_vP^{-1}, v = 1,2$, $\bar{Q}_j = P^{-1}Q_jP^{-1}$, $(j = 1,\ldots,4,6)$, $\bar{S}_n$, and $= P^{-1}S_nP^{-1}$, $(n = 3,4,5,6,7)$

$\Phi_{d_{iz}} = [A_i\, A_{di}\, 0\,0\,0\, B_iK_z\, \underbrace{0\,0\,0}_{5\,\text{times}} - B_iK_z\, D_i\, \underbrace{0\,0\,0}_{8\,\text{times}}]$.

Pre and post multiplying (30) by J yields

$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\Xi + \Sigma_{iz}\hat{\Psi}^{-1}\Sigma_{iz}^{\mathrm{T}} - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{23}\kappa_1^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}\} < 0,$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\Xi + \Sigma_{iz}\hat{\Psi}^{-1}\Sigma_{iz}^{\mathrm{T}} - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2\tilde{h}^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}}\} < 0, \quad (32)$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\Xi + \Sigma_{iz}\hat{\Psi}^{-1}\Sigma_{iz}^{\mathrm{T}} - \hbar_{24}\kappa_1\tilde{h}^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}\} < 0,$$
$$\sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\{\Xi + \Sigma_{iz}\hat{\Psi}^{-1}\Sigma_{iz}^{\mathrm{T}} - \hbar_{24}\tilde{h}^{-1}\bar{R}_2\hbar_{24}^{\mathrm{T}}\} < 0,$$

where

$$\hat{\Psi} = \mathrm{diag}\Big\{-\bar{R}_1^{-1}, -\bar{R}_2^{-1}, -\bar{Q}_6^{-1}, -(\bar{S}_4 + \bar{S}_5)^{-1}, -\bar{S}_3^{-1}, -\bar{S}_6^{-1}\Big\}, \ \Xi = \hat{\Theta}_{1_{iz}} + \sum_{j=1}^{4}\hat{\Theta}_j.$$

And the remaining terms are referred in Theorem 1. Then by Schur complement Lemma, we have

$$\begin{bmatrix} \Xi_j & \Sigma_{iz} \\ \Sigma_{iz}^{\mathrm{T}} & -\hat{\Psi} \end{bmatrix} < 0, j = 1,2,3,4. \quad (33)$$

Since the term $\hat{R}_i^{-1}$, Q_6^{-1}, S_n^{-1}, $\bar{R}_v = \bar{P}\hat{R}_v\bar{P}$, $\bar{Q}_6 = \bar{P}Q_6\bar{P}$

and $\bar{S}_n = \bar{P}S_n\bar{P}$ are both in (33), which is hard to comprehend, in order to facilitate the model of adaptive event-triggered controller, we transform $\hat{R}_v^{-1}$, Q_6^{-1}, S_n^{-1} into the following inequality:

$$-\bar{R}_v^{-1} \le \hat{R}_v - 2\bar{P}, \ -Q_6^{-1} \le \bar{Q}_6 - 2\bar{P}, \ -S_n^{-1} \le \bar{S}_n - 2\bar{P}. \quad (34)$$

Then substituting $\hat{R}_v^{-1}$, Q_6^{-1}, S_n^{-1} with $\bar{R}_i - 2\bar{P}$, $\bar{Q}_6 - 2\bar{P}$, $\bar{S}_n - 2\bar{P}$ in (33), inequality (17) holds. Meanwhile, we are developed the extended dissipative condition for the consider T–S fuzzy system. Based on (17), it is anything but difficult to reach the end that

$$\dot{V}(t) - J(t) \le 0.$$

After the integration of above inequality from 0 to t gives

$$\int_0^t J(\theta)\,\mathrm{d}\theta \ge V(t) - V(0) \ge m^{\mathrm{T}}(t)Pm(t) + \omega. \quad (35)$$

The accompanying lines are concentrated to show that the inequality in Definition 1 is valid, thus, double cases are required, i.e., $\| \Theta_4 \| = 0$, and $\| \Theta_4 \| \ne 0$. To begin with, if $\| \Theta_4 \| = 0$, at that point (35) suggests that for any $t_f \ge 0$

$$\int_0^{t_f} J(\theta)\,\mathrm{d}\theta \ge m^{\mathrm{T}}(t_f)Pm(t_f) + \omega \ge \omega, \quad (36)$$

this implies that Definition 1 is true. If $\| \Theta_4 \| \ne 0$, as specified in Assumption 2.1, we can finish up that the matrices $\Theta_1 = 0$, $\Theta_2 = 0$, and $\Theta_3 > 0$, thus for any $t_f \ge t \ge 0$, we have

$$\int_0^{t_f} J(\theta)\,\mathrm{d}\theta \ge \int_0^{t} J(\theta)\,\mathrm{d}\theta \ge m^{\mathrm{T}}(t)Pm(t) + \omega, \quad (37)$$

while, if $t \le \kappa(t)$, then it can be verified that

$$\begin{aligned} &\omega + m^{\mathrm{T}}(t - \kappa(t))Pm(t - \kappa(t)) \\ &\le \omega + \| P \| \left| m(t - \kappa(t)) \right|^2 \le \omega + \| P \| \sup_{-h \le \theta \le 0} \left| \phi(\theta) \right|^2 \\ &= -V(0) \le \int_0^{t_f} J(\alpha)\,\mathrm{d}\alpha. \end{aligned}$$

This suggests (37) holds for any $t_f \ge t \ge 0$. Thus, as indicated by (36) and (37), we realise that there exists a scalar $0 < \epsilon < 1$, such that

$$\int_0^{t_f} J(\theta)\,\mathrm{d}\theta \ge \omega + \epsilon m^{\mathrm{T}}(t)Pm(t) + (1 - \epsilon)m^{\mathrm{T}}(t - \kappa(t))Pm(t - \kappa(t)).$$

Noting the fact that

$$y^{\mathrm{T}}(t)\Theta_4 y(t) = -\begin{bmatrix} m(t) \\ m(t - \kappa(t)) \end{bmatrix}^{\mathrm{T}} \Pi \begin{bmatrix} m(t) \\ m(t - \kappa(t)) \end{bmatrix} + \epsilon m^{\mathrm{T}}(t)Pm(t) + (1 - \epsilon)m^{\mathrm{T}}(t - \kappa(t))Pm(t - \kappa(t)),$$

for $\Pi > 0$, then

$$y^{\mathrm{T}}(t)\Theta_4 y(t) \le \epsilon m^{\mathrm{T}}(t)Pm(t) + (1 - \epsilon)m^{\mathrm{T}}(t - \kappa(t))Pm(t - \kappa(t)).$$

Obviously, for any $t \ge 0$, $t_f \ge 0$ with $t_f \ge t$

$$\int_0^{t_f} J(\theta)\,\mathrm{d}\theta \ge y^{\mathrm{T}}(t)\Theta_4 y(t) + \omega.$$

Thus, the inequality in Definition 1 holds for any $t_f \ge 0$. As per the above examination, regardless $\| \Theta_4 \| = 0$ or $\| \Theta_4 \| \ne 0$, system (15) with (14) is extended dissipative with the effect of Definition 1.

Since LMI $(17) < 0$, thus there always exists a sufficiently small scalar $\nu > 0$, such that

$$\dot{V}(t) - J(t) \le -\nu\left|\zeta(t)\right|^2 \le -\nu\left|m(t)\right|^2,$$

i.e.,

$$\dot{V}(t) \le J(t) - \nu \left| m(t) \right|^2 .$$

When considering $v(t) = 0$, then $J(t) = y^{\mathrm{T}}(t)\Theta_1 y(t)$. Noticing that $\Theta_1 \le 0$ under Assumption 2.1, it yields that

$$\dot{V}(t) \le -\nu \left| m(t) \right|^2 .$$

This suggests that the consider system (15) is quadratically stable with the effect of Definition 2. □

Remark 1: It ought to be said that Theorem 2 is valid only when the appropriate control gain matrix $\bar{K}_z$ acquired from LMI (17). Taking note of that the inequality (30) in Theorem 1 is not an LMI since, it contains non-linearities like products of the control gain matrix K_z and the matrix P, it has been linearised by a change of variable $\bar{K}_z = K_z\bar{P}$ to convert over the type of LMI (17).

Remark 2: In view of the methodology (6), one can check that, initially, our plan does not only include the information on triggering the error, yet in addition depends upon the most recent refreshed data and current sampling one; besides, the triggering thresholds $\tilde{\beta}(t), \delta(t)$ are not fixed constants but two time-varying functions managed by the adaptive laws in (7) and (8), in which the triggering error, the effectively transmitted information, and the current one are totally analysed. Especially $\tilde{\beta}(t)$ or $\delta(t)$ is a constant, the event-triggered condition in (6) can be reduced to the traditional one.

B. Robust extended dissipative criteria for fuzzy ETC:

With the results of Theorem 1, we establish robust extended dissipative analysis for T–S fuzzy system with parameter uncertainties in the outline.

$$\begin{aligned} \dot{m}(t) = \sum_{i=1}^{r}\sum_{z=1}^{r} w_i(h(t))w_z(h(t))\big[\bar{A}_i m(t) + \bar{A}_{di} m(t-\kappa(t)) \\ +\bar{B}_i[K_z(m(t-\eta_2(t)) - s(t-\eta_2(t)))] + \bar{D}_i v(t)\big] . \end{aligned} \tag{38}$$

The parameter uncertainties in this paper as stated in (3) occurs randomly, which was initially presented in [13].

Assumption 2: The stochastic variables α, ϱ, γ, and σ are mutually independent Bernoulli-distributed white sequences taking the value of zero or one and obey the following probability distribution laws

$$\begin{aligned} &\mathrm{Prob}\{\alpha(t) = 1\} = \alpha, &&\mathrm{Prob}\{\alpha(t) = 0\} = 1 - \alpha \\ &\mathrm{Prob}\{\varrho(t) = 1\} = \varrho, &&\mathrm{Prob}\{\varrho(t) = 0\} = 1 - \varrho \\ &\mathrm{Prob}\{\gamma(t) = 1\} = \gamma, &&\mathrm{Prob}\{\gamma(t) = 0\} = 1 - \gamma \\ &\mathrm{Prob}\{\sigma(t) = 1\} = \sigma, &&\mathrm{Prob}\{\sigma(t) = 0\} = 1 - \sigma \end{aligned}$$

where $\alpha \in [0,1], \varrho \in [0,1], \gamma \in [0,1]$, and $\sigma \in [0,1]$ are known constants.

In the view of Theorem 1, the subsequent Theorem can be obtained for the uncertain T–S fuzzy model:

Theorem 2: Assume that Assumption 1 is hold. At that point for given positive scalars

$\kappa_1, \kappa_2, \mu_1, \mu_2, h, \alpha, \varrho, \gamma, \sigma$, and $0 < \epsilon < 1$, the T–S fuzzy system (38) is extended dissipative in terms of Definition 1, if there exist scalar $\lambda > 0$ and positive definite matrices $P, \bar{Q}_l, l = 1,2,3,4, \bar{S}_k, k = 1,2,3,\ldots,7, \bar{Q}_6, \bar{R}_j, j = 1,2, \bar{\Phi}_1 > 0, \bar{\Phi}_2 > 0, \bar{K}_z$ and $\bar{P}$, such that the following matrix inequalities hold:

$$\Pi = \begin{bmatrix} \epsilon P - \Theta_4 & -\Theta_4 \\ \star & (1-\epsilon)P - \Theta_4 \end{bmatrix} > 0,$$

$$\begin{bmatrix} \Xi_l & \Sigma_{iz} & \Gamma_d & \lambda\Gamma_e^{\mathrm{T}} \\ \Sigma_{iz}^{\mathrm{T}} & \Psi & 0 & 0 \\ * & * & -\lambda I & 0 \\ * & * & * & -\lambda I \end{bmatrix} < 0, \quad l = 1,2,3,4. \tag{39}$$

The remaining elements of Ξ_l, Σ_{iz} and Ψ are the same as in Theorem 1. Furthermore, if the foregoing condition holds, a gain matrix is described by $K_z = \bar{K}_z\bar{P}^{-1}$.

Proof: Replace A_i, A_{di}, B_i, and D_i in (17) with $A_i + \alpha(t)\Delta A_i(t)$, $A_{di} + \varrho(t)\Delta A_{di}(t)$, $B_i + \gamma(t)\Delta B_i(t)$, and $D_i + \sigma(t)\Delta D_i(t)$ mutually. Then, the above close-loop system (38) is similar to the below-stated condition and we obtain the equivalent lines in the proof of Theorem 1.

$$\tilde{\Omega}_{iz} + \Gamma_d F_i(t)\Gamma_e + \Gamma_e^{\mathrm{T}} F_i(t)^{\mathrm{T}} \Gamma_d^{\mathrm{T}} < 0, \tag{40}$$

where

$$\Gamma_d = [PH_{1i}\underbrace{0\ldots0}_{20\,\text{times}}\underbrace{H_{1i}\ldots H_{1i}}_{7\,\text{times}}] \text{ and}$$

$$\Gamma_e = [\alpha E_{1i}\,\beta E_{2i}\,0\,0\,0\,\gamma K_z E_{3i}\underbrace{0\ldots0}_{5\,\text{times}} - \gamma K_z E_{3i}\,\sigma E_{4i}\underbrace{0\ldots0}_{15\,\text{times}}] .$$

Utilising Lemma 1, a necessary and sufficient condition to fulfil the inequality (40) and there exists a scalar $\lambda > 0$, such that

$$\tilde{\Omega}_{iz} + \lambda^{-1}\Gamma_d\Gamma_d^{\mathrm{T}} + \lambda\Gamma_e^{\mathrm{T}}\Gamma_e . \tag{41}$$

By applying Schur complement Lemma in (41), we get

$$\begin{bmatrix} \tilde{\Omega}_{iz} & \Gamma_d & \lambda\Gamma_e^{\mathrm{T}} \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0. \tag{42}$$

Then pre and post multiply (42) by $\mathrm{diag}\{\overbrace{\bar{P}, \ldots, \bar{P}}^{12\,\text{times}}, I, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \overbrace{I, \ldots, I}^{9\,\text{times}}\}$ and its transpose and letting

$$\begin{aligned} &\bar{P} = P^{-1}, \bar{R}_j = \bar{P}\hat{R}_j\bar{P}, \bar{S}_k = \bar{P}S_k\bar{P}, \\ &\bar{Q}_i = \bar{P}Q_f\bar{P}, (f = 1,2,3,4,6), (j = 1,2), (k = 1,2,3,4,5,6,7) \end{aligned} \tag{43}$$

it is straightforward to get the inequality (39) by analysing the above condition. □

Remark 3: Definition 1 implies that the new presentation contains more general solution by set up the weighting matrices Θ_i, i = 1, 2, 3, 4. i.e.

(i) If $\Theta_1 = 0, \Theta_2 = 0, \Theta_3 = \tilde{\gamma}^2 I, \Theta_4 = I$ and $\omega = 0$, then the expression in (15) equivalent to $\mathscr{L}_2 - \mathscr{L}_\infty$ performance.

(ii) If $\Theta_1 = -I, \Theta_2 = 0, \Theta_3 = \tilde{\gamma}^2 I, \Theta_4 = 0$ and $\omega = 0$, then the expression in (15) equivalent to $\mathscr{H}_\infty$ performance.

(iii) If $\Theta_1 = 0, \Theta_2 = 0, \Theta_3 = \tilde{\gamma}^2 I, \Theta_4 = I$ and $\omega = 0$, then the expression in (15) yields passivity performance.

(iv) If $\Theta_1 = \mathscr{Q}, \Theta_2 = \mathscr{S}, \Theta_3 = \mathscr{R} - \tilde{\alpha}I, \Theta_4 = 0$ and $\omega = 0$, then the expression in (15) promote $(\mathscr{Q} - \mathscr{S} - \mathscr{R})-$ dissipative performance.

Thus the extended dissipative performance can provide more design flexibility by choosing the desired parameters.

Remark 4: Recently the considered, adaptive event-triggered scheme (AETS) has attracted a lot of attention and has been widely used in numerous practical systems, since it is easy to design and maintain. However, the other ETS like time-triggered schemes (TTS) etc. may increase the load of the network and waste the limited resource of the network for the reason that it transmits numerous unnecessary data over the network. In addition, a lot of research results have been tackled on the hybrid driven scheme (TTS and the traditional event-triggered scheme (TETS)) in several literature, the threshold of TETS has a great influence on the total number of released data into the network. However, the threshold of TETS is given in advance and difficult to adapt to the variation of the considered system. To overcome this issue mentioned above, we consider the AETM which can be effectively reduce the total number of released data into the network.

Remark 5: Consider the system (15) with absence of T–S fuzzy, uncertainties ($\text{i.e.}\ \Delta A = \Delta A_d = \Delta B = \Delta D = 0$), and there is no extended dissipative. Moreover, for the general system one has accomplished a few results in [8, 10], we get

$$\dot{m}(t) = Am(t) + A_d m(t - \kappa(t)) + Bu(t). \tag{44}$$

As per Theorem 1, the delay-dependent stability of system (44) is shown in the subsequent Corollary 1.

Corollary 1: For a given scalars κ_1, κ_2, μ_1, μ_2 and h, the system (44) is asymptotically stable, if there exist matrices $P > 0,\ \bar{Q}_i > 0, i = 1, 2, 3, 4,\ \hat{R}_1 > 0,\ \hat{R}_2 > 0,\ S_j > 0, j = 1, 2, 3, \ldots,$ 7, $\bar{Q}_6 > 0,\ \bar{\Phi}_1 > 0,\ \bar{\Phi}_2 > 0,\ \bar{K}$ and $\bar{P}$, such that the following LMI hold:

$$\begin{bmatrix} \Xi_j & \Sigma \\ \Sigma^{\mathrm{T}} & \Psi \end{bmatrix} < 0, \quad j = 1, 2, 3, 4, \tag{45}$$

and the non-zero elements in $\Xi_1, \Xi_2, \Xi_3, \Xi_4$ can be listed as

$$\begin{aligned}
\Xi_1 &= \Xi - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{23}\kappa_1^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}, \\
\Xi_2 &= \Xi - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2\tilde{h}^{-1}\bar{R}_2\hbar_{24}^{\mathrm{T}}, \\
\Xi_3 &= \Xi - \hbar_{24}\kappa_1\tilde{h}^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}, \\
\Xi_4 &= \Xi - \hbar_{24}\tilde{h}^{-1}\bar{R}_2\hbar_{24}^{\mathrm{T}}, \text{ and } \Xi = \sum_{\mathrm{j}=1}^{4}\hat{\Theta}_{\mathrm{j}},
\end{aligned}$$

and the other elements are the same as in Theorem 1. Additionally, if the foregoing condition holds, the designed gain matrix is stated by $K = \bar{K}\bar{P}^{-1}$.

Proof: Define a vector $\hat{\xi}^{\mathrm{T}}(t) \in \mathbb{R}^{20n}$ as follows:

$$\begin{aligned}
\hat{\xi}^{\mathrm{T}}(t) = \big[&m^{\mathrm{T}}(t)\, m^{\mathrm{T}}(t - \kappa(t))\, m^{\mathrm{T}}(t - \kappa_1(t))\, m^{\mathrm{T}}(t - \tilde{h}) \\
&\times m^{\mathrm{T}}(t - \kappa_1) \\
&\times m^{\mathrm{T}}(t - \eta_2(t))\, m^{\mathrm{T}}(t - h) \\
&\upsilon_1^{\mathrm{T}}(t)\, \upsilon_2^{\mathrm{T}}(t) \\
&\times \upsilon_3^{\mathrm{T}}(t)\, \upsilon_4^{\mathrm{T}}(t)\, s^{\mathrm{T}}(t - \eta_2(t))\, m^{\mathrm{T}}(t - \kappa_2(t)) \\
&\times m^{\mathrm{T}}(t - \kappa_2)\, \hat{\xi}_1^{\mathrm{T}}(t)\, \hat{\xi}_2^{\mathrm{T}}(t)\big].
\end{aligned}$$

Choose the LKF and engage same procedure in Theorem 1, then the system (44) is asymptotically stable. □

Remark 6: In Corollary 1, from (44), we consider the subsequent STD system

$$\dot{m}(t) = Am(t) + A_d m(t - \kappa(t)). \tag{46}$$

The related result is outlined in the following Corollary 2.

Corollary 2: For a given scalars $\kappa_1, \kappa_2, \mu_1$, and μ_2, the system (46) is asymptotically stable, if there exist matrices $P > 0,\ Q_i > 0, i = 1, 2, 3, 4,\ \hat{R}_1 > 0,\ \hat{R}_2 > 0,\ S_1 > 0,\ S_2 > 0,\ S_3$ and $> 0,\ S_4 > 0,\ S_5 > 0$
$S_6 > 0$, such that the following LMI hold:

$$\begin{bmatrix} \Phi_j & \Gamma_1 \\ * & \Gamma_2 \end{bmatrix} < 0, \quad j = 1, 2, 3, 4, \tag{47}$$

where

$$\begin{aligned}
\Phi_1 &= \Phi - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{23}\kappa_1^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}, \\
\Phi_2 &= \Phi - \hbar_{13}\kappa_1^{-1}\bar{R}_2\hbar_{13}^{\mathrm{T}} - \hbar_{24}\kappa_2\tilde{h}^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}}, \\
\Phi_3 &= \Phi - \hbar_{24}\kappa_1 h^{-2}\bar{R}_2\hbar_{24}^{\mathrm{T}} - \hbar_{23}\kappa_2^{-1}\bar{R}_2\hbar_{23}^{\mathrm{T}}, \\
\Phi_4 &= \Phi - \hbar_{24}h^{-1}\bar{R}_2\hbar_{24}^{\mathrm{T}},\ \Phi = \hat{\Theta}_1 + \hat{\Theta}_2 + \hat{\Theta}_4, \\
\Gamma_1 &= [\sqrt{\kappa_1}\Lambda^{\mathrm{T}}\sqrt{\tilde{h}}\Lambda^{\mathrm{T}}\frac{\kappa_1}{\sqrt{2}}\Lambda^{\mathrm{T}}\frac{\kappa_1}{\sqrt{2}}\Lambda^{\mathrm{T}}\kappa_1\Lambda^{\mathrm{T}}\frac{\tilde{h}}{\sqrt{2}}\Lambda^{\mathrm{T}}], \\
\Gamma_2 &= \mathrm{diag}\{-\hat{R}_1, -\hat{R}_2, -S_4, -S_5, -S_3, -S_6\}, \\
\Lambda^{\mathrm{T}} &= [\bar{P}A_i\,\bar{P}A_{di}\,\overbrace{0\,0\,0}^{11\text{ times}}]^{\mathrm{T}}, \hat{\Theta}_1 = sym\{e_1\bar{P}(A_ie_1^{\mathrm{T}} + A_{di}e_2^{\mathrm{T}})\} \\
&\quad -(1 - \mu_1)e_3Q_1e_3^T + e_1\Big(\sum_{i=1}^{4}Q_i + S_1 + S_2\Big)e_1^T \\
&\quad -e_4Q_2e_4^T - (1 - \mu)e_2Q_3e_2^T - e_5Q_4e_5^T \\
&\quad -(1 - \mu_2)e_6S_1e_6^T - e_7S_2e_7^T, \\
\hat{\Theta}_2 &= -(e_1 - e_3)\kappa_1^{-1}\hat{R}_2(e_1 - e_3)^T - (e_2 - e_4)\tilde{h}^{-1}\hat{R}_2 \\
&\quad \times (e_2 - e_4)^T - (e_2 - e_3)\kappa_2^{-1}\hat{R}_2(e_2 - e_3)^T - (e_1 - e_3) \\
&\quad \times \kappa_1^{-1}\hat{R}_1(e1 - e_3)^T - (e_3 - e_4)\kappa_1^{-1}\hat{R}_1(e_3 - e_4)^T, \\
\hat{\Theta}_4 &= -2(e_5 - e_{12})S_4(e_5 - e_{12})^T - 2(e_1 - e_{12})S_5(e_1 - e_{12})^T \\
&\quad -4(e_5 - 4e_{12} + 6e_{13})S_4(e_5 - 4e_{12} + 6e_{13})^T \\
&\quad -4(e_1 + 2e_{12} - 6e_{13})S_4(e_1 + 2e_{12} - 6e_{13})^T \\
&\quad -4(e_1 + 2e_8 - 6e_{10})S_6(e_1 + 2e_8 - 6e_{10})^T \\
&\quad -2(e_6 - e_9)S_6(e_6 - e_9)^T - 2(e_1 - e_8)S_6(e_1 - e_8)^T \\
&\quad -4(e_6 + 2e_9 - 6e_{11})S_6(e_6 + 2e_9 - 6e_{11})^T \\
&\quad -3(e_1 + e_6 - 2e_8)S_6(e_1 + e_6 - 2e_8)^T \\
&\quad -(e_1 - e_6)S_6(e_1 - e_6)^T - 5(e_1 - e_6 - 6e_8 + 12e_{10}) \\
&\quad \times S_6(e_1 - e_6 - 6e_8 + 12e_{10})^T, \hbar_{13} = [I0 - I\underbrace{0\,0\,0}_{10\text{ times}}]^T, \\
&\quad \hbar_{23}, \text{ and } \hbar_{24} \text{ follow similarly}.
\end{aligned}$$

and corresponding terms are described in Theorem 1.

Proof: Choose $V_1(t)$, $V_2(t)$, $V_4(t)$ remove $V_3(t)$ in (18) and construct

$$\begin{aligned}
\hat{\xi}^{\mathrm{T}}(t) = \big[&m^{\mathrm{T}}(t)\, m^{\mathrm{T}}(t - \kappa(t))\, m^{\mathrm{T}}(t - \kappa_1(t))\, m^{\mathrm{T}}(t - \tilde{h}) \\
&m^{\mathrm{T}}(t - \kappa_1)m^{\mathrm{T}}(t - \kappa_2(t))\, m^{\mathrm{T}}(t - \kappa_2)\, \hat{\xi}_1^{\mathrm{T}}(t)\, \hat{\xi}_2^{\mathrm{T}}(t)\big].
\end{aligned} \tag{48}$$

Utilising the procedures of Theorem 1 and we get the subsequent results.□

Remark 7: In this paper, to tackle stabilisation of T–S fuzzy system, a fuzzy event-triggered control (FETC) method with successive time-varying delay (STVD) has been introduced. Compared with the conventional event-triggered mechanism in the [21–24], the designed control technique and STVD in this paper are more general and more practical. It is clear from the below simulation studies that the modelled controller design method is more effective.

Table 1 MAUBs κ_2 for various values κ_1 in Example 2

κ_2	1	1.5
Theorem 7 by [10]	5.334	—
Theorem 2 by [8]	—	1.1207
Corollary 1	6.0213	1.5712

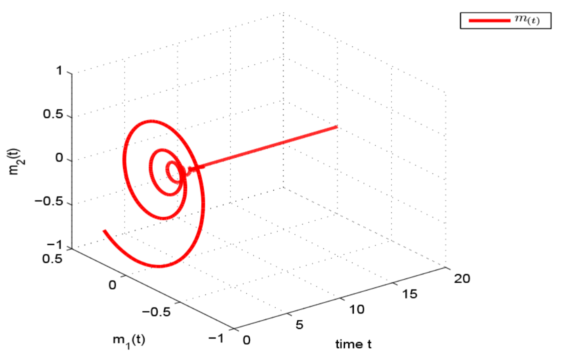

Fig. 1 *Phase trajectories based on $l_2 - l_\infty$ analysis in Example 1*

Remark 8: It is noteworthy that, in many industrial processes, the dynamical behaviours are generally complex and non-linear, and their genuine mathematical models are always difficult to obtain. How to model the robust event-triggered mechanism for the T–S fuzzy framework with respect to additive time-varying delay approach for finding the dissipativity performance has become one of the primary focus in our research work. More especially, some remarkable works have been done in the event-triggered mechanism for T–S fuzzy system by using simple time-varying delay. In [23], robust event-triggered reliable control for T–S fuzzy uncertain systems have been studied with weighted based inequality. Event- triggered control has been designed for T–S fuzzy networked systems with distributed delay method and transmission delay in [24] and network-based H_∞ control for T–S fuzzy systems with an adaptive event-triggered communication scheme has been discussed in [32]. Recently, event-triggered synchronisation control has been proposed in [31] for the T–S fuzzy neural networked systems based on simple time-delay method. The model considered in the present study is more practical than that proposed by [23, 24, 31, 32], in light of the fact that they consider usual ETM has been studied with T–S fuzzy system based on the simple time-varying delay approach, but in this paper, we consider a new adaptive event-triggered mechanism for successive time delay method with the combination of dissipativity performance. In addition, the proposed dissipative analysis is the relation of applied energy to the system with energy started in the system, that is why we analyse ETM this issue in our paper to save the communication resources and have many real-life application, which is another advantage of our paper. Additionally, it is mentioned that we utilising (RII, SAFBII, DAFBII) to estimate the derivative of an LKF such as $\dot{V}_2(t), \dot{V}_4(t)$, which can induce tighter information on the successive time delay of the considered system and can be provided in the numerical example section.

Remark 9: Computational complexity will be a fundamental issue in line with larger LMIs size and more the decision variables. In our LMIs maximum number of decision variables used in Theorems 3.1 and 3.2. Moreover, larger the LMIs size yield better performance. The newly introduced integral techniques used in the construction of proper LKF to derive the results in Theorems, which produces tighter bounds than the existing one like the reciprocally convex approach and so on. Maximum allowable upper bounds κ_2 are less conservative than the existing one in the literature as seen in Table 1. Also, the relaxation of the derived results is obtained at the cost of multiple decision variables. Having, maximum allowable bounds κ_2 obtains the efficient result but to minimise computation complexity burden and time computation we will be using Finsler's Lemma in our future work to reduce the number of decision variables.

4 Simulation results

In this part, in view of the conditions acquired in the previous section, we introduce several simulation studies are represent the adequacy of the suggested control scheme and the merits of our methodology.

Example 1: Consider the subsequent two rule fuzzy system with randomly occurring uncertainties:

$$\dot{m}(t) = \sum_{i=1}^{r} w_i(h(t))\big[\bar{A}_i m(t) + \bar{A}_{di} m(t-\kappa(t)) + \bar{B}_i u(t) + \bar{D}_i v(t)\big] \quad (49)$$

where

$$A_1 = \begin{bmatrix} -0.5 & 0 \\ -1 & -0.2 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0 & 1 \\ -0.5 & 1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.1 & -0.1 \\ 0.2 & -0.1 \end{bmatrix}, D_1 = \begin{bmatrix} 0.04 \\ 0.3 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.3 & 0 \\ -1.5 & -0.5 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0 & 1.5 \\ -0.9 & 1.5 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 & -0.3 \\ 0.4 & -1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.56 \\ -0.25 \end{bmatrix},$$
$$H_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, H_{12} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$\Lambda_1 = \Lambda_2 = \mathrm{diag}(2, 2.5), \chi_1 = \chi_2 = \mathrm{diag}(0.2, 0.3),$$
$$E_{11} = E_{21} = E_{31} = E_{41} = \mathrm{diag}\{0.1, 0.1\},$$
$$E_{12} = E_{22} = E_{32} = E_{42} = \mathrm{diag}\{0.2, 0.2\}.$$

The membership functions for rules 1 and 2 are $w_1(h_1(t)) = 1/\exp(-2h_1(t)), w_2(h_1(t)) = 1 - w_1(h_1(t))$. Assume that $\alpha = 0.6, \varrho = 0.7, \gamma = 0.2$, and $\sigma = 0.1$. Let us pick the constant values are $\kappa_1 = 0.2$, $\kappa_2 = 0.25$, $\mu_1 = 0.1$, and $\mu_2 = 0.15$. Then the corresponding time-delays are taken as $\epsilon = 0.5$, $\kappa_1(t) = 0.1\sin t + 0.1$, $\kappa_2(t) = 0.1 + 0.15\cos t$ with these parameters using MATLAB LMI toolbox, to solve the LMIs in Theorem 2 and we establish the subsequent parts of the extended dissipative conditions for the system (49). Moreover, the extended dissipative condition contains $\mathscr{L}_2 - \mathscr{L}_\infty$ performance, passivity, H_∞ performance, mixed passivity and H_∞ performance as well as $(\mathscr{Q} - \mathscr{S} - \mathscr{R})$-dissipativity as special cases. Along these lines, extended dissipative examination of system (49) is focused here with the weighting matrices $\Theta_1, \Theta_2, \Theta_3$, and Θ_4.

$\mathscr{L}_2 - \mathscr{L}_\infty$ performance: $\Theta_1 = 0$, $\Theta_2 = 0$, $\Theta_3 = \tilde{\gamma}^2 I$, $\Theta_4 = I$, and $\omega = 0$. With the utilisation of the above parameters, then by means of working out the feasibility problem for the LMIS in Theorem 2 and MATLAB LMI control toolbox, we can obtain the following controller gain matrix K_z and triggering parameters as:

$$\bar{\Phi}_1 = 10^3 \times \begin{bmatrix} 1.5628 & 0.0019 \\ 0.0019 & 1.5793 \end{bmatrix}, \bar{\Phi}_2 = \begin{bmatrix} 4.1190 & -0.0028 \\ -0.0028 & 3.6654 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.2630 & 0.0072 \\ 0.0103 & 0.0422 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1352 & 0.0014 \\ 0.0142 & 0.0654 \end{bmatrix}.$$

under the randomised initial conditions, the numerical simulation of state trajectories and using the above-mentioned controller gains the control inputs of the system (49) is shown in Figs. 1 and 6*a*, respectively. Noted from Figs. 1 and 6(a), the responses of the state and control inputs of the system (49) can really keep stable (converges to zero) and behaves $\mathscr{L}_2 - \mathscr{L}_\infty$ performance under the above parameter values, which indicates that the designed controller is effective. Furthermore, the release instants and release intervals are depicted in Fig. 7*a* with $t \in (0, 15]$, which decrease the number of transmissions on the network significantly. Obviously, it is demonstrated in the simulation result that the proposed method is feasible and effective.

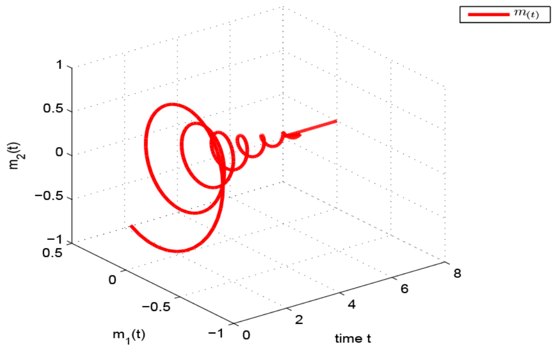

Fig. 2 *Phase trajectories based on H_∞ analysis in Example 1*

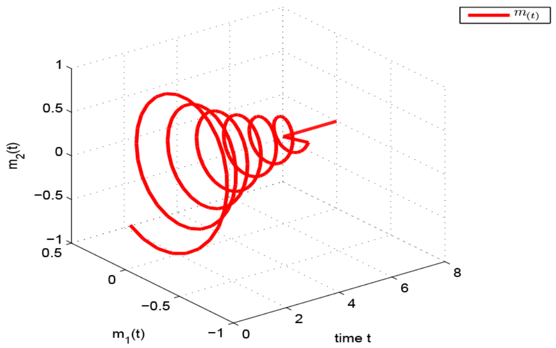

Fig. 3 *Evolution of curves via passivity analysis in Example 1*

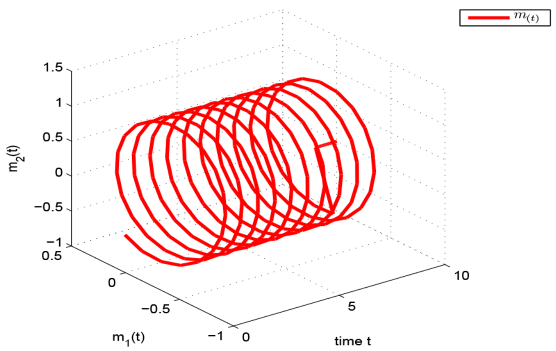

Fig. 4 *Evolution of curves via mixed H_∞ and passivity analysis in Example 1*

Table 2 Allowable minimum $\tilde{\gamma}$ for different κ_2

κ_2	0.2	0.5	0.8	1.0	1.3
$\tilde{\gamma}_{\min}$	0.5631	0.4012	0.3682	0.3106	0.2563

H_∞ performance: $\Theta_1 = -I$, $\Theta_2 = 0$, $\Theta_3 = \tilde{\gamma}^2 I$, $\Theta_4 = 0$, and $\omega = 0$, it can be easily estimated the LMIs stated in Theorem 2, and the results are

$$\bar{\Phi}_1 = 10^3 \times \begin{bmatrix} 1.5810 & 0.0016 \\ 0.0016 & 1.5909 \end{bmatrix}, \bar{\Phi}_2 = \begin{bmatrix} 4.2266 & -0.0024 \\ -0.0024 & 3.7688 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.2581 & 0.0070 \\ 0.0100 & 0.0421 \end{bmatrix}, K_2 = \begin{bmatrix} 0.3487 & 0.0032 \\ 0.0142 & 0.0533 \end{bmatrix}.$$

Simultaneously, relative simulation is drawn to demonstrate the acquired result from Fig. 2, which describes the corresponding evolution of curves of the state responses in respect to the role of control gain matrix, Fig. 6*b* depicts that control inputs of the system (49) and exhibits H_∞ performance under the known parameters. In addition, the release instants and release intervals are depicted in Fig. 7*b* with $t \in (0, 20]$. Based on the system disturbance and delay, the system under consideration well behaves with managed communication resources. Moreover, the calculated allowable minimum $\tilde{\gamma}$ for different κ_2, when $\kappa_1 = 0.2, \mu_1 = 0.1, \mu_2 = 0.15$ is listed in Table 2.

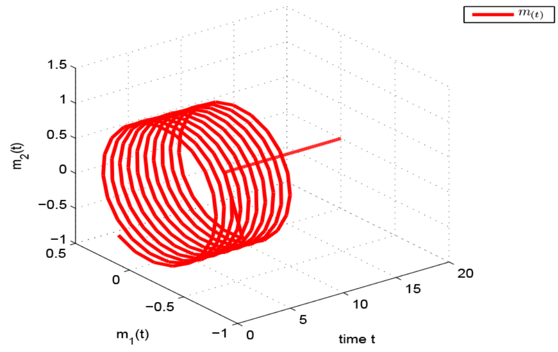

Fig. 5 *Evolution of curves via $(\mathcal{Q} - \mathcal{S} - \mathcal{R})$ Dissipativity analysis in Example 1*

Table 3 Allowable minimum $\tilde{\gamma}$ for different κ_2

κ_2	0.2	0.5	0.8	1.0	1.3
$\tilde{\gamma}_{\min}$	1.6718	1.5321	1.4723	1.2816	1.0981

Passivity performance: $\Theta_1 = 0$, $\Theta_2 = I$, $\Theta_3 = \tilde{\gamma}$, $\Theta_4 = 0$, and $\omega = 0$. Then, the unified system examination turns to the passivity performance. By employing the MATLAB LMI toolbox to verify the LMIs in Theorem 2, we get the following control gain matrix and triggered matrices:

$$\bar{\Phi}_1 = 10^3 \times \begin{bmatrix} 3.1383 & 0.1427 \\ 0.1427 & 3.9973 \end{bmatrix}, \bar{\Phi}_2 = \begin{bmatrix} 0.7732 & -0.0051 \\ -0.0051 & 0.7843 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.1541 & 0.0164 \\ 0.0126 & 0.0315 \end{bmatrix}, K_2 = \begin{bmatrix} 0.4511 & 0.0143 \\ 0.0326 & 0.0431 \end{bmatrix}.$$

Likewise, the corresponding simulation is illustrated to verify the obtained consequence from Figs. 3 and 6*c*. Fig. 3 represents the corresponding state responses under the random initial conditions with the distraction $v(t)$. Fig. 6*c* shows the evolution of the control inputs which converges to zero and both the figures behaves passivity performance with the avail parameters. Fig. 7*c* illustrates the release instants and intervals of the event-triggered scheme with $t \in (0, 30]$. All these results demonstrate that the designed controller and event-triggered mechanism is effective. Moreover, the calculated allowable minimum γ for different κ_2, when $\kappa_1 = 0.2, \mu_1 = 0.1, \mu_2 = 0.15$ is listed in Table 3 under the passivity performance.

Mixed H_∞ and Passivity performance: $\Theta_1 = -\tilde{\gamma}^2 \tilde{\alpha} I$, $\Theta_2 = (1 - \tilde{\alpha})I$, $\Theta_3 = \tilde{\gamma} I$, $\Theta_4 = 0$, and $\omega = 0$. At that point, the extended dissipativity performance decreases to the mixed H_∞ and passivity performance. By using the MATLAB LMI toolbox to solve the LMIs in Theorem 2, we get both control gain matrix and triggered parameters:

$$\bar{\Phi}_1 = \begin{bmatrix} 0.1738 & 0.0001 \\ 0.0147 & 0.0310 \end{bmatrix}, \bar{\Phi}_2 = \begin{bmatrix} 2.5928 & 0.0633 \\ 0.0633 & 2.5953 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 0.0274 & 0 \\ 0 & 0.6811 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0854 & 0.1123 \\ 0.1431 & 0.3267 \end{bmatrix}.$$

Under the initial conditions, $m(0) = [0.3, -0.7]^{\mathrm{T}}$, the dynamical responses of the state trajectories and control input of the system (49) with the above control gain matrix is demonstrates in Figs. 4 and 6*d* and shows mixed H_∞ and passivity performance under the stated parameters. Meanwhile, the event-triggered release instants and the corresponding release intervals are shown in Fig. 7*d* with the period $t \in (0, 50]$, which reveals that the amount of transmitted data is reduced obviously. It can obtained that the designed controller performs well.

$(\mathcal{Q} - \mathcal{S} - \mathcal{R})$ Dissipativity: $\Theta_1 = \mathcal{Q}$, $\Theta_2 = \mathcal{S}$, $\Theta_3 = \mathcal{R} - \tilde{\alpha} I$, and $\Theta_4 = 0$ with

$$\mathscr{Q} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \mathscr{S} = \begin{bmatrix} 0.3 & 0 \\ 0.4 & 0.25 \end{bmatrix}, \mathscr{R} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

Likewise, by applying the LMIs in Theorem 2, using the above parameters found feasible, at that point the following triggered parameters and gain matrix are:

$$\bar{\Phi}_1 = \begin{bmatrix} 5.0808 & 0.0741 \\ 0.0741 & 4.5093 \end{bmatrix}, \bar{\Phi}_2 = \begin{bmatrix} 2.4623 & 0 \\ 0 & 2.1724 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} 2.2494 & 0.0001 \\ 0.0002 & 2.0368 \end{bmatrix}, K_2 = \begin{bmatrix} 3.2156 & 0.0248 \\ 0.0145 & 4.1271 \end{bmatrix},$$

and the dissipativity performance is $\tilde{\alpha} = 0.0072$. Simultaneously, the relative simulation is demonstrated to verify the acquired result from Fig. 5 and Fig. 6*e*. Fig. 5 represents the corresponding state trajectories under the randomised initial condition. In order to explore the simulation results of the controller input $u(t)$, the initial condition is taken as $[-3, 3]^{\mathrm{T}}$, then the control input for the dynamical system (49) has been displayed in Fig. 6*e*. Therefore from the simulation results, we confirm that the state trajectories and control input converges to zero. Moreover, the release instants and release intervals are shown in Fig. 7*e*. In order to determine, if the $(\mathscr{Q} - \mathscr{S} - \mathscr{R})$ dissipativity performance requirement is satisfied, the period is taken as $t \in (0, 70]$, which shows that the required transmission can save limited network resources. From Figs. 5, 6*e* and 7*e*, one can check the event-triggered mechanism cannot just mitigate the issue of resource constraints but also make the data in the transmission process faster and more stable, so the method proposed in this paper is effective. Essentially, the dissipative analysis is the connection of applied energy to the framework with energy stored in the system, that is the reason we investigate this issue in our paper.

Therefore, from Figs. 1–7 do not only validate the stability region of the system (49), yet in addition show the superiorities of our event-triggered mechanism by resorting to release less transmission data.

5 Comparison example

To exhibit the advantage of our technique, we consider system (44) with the subsequent parameters:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 0.9 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1.2 & 0.8 \end{bmatrix},$$
$$\dot{k}_1(t) \le 0.1,\ \dot{k}_2(t) \le 0.8.$$

In this paper, κ_1 and κ_2 represent the delay upper bound of $\kappa_1(t)$ and $\kappa_2(t)$, respectively. We calculated delay bounds for various cases by utilising Corollary 1, and the stability criteria in [8, 10], MAUBs are listed in Table 1. From this Table 1, one can clearly observe that the strategy and procedures (RII, SAFBII, and DAFBII) of this paper can give less conservative results than in [8, 10]. Simulation results are depicted in Figs. 8 and 9 with the initial condition $[-5, 5]^{\mathrm{T}}$ and $[-1, 1]^{\mathrm{T}}$, we can see that all the state responses converges to zero as time mutually increases, which explained that the system is stable. Along these lines, compared with the method mentioned in [8, 10], the method in this paper provides better control performance. In addition, compared with the results of the proposed event-triggered mechanism and the ETS in [29], to show the advantage of our proposed method. By designing $t \in (0, 20]$, Fig. 10 shows the release instants and release intervals under the proposed event-triggered mechanism. Obviously, the transmission trigger times of the two event-triggered mechanisms are listed in Table 4. One can check that the proposed ETS releases less transmitted data than those in [29]. As the state of the system converges to zero, the transmission trigger times is lower than that by the TTS. Thus, it should be pointed out that the proposed event-triggered mechanism can more effectively decrease the number of data transmission during the time intervals.

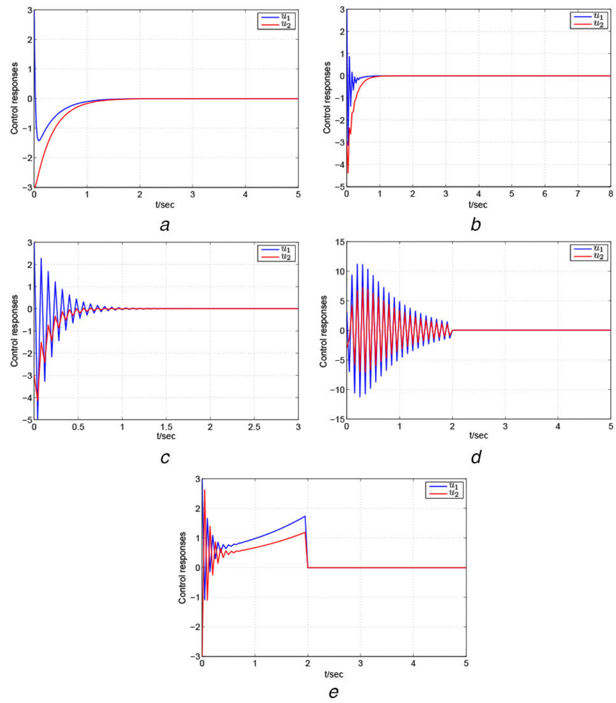

Fig. 6 *Panels (a)-(e) contain the evolution of curves of the control responses in terms of* $\mathscr{L}_2 - \mathscr{L}_\infty$, H_∞, *passivity, mixed* H_∞ *and Passivity performance, and* $(\mathscr{Q} - \mathscr{S} - \mathscr{R})$ *dissipativity analysis in Example 1*

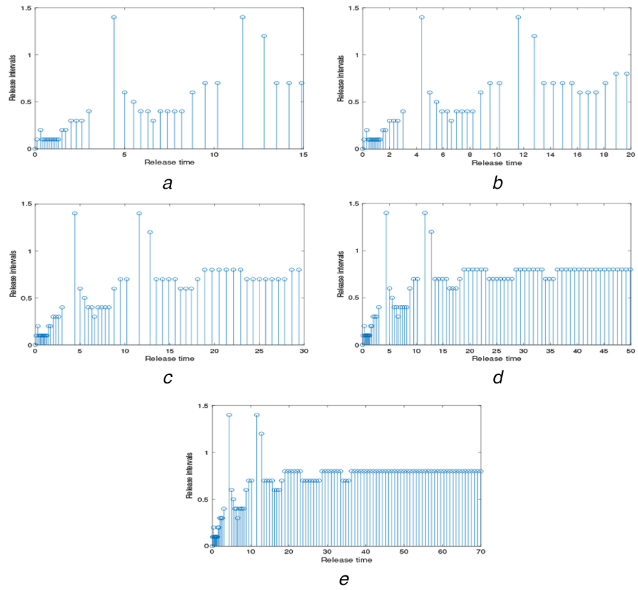

Fig. 7 *Panels (a)–(e) contain the release time and release interval in terms of* $\mathscr{L}_2 - \mathscr{L}_\infty$, H_∞, *passivity, mixed* H_∞ *and Passivity performance, and* $(\mathscr{Q} - \mathscr{S} - \mathscr{R})$ *dissipativity analysis in Example 1*

6 Applications

In this section, single-area load frequency control (SALFC) system to examine the real-world application problem in the sense of STD and simulation responses are verify to demonstrate the performance and less conservativeness of the developed theoretical results.

Example 2: In this example, the proposed delay-dependent stability criterion is approved on SALFC system is explained to demonstrate the effectiveness of the designed methodology, and the notations are listed in Nomenclature section. The maximum delay bounds provided by the delay-dependent stability criterion displayed in Corollary 2 for different delay bound of κ_1 and κ_2 are provided in Tables 5, 6 and compared the existing result with [5].

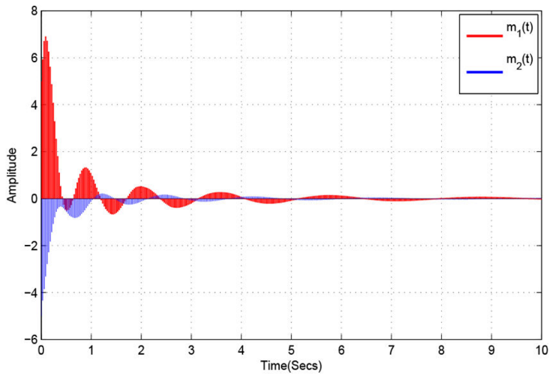

Fig. 8 *For $\kappa_1 = 1$ and $\kappa_2 = 6.0213$, the evolution of system (44) in Corollary 1 with the presence of controller*

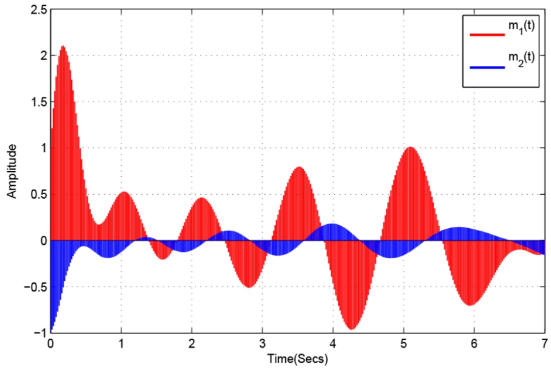

Fig. 9 *For $\kappa_2 = 1.5712$ and $\kappa_1 = 1.5$, the evolution of system (44) in Corollary 1 with the presence of controller*

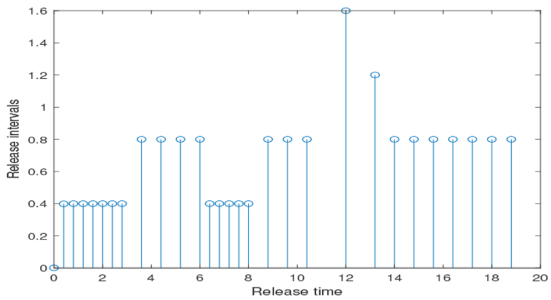

Fig. 10 *Release instant and release interval*

Table 4 Number of transmission trigger times during time intervals

Methods/ Time intervals	[0, 20]
[29]	51
Corollary 1	29

Table 5 MAUBs for κ_2 for given different values of κ_1

κ_1	1.0	1.2	1.5
[5]	4.803	4.603	4.303
Corollary 2	5.413	5.102	4.821

Table 6 MAUBs for κ_1 for given different values of κ_2

κ_2	2.0	3.0	4.0
[5]	3.803	2.803	1.503
Corollary 2	4.287	3.715	3.102

The passionate model of single-area proportional-integral (PI) LFC system with two successive time-delay components, appeared in Fig. 11. Delay-dependent stability analysis and control configuration are examined by using a single delay method [5, 6]. For simplicity, delay existing at the transmission of control signal between the control centre and the plant is aggregated with the delay existing in the transmission of ACE, as shown by an exponential block $e^{-s\kappa_1(t)}$ and $e^{-s\kappa_2(t)}$ in Fig. 11. Based on LFC, the mechanical structure of turbine is shown in Fig. 13.

The open-loop system can be communicated as follows:

$$\dot{\hat{m}} = \hat{A}\hat{m}(t) + \hat{B}\triangle P_c(t), \tag{50}$$

where

$$\hat{m}(t) = \begin{bmatrix} \triangle_f \\ \triangle \hat{P}_m \\ \triangle \hat{P}_v \end{bmatrix}, \hat{A} = \begin{bmatrix} -\frac{\bar{D}}{\bar{M}} & \frac{1}{\bar{M}} & 0 \\ 0 & -\frac{1}{\hat{T}_{ch}} & \frac{1}{\hat{T}_{ch}} \\ -\frac{1}{R\hat{T}_g} & 0 & -\frac{1}{\hat{T}_g} \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} \hat{\theta} & 0 & 0 \end{bmatrix}.$$

Because of no net tie-line power exchange in the one-area LFC scheme, the area control error ACE is defined as

$$ACE = \hat{\theta}\triangle_f,$$

where $\hat{\theta}$ is the frequency bias factor. Using ACE as the input of the load frequency controller, a PI controller is designed as

$$u(t) = -\hat{K}_p ACE - \hat{K}_I \int ACE. \tag{51}$$

Moreover, time-varying delays are represented by $\kappa_1(t)$ and $\kappa_2(t)$, we have

$$\triangle P_c(t) = u(t)(t - \kappa_2(t)),\ ACE(t) = \hat{\theta}\triangle_f(t - \kappa_1(t)). \tag{52}$$

Define the following new virtual state vectors as $m(t) = [\triangle_f, \triangle\hat{P}_m, \triangle\hat{P}_v, \int ACE]^{\mathrm{T}}$. Combining (50)–(52), the closed-loop model of SALFC system can be defined as follows:

$$\dot{m}(t) = Am(t) + A_d m(t - \kappa(t)), \tag{53}$$

where

$$m(t) = \begin{bmatrix} \triangle_f \\ \triangle \hat{P}_m \\ \triangle \hat{P}_v \\ \int ACE \end{bmatrix}, A = \begin{bmatrix} -\frac{\bar{D}}{\bar{M}} & \frac{1}{\bar{M}} & 0 & 0 \\ 0 & -\frac{1}{\hat{T}_{ch}} & \frac{1}{\hat{T}_{ch}} & 0 \\ -\frac{1}{R\hat{T}_g} & 0 & -\frac{1}{\hat{T}_g} & 0 \\ \hat{\theta} & 0 & 0 & 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\hat{K}_p\hat{\theta}}{\hat{T}_g} & 0 & 0 & -\frac{\hat{K}_I}{\hat{T}_g} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\int ACE$ is integration of the area control error. The output $y(t)$ is only a virtual vector and ACE denotes the practical measurement output. The parameters of SALFC are displayed in Table 7. Based on the analysis of system (53) and a similar method of Corollary 2. The maximum bounds of time-delays of the system obtained by different methods and can be listed in Tables 5 and 6. From the tables, most of the results in this paper are less conservative than the results stated in [5], which shows the effectiveness of the

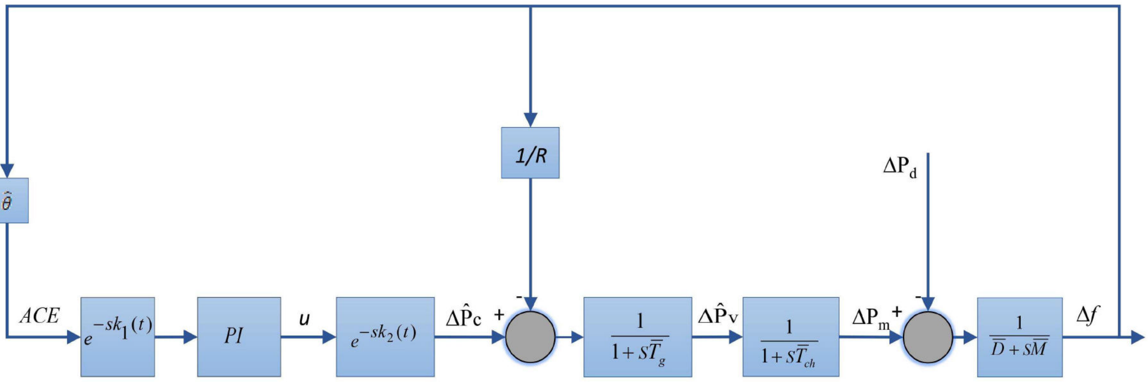

Fig. 11 *Single area LFC scheme with two successive delays*

Table 7 Parameters of SALFC

Parameter	$\hat{T}_{ch}$	$\hat{T}_g$	$\hat{\theta}$	$\bar{M}$	$\bar{D}$	R
Value	0.3	0.1	21	10	1	0.05

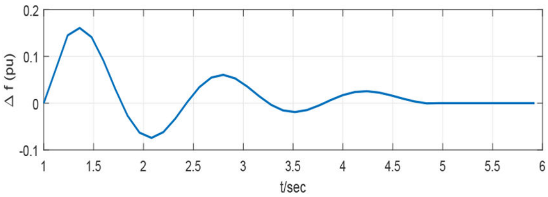

Fig. 12 *Simulation result for $\hat{K}_P = 0.2$, $\hat{K}_I = 0.07$*

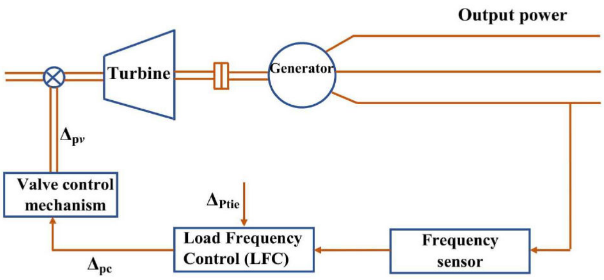

Fig. 13 *Mechanical structure of turbine*

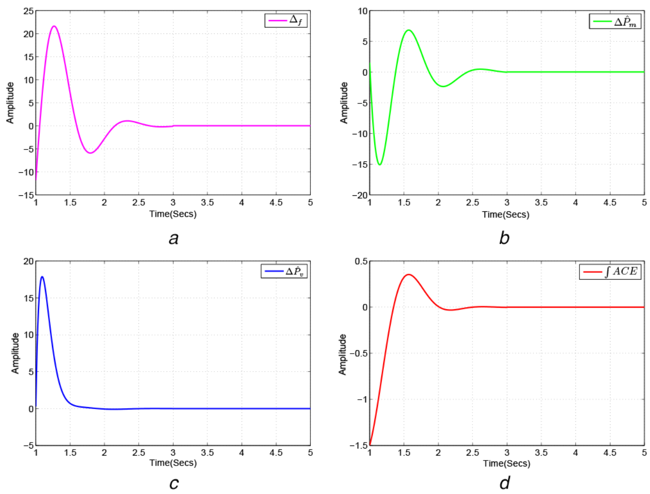

Fig. 14 *State trajectories for system (53) in Example 2*

methods presented in this paper. Likewise, the simulation result is given to check the adequacy of the proposed method. The reply of LFC scheme provided with a PI controller ($\hat{K}_P = 0.2$ and $\hat{K}_I = 0.07$) with step load 0.7 s is appeared in Fig. 12 and the simulation results on the state responses of system (53) are depicted in Fig. 14, in this connection SALFC has accomplished its target and the control system is stable, which confirms the adequacy of the suggested strategy.

7 Conclusion and future directions

In this paper, we have solved the extended dissipativity problem for the T–S fuzzy system with randomly occurring uncertainties and STD components via event-triggered approach. An adaptive event-triggered procedure has been utilised to additionally reduce the number of transmissions over the network. By constructing a proper LKF and handling integral inequality techniques like RII, SAFBII and DAFBII, STD signals, we have established extended dissipativity criteria for the considered T–S fuzzy system. Considering the adaptive event-triggered mechanism, network transmission delays and LKF approach is developed for the subsequent T–S fuzzy system to ensure quadratic stability conditions and derived the controller gains in terms of an arrangement of LMIs, which can be solved by MATLAB toolbox. Finally, simulation studies are stated to verify the potency of the developed technique. Additionally, the proposed work can be also extended adaptive event-triggered mechanism to the coupled system with imperfect communication, for example, packet dropouts and quantisation. We will also focus on the complex phenomena like the state estimation or filtering problem with incomplete measurements, Markovian jump systems under network-induced delays, T–S fuzzy-based piecewise Lyapunov function, sliding mode fault-tolerant control design for finite-time approach and adaptive event-triggered with asynchronous sampling. These research topics make more practical and will be investigated in our future work.

8 Acknowledgments

This work was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1A6A1A03013567) and by the Korea government (MEST) and by Korean Electric Power Corporation (grant no. R18XA04).

9 References

[1] Gu, K., Chen, J., Kharitonov, V.L.: '*Stability of time-delay systems*' (Springer Science & Business Media, Birkhäuser Basel (Switzerland), 2003)

[2] Fridman, E.: '*Introduction to time-delay systems: analysis and control*' (Springer, 2014)

[3] Briat, C.: 'Linear parameter-varying and time-delay systems', *Anal. Obser. Filter Control*, 2014, **3**, pp. 5–7

[4] Lam, J., Gao, H., Wang, C.: 'Stability analysis for continuous systems with two additive time-varying delay components', *Syst. Control Lett.*, 2007, **56**, (1), pp. 16–24

[5] Jiang, L., Yao, W., Wu, Q., *et al.*: 'Delay-dependent stability for load frequency control with constant and time-varying delays', *IEEE Trans. Power Syst.*, 2012, **27**, (2), pp. 932–941

[6] Peng, C., Zhang, J., Yan, H.: 'Adaptive event-triggering H_∞ load frequency control for network-based power systems', *IEEE Trans. Ind. Electron.*, 2018, **65**, (2), pp. 1685–1694

[7] Muthukumar, P., Arunagirinathan, S., Lakshmanan, S.: 'Nonfragile sampled-data control for uncertain networked control systems with additive time-varying delays', *IEEE Trans. Cybern.*, 2019, **49**, (4), pp. 1512–1523

[8] Xiong, L., Li, Y., Zhou, W.: 'Improved stabilization for continuous dynamical systems with two additive time-varying delays', *Asian J. Control*, 2015, **17**, (6), pp. 2229–2240

[9] Xu, H.T., Zhang, C.K., Jiang, L., *et al.*: 'Stability analysis of linear systems with two additive time-varying delays via delay-product-type lyapunov functional', *Appl. Math. Modell.*, 2017, **45**, pp. 955–964

[10] Xiong, L., Yang, F., Chen, X.: 'New stabilization for dynamical system with two additive time-varying delays', *Sci. World J.*, 2014, Article ID 315817, 8 pages

[11] Zhang, C.K., He, Y., Jiang, L., *et al.*: 'Stability analysis of systems with time-varying delay via relaxed integral inequalities', *Syst. Control Lett.*, 2016, **92**, pp. 52–61

[12] Park, P., Lee, W.I., Lee, S.Y.: 'Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems', *J. Franklin Inst.*, 2015, **352**, (4), pp. 1378–1396

[13] Hu, J., Wang, Z., Gao, H., *et al.*: 'Robust sliding mode control for discrete stochastic systems with mixed time delays, randomly occurring uncertainties, and randomly occurring nonlinearities', *IEEE Trans. Ind. Electron.*, 2011, **59**, (7), pp. 3008–3015

[14] Selvi, S., Sakthivel, R., Mathiyalagan, K.: 'Robust $L_2 - L_\infty$ control for uncertain systems with additive delay components', *Circuits Syst. Signal Process.*, 2015, **34**, (9), pp. 2819–2838

[15] Li, J., Hu, M., Guo, L., *et al.*: 'Stability of uncertain impulsive stochastic fuzzy neural networks with two additive time delays in the leakage term', *Neural Comput. Appl.*, 2015, **26**, (2), pp. 417–427

[16] Naamane, K., Tissir, E.H.: 'H_∞ filter design for T–S fuzzy nonlinear quadratic systems with time- varying delay', *Circuits Syst. Signal Process.*, 2019, **38**, (11), pp. 4999–5017

[17] Naamane, K., Chaibi, R., Tissir, E.H., *et al.*: 'Stabilization of discrete-time T–S fuzzy systems with saturating actuators'. 2017 Int. Conf. on Advanced Technologies for Signal and Image Processing, Fez, Morocco, 2017, pp. 1–5

[18] Karthick, S., Sakthivel, R., Ma, Y.K., *et al.*: 'Disturbance rejection of fractional-order T–S fuzzy neural networks based on quantized dynamic output feedback controller', *Appl. Math. Comput.*, 2019, **361**, pp. 846–857

[19] Sathishkumar, M., Sakthivel, R., Kwon, O., *et al.*: 'Finite-time mixed H_∞ and passive filtering for Takagi–Sugeno fuzzy nonhomogeneous markovian jump systems', *Int. J. Syst. Sci.*, 2017, **48**, (7), pp. 1416–1427

[20] Lian, Z., He, Y., Zhang, C.K., *et al.*: 'Robust H_∞ control for T–S fuzzy systems with state and input time-varying delays via delay-product-type functional method', *IEEE Trans. Fuzzy Syst.*, 2019, **27**, (10), pp. 1917–1930

[21] Naamane, K., Tissir, E.H.: 'Improved delay-dependent stability of nonlinear quadratic T–S fuzzy systems', *J. Circuits Syst. Comput.*, 2019, **29**, (9), p. 2050134

[22] Yu, H., Ma, Y., Liu, J.: 'Extended dissipative analysis for uncertain T–S fuzzy system with time-varying delay and randomly occurring gain variations', *J. Franklin Inst.*, 2019, **356**, (15), pp. 8542–8568

[23] Nagamani, G., Joo, Y.H., Soundararajan, G., *et al.*: 'Robust event-triggered reliable control for T–S fuzzy uncertain systems via weighted based inequality', *Inf. Sci.*, 2020, **512**, pp. 31–49

[24] Yan, S., Shen, M., Nguang, S.K., *et al.*: 'A distributed delay method for event-triggered control of T–S fuzzy networked systems with transmission delay', *IEEE Trans. Fuzzy Syst.*, 2019, **27**, (10), pp. 1963–1973

[25] Sathishkumar, M., Liu, Y.C.: 'Hybrid-triggered reliable dissipative control for singular networked cascade control systems with cyber-attacks', *J. Franklin Inst.*, 2020, **357**, (7), pp. 4008–4033

[26] Zeng, D., Wu, K.T., Liu, Y., *et al.*: 'Event-triggered sampling control for exponential synchronization of chaotic lur'e systems with time-varying communication delays', *Nonlinear Dyn.*, 2018, **91**, (2), pp. 905–921

[27] Wu, Y., Jiang, B., Lu, N.: 'A descriptor system approach for estimation of incipient faults with application to high-speed railway traction devices', *IEEE Trans. Syst. Man Cybern., Syst.*, 2019, **49**, (10), pp. 2108–2118

[28] Wu, Y., Jiang, B., Wang, Y.: 'Incipient winding fault detection and diagnosis for squirrel-cage induction motors equipped on CRH trains', *ISA Trans.*, 2020, **99**, pp. 488–495

[29] Li, T., Yuan, R., Fei, S., *et al.*: 'Sampled-data synchronization of chaotic lur'e systems via an adaptive event-triggered approach', *Inf. Sci.*, 2018, **462**, pp. 40–54

[30] Li, Y.X., Yang, G.H.: 'Observer-based fuzzy adaptive event-triggered control codesign for a class of uncertain nonlinear systems', *IEEE Trans. Fuzzy Syst.*, 2017, **26**, (3), pp. 1589–1599

[31] Tan, Y., Liu, Y., Niu, B., *et al.*: 'Event-triggered synchronization control for T–S fuzzy neural networked systems with time delay', *J. Franklin Inst.*, 2020, **357**, pp. 5934–5953

[32] Peng, C., Yang, M., Zhang, J., *et al.*: 'Network-based H_∞ control for T–S fuzzy systems with an adaptive event-triggered communication scheme', *Fuzzy Sets Syst.*, 2017, **329**, pp. 61–76

[33] Lu, A.Y., Zhai, D., Dong, J., *et al.*: 'Network-based fuzzy H_∞ controller design for T–S fuzzy systems via a new event-triggered communication scheme', *Neurocomputing*, 2018, **273**, pp. 403–413

[34] Gnaneswaran, N., Joo, Y.H.: 'Event-triggered stabilisation for T–S fuzzy systems with asynchronous premise constraints and its application to wind turbine system', *IET Control Theory Applic.*, 2019, **13**, (10), pp. 1532–1542

[35] Lozano, R., Brogliato, B., Egeland, O., *et al.*: 'Dissipative systems analysis and control. Theory and applications', *Meas. Sci. Technol.*, 2001, **12**, (12), pp. 2211–2211

[36] Willems, J.C.: 'Dissipative dynamical systems part I: general theory', *Arch. Ration. Mech. Anal.*, 1972, **45**, (5), pp. 321–351

[37] Mathiyalagan, K., Park, J.H., Sakthivel, R.: 'Robust reliable dissipative filtering for networked control systems with sensor failure', *IET Signal Process.*, 2014, **8**, (8), pp. 809–822

[38] Wei, H., Ruoxia, L., Chunrong, C., *et al.*: 'Extended dissipative analysis for memristive neural networks with two additive time-varying delay components', *Neurocomputing*, 2016, **216**, pp. 429–438

[39] Ma, C., Qiao, H.: 'Distributed asynchronous event-triggered consensus of nonlinear multi-agent systems with disturbances: an extended dissipative approach', *Neurocomputing*, 2017, **243**, pp. 103–114

[40] Li, M., Zhao, J., Xia, J., *et al.*: 'Extended dissipative analysis and synthesis for network control systems with an event-triggered scheme', *Neurocomputing*, 2018, **312**, pp. 34–40

[41] Gao, H., Zhang, H., Zheng, D., *et al.*: 'Finite-time event-triggered extended dissipative control for discrete time switched linear systems', *Int. J. Gener. Syst.*, 2019, **48**, (5), pp. 476–491

[42] Wang, J., Xing, M., Sun, Y., *et al.*: 'Event-triggered dissipative state estimation for markov jump neural networks with random uncertainties', *J. Franklin Inst.*, 2019, **356**, (17), pp. 10155–10178