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Abstract: This study is concerned about the stabilization for delayed fuzzy neutral-type system (DFNTS) with uncertain parameters under intermittent control. Firstly, by constructing the augmented Lyapunov-Krasovskii functional (LKF) about different time delays along with single and double auxillary function-based integral inequalities (SAFBII, and DAFBII, respectively), a new class of delay-dependent adequate conditions are proposed, so that the robust fuzzy neutral-type system under consideration is guaranteed to be globally asymptotically stable (GAS). Secondly, the intermittent control (IC) is introduced to stabilize the system with mixed time-varying delays. In the view of inferred adequate conditions, the IC parameters are determined as for the arrangement of linear matrix inequalities (LMIs). It is noted that the strategies exploited in this work are apart from the other methods engaged in the literature, and the proposed conditions are less conservative. Finally, numerical examples are given to demonstrate the effectiveness of the developed techniques in this work. One of the practical applications is single-link robot arm (SLRA) model to show the viability and benefits of the structured intermittent control.

Keywords: Global asymptotic stability, intermittent control, linear matrix inequality, Lyapunov-Krasovskii functional, time-varying delay.

1. INTRODUCTION

It is notable that delayed time-varying systems have gotten a powerful research zone in the earlier years, especially for the time-delayed neutral-type system. This time-delayed system contains time-delays both in its state and in the derivatives of the states. This kind of systems have been referred to as time-delay neutral-type system. Neutral-type systems are frequently encountered in many dynamics, such that automatic control, in-stability, oscillations, stability issue on time-delay systems has become a topic of the great theoretic and practical importance [1–7]. Thus, the research on stability of such neutral system proves to be of great significance [8–11]. For example, in [12], $\mathscr{L}_2-\mathscr{L}_\infty$ filtering was adopted and a delay-dependent stability condition for fuzzy neutral-type stochastic system was proposed. Robust reliable guaranteed cost control was introduced to uncertain fuzzy neutral-type system in [13]. Recently, H_∞ filtering technique has been adopted to T-S fuzzy neutral-type stochastic system in [14]. The authors in [16] discussed the problem of impulsive neutral-type systems based on the exponential stabilization condition. In addition, the parameter uncertainties in consideration of the model errors are unavoidable and lead to stochastic failures and abruption besides developing inconveniences in the usage of models [8,13,14].

On the other hand, fuzzy logic theory has been generally chosen for modelling complex nonlinear systems. Among the various fuzzy logic models, the Takagi-Sugeno (T-S) fuzzy pattern is popular because of its capability to model nonlinear systems as a group of linear subsystems with the assistance of IF-THEN fuzzy rules and fuzzy membership functions [12–15]. In addition, this strategy has been quite specific and well-known for handling the issues of some difficult nonlinear systems. Thus, many scholars in the field of control carry out research on T-S fuzzy systems from various aspects [17–19]. Researchers in [20] studied robust static output feedback H_∞ control for uncertain fuzzy systems. In [21], robust control of electrically driven robots has been discussed for adaptive fuzzy estimation of uncertainty. In [22], authors proposed finite-time L_2-gain asynchronous control for continuous-time positive hidden markov jump systems

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via T-S fuzzy model approach. Quite recently, in [23], the stability and stabilization of T-S fuzzy systems have been studied for two additive time-varying delays. In any case, how to design delay terms in the intermittent control has never been examined thoroughly in previously works, particularly in response to T-S fuzzy neutral-type model approach.

Meanwhile, many control approaches have been proposed to stabilize nonlinear systems such as adaptive control [24], impulsive control [25], and intermittent control [26–28] and so on. Intermittent control, as a discontinuous control method, has been adopted in various engineering applications in view of its convenient implementation in engineering control [29,30]. Recently, an intermittent control scheme with both fixed control period and control width, namely periodically intermittent control scheme, has been developed to study various types of systems as well as dynamical networks. For example, in [31], H_∞ synchronization of uncertain stochastic time-varying delay systems with exogenous disturbance through intermittent control has been investigated. In [32] and [33], the authors designed the aperiodic intermittent pinning control and intermittent control for the synchronization of chaotic systems. Authors in [34] investigated stabilization problem of stochastic uncertain complex-valued delayed networks with intermittent nonlinear control and references cited therein. However, in practical applications, the requirement that both control period and control width be fixed is evidently unreasonable. It is obvious that aperiodically intermittent control is more feasible than the periodically intermittent one. Intermittent control, which was first introduced to control linear econometric models in [26], has been widely used for a variety of purposes such as manufacturing, transportation, and communication in practice. After several decades, numerous studies with respect to intermittent control have been carried out [26,29,31,33]. Moreover, modeling time-varying property in control schemes can procure superior control execution including time-varying delay state-feedback control [35]. It may change the estimation of delay in the controller with the absence of control gain, which leads to extensive consideration for the benefit of control execution in recent years [36–38]. Thus, the results by utilizing intermittent control (IC) method to explore the delayed fuzzy neutral-type system (DFNTS) have not been thoroughly studied at this point of research. This structural model is quite difficult to achieve but of great interests and attention, so that it motivates this study.

In view of previously mentioned discussions, we focus on the stabilization of delayed fuzzy neutral-type systems with uncertain parameters under intermittent control. The core contributions of this study can be summarized as follows:

(Q1) The neutral-type system is studied in terms of the T-S fuzzy model under the consideration of mixed time-varying delays and uncertainties to build up delay-dependent criterion, such that the obtained closed-loop system is GAS.

(Q2) By exploiting the augmented Lyapunov-Krasovskii functional (LKF) technique, adequate conditions for the stability analysis and controller plan of the system under consideration are derived by means of well established LMI approach.

(Q3) The less conservativeness of the proposed stability criteria is achieved by incorporating the latest integral inequality approaches like single and double auxiliary function-based integral inequalities (SAFBII, DAFBII) to deal with the LKF derivatives (double, triple and four integral terms), so that the obtained outcomes are more applicable than the existing results in [8,9]. The number of decision variables (NDVs) in this paper is $17*n(n+1)/2$, which is significantly smaller than that of [9], $35*n(n+1)/2$. In addition, free weighting matrices using the Finsler's lemma are employed to make the proposed technique less conservative.

(Q4) The intermittent controller is designed to stabilize the considered DFNTS. In addition, the proposed stability criteria suggest a connection between mixed delay in the system and time delay in the controller. The obtained conditions can be converted into LMIs, which is verified by MATLAB LMI toolbox.

(Q5) Simulation examples are presented to demonstrate the viability and less conservatism of the proposed approaches. In the application perspective, the obtained theoretical results and organized delayed intermittent controller is validated through the single-link robot arm model system.

This paper is organized as follows: Section 2 describes the T-S fuzzy neutral-type model description and gives theoretical background. The primary results for robust neutral-type systems and the constructed controller are summarized in Section 3. Simulation, application, and comparison are described in Sections 4, 5, and 6, respectively to demonstrate the adequacy and less conservatism of the proposed approaches. Lastly, some conclusions are given in Section 7.

Notation: Let $\mathbb{R}^{n\times q}$ and $\mathbb{R}^n$ represent the set of all $n\times q$ real valued matrices and $n-$ dimensional Euclidean space. The expressions $X>0$ and $X\geq 0$ denotes a positive definite and positive semi-definite matrices X, respectively; the superscripts T and -1 indicate that the transpose and inverse of a matrix. $*$ denotes the elements that are introduced due to corresponding symmetry. Given a matrix G of proper dimensions, $H(G)=G+G^T$. Given any set of matrices $\tilde{N}_i$ and $\tilde{M}_i$ of corresponding dimensions and scalar functions $\tilde{h}_i$, means that $\tilde{M}_h=\sum_r^{i=1}\tilde{h}_i\tilde{M}_i$ and $\tilde{M}_{hh}=\sum_r^{i=1}\sum_r^{j=1}\tilde{h}_i\tilde{h}_j\tilde{M}_{ij}$. Given any $F\subseteq\mathbb{Q}$ and $Z\subseteq\mathbb{Q}^j(1\neq j\neq k)$, set $\mathfrak{C}(F,Z)=\{\phi:F\to Z \text{ is continuous}\}$ and $\mathfrak{C}^1(F,Z)=\{\phi:F\to Z \text{ is continuously differentiable}\}$. The norm $\|\phi\|_\rho=\sup_{s\in[-\rho,0]}\{|\phi(s)|,|\dot{\phi}(s)|\}$ and AMD noted as admissible maximum delay.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the following delayed neutral-type uncertain system, that can be approximated by the following IF-THEN rules.

Plant Rule i:

IF $z_1(t)$ is η_{i1}, $z_2(t)$ is η_{i2} and $\cdots$ and $z_p(t)$ is η_{ip}, **THEN**

$$\begin{cases} \dot{u}(t) - (\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) \\ \quad = (\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ \qquad + (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) + \bar{E}_i w(t), \\ u(t) = \phi(t),\ \forall t \in [-\tau, 0], \end{cases} \tag{1}$$

where $i \in I = \{1, 2, ..., r\}$ denotes the i^{th} fuzzy rule and r is the total number of rules; $\eta_{i1}, ..., \eta_{ip}$ denotes fuzzy sets and $z_1(t), z_2(t), ..., z_p(t)$ indicates premise variables, respectively. $u(t) \in \mathbb{R}^n$ is the state vector and $u(t-\rho(t)) \in \mathbb{R}^n$ is the delayed state vector; $w(t) \in \mathbb{R}^n$ denotes the control input of the system; $\phi(t)$ indicates the initial condition, where $\tau \in \max[\rho_2, \mu_2]$. $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{D}_i$, and $\bar{E}_i$ are the known real valued matrices of proper dimensions. Furthermore, $\triangle\mathcal{A}_i(t)$, $\triangle\mathcal{B}_i(t)$, and $\triangle\mathcal{D}_i(t)$ are time-varying parameter uncertainties and defined as

$$[\triangle\mathcal{A}_i(t),\ \triangle\mathcal{B}_i(t),\ \triangle\mathcal{D}_i(t)] = E_{bi}F_i(t)[\hat{\chi}_{1i},\ \hat{\chi}_{2i},\ \hat{\chi}_{4i}], \tag{2}$$

where E_{bi}, $\hat{\chi}_{1i}$, $\hat{\chi}_{2i}$, and $\hat{\chi}_{4i}$ are known matrices, and the uncertain matrix $F_i(t)$ satisfies

$$F_i^T(t)F_i(t) \le I,\ \ \forall t \ge 0. \tag{3}$$

Denote

$$\hat{\mu}_i(z(t)) = \prod_{j=1}^{p} \eta_{ij}(z_j(t)),$$
$$h_i(z(t)) = \frac{\hat{\mu}_i(z(t))}{\sum_{i=1}^{r}\hat{\mu}_i(z(t))} \ge 0, \tag{4}$$

where $z(t) = [z_1(t),\ ...,\ z_p(t)]^T$ and $\eta_{ij}(z_j(t))$ are the grades of membership $z_j(t)$ in η_{ij}. Then, $\hat{\mu}_i(z(t)) \ge 0$ and $\sum_{i=1}^{r}\hat{\mu}_i(z(t)) \ge 0$. Hence, we have $h_i(z(t)) \ge 0$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$. Moreover, by fuzzy blending, the neutral-type uncertain model can be approximated at a desirable accuracy by

$$\begin{aligned} \dot{u}(t) = &\sum_{i=1}^{r} h_i(z(t))\Big\{(\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ &+ (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) \\ &+ (\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) + \bar{E}_i w(t)\Big\}. \end{aligned} \tag{5}$$

The parametric uncertainties $\triangle\mathcal{A}_i(t)$, $\triangle\mathcal{B}_i(t)$, and $\triangle\mathcal{D}_i(t)$ are said to be admissible if both (2) and (3) hold. The delays $\rho(t)$ and $\mu(t)$ represent the time-varying delay and the neutral delay, respectively, and satisfying $0 \le \rho_1 \le \rho(t) \le \rho_2$, $\dot{\rho}(t) \le \rho$, $\mu_1 \le \mu(t) \le \mu_2$, $\dot{\mu}(t) \le \mu$, $\rho_a = \frac{\rho_2^3 - \rho_1^3}{6}$ and $\rho_b = \frac{\rho_2^2 - \rho_1^2}{2}$. With the presence of initial values in system (5), the following intermittent controller with delay term is defined as

$$w(t) = \begin{cases} \hat{K}u(t-\hbar), & t_k \le t < t_k + d_k, \\ 0, & t \in t_k + d_k \le t < t_{k+1}, \end{cases}$$

where $\hbar > 0$ indicates the constant delay, $\hat{K} \in \mathbb{R}^{m \times n}$ represent the gain matrix of the controller to be computed afterwards, in which d_k and $t_{k+1} - t_k$ denotes control width and control period respectively, which subjected to the following restrictions

$$0 = t_1 < t_2 < \cdots < t_k < \cdots,\ \ \lim_{k\to\infty} t_k = +\infty, k \in \mathbb{Z}^+.$$

At that point, the fuzzy intermittent controller is expressed in the accompanying structure

Controller rule j:

IF $z_1(t)$ is η_{i1}, $z_2(t)$ is η_{i2} and $\cdots$ and $z_p(t)$ is η_{ip}, **THEN**

$$w(t) = \begin{cases} \sum_{j=1}^{r} h_j(z(t))\hat{K}_j u(t-\hbar), & t_k \le t < t_k + d_k, \\ 0, \ \ t \in t_k + d_k \le t < t_{k+1}. \end{cases} \tag{6}$$

When the fuzzy intermittent control (6) is applied to (5), therefore the above system can be rewritten as follows:

$$\begin{cases} \dot{u}(t) = \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_j(z(t))\Big\{(\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ \qquad + (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) + (\mathcal{D}_i + \triangle\mathcal{D}_i(t)) \\ \qquad \times \dot{u}(t-\mu(t)) + \bar{E}_i\hat{K}_j u(t-\hbar)\Big\},\ t_k \le t < t_k + d_k, \\ \dot{u}(t) = \sum_{i=1}^{r} h_i(z(t))\Big\{(\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ \qquad + (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) \\ \qquad + (\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t))\Big\},\ t_k + d_k \le t < t_{k+1}. \end{cases} \tag{7}$$

Before presenting our main results, the following definitions and instrumental lemmas are introduced to verify the stability criteria for the closed-loop system.

Definition 1 [39]**:** DFNTS (7) is said to be globally asymptotically stable (GAS) with respect to Lyapunov, $\lim_{t\to\infty} u(t) = 0$ and $\phi \in \mathfrak{C}([-\tau, 0], \mathbb{R}^n)$.

Definition 2 [28]**:** Given a constants $\varepsilon > 0$ and $\mathfrak{A} > 0$. The DFNTS (7) is called exponentially stable, for any $\|u(t)\| \le \mathfrak{A}\|\phi\|_\tau e^{-\varepsilon t}, \forall\, t \ge 0,\ \phi \in \mathfrak{C}([-\tau, 0], \mathbb{R}^n)$, then the solution $u(t)$ of (7) is satisfied.

Lemma 1 [2]**:** Let $Z>0$ and for given scalars α and β, the following relation is well defined for any differentiable function u in $[\alpha,\beta]\to\mathbb{R}^n$.

$$-\int_{\alpha}^{\beta}\dot{u}^T(s)Z\dot{u}(s)ds\leq-\frac{1}{\beta-\alpha}\varphi_1^TZ\varphi_1-\frac{3}{\beta-\alpha}\varphi_2^TZ\varphi_2,$$
$$-\int_{\alpha}^{\beta}\int_{\lambda}^{\beta}\dot{u}^T(s)Z\dot{u}(s)dsd\lambda\leq-2\varphi_3^TZ\varphi_3-4\varphi_4^TZ\varphi_4,$$
$$-\int_{\alpha}^{\beta}\int_{\alpha}^{\lambda}\dot{u}^T(s)Z\dot{u}(s)dsd\lambda\leq-2\varphi_5^TZ\varphi_5-4\varphi_6^TZ\varphi_6,$$

where

$$\varphi_1=u(\beta)-u(\alpha),$$
$$\varphi_2=u(\beta)+u(\alpha)-\frac{2}{\beta-\alpha}\int_{\alpha}^{\beta}u(s)ds,$$
$$\varphi_3=u(\beta)-\frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}u(s)ds,$$
$$\varphi_5=u(\alpha)-\frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}u(s)ds,$$
$$\varphi_4=u(\beta)+\frac{2}{\beta-\alpha}\int_{\alpha}^{\beta}u(s)ds-\frac{6}{(\beta-\alpha)^2}\int_{\alpha}^{\beta}\int_{\lambda}^{\beta}u(s)dsd\lambda,$$
$$\varphi_6=u(\alpha)-\frac{4}{\beta-\alpha}\int_{\alpha}^{\beta}u(s)ds+\frac{6}{(\beta-\alpha)^2}\int_{\alpha}^{\beta}\int_{\lambda}^{\beta}u(s)dsd\lambda.$$

Lemma 2 [27]**:** Let $\tilde{\xi}\in\mathbb{R}^n$, $\tilde{G}\in\mathbb{R}^{m\times n}$ and $\tilde{Q}=\tilde{Q}^T\in\mathbb{R}^{n\times n}$ such that $rank(G)<n$. Then the following statements are equivalent.

$$\tilde{\xi}^T\tilde{Q}\tilde{\xi}<0,\ \forall\tilde{\xi}\in\{\xi\in\mathbb{R}^n:\tilde{\xi}\neq0,\tilde{G}\tilde{\xi}=0\}$$
$$\exists\tilde{R}\in\mathbb{R}^{n\times m}:\tilde{Q}+H(\tilde{R}\tilde{G})<0.$$

Lemma 3 [3]**:** Let the matrices $\hat{\Sigma}_1,\hat{\Sigma}_2$ and $\hat{\Omega}$ of proper dimensions be constant and $0\leq\rho_1\leq\rho(t)\leq\rho_2$. Then the inequality becomes

$$(\rho_1-\rho(t))\hat{\Sigma}_1+(\rho(t)-\rho_2)\hat{\Sigma}_2+\hat{\Omega}<0,$$

and holds if both the following relations hold:

$$(\rho_2-\rho_1)\hat{\Sigma}_1+\hat{\Omega}<0,$$
$$(\rho_2-\rho_1)\hat{\Sigma}_2+\hat{\Omega}<0.$$

Lemma 4 [17]**:** For $(i,j)\in I_r^2$, let $\hat{\Gamma}_{ij}$ be matrices of appropriate dimensions, $\hat{\Gamma}_{ij}<0$ is satisfied if both the following conditions hold:

$$\hat{\Gamma}_{ii}<0,\ \ \forall i\in\mathcal{I}_r,$$
$$\frac{2}{r-1}\hat{\Gamma}_{ii}+\hat{\Gamma}_{ij}+\hat{\Gamma}_{ji}<0,\ \ \forall(i,j)\in\mathcal{I}_r^2,\ i\neq j.$$

3. MAIN RESULTS

In this section, we will present GAS criteria for DFNTS (7) with intermittent controller. Based on the framework of LKF and the newly improved integral inequality approaches, we give the robust stability conditions in the following Theorems 1 and 2. Moreover, we consider the block entry matrices $\rho_j=[0_{n\times(j-1)n}\ I_{n\times n}\ 0_{n\times(14n-j)n}]^T\in\mathbb{R}^{14n\times n}$, for example: $\rho_5=[0\ 0\ 0\ 0\ I\ \underbrace{0\ 0\ 0}_{9\text{ times}}]$. For the convenience of presentation, we denote

$$\check{\theta}(t)=\Big[u(t)\ \int_{t-\rho_1}^{t}u(s)ds\ \int_{t-\rho_2}^{t-\rho_1}u(s)ds\ \frac{1}{\rho_1}\int_{-\rho_1}^{0}\int_{t+\beta}^{t}u(s)dsd\beta\ \frac{1}{\rho_2-\rho_1}\int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^{t}u(s)dsd\beta\Big]^T,$$

$$\zeta^T(t)=\Big[u(t)\ u(t-\rho_1)\ u(t-\rho(t))\ u(t-\rho_2)\ \frac{1}{\rho_1}\int_{t-\rho_1}^{t}u(s)ds\ \frac{1}{\rho_2-\rho_1}\int_{t-\rho_2}^{t-\rho_1}u(s)ds\ \frac{1}{\rho_1^2}\int_{-\rho_1}^{0}\int_{t+\beta}^{t}u(s)dsd\beta\ \frac{1}{(\rho_2-\rho_1)^2}\int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^{t}u(s)dsd\beta\ \dot{u}(t)\ \dot{u}(t-\mu(t))\ u(t-\hbar)\ \int_{-\rho_2}^{0}\int_{t+w}^{t}u^T(s)dsdw\ \int_{-\rho(t)}^{-\rho_1}\int_{t+w}^{t}u^T(s)dsdw\ \int_{-\rho_2}^{-\rho(t)}\int_{t+w}^{t}u^T(s)dsdw\Big]^T.$$

Theorem 1: For given scalars $\rho>0$, $\rho_1>0$, $\rho_2>0$, $\mu_2>0$, and $\mu>0$, the DFNTS (7) is GAS, if for given constants $\hat{\gamma}\geq\gamma,\gamma>0,\hat{c}\geq0$, there exist positive definite matrices $\mathbb{P}\in\mathbb{R}^{5n\times5n}$, $\mathbb{Q}_1$, $\mathbb{Q}_2$, $\mathbb{Q}_3$, $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{R}_1$, $\mathbb{R}_2$, $\mathbb{Z}_1$, $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{W}_1$, $\mathbb{W}_2$, $\mathbb{W}_3$ $\mathcal{T}_i$, and $\mathcal{N}_{ij}^q\in\mathbb{R}^{14n\times n}$, such that the following LMI conditions hold for both $\ell=1,2$:

$$\begin{cases}\hat{\Gamma}_{ii}^{\ell}<0,\\ \frac{2}{r-1}\hat{\Gamma}_{ii}^{\ell}+\hat{\Gamma}_{ij}^{\ell}+\hat{\Gamma}_{ji}^{\ell}<0,\end{cases}$$
$$\begin{cases}\check{\Gamma}_{ii}^{\ell}<0,\ \ \forall i\in\mathcal{I}_r,\\ \frac{2}{r-1}\check{\Gamma}_{ii}^{\ell}+\check{\Gamma}_{ij}^{\ell}+\check{\Gamma}_{ji}^{\ell}\ \ <0,\ \ \forall(i,j)\in\mathcal{I}_r^2,i\neq j\end{cases}\tag{8}$$

with

$$\hat{\Gamma}_{ij}^{\ell}=\begin{bmatrix}\mathbf{A}_{11}&(\rho_2-\rho_1)\mathcal{N}_{ij}^{\ell}&\Upsilon_1^T&\Upsilon_2^T&\Upsilon_3^T\\ *&-(\rho_2-\rho_1)\mathbb{Q}_3&0&0&0\\ *&*&-\varepsilon_1I&0&0\\ *&*&*&-\varepsilon_2I&0\\ *&*&*&*&-\varepsilon_3\end{bmatrix},$$

$$\check{\Gamma}_{ij}^{\ell}=\begin{bmatrix}\mathbf{B}_{11} & (\rho_2-\rho_1)\mathcal{N}_{ij}^{\ell} & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T\\ * & -(\rho_2-\rho_1)\mathbb{Q}_3 & 0 & 0 & 0\\ * & * & -\varepsilon_1 I & 0 & 0\\ * & * & * & -\varepsilon_2 I & 0\\ * & * & * & * & -\varepsilon_3\end{bmatrix},$$

and satisfy the subsequent relations for control width and period

$$\sup_{k\in\mathbb{Z}^+} t_{k+1}-t_k-d_k\le\hat{c},$$
$$\lim_{k\to\infty}(\hat{\gamma}t_k-(\gamma+\hat{\gamma})\sum_{l=1}^{k-1}d_l)=-\infty,\quad k\in\mathbb{Z}^+, \tag{9}$$

where

$$\mathbf{A}_{11}=\sum_{l=1}^{8}\Phi_l+\mathbf{H}(\mathcal{T}_i\mathcal{G}_i)+\tilde{\Pi},$$
$$\mathbf{B}_{11}=\hat{\Phi}_l+\mathbf{H}(\mathcal{T}_i\mathcal{G}_i)+\tilde{\Pi},$$
$$\mathcal{G}_i=\Big[\mathcal{A}_i\ 0\ \mathcal{B}_i\ 0\ 0\ 0\ 0\ 0\ 0\ \mathcal{D}_i\ \bar{E}_i\hat{K}_j\ 0\ 0\ 0\Big],$$
$$\Phi_1=\mathbf{H}(\Upsilon_1^T\mathbb{P}\Upsilon_2)+\gamma(\Upsilon_1^T\mathbb{P}\Upsilon_1),$$
$$\Upsilon_1=\Big[\rho_1\ \ \rho_1\rho_5\ \ (\rho_2-\rho_1)\rho_6\ \ \rho_1\rho_7\ \ (\rho_2-\rho_1)\rho_8\Big]^T,$$
$$\Upsilon_2=\Big[\rho_9\ \rho_1-\rho_2\ \rho_2-\rho_4\ \rho_1-\rho_5\ \rho_2-\rho_6\Big]^T,$$
$$\begin{aligned}\Phi_2=&\mathbf{H}(\rho_1\mathbb{Z}_1\rho_9^T)+\gamma\rho_1\mathbb{Z}_1\rho_1^T+\rho^{\gamma\rho_1}\rho_1\mathbb{Z}_2\rho_1^T\\&+\gamma\rho_1\mathbb{Z}_2\rho_1^T-\rho_2(\mathbb{Z}_4-\mathbb{Z}_2)\rho_2^T\\&+(1-\rho)e^{\gamma\rho_1}\rho_3(\mathbb{Z}_3-\mathbb{Z}_4)\rho_3^T\\&-e^{\gamma\rho_1}\rho_4\mathbb{Z}_2\rho_4^T+\rho_9\mathbb{Z}_5\rho_9^T-(1-\mu)e^{\gamma\rho}\rho_{10}\mathbb{Z}_5\rho_{10}^T\\&+2\rho_9^T\hat{\mathbb{R}}_1\rho_9+\rho_1^T\mathcal{A}_i\mathbb{R}_1\rho_9+\rho_3^T\mathcal{B}_i\hat{\mathbb{R}}_1\rho_9\\&+\rho_9^T\mathcal{D}_i\hat{\mathbb{R}}_1\rho_{10}+\rho_9^T\tilde{\mathbb{R}}_1\bar{E}_i\hat{K}\rho_{11},\end{aligned}$$
$$\begin{aligned}\Phi_3=&\rho_9e^{\gamma\rho_1}(\rho_1^2\mathbb{Q}_1+(\rho_2-\rho_1)^2\mathbb{Q}_2+(\rho_2-\rho_1)\mathbb{Q}_3)\rho_9^T\\&-e^{\gamma\rho_1}(\rho_1-\rho_2)\mathbb{Q}_1(\rho_1-\rho_2)^T\\&-3e^{\gamma\rho_1}(\rho_1+\rho_2-2\rho_5)\mathbb{Q}_1(\rho_1+\rho_2-2\rho_5)^T\\&-e^{\gamma\rho_{21}}(\rho_2-\rho_4)\mathbb{Q}_2(\rho_2-\rho_4)^T\\&-3(\rho_2+e^{\gamma\rho_{21}}\rho_4-2\rho_6)\mathbb{Q}_2(\rho_2+\rho_4-2\rho_6)^T,\end{aligned}$$
$$\begin{aligned}\Phi_4=&\rho_9e^{\gamma\rho_1}(\frac{\rho_1^2}{2}(\mathbb{S}_1+\mathbb{S}_2))\rho_9^T\\&-2e^{\gamma\rho_1}(\rho_2-\rho_5)\mathbb{S}_1(\rho_2-\rho_5)^T\\&-4e^{\gamma\rho_1}(\rho_2-4\rho_5+6\rho_7)\mathbb{S}_1(\rho_2-4\rho_5+6\rho_7)^T\\&-2e^{\gamma\rho_1}(\rho_1-\rho_5)\mathbb{S}_2(\rho_1-\rho_5)^T\\&-4e^{\gamma\rho_1}(\rho_1+2\rho_5-6\rho_7)\mathbb{S}_2(\rho_1+2\rho_5-6\rho_7)^T,\end{aligned}$$
$$\begin{aligned}\Phi_5=&\rho_9e^{\gamma\rho_1}(\frac{\rho_2-\rho_1}{2}(\mathbb{R}_1+\mathbb{R}_2))\rho_9^T\\&-2e^{\gamma\rho_{21}}(\rho_4-\rho_6)\mathbb{R}_1(\rho_4-\rho_6)^T\\&-4e^{\gamma\rho_{21}}(\rho_4-4\rho_6+6\rho_8)\mathbb{R}_1(\rho_4-4\rho_6+6\rho_8)^T\\&-2e^{\gamma\rho_{21}}(\rho_2-\rho_6)\mathbb{R}_2(\rho_2-\rho_6)^T\\&-4e^{\gamma\rho_{21}}(\rho_2+2e_6-6\rho_8)\mathbb{R}_2(\rho_2+2\rho_6-6\rho_8)^T,\end{aligned}$$
$$\begin{aligned}\Phi_6=&e^{\gamma\rho_1}\rho_1\Big[-\frac{\rho_1^4}{4}\mathbb{W}_1-\frac{\rho_2^4}{4}\mathbb{W}_2-\rho_b^2\mathbb{W}_3-\rho_b^2\mathbb{W}_3\Big]\rho_1^T\\&+e^{\gamma\rho_1}\Big[\rho_1\Big(\frac{\rho_1^2}{2}\mathbb{W}_1\Big)\rho_7^T-\rho_7\mathbb{W}_1\rho_7^T\\&+\rho_1\Big(\frac{\rho_2^2}{2}\mathbb{W}_2\Big)\rho_{12}^T-\rho_{12}\mathbb{W}_2\rho_{12}^T+\rho_1(\rho_b\mathbb{W}_3)\rho_{13}^T\\&-\rho_{13}\mathbb{W}_3\rho_{13}^T+\rho_1(\rho_b\mathbb{W}_3)\rho_{14}^T-\rho_{14}\mathbb{W}_3\rho_{14}^T\Big],\end{aligned}$$
$$\Phi_7=\mathbf{H}(N_{hh}^1(\rho_3^T-\rho_4^T)+N_{hh}^2(\rho_2^T-\rho_3^T)),$$
$$\Phi_8=2\rho_9^T\hat{\mathbb{R}}_1[\rho_9+\mathcal{A}_i\rho_1+\mathcal{B}_i\rho_3+\mathcal{D}_i\rho_{10}+\bar{E}_i\hat{K}\rho_{11}],$$
$$\hat{\Phi}_l=\hat{\Phi}_1+\sum_{i=2}^{7}\Phi_i,$$
$$\hat{\Phi}_1=\mathbf{H}(\Upsilon_1^T\mathbb{P}\Upsilon_2)+\hat{\gamma}(\Upsilon_1^T\mathbb{P}\Upsilon_1),$$
$$\hat{\Phi}_8=2\rho_9^T\hat{\mathbb{R}}_1[\rho_9+\mathcal{A}_i\rho_1+\mathcal{B}_i\rho_3+\mathcal{D}_i\rho_{10}],$$
$$\tilde{\Pi}=\varepsilon_1\tilde{\Gamma}_1^T\tilde{\Gamma}_1+\varepsilon_2\tilde{\Gamma}_2^T\tilde{\Gamma}_2+\varepsilon_3\tilde{\Gamma}_3^T\tilde{\Gamma}_3.$$

Proof: Choose an LKF candidate for system (7) as follows:

$$V(t)=\sum_{r=1}^{6}V_r(t), \tag{10}$$
$$V_1(t)=\check{\theta}^T(t)\mathbb{P}\check{\theta}(t),$$
$$\begin{aligned}V_2(t)&\\=&u^T(t)\mathbb{Z}_1u(t)+e^{\gamma(s-t+\rho_1)}\Big[\int_{t-\rho_1}^{t}u^T(s)\mathbb{Z}_2u(s)ds\\&+\int_{t-\rho_2}^{t-\rho(t)}u^T(s)\mathbb{Z}_3u(s)ds+\int_{t-\rho(t)}^{t-\rho_1}u^T(s)\mathbb{Z}_4u(s)ds\Big]\\&+\int_{t-\mu(t)}^{t}e^{\gamma(s-t)}\dot{u}^T(s)\mathbb{Z}_5\dot{u}(s)ds,\end{aligned}$$
$$\begin{aligned}V_3(t)&\\=&e^{\gamma(s-t+\rho_1)}\Bigg[\rho_1\int_{-\rho_1}^{0}\int_{t+\beta}^{t}\dot{u}^T(s)\mathbb{Q}_1\dot{u}(s)dsd\beta\\&+\int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^{t}\dot{u}^T(s)((\rho_2-\rho_1)\mathbb{Q}_2+\mathbb{Q}_3)\dot{u}(s)dsd\beta\Bigg],\end{aligned}$$
$$\begin{aligned}V_4(t)&\\=&e^{\gamma(s-t+\rho_1)}\Big[\int_{-\rho_1}^{0}\int_{-\rho_1}^{\lambda}\int_{t+\beta}^{t}\dot{u}^T(s)\mathbb{S}_1\dot{u}(s)dsd\beta d\lambda\\&+\int_{-\rho_1}^{0}\int_{\lambda}^{0}\int_{t+\beta}^{t}\dot{u}^T(s)\mathbb{S}_2\dot{u}(s)dsd\beta d\lambda\Big],\end{aligned}$$
$$\begin{aligned}V_5(t)&\\=&e^{\gamma(s-t+\rho_1)}\Big[\int_{-\rho_2}^{-\rho_1}\int_{-\rho_2}^{\lambda}\int_{t+\beta}^{t}\dot{u}^T(s)\mathbb{R}_1\dot{u}(s)dsd\beta d\lambda\\&+\int_{-\rho_2}^{-\rho_1}\int_{\lambda}^{-\rho_1}\int_{t+\beta}^{t}\dot{u}^T(s)\mathbb{R}_2\dot{u}(s)dsd\beta d\lambda\Big],\end{aligned}$$
$$\begin{aligned}V_6(t)&\\=&\frac{\rho_1^3}{6}\int_{-\rho_1}^{0}\int_{w}^{0}\int_{\theta}^{0}\int_{t+\xi}^{t}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{W}_1\dot{u}(s)dsd\xi d\theta dw\end{aligned}$$

$$+\frac{\rho_2^3}{6}\int_{-\rho_2}^{0}\int_{w}^{0}\int_{\theta}^{0}\int_{t+\xi}^{t}e^{\gamma(s-t+\rho_2)}\dot{u}^T(s)\mathbb{W}_2\dot{u}(s)dsd\xi d\theta dw$$
$$+\rho_a\int_{-\rho_2}^{-\rho_1}\int_{w}^{0}\int_{\theta}^{0}\int_{t+\xi}^{t}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{W}_3\dot{u}(s)dsd\xi d\theta dw.$$

The derivatives of $V_l(t)$ with respect to t along the trajectories of system (7), where $l = 1, 2, ..., 6$ yield

$$\dot{V}(t) = \sum_{l=1}^{6}\dot{V}_l(t) < 0,$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2\dot{\theta}(t)\mathbb{P}\theta^T(t) = -\gamma V_1(t) + \gamma\theta^T(t)\mathbb{P}\theta(t)\\
&\quad + 2\dot{\theta}(t)\mathbb{P}\theta^T(t)\\
&= \zeta^T(t)\Phi_1\zeta(t),\\
\dot{V}_2(t) &\leq 2u^T(t)\mathbb{Z}_1\dot{u}(t) + \gamma u^T(t)Z_1u(t) + e^{\gamma\rho_1}u^T(t)\mathbb{Z}_2u(t)\\
&\quad - e^{\gamma\rho_1}u^T(t-\rho_2)\mathbb{Z}_3u(t-\rho_2)\\
&\quad + u^T(t-\rho_1)(\mathbb{Z}_4-\mathbb{Z}_2)u(t-\rho_1)\\
&\quad + (1-\rho)e^{\gamma\rho_1}u^T(t-\rho(t))(\mathbb{Z}_3-\mathbb{Z}_4)u(t-\rho(t))\\
&\quad + \dot{u}^T(t)\mathbb{Z}_5\dot{u}(t)\\
&\quad - (1-\mu_3)e^{\gamma\rho}\dot{u}^T(t-\mu(t))\mathbb{Z}_5\dot{u}(t-\mu(t))\\
&\quad - \gamma V_2(t),\\
&= \zeta^T(t)\Phi_2\zeta(t),\\
\dot{V}_3(t) &= e^{\gamma\rho_1}\dot{u}^T(t)(\rho_1^2\mathbb{Q}_1 + (\rho_2-\rho_1)\mathbb{Q}_2\\
&\quad + (\rho_2-\rho_1)\mathbb{Q}_3)\dot{u}(t)\\
&\quad - e^{\gamma(s-t+\rho_1)}\Big[\rho_1\int_{t-\rho_1}^{t}\dot{u}^T(s)\mathbb{Q}_1\dot{u}(s)ds - (\rho_2-\rho_1)\\
&\quad \times\int_{t-\rho_2}^{t-\rho_1}\dot{u}^T(s)\mathbb{Q}_2\dot{u}(s)ds\\
&\quad - \int_{t-\rho_2}^{t-\rho_1}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds\Big] - \gamma V_3(t).
\end{aligned}\tag{11}$$

From Lemma 1, we have

$$\begin{aligned}
&-\rho_1\int_{t-\rho_1}^{t}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_1\dot{u}(s)ds\\
&\quad - (\rho_2-\rho_1)\int_{t-\rho_2}^{t-\rho_1}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_2\dot{u}(s)ds\\
&\leq -e^{\gamma\rho_1}[(\rho_1-\rho_2)\mathbb{Q}_1(\rho_1-\rho_2)^T\\
&\quad + 3(\rho_1+\rho_2-2\rho_5)\mathbb{Q}_1(\rho_1+\rho_2-2\rho_5)^T]\\
&\quad - e^{\gamma\rho_{21}}[(\rho_2-\rho_4)\mathbb{Q}_2(\rho_2-\rho_4)^T\\
&\quad + 3(\rho_2+\rho_4-2\rho_6)\mathbb{Q}_2(\rho_2+\rho_4-2\rho_6)^T].
\end{aligned}\tag{12}$$

Therefore, from (11)-(12), we get

$$\begin{aligned}
\dot{V}_3(t) &= e^{\gamma\rho_1}\dot{u}^T(t)(\rho_1^2\mathbb{Q}_1 + (\rho_2-\rho_1)\mathbb{Q}_2\\
&\quad + (\rho_2-\rho_1)\mathbb{Q}_3)\dot{u}(t)\\
&\quad - e^{\gamma\rho_1}[(\rho_1-\rho_2)\mathbb{Q}_1(\rho_1-\rho_2)^T\\
&\quad + 3(\rho_1+\rho_2-2\rho_5)\mathbb{Q}_1(\rho_1+\rho_2-2\rho_5)^T]\\
&\quad - e^{\gamma\rho_{21}}[(\rho_2-\rho_4)\mathbb{Q}_2(\rho_2-\rho_4)^T\\
&\quad + 3(\rho_2+\rho_4-2\rho_6)\mathbb{Q}_2(\rho_2+\rho_4-2\rho_6)^T]\\
&\quad + e^{\gamma(s-t+\rho_1)}\int_{t-\rho_2}^{t-\rho_1}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds - \gamma V_3(t),\\
\dot{V}_3(t) &= \zeta^T(t)\Phi_3\zeta(t)\\
&\quad - \int_{t-\rho_2}^{t-\rho_1}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds - \gamma V_3(t),
\end{aligned}\tag{13}$$

$$\begin{aligned}
\dot{V}_4(t) &= \frac{\rho_1^2}{2}e^{\gamma\rho_1}\dot{u}^T(t)(\mathbb{S}_1+\mathbb{S}_2)\dot{u}(t)\\
&\quad - \int_{-\rho_1}^{0}\int_{t-\rho_1}^{\beta}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{S}_1\dot{u}(s)dsd\beta\\
&\quad - \int_{-\rho_1}^{0}\int_{\beta}^{t}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{S}_2\dot{u}(s)dsd\beta - \gamma V_4(t).
\end{aligned}\tag{14}$$

Applying Lemma 1 in (14), we have

$$\begin{aligned}
&\int_{-\rho_1}^{0}\int_{t-\rho_1}^{t+\beta}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{S}_1\dot{u}(s)dsd\beta\\
&\quad + \int_{-\rho_1}^{0}\int_{t+\beta}^{t}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)S_2\dot{u}(s)dsd\beta\\
&\leq 2e^{\gamma\rho_1}(\rho_2-\rho_5)\mathbb{S}_1(\rho_2-\rho_5)^T\\
&\quad + 2e^{\gamma\rho_1}(\rho_1-\rho_5)\mathbb{S}_2(\rho_1-\rho_5)^T\\
&\quad + 4e^{\gamma\rho_1}(\rho_1-4\rho_5+6\rho_7)\mathbb{S}_1(\rho_1-4\rho_5+6\rho_7)^T\\
&\quad + 4e^{\gamma\rho_1}(\rho_1+2\rho_5-6\rho_7)\mathbb{S}_2(\rho_1+2\rho_5-6\rho_7)^T.
\end{aligned}$$

Such that, we get

$$\begin{aligned}
\dot{V}_4(t) &\leq \frac{\rho_1^2}{2}e^{\gamma\rho_1}\dot{u}^T(t)(\mathbb{S}_1+\mathbb{S}_2)\dot{u}(t) - 2e^{\gamma\rho_1}\Big[(\rho_2-\rho_5)\\
&\quad \times\mathbb{S}_1(\rho_2-\rho_5)^T + (\rho_1-\rho_5)\mathbb{S}_2(\rho_1-\rho_5)^T\\
&\quad + 2(\rho_1-4\rho_5+6\rho_7)\mathbb{S}_1(\rho_1-4\rho_5+6\rho_7)^T\\
&\quad + 2(\rho_1+2\rho_5-6\rho_7)\mathbb{S}_2(\rho_1+2\rho_5-6\rho_7)^T\Big]\\
&\quad - \gamma V_4(t),\\
\dot{V}_4(t) &\leq \zeta^T(t)\Phi_4\zeta(t) - \gamma V_4(t),
\end{aligned}\tag{15}$$

$$\begin{aligned}
\dot{V}_5(t) &= \frac{(\rho_2-\rho_1)^2}{2}e^{\gamma\rho_1}\dot{u}^T(t)(\mathbb{R}_1+\mathbb{R}_2)\dot{u}(t)\\
&\quad - \int_{-\rho_2}^{-\rho_1}\int_{t-\rho_2}^{t+\beta}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{R}_1\dot{u}(s)dsd\beta\\
&\quad - \int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^{t-\rho_1}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{R}_2\dot{u}(s)dsd\beta\\
&\quad - \gamma V_5(t).
\end{aligned}\tag{16}$$

Utilizing Lemma 1, it can be verified that

$$\begin{aligned}
&-\int_{-\rho_2}^{-\rho_1}\int_{t-\rho_2}^{t+\beta}e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{R}_1\dot{u}(s)dsd\beta\\
&\leq -2e^{\gamma\rho_{21}}(\rho_4-\rho_6)\mathbb{R}_1(\rho_4-\rho_6)^T\\
&\quad - 4e^{\gamma\rho_{21}}(\rho_4-4\rho_6+6\rho_8)\mathbb{R}_1(\rho_4-4\rho_6+6\rho_8)^T,
\end{aligned}$$

$$
\begin{aligned}
&-\int_{-\rho_2}^{-\rho_1}\int_{t+\beta}^{t-\rho_1} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{R}_2\dot{u}(s)dsd\beta \\
&\leq -2e^{\gamma\rho_{21}}(\rho_2-\rho_6)\mathbb{R}_2(\rho_2-\rho_6)^T \\
&\quad -4e^{\gamma\rho_{21}}(\rho_2+2\rho_6-6\rho_8)\mathbb{R}_2(\rho_2+2\rho_6-6\rho_8)^T.
\end{aligned} \tag{17}
$$

Therefore from (16)-(17), we get

$$
\begin{aligned}
\dot{V}_5(t) \leq & \frac{(\rho_2-\rho_1)^2}{2}e^{\gamma\rho_1}\dot{u}^T(t)(\mathbb{R}_1+\mathbb{R}_2)\dot{u}(t) \\
& -2e^{\gamma\rho_{21}}(\rho_4-\rho_6)\mathbb{R}_1(\rho_4-\rho_6)^T \\
& -4e^{\gamma\rho_{21}}(\rho_4-4\rho_6+6\rho_8)\mathbb{R}_1(\rho_4-4\rho_6+6\rho_8)^T \\
& -2e^{\gamma\rho_{21}}(\rho_2-\rho_6)\mathbb{R}_2(\rho_2-\rho_6)^T \\
& -4e^{\gamma\rho_{21}}(\rho_2+2\rho_6-6\rho_8)\mathbb{R}_2(\rho_2+2\rho_6-6\rho_8)^T \\
& -\gamma V_5(t), \\
\dot{V}_5(t) \leq & \zeta^T(t)\Phi_5\zeta(t)-\gamma V_5(t), \\
\dot{V}_6(t) = & \dot{u}^T(t)e^{\gamma\rho_1}\Big(\Big(\frac{\rho_1^3}{6}\Big)^2\mathbb{W}_1+\Big(\frac{\rho_2^3}{6}\Big)^2\mathbb{W}_2+\rho_a^2\mathbb{W}_3\Big)\dot{u}(t) \\
& -\frac{\rho_1^3}{6}\int_{-\rho_1}^0\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_1)}\dot{u}^T(t)\mathbb{W}_1\dot{u}(t)dsd\theta dw \\
& -\frac{\rho_2^3}{6}\int_{-\rho_2}^0\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_2)}\dot{u}^T(t)\mathbb{W}_2\dot{u}(t)dsd\theta dw \\
& -e^{\gamma(s-t+\rho_1)} \\
& \times[\rho_a\int_{-\rho(t)}^{-\rho_1}\int_w^0\int_{t+\theta}^t \dot{u}^T(t)\mathbb{W}_3\dot{u}(t)dsd\theta dw \\
& +\rho_a\int_{-\rho_2}^{-\rho(t)}\int_w^0\int_{t+\theta}^t \dot{u}^T(t)\mathbb{W}_3\dot{u}(t)dsd\theta dw] \\
& -\gamma V_6(t).
\end{aligned} \tag{18}
$$

Utilizing Jensen's inequality techniques, the above integral term (18) could be rewritten as

$$
\begin{aligned}
&-\frac{\rho_1^3}{6}\int_{-\rho_1}^0\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_1)}\dot{u}^T(t)\mathbb{W}_1\dot{u}(t)dsd\theta dw \\
&\leq -e^{\gamma\rho_1}\left(\frac{\rho_1^2}{2}u(t)-\int_{-\rho_1}^0\int_{t+w}^t u(s)dsdw\right)^T\mathbb{W}_1 \\
&\quad\times\left(\frac{\rho_1^2}{2}u(t)-\int_{-\rho_1}^0\int_{t+w}^t u(s)dsdw\right) \\
&\leq \zeta^T(t)e^{\gamma\rho_1}(\rho_1\left[-\frac{\rho_1^4}{4}\mathbb{W}_1\right]\rho_1^T \\
&\quad +\rho_1\Big(\frac{\rho_1^2}{2}\mathbb{W}_1\Big)\rho_7^T-\rho_7\mathbb{W}_1\rho_7^T)\zeta(t), \\
&-\frac{\rho_2^3}{6}\int_{-\rho_2}^0\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_2)}\dot{u}^T(t)\mathbb{W}_2\dot{u}(t)dsd\theta dw \\
&\leq \zeta^T(t)e^{\gamma\rho_2}(\rho_1\left[-\frac{\rho_2^4}{4}\mathbb{W}_2\right]\rho_1^T+\rho_1\Big(\frac{\rho_2^2}{2}\mathbb{W}_2\Big)\rho_{12}^T \\
&\quad -\rho_{12}\mathbb{W}_2\rho_{12}^T)\zeta(t), \\
&-\rho_a\int_{-\rho(t)}^{-\rho_1}\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_1)}\dot{u}^T(t)\mathbb{W}_3\dot{u}(t)dsd\theta dw \\
&\leq \zeta^T(t)e^{\gamma\rho_1}(\rho_1(-\rho_b^2\mathbb{W}_3)\rho_1^T+\rho_1(\rho_b W_3)\rho_{13}^T \\
&\quad -\rho_{13}\mathbb{W}_3\rho_{13}^T)\zeta(t), \\
&-\rho_a\int_{-\rho_2}^{-\rho(t)}\int_w^0\int_{t+\theta}^t e^{\gamma(s-t+\rho_1)}\dot{u}^T(t)\mathbb{W}_3\dot{u}(t)dsd\theta dw \\
&\leq \zeta^T(t)e^{\gamma\rho_1}(\rho_1(-\rho_b^2\mathbb{W}_3)\rho_1^T+\rho_1(\rho_b\mathbb{W}_3)\rho_{14}^T \\
&\quad -\rho_{14}\mathbb{W}_3\rho_{14}^T)\zeta(t).
\end{aligned}
$$

From (18) one can get

$$
\dot{V}_6(t)=\zeta^T(t)\Phi_6\zeta(t)-\gamma V_6(t). \tag{19}
$$

Combining (11)-(19), we can obtain

$$
\begin{aligned}
&\zeta^T(t)\sum_{i=0}^6\Phi_i\zeta(t) \\
&\quad -\int_{t-\rho_2}^{t-\rho_1} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds-\gamma V(t)<0.
\end{aligned} \tag{20}
$$

Furthermore, $\int_{t-\rho_2}^{t-\rho_1}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds$ can be written as with the additive property of integration

$$
\begin{aligned}
&-\int_{t-\rho_2}^{t-\rho_1} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds \\
&\leq -e^{\gamma(s-t+\rho_1)} \\
&\quad\times\Big[\int_{t-\rho_2}^{t-\rho(t)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds+\int_{t-\rho(t)}^{t-\rho_1}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds\Big].
\end{aligned}
$$

Introduce the following equations:

$$
\begin{aligned}
&u(t-\rho(t))-u(t-\rho_2)-\int_{t-\rho_2}^{t-\rho(t)}\dot{u}(s)ds=0, \\
&u(t-\rho_1)-u(t-\rho(t))-\int_{t-\rho(t)}^{t-\rho_1}\dot{u}(s)ds=0.
\end{aligned}
$$

For appropriate dimensional matrices $\mathcal{N}_{hh}^q\in\mathbb{R}^{14n\times n}$ $(q\in 1,2)$, (20) leads to

$$
\begin{aligned}
&\zeta^T(t)\sum_{i=0}^6\Phi_i\zeta(t)-\int_{t-\rho_2}^{t-\rho(t)} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds \\
&\quad -\int_{t-\rho(t)}^{t-\rho_1} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds \\
&\quad +2\zeta^T\mathcal{N}_{hh}^1\Big(u(t-\rho(t))-u(t-\rho_2)-\int_{t-\rho_2}^{t-\rho(t)}\dot{u}(s)ds\Big) \\
&\quad +2\zeta^T\mathcal{N}_{hh}^2\Big(u(t-\rho_1)-u(t-\rho(t))-\int_{t-\rho(t)}^{t-\rho_1}\dot{u}(s)ds\Big) \\
&<0.
\end{aligned} \tag{21}
$$

For any matrices $\mathbb{Q}_3=\mathbb{Q}_3^T>0$ the following inequalities hold:

$$
-2\zeta^T(t)\mathcal{N}_{hh}^1\int_{t-\rho_2}^{t-\rho(t)}\dot{u}(s)ds)
$$

$$\leq (\rho_2-\rho(t))\zeta^T(t)\mathcal{N}_{hh}^1\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{1T}\zeta(t) + \int_{t-\rho_2}^{t-\rho(t)} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds,$$

$$-2\zeta^T(t)\mathcal{N}_{hh}^2\int_{t-\rho(t)}^{t-\rho_1}\dot{u}(s)ds \leq (\rho(t)-\rho_1)\zeta^T(t)\mathcal{N}_{hh}^2\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{2T}\zeta(t) + \int_{t-\rho(t)}^{t-\rho_1} e^{\gamma(s-t+\rho_1)}\dot{u}^T(s)\mathbb{Q}_3\dot{u}(s)ds. \quad (22)$$

Case I: $\hat{K}_j \neq 0$.

In addition, when $t_k \leq t < t_k+d_k$, we establish the following equation

$$\sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))\{2\dot{u}^T(t)\hat{\mathbb{R}}_1(-\dot{u}(t)+\dot{u}(t))\}=0,$$

$$\sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))\{-2\dot{u}^T(t)\hat{\mathbb{R}}_1\dot{u}(t) + 2\dot{u}^T(t)\hat{\mathbb{R}}_1(\mathcal{A}_i+\triangle\mathcal{A}_i(t))u(t) + 2\dot{u}^T(t)\hat{\mathbb{R}}_1(\mathcal{B}_i+\triangle\mathcal{B}_i(t))u(t-\rho(t)) + 2\dot{u}^T(t)\hat{\mathbb{R}}_1(\mathcal{D}_i+\triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) + 2\dot{u}^T(t)\hat{\mathbb{R}}_1\bar{E}_i\hat{K}_j u(t-\hbar)\}=0. \quad (23)$$

From (21), (22), and (23), the following inequality is written as follows:

$$\dot{V}(t) \leq -\gamma V(t)+\zeta^T(t)\sum_{l=0}^{6}\Phi_l\zeta(t) + \zeta^T(t)\mathbf{H}(\mathcal{N}_{hh}^1(\rho_3^T-\rho_4^T))\zeta(t) + \zeta^T(t)\mathbf{H}(\mathcal{N}_{hh}^2(\rho_2^T-\rho_3^T))\zeta(t) + (\rho_2-\rho(t))\zeta^T(t)\mathcal{N}_{hh}^1 e^{\gamma\rho_1}\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{1\,T}\zeta(t) + (\rho(t)-\rho_1)\zeta^T(t)\mathcal{N}_{hh}^2 e^{\gamma\rho_1}\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{2\,T}\zeta(t) + \sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))2\rho_9^T\hat{\mathbb{R}}_1\big[\rho_9 + \mathcal{A}_i(t)\rho_1+\mathcal{B}_i(t)\rho_3+\mathcal{D}_i(t)\rho_{10}+\bar{E}_i\hat{K}_j\rho_{11}\big]. \quad (24)$$

which implies

$$\dot{V}(t) = \sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))\zeta^T(t)\tilde{\Psi}_{ij}(t)\zeta(t)+\tilde{\Xi} - \gamma V(t). \quad (25)$$

with

$$\tilde{\Psi}_{ij}(t) = \sum_{l=1}^{8}\Phi_l + (\rho_2-\rho(t))\zeta^t(t)\mathcal{N}_{hh}^1\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{1\,T}\zeta(t) + (\rho(t)-\rho_1)\zeta^T(t)\mathcal{N}_{hh}^2\mathbb{Q}_3^{-1}\mathcal{N}_{hh}^{2\,T}\zeta(t),$$

$$\tilde{\Xi} = [\hat{\delta}_1\ \hat{\delta}_2\ \hat{\delta}_3\ \hat{\delta}_4\ \hat{\delta}_5\ \hat{\delta}_6\ \hat{\delta}_7\ \hat{\delta}_8\ \hat{\delta}_9\ \hat{\delta}_{10}\ \hat{\delta}_{11}\ \hat{\delta}_{12}\ \hat{\delta}_{13}\ \hat{\delta}_{14}],$$

$$\hat{\delta}_v = [\overbrace{0\,0\,0}^{14\text{ times}}]^T, v=1,2,...,8,11,12,13,14,$$

$$\hat{\delta}_v = [(\mathbb{R}_1\triangle\mathcal{A}_i(t))^T\ 0\ (\mathbb{R}_1\triangle\mathcal{B}_i(t))^T\ \overbrace{0\,0\,0}^{11\text{ times}}]^T,$$

$$\hat{\delta}_v = [\overbrace{0\,0\,0}^{9\text{ times}}\ (\mathbb{R}_1\triangle\mathcal{D}_i(t))^T\ 0\,0\,0\,0]^T,$$

and Φ_7, Φ_8, as given in (8). From (25), we can obtain

$$\dot{V}(t) = \sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))\zeta^T(t)\tilde{\Psi}_{ij}(t)\zeta(t) + \tilde{\Gamma}_1^T F_i(t)\tilde{\Upsilon}_1+\tilde{\Upsilon}_1^T F_i^T(t)\tilde{\Gamma}_1+\tilde{\Gamma}_2^T F_i(t)\tilde{\Upsilon}_2 + \tilde{\Upsilon}_2^T F_i^T(t)\tilde{\Gamma}_2+\tilde{\Gamma}_3^T F_i(t)\tilde{\Upsilon}_3+\tilde{\Upsilon}_3^T F_i^T(t)\tilde{\Gamma}_3-\gamma V(t) < 0, \quad (26)$$

where

$$\tilde{\Gamma}_1^T = [E_{bi}^T\hat{\mathbb{R}}_1^T\overbrace{0\quad 0\quad 0}^{13\ times}],$$

$$\tilde{\Gamma}_2^T = [0\quad 0\quad E_{bi}^T\hat{\mathbb{R}}_1^T\overbrace{0\quad 0\quad 0}^{11\ times}],$$

$$\tilde{\Gamma}_3^T = [\overbrace{0\,0\,0}^{9\ times}\ E_{bi}\hat{\mathbb{R}}_1\ 0\,0\,0\,0],$$

$$\tilde{\Upsilon}_1 = [\overbrace{0\,0\,0}^{8\ times}\ \hat{\chi}_{1i}\ 0\,0\,0\,0\,0],$$

$$\tilde{\Upsilon}_2 = [\overbrace{0\,0\,0}^{8\ times}\ \hat{\chi}_{2i}\ 0\,0\,0\,0\,0],$$

$$\tilde{\Upsilon}_3 = [\overbrace{0\,0\,0}^{9\ times}\ \hat{\chi}_{3i}\ 0\,0\,0\,0].$$

Moreover, let the neutral type T-S fuzzy model with interval time-varying delays (7) can be written as

$$\mathcal{G}_i\zeta(t)=0. \quad (27)$$

From (26) and (27), considering Lemma 2 and DFNTS (7) together leads to stability condition only if there exists $\mathcal{T}_h \in R^{14n\times n}$ such that

$$\dot{V}(t) = \sum_{i=1}^r h_i(z(t))\sum_{j=1}^r h_j(z(t))\tilde{\Psi}_{ij}(t)+\tilde{\Gamma}_1^T F_i(t)\tilde{\Upsilon}_1 + \tilde{\Upsilon}_1^T F_i^T(t)\tilde{\Gamma}_1+\tilde{\Gamma}_2^T F_i(t)\tilde{\Upsilon}_2+\tilde{\Upsilon}_2^T F_i^T(t)\tilde{\Gamma}_2 + \tilde{\Gamma}_3^T F_i(t)\tilde{\Upsilon}_3+\tilde{\Upsilon}_3^T F_i^T(t)\tilde{\Gamma}_3+\mathbf{H}(\mathcal{T}_i\mathcal{G}_i)-\gamma V(t). \quad (28)$$

Utilizing Lemma 3, the above inequality (28) is satisfied if and only if, $\forall q \in \mathcal{I}_2$

$$\dot{V}(t) = \sum_{l=1}^{8}\Phi_l+\varepsilon_1\tilde{\Gamma}_1^T\tilde{\Gamma}_1+\varepsilon_2\tilde{\Gamma}_2^T\tilde{\Gamma}_2+\varepsilon_3\tilde{\Gamma}_3^T\tilde{\Gamma}_3+\varepsilon_1^{-1}\tilde{\Upsilon}_1^T\tilde{\Upsilon}_1 + \varepsilon_2^{-1}\tilde{\Upsilon}_2^T\tilde{\Upsilon}_2+\varepsilon_3^{-1}\tilde{\Upsilon}_3^T\tilde{\Upsilon}_3+\mathbf{H}(\mathcal{T}_i\mathcal{G}_i) + (\rho_2-\rho_1)\mathcal{N}_{hh}^q\mathbb{Q}_3^{-1}N_{hh}^{qT}-\gamma V(t). \quad (29)$$

Hence, using the Schur complement Lemma, the relation (29) is similar to

$$\dot{V}(t) = \begin{bmatrix} \mathbf{A}_{11} & (\rho_2-\rho_1)\mathcal{N}^q_{hh} & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\mathbb{Q}_3 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix} - \gamma V(t), \tag{30}$$

where $\mathbf{A}_{11} = \sum_{l=1}^{8}\Phi_l + \mathbf{H}(\mathcal{T}_i\mathcal{G}_i) + \tilde{\Pi}$, $\tilde{\Pi} = \varepsilon_1\tilde{\Gamma}_1^T\tilde{\Gamma}_1 + \varepsilon_2\tilde{\Gamma}_2^T\tilde{\Gamma}_2 + \varepsilon_3\tilde{\Gamma}_3^T\tilde{\Gamma}_3$ and utilizing lemma 4, together with (8) yields

$$\dot{V}(t) = -\gamma V(t), t \in [t_k, t_k + d_k). \tag{31}$$

Let $H_1(t) = e^{\gamma t}V(t)$. By (31), one can see that $H_1(t)$ is a monotone decreasing function on $t \in [t_k, t_k + d_k)$. Then,

$$H_1(t) \le H_1(t_k),\ H_1(t_k + d_k) \le H_1(t_k). \tag{32}$$

Thus, we get

$$\begin{aligned} &V(t) \le e^{-\gamma(t-t_k)}V(t_k),\ t \in [t_k, t_k + d_k), \\ &V(t_k + d_k) \le e^{-\gamma d_k}V(t_k). \end{aligned} \tag{33}$$

Case II: $\hat{K}_j = 0$.

When $t_k + d_k \le t < t_{k+1}$, from the second equation of system (7), it follows that

$$\begin{aligned} &2\dot{u}^T(t)\hat{\mathbb{R}}_1(-\dot{u}(t) + \dot{u}(t)) = 0, \\ &-2\dot{u}^T(t)\hat{\mathbb{R}}_1\dot{u}(t) - 2\dot{u}^T(t)\hat{\mathbb{R}}_1(\mathcal{A}_i(t) + \triangle\mathcal{A}_i(t))u(t) \\ &\quad + 2\dot{u}^T(t)\hat{\mathbb{R}}_1(\mathcal{B}_i(t) + \triangle\mathcal{B}_i(t))u(t-\rho(t)) \\ &\quad + 2\dot{u}^T(t)\hat{R}_1(\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) = 0. \end{aligned} \tag{34}$$

Since $\gamma + \hat{\gamma} > 0$, from (8), (21), (22) and (34), we obtain

$$\begin{aligned} \dot{V}(t) &\le -\gamma V(t) + \tilde{\tilde{\Sigma}}_{aa} + (\gamma+\hat{\gamma})u^T(t)\mathbb{P}u(t) \\ &\le -\gamma V(t) + \tilde{\tilde{\Sigma}}_{aa} + (\gamma+\hat{\gamma})V(t) \le \hat{\gamma}V(t). \end{aligned} \tag{35}$$

Here,

$$\tilde{\tilde{\Sigma}}_{aa} = \begin{bmatrix} \mathbf{B}_{11} & (\rho_2-\rho_1)\mathcal{N}^q_{hh} & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\mathbb{Q}_3 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix},$$

$\mathbf{B}_{11} = \hat{\Phi}_l + \mathbf{H}(\mathcal{T}_h\mathcal{G}_h) + \tilde{\Pi}$, $\hat{\Phi}_1 = \mathbf{H}(\Upsilon_1^T\mathbb{P}\Upsilon_2) + \hat{\gamma}(\Upsilon_1^T\mathbb{P}\Upsilon_1)$,

$\hat{\Phi}_l = \sum_{l=2}^{8}\Phi_l$ are defined in $\mathbf{A}_{11}$.

Let $H_2(t) = e^{\hat{\gamma}(t)V(t)}$. Then $H_2(t)$ is a monotone decreasing function on $t \in [t_k + d_k, t_{k+1})$ and we have

$$H_2(t) = H_2(t_k + d_k),\ H_2(t_{k+1}) \le H_2(t_k + d_k), \tag{36}$$

which implies that

$$\begin{aligned} &V(t) \le V(t_k + d_k)e^{\hat{\gamma}(t-t_k-d_k)},\ t \in [t_k + d_k, t_{k+1}), \\ &V(t_{k+1}) \le V(t_k + d_k)e^{\hat{\gamma}(t_{k+1}-t_k-d_k)}. \end{aligned} \tag{37}$$

Thus combining (33) and (37), one may deduce that

$$\begin{aligned} V(t_{k+1}) &\le V(0)e^{-\gamma\sum_{l=1}^{k}d_l + \hat{\gamma}(t_{k+1}-\sum_{l=1}^{k}d_l)} \\ &= V(0)e^{\hat{\gamma}_{t_{k+1}} - (\gamma+\hat{\gamma})\sum_{l=1}^{k}d_l}, \\ V(t_k + d_k) &\le V(0)e^{-\gamma\sum_{l=1}^{k}d_l + \hat{\gamma}(t_k-\sum_{l=1}^{k}d_l)} \\ &\le V(0)e^{\hat{\gamma}_{t_k} - (\gamma+\hat{\gamma})\sum_{l=1}^{k-1}d_l}. \end{aligned} \tag{38}$$

When $t_k \le t \le t_k + d_k$, it follows that from (33) and (38)

$$V(t) \le e^{-\gamma(t-t_k)}V(t_k) \le V(0)e^{\hat{\gamma}_{t_k} - (\gamma+\hat{\gamma})\sum_{l=1}^{k-1}d_l}. \tag{39}$$

When $t_k + d_k \le t < t_{k+1}$, it follows from (9), (37), and (38) that

$$\begin{aligned} V(t) &\le e^{-\gamma(t-t_k)}V(t_k) \le V(0)e^{\hat{\gamma}(t-t_k-d_k)}V(t_k + d_k) \\ &\le e^{|\hat{\gamma}|\hat{c}}V(0)e^{\hat{\gamma}_{t_k} - (\gamma+\hat{\gamma})\sum_{l=1}^{k-1}d_l}. \end{aligned} \tag{40}$$

From (39) and (40), it can be verified that

$$V(t) \le e^{|\hat{\gamma}|\hat{c}}V(0)e^{\hat{\gamma}_{t_k} - (\gamma+\hat{\gamma})\sum_{i=1}^{k-1}d_i},\ k \in \mathbb{Z}^+,\ t \ge 0. \tag{41}$$

By the relation (9), we have $\lim_{t\to\infty}V(t) = 0$, which implies that $\lim_{t\to\infty}u(t) = 0$. In what follows, we will derive the GAS for system (7). Thus the proof is completed. □

Theorem 2: For given scalars $\rho > 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\mu_2 > 0$ and $\mu > 0$, such that the robust DFNTS (7) is GAS, if for given constants τ_k, $k = 1, 2, ..., 17$, $\hat{\gamma} \ge \gamma$, $\gamma > 0$, and $\hat{c} \ge 0$, there exist positive definite matrices S, Z, and $\mathcal{N}^q_{ij} \in \mathbb{R}^{14n\times n}$, such that the following conditions hold, for $\ell = 1, 2$:

$$\begin{cases} \hat{\Gamma}^\ell_{ii} < 0, \\ \dfrac{2}{r-1}\hat{\Gamma}^\ell_{ii} + \hat{\Gamma}^\ell_{ij} + \hat{\Gamma}^\ell_{ji} < 0, \end{cases}$$
$$\begin{cases} \check{\Gamma}^\ell_{ii} < 0,\ \forall\, i \in \mathcal{I}_r, \\ \dfrac{2}{r-1}\check{\Gamma}^\ell_{ii} + \check{\Gamma}^\ell_{ij} + \check{\Gamma}^\ell_{ji} < 0,\ (i,j) \in \mathcal{I}_r^2,\ i \ne j \end{cases} \tag{42}$$

with

$$\hat{\Gamma}^q_{ij} = \begin{bmatrix} \tilde{\Xi}_{11} & (\rho_2-\rho_1)\mathcal{N}^\ell_{ij} & \check{\Upsilon}_1^T & \check{\Upsilon}_2^T & \check{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\tau_3 S & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix},$$

$$\check{\Gamma}^q_{ij} = \begin{bmatrix} \tilde{\tilde{\Xi}}_{11} & (\rho_2-\rho_1)\mathcal{N}^\ell_{ij} & \check{\Upsilon}_1^T & \check{\Upsilon}_2^T & \check{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\tau_3 S & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3 I \end{bmatrix},$$

where

$$\tilde{\Xi}_{11} = \sum_{i=1}^{7}\Phi_i + \mathbf{H}(\tau_{13}S_i\mathcal{G}_i) + \tilde{\Pi}, \tilde{\tilde{\Xi}}_{11} = \hat{\Phi}_l + \mathbf{H}(\tau_{13}S_i\mathcal{G}_i) + \tilde{\Pi},$$

$$\mathcal{G}_i = \Big[\mathcal{A}_i \;\; 0 \;\; \mathcal{B}_i \;\; 0\;0\;0\;0\;0\;0 \;\; \mathcal{D}_i \;\; \bar{E}_iZ_j \;\; 0\;0\;0\Big],$$

$$\Phi_1 = \mathbf{H}(\Upsilon_1^T\mathbb{P}\Upsilon_2) + \gamma(\Upsilon_1^T\mathbb{P}\Upsilon_1),$$

$$\Upsilon_1 = \Big[\rho_1 \;\; \rho_1\rho_5 \;\; (\rho_2-\rho_1)\rho_6 \;\; \rho_1\rho_7 \;\; (\rho_2-\rho_1)\rho_8\Big]^T,$$

$$\breve{\Upsilon}_1 = [\overbrace{0 \;\; 0 \;\; 0}^{8\ times} \;\; S^T\hat{\chi}_{1i} \;\; 0\;\;0\;\;0\;\;0\;\;0],$$

$$\Upsilon_2 = \Big[\rho_9 \;\; \rho_1-\rho_2 \;\; \rho_2-\rho_4 \;\; \rho_1-\rho_5 \;\; \rho_2-\rho_6\Big]^T,$$

$$\breve{\Upsilon}_2 = [\overbrace{0 \;\; 0 \;\; 0}^{8\ times} \;\; S^T\hat{\chi}_{2i} \;\; 0\;\;0\;\;0\;\;0\;\;0],$$

$$\begin{aligned}\Phi_2 =\ & \mathbf{H}(\rho_1\tau_9S\rho_9^T) + \rho_1\tau_{10}S\rho_1^T - \rho_2(\tau_{12}S-\tau_{10}S)\rho_2^T \\ & + (1-\rho)\rho_3(\tau_{11}S-\tau_{12}S)\rho_3^T - \rho_4\tau_{10}S\rho_4^T \\ & + \rho_9\tau_8S\rho_9^T - (1-\mu_3)\rho_{10}\tau_8S\rho_{10}^T + 2\rho_9^T\tau_{14}S\rho_9 \\ & + \rho_1^T\mathcal{A}_i\tau_{14}S\rho_9 + \rho_3^T\mathcal{B}_i\tau_{14}S\rho_9 + \rho_9^T\mathcal{D}_i\tau_{14}S\rho_{10} \\ & + \rho_9^T\tau_{14}\bar{E}_iZ\rho_{11},\end{aligned}$$

$$\begin{aligned}\Phi_3 =\ & \rho_1^2\tau_1S + (\rho_2-\rho_1)^2\tau_2S + (\rho_2-\rho_1)\tau_3S \\ & - (\rho_1-\rho_2)\tau_1S(\rho_1-\rho_2)^T \\ & - 3(\rho_1+\rho_2-2\rho_5)\tau_1S(\rho_1+\rho_2-2\rho_5)^T \\ & - (\rho_2-\rho_4)\tau_2S(\rho_2-\rho_4)^T \\ & - 3(\rho_2+\rho_4-2\rho_6)\tau_2S(\rho_2+\rho_4-2\rho_6)^T,\end{aligned}$$

$$\begin{aligned}\Phi_4 =\ & \frac{\rho_1^2}{2}(\tau_4S+\tau_5S) - 2(\rho_2-\rho_5)\tau_4S(\rho_2-\rho_5)^T \\ & - 4(\rho_2-4\rho_5+6\rho_7)\tau_4S(\rho_2-4\rho_5+6\rho_7)^T \\ & - 2(\rho_1-\rho_5)\tau_5S(\rho_1-\rho_5)^T \\ & - 4(\rho_1+2\rho_5-6\rho_7)\tau_5S(\rho_1+2\rho_5-6\rho_7)^T,\end{aligned}$$

$$\begin{aligned}\Phi_5 =\ & \frac{\rho_2-\rho_1}{2}(\tau_6S+\tau_7S) - 2(\rho_4-\rho_6)\tau_6S(\rho_4-\rho_6)^T \\ & - 4(\rho_4-4\rho_6+6\rho_8)\tau_6S(\rho_4-4\rho_6+6\rho_8)^T \\ & - 2(\rho_2-\rho_6)\tau_7S(\rho_2-\rho_6)^T \\ & - 4(\rho_2+2\rho_6-6\rho_8)\tau_7S(\rho_2+2\rho_6-6\rho_8)^T,\end{aligned}$$

$$\begin{aligned}\Phi_6 =\ & e^{\gamma\rho_1}\rho_1\Big[-\frac{\rho_1^4}{4}\tau_{15}S - \frac{\rho_2^4}{4}\tau_{16}S - \rho_b^2\tau_{17}S - \rho_b^2\tau_{17}S\Big]\rho_1^T \\ & + e^{\gamma\rho_1}\Big[\rho_1\Big(\frac{\rho_1^2}{2}\tau_{15}S\Big)\rho_7^T - \rho_7\tau_{15}S\rho_7^T \\ & + \rho_1\Big(\frac{\rho_2^2}{2}\tau_{16}S\Big)\rho_{12}^T - \rho_{12}\tau_{16}S\rho_{12}^T + \rho_1(\rho_b\tau_{17}S)\rho_{13}^T \\ & - \rho_{13}\tau_{17}S\rho_{13}^T + \rho_1(\rho_b\tau_{17}S)\rho_{14}^T - \rho_{14}\tau_{17}S\rho_{14}^T\Big],\end{aligned}$$

$$\Phi_7 = \mathbf{H}(\mathcal{N}_{hh}^1(\rho_3^T-\rho_4^T) + \mathbb{N}_{hh}^2(\rho_2^T-\rho_3^T)),$$

$$\breve{\Upsilon}_3 = [\overbrace{0 \;\; 0 \;\; 0}^{9\ times} \;\; S^T\hat{\chi}_{3i} \;\; 0\;\;0\;\;0\;\;0],$$

$$\Phi_8 = 2\rho_9^T\tau_{14}[\rho_9 + \mathcal{A}_i\rho_1 + \mathcal{B}_i\rho_3 + \mathcal{D}_i\rho_{10} + \bar{E}_iZ_j\rho_{11}],$$

$$\hat{\Phi}_l = \sum_{i=2}^{7}\Phi_i, \hat{\Phi}_1 = \mathbf{H}(\Upsilon_1^T\mathbb{P}\Upsilon_2) + \hat{\gamma}(\Upsilon_1^T\mathbb{P}\Upsilon_1),$$

$$\hat{\Phi}_8 = 2\rho_9^T\hat{\mathbb{R}}_1[\rho_9 + \mathcal{A}_i\rho_1 + \mathcal{B}_i\rho_3 + \mathcal{D}_i\rho_{10}].$$

Moreover, the gain matrices $\hat{K}_j = Z_jS^{-1}$.

Proof: For $t_k \le t \le t_k + d_k$ and $t_k + d_k \le t < t_{k+1}$, the following conditions hold:

$$\begin{bmatrix}\mathbf{A}_{11} & (\rho_2-\rho_1)\mathcal{N}_{hh}^q & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\mathbb{Q}_3 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3\end{bmatrix} < 0, \quad (43)$$

$$\begin{bmatrix}\mathbf{B}_{11} & (\rho_2-\rho_1)\mathcal{N}_{hh}^q & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)\mathbb{Q}_3 & 0 & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & -\varepsilon_3\end{bmatrix} < 0. \quad (44)$$

Let $\mathbb{Q}_1 = \tau_1\mathbb{P}$, $\mathbb{Q}_2 = \tau_2\mathbb{P}$, $\mathbb{Q}_3 = \tau_3\mathbb{P}$, $\mathbb{S}_1 = \tau_4\mathbb{P}$, $\mathbb{S}_2 = \tau_5\mathbb{P}, \mathbb{R}_1 = \tau_6\mathbb{P}$, $\mathbb{R}_2 = \tau_7\mathbb{P}$, $\mathbb{R}_3 = \tau_8\mathbb{P}$, $\mathbb{Z}_1 = \tau_9\mathbb{P}$, $\mathbb{Z}_2 = \tau_{10}\mathbb{P}$, $\mathbb{Z}_3 = \tau_{11}\mathbb{P}, \mathbb{Z}_4 = \tau_{12}\mathbb{P}$, $\mathbb{T}_i = \tau_{13}\mathbb{P}$, $\hat{\mathbb{R}}_1 = \tau_{14}\mathbb{P}$, $\mathbb{W}_1 = \tau_{15}\mathbb{P}$, $\mathbb{W}_2 = \tau_{16}\mathbb{P}$, $\mathbb{W}_3 = \tau_{17}\mathbb{P}$. Moreover $S = \mathbb{P}^{-1}, Z_j = S\hat{K}_j$. Then performing congruence transformation to (43) and (44) with $diag\{\overbrace{\mathbb{P}^{-1},\mathbb{P}^{-1},\mathbb{P}^{-1}}^{15\ times},I,I,I\}$, we obtain the condition (42), which completes the proof. □

Remark 1: It ought to be noted that the suggested procedure in this article is easy to examine the stability of designed system (7), can be possible to the designed controller is also periodic.

Moreover, the control scheme $w(t)$ can be treated in the following way

$$w(t) = \begin{cases}\sum_{j=1}^{r} h_j(z(t))\hat{K}_ju(t-\hbar), \ k\digamma \le t < k\digamma + \tilde{\delta}, \\ 0, \ t \in k\digamma + \tilde{\delta} \le t < (k+1)\digamma,\end{cases} \quad (45)$$

where $\tilde{\delta}$ and $\digamma > 0$ indicate the width and period in the controller approach, respectively. Therefore, system (7) is described by

$$\begin{cases}\dot{u}(t) = \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_j(z(t))\Big\{(\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ \qquad + (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) \\ \qquad + (\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) \\ \qquad + \bar{E}_i\hat{K}_ju(t-\hbar)\Big\}, \ \ k\digamma \le t < k\digamma + \tilde{\delta},\end{cases}$$

$$\begin{cases} \dot{u}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big\{ (\mathcal{A}_i + \triangle\mathcal{A}_i(t))u(t) \\ \quad + (\mathcal{B}_i + \triangle\mathcal{B}_i(t))u(t-\rho(t)) \\ \quad + (\mathcal{D}_i + \triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t)) \Big\}, \\ \quad k\digamma + \tilde{\delta} \le t < (k+1)\digamma. \end{cases} \tag{46}$$

Corollary 1: For given scalars $\rho > 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\mu_2 > 0$, and μ, such that robust DFNTS (46) is exponentially stable, if for given constants τ_k, $k = 1, 2, ..., 17$, $\hat{\gamma} \ge \gamma$, $\gamma > 0$, and $\hat{c} \ge 0$, there exist positive definite matrices S, Z, and $\mathcal{N}_{ij}^{q} \in \mathbb{R}^{14n\times n}$, such that inequalities (42) hold for both $\ell = 1, 2$ and satisfy both control period and width as

$$(\gamma + \tilde{\gamma})\tilde{\delta} - \tilde{\gamma}\digamma > 0. \tag{47}$$

Proof: When $k\digamma \le t < k\digamma + \tilde{\delta}$, based on (39) and (47)

$$\begin{aligned} V(t) &\le V(0)e^{\beta k\digamma - (\gamma+\tilde{\gamma})(k-1)\tilde{\delta}} \\ &= V(0)e^{(\gamma+\tilde{\gamma})\tilde{\delta} + (\tilde{\gamma}\digamma - (\gamma+\tilde{\gamma})\tilde{\delta})k} \\ &= C_1 V(0)e^{(\tilde{\gamma}\digamma - (\gamma+\tilde{\gamma})\tilde{\delta})\frac{t-\tilde{\gamma}}{\digamma}}, \end{aligned} \tag{48}$$

where $C_1 = e^{(\gamma+\tilde{\gamma})\tilde{\delta}}$. When $k\digamma + \tilde{\delta} \le t < (k+1)\digamma$, based on (40) and (47), we get

$$\begin{aligned} V(t) &\le e^{|\tilde{\gamma}|\hat{c}} V(0)e^{\beta k\digamma - (\gamma+\tilde{\gamma})(k-1)\tilde{\delta}} \\ &= V(0)e^{|\tilde{\gamma}|\hat{c} + (\gamma+\tilde{\gamma})\tilde{\delta} + (\tilde{\gamma}\digamma - (\gamma+\tilde{\gamma})\tilde{\delta})k} \\ &= C_2 V(0)e^{(\tilde{\gamma}\digamma - (\gamma+\tilde{\gamma})\tilde{\delta})\frac{t}{\digamma}}, \end{aligned} \tag{49}$$

where $C_1 = e^{|\tilde{\gamma}|\hat{c} - \tilde{\gamma}\digamma + 2(\gamma+\tilde{\gamma})\tilde{\delta}}$. Together with (48) and (49), it implies that

$$V(t) \le C_2 V(0)e^{-2\varsigma(t-\tilde{\delta})},$$

where $\varsigma = \frac{(\gamma+\tilde{\gamma})\tilde{\delta} + \tilde{\gamma}\digamma}{2\digamma} > 0$. Let

$$\begin{aligned} C_3 =& \lambda_{\max}(\mathbb{P}) + \lambda_{\max}(\mathbb{Z}_1) + \rho_1 e^{2\gamma\rho_1}\lambda_{\max}(\mathbb{Z}_2) \\ &+ \rho_2 e^{\gamma\rho_1} e^{\gamma\rho_2}\lambda_{\max}(\mathbb{Z}_3) + \rho_1 e^{2\gamma\rho_1}\lambda_{\max}(\mathbb{Z}_4) \\ &+ \mu e^{\gamma\mu}\lambda_{\max}(\mathbb{Z}_5) + \rho_1^2 e^{2\gamma\rho_1}\lambda_{\max}(\mathbb{Q}_1) \\ &+ \frac{(\rho_2-\rho_1)^3}{2} e^{\gamma\rho_1} e^{\gamma\rho_{21}}\lambda_{\max}(\mathbb{Q}_{\not{\in}} + \mathbb{Q}_3) \\ &+ \frac{\rho_1^3}{3} e^{2\gamma\rho_1}\lambda_{\max}(\mathbb{S}_1) + \frac{\rho_1^3}{3} e^{\gamma\rho_1}\lambda_{\max}(\mathbb{S}_2) \\ &+ \frac{(\rho_2-\rho_1)^3}{3} e^{\gamma\rho_1} e^{\gamma\rho_{21}}\lambda_{\max}(\mathbb{R}_1) \\ &+ \frac{(\rho_2-\rho_1)^3}{3} e^{\gamma\rho_1} e^{\gamma\rho_{21}}\lambda_{\max}(\mathbb{R}_2) \\ &+ \frac{(\rho_1)^3}{6}\frac{(\rho_1)^4}{4} e^{2\gamma\rho_1}\lambda_{\max}(\mathbb{W}_1) \\ &+ \frac{(\rho_2)^3}{6}\frac{(\rho_2)^4}{4} e^{2\gamma\rho_2}\lambda_{\max}(\mathbb{W}_2) \\ &+ \frac{(\rho_2-\rho_1)^4}{4}\rho_a e^{\gamma\rho_1} e^{\gamma\rho_{21}}\lambda_{\max}(\mathbb{W}_3), \end{aligned}$$

we get

$$V(t) \ge \lambda_{\min}(\mathbb{P})||y(t)||^2, \;\; V(0) \le C_3||\phi||_{\rho}^2.$$

Thus, $||y(t)|| \le C_4||\phi||_{\rho}e^{-\varsigma t}$, where $C_4 = e^{\varsigma\tilde{\delta}}\sqrt{C_2C_3/\lambda_{\min}(\mathbb{P})}$. This completes the proof. □

If there is no control input in (7) and without fuzzy rules, that is to say, $\bar{E} = 0$; then the following Corollary 2 can be obtained easily.

Corollary 2: For given scalars $\rho > 0$, $\rho_1 > 0$, $\rho_2 > 0$, $\mu_2 > 0$ and $\mu > 0$, the neutral-type system (7) without control input is GAS, if for given constants $\hat{\gamma} \ge \gamma$, $\gamma > 0$, and $\hat{c} \ge 0$, there exist real positive definite matrices $\mathbb{P} \in \mathbb{R}^{5n\times 5n}$, $\mathbb{Q}_1$, $\mathbb{Q}_2$, $\mathbb{Q}_3$, $\mathbb{S}_1$, $\mathbb{S}_2$, $\mathbb{R}_1$, $\mathbb{R}_2$, $\mathbb{Z}_1$, $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{T}$, and $\mathcal{N} \in \mathbb{R}^{9n\times n}$, such that the subsequent inequalities are satisfied

$$\begin{bmatrix} \mathbf{A}_{11} & (\rho_2-\rho_1)\mathcal{N} & \tilde{\Upsilon}_1^T & \tilde{\Upsilon}_2^T & \tilde{\Upsilon}_3^T \\ * & -(\rho_2-\rho_1)Q_3 & 0 & 0 & 0 \\ * & * & \varepsilon_1 I & 0 & 0 \\ * & * & * & \varepsilon_2 I & 0 \\ * & * & * & * & \varepsilon_3 \end{bmatrix} < 0. \tag{50}$$

Proof: Let $\bar{E} = 0$ in Theorem 1, we get the asymptotic stability criterion for system (7). The proof is similar to that of Theorem 1 and is omitted for brevity. □

Remark 2: The intermittent controller configuration is formulated by LMIs in Theorem 2 and these conditions might be solved proficiently via MATLAB Toolbox. Subsequently, the mode-dependent fuzzy intermittent controller is simple to be constructed as in Theorem 2.

Remark 3: Compared with the earlier works [30], [32], and [34], whose work/rest time is periodic, it is adopted to achieve the objective of stability. However, the limitation of periodicity is very short so that the feasibility is limited in the real life problem. In this paper, we focus on the delayed intermittent control with period for the stability of the T-S fuzzy system. Therefore, the analysis technique and system model proposed in this paper is more general than [30,32,34]

Remark 4: The computational complexity is a primary issue that is subject to how large the LMIs are and how many decision variables are. However, LMIs of large size yield better performance. The results in Theorems are derived by constructing proper L-K functionals with double, triple, and four integral terms, and using a newly introduced integral inequality technique which produces tighter bounds than the existing ones including reciprocally convex approach and etc. In this paper, the proposed criteria employs several integral inequalities; as a result, some degree of high computational complexity can occur in the proposed criterion. It is noted that the obtained maximum allowable bounds ρ_2 are less conservative than the existing ones in the literature, as shown in Table 4 and Table 5. Finsler's lemma was applied in the proof of the main results, which in turn to reduce the computational burden. Moreover, in the future work we will focus on lower computational complexity of the stability problems while maintaining the desired system performances.

4. SIMULATION RESULTS

In the view of the conditions acquired in the previous section, we present some simulation examples to demonstrate the adequacy of the proposed control scheme and the merits of our approach in this section.

Example 1: Consider the following T-S fuzzy uncertain neutral-type system with time-varying delays

$$\dot{u}(t)=\sum_{i=1}^{r}h_i(z(t))\Big\{(\mathcal{A}_i+\triangle\mathcal{A}_i(t))u(t)+(\mathcal{B}_i+\triangle\mathcal{B}_i(t))g(u(t-\rho(t)))+(\mathcal{D}_i+\triangle\mathcal{D}_i(t))\dot{u}(t-\mu(t))+\bar{E}_i w(t)\Big\},$$

with two IF-THEN rules ($r=2$) and the following parameters:

$$\mathcal{A}_1=\begin{bmatrix}-1 & -3.1321\\ 1 & -2\end{bmatrix},\ \mathcal{A}_2=\begin{bmatrix}-3.6 & -1.1321\\ 1 & -2.8\end{bmatrix},$$
$$\mathcal{B}_1=\begin{bmatrix}0.2 & -0.4\\ 0.5 & 1.1\end{bmatrix},\ \mathcal{B}_2=\begin{bmatrix}0.2 & -0.3\\ 0.6 & 1.3\end{bmatrix},\ \bar{E}_1=\begin{bmatrix}0\\ 0.25\end{bmatrix},$$
$$\mathcal{D}_1=\begin{bmatrix}0.9 & 0.2\\ 0.1 & 0.4\end{bmatrix},\ \mathcal{D}_2=\begin{bmatrix}0.2 & 0.5\\ 0.2 & 0.3\end{bmatrix},\ \bar{E}_2=\begin{bmatrix}0\\ 0.3\end{bmatrix},$$
$$\hat{\chi}_{11}=\hat{\chi}_{12}=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},\ \hat{\chi}_{21}=\hat{\chi}_{22}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},$$
$$\hat{\chi}_{41}=\hat{\chi}_{42}=\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix},\ E_{b1}=E_{b2}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}.$$

The membership functions of rules 1 and 2 are $h_1(u)=\frac{1}{\exp(-2u(t))}$ and $h_2(u)=1-h_1(u)$, respectively. Moreover, the other parameters involved in this simulation are selected as follows: $\rho_1=0.1$, $\rho_2=0.25$, $\rho=0.15$, $\mu_2=0.4$, $\mu=0.11$, $\gamma=\tilde{\gamma}=0.1$, $\hat{c}=0.2$, $d_k=1.85$ and $\tau_k=0.2$ ($k=1,\dots,17$). In order to stabilize the concerned uncertain T-S fuzzy uncertain neutral-type system (7) under consideration, we design the more generalized intermittent controller as mentioned in (6) with the above given parameters. For this purpose, solving the LMIs in Theorem 2 with the above parameters utilizing Matlab LMI tool box, we obtain the following gain matrix of the control law

$$\hat{K}_1=\begin{bmatrix}0.3577 & -2.5342\end{bmatrix},$$
$$\hat{K}_2=\begin{bmatrix}2.1324 & -6.8743\end{bmatrix}.$$

To further show the effectiveness of the designed intermittent control, simulations have been carried out. The corresponding state responses of the neutral-type system (7) are presented in Figs. 1 and 2. The numerical simulation of system neutral-type (7) with controller $w(t)=0$ is depicted in Fig. 1, which shows the performance of unstable system. Based on the modeled IC (6), Fig. 2 exhibits the evolution of the system state under the initial condition $[-3,\ 5]^T$, from which we can see that the closed loop fuzzy system (7) is GAS. The response curve of the control input is plotted in Fig. 3. Fig. 4 displays a flowchart to calculate the controller gains. Moreover, the allowable bound of ρ_2 for different values of $\hbar$ and allowable bound of ρ_2 for different values of ρ respectively, are displayed in Tables 1 and 2. Based on the simulation results, it is demonstrated that the proposed intermittent controller performs well.

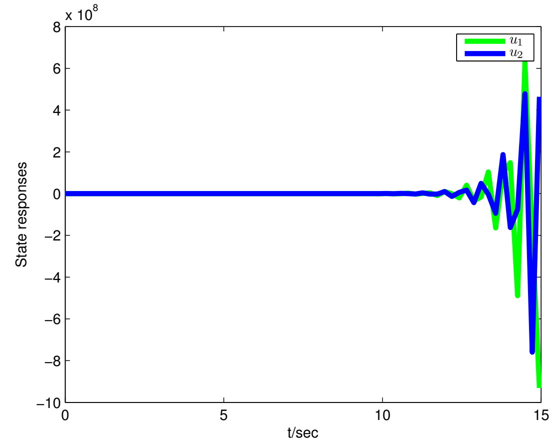

Fig. 1. Evolution of open-loop system in Example 1.

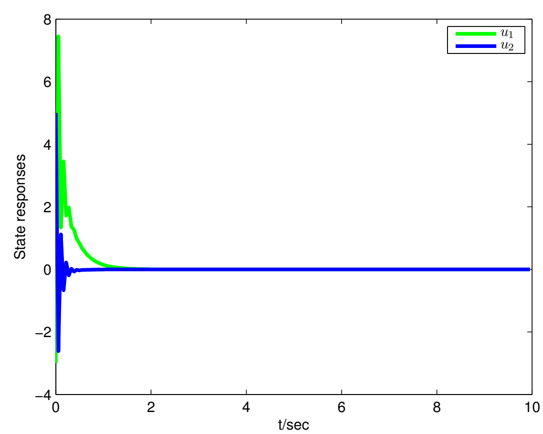

Fig. 2. Evolution of the system state in Example 1.

Table 1. AMD ρ_2 for various values of $\hbar$ in Example 1.

$\hbar$	0.01	0.03	0.05	0.06	0.07
ρ_2	0.2314	0.1752	0.0553	0.0242	0.0031

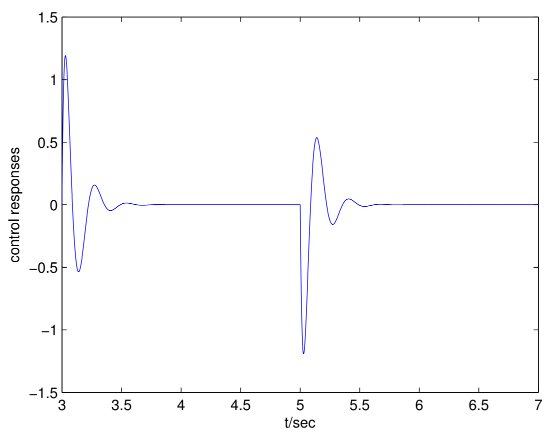

Fig. 3. Evolution of the control responses in Example 1.

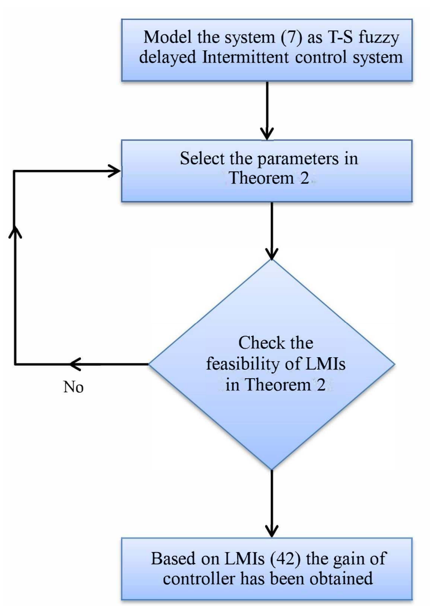

Fig. 4. A flowchart to calculate the controller gains.

Table 2. AMD ρ_2 for various values of ρ in Example 1.

ρ	0.01	0.03	0.05	0.07	0.1
ρ_2	0.2163	0.1935	0.1572	0.0931	0.0563

5. APPLICATIONS

In this section, an illustrative model of single-link robot arm (SLRA) system in [40] and [41] is explained to demonstrate the effectiveness of the designed methodology. Furthermore, for the simulation purpose, the certain parameters of the SLRA are summarized in the following Table 3.

1) Design example of single-link robot arm system:

$$\ddot{\eta}(t) = -\frac{\hat{g}\hat{L}\hat{M}}{\hat{J}}\sin(\eta(t)) - \frac{\hat{R}}{\hat{J}}\dot{\eta}(t) + \frac{1}{\hat{J}}w(t). \tag{51}$$

Table 3. Parameters of the SLRA model.

Symbol	Value
$\hat{M}$ (mass of the load)	1 kg
$\hat{J}$ (moment of inertia)	1 kg m^2
$\hat{R}$ (damping coefficient)	2 N m/rad
$\hat{L}$ (value of the length)	0.5 m
$\hat{g}$ (acceleration of gravity)	9.81 m/s^2
λ (retarded coefficient)	0.8 N m/rad

Defining $u_1(t) = \eta(t)$ and $u_2(t) = \dot{\eta}(t)$, to deal with the time delay in the system, we get

$$\begin{aligned}\dot{u}_1(t) &= \lambda u_2(t) + (1-\lambda)u_2(t-\rho(t)),\\ \dot{u}_2(t) &= -\frac{\hat{g}\hat{L}\hat{M}}{\hat{J}}\sin(u_1(t)) - \lambda\frac{\hat{R}}{\hat{J}}u_2(t)\\ &\quad - \frac{(1-\lambda)\hat{R}}{\hat{J}}u_2(t-\rho(t)) + \frac{1}{\hat{J}}w(t),\end{aligned}$$

where $\eta(t)$ and $\dot{\eta}(t)$ represent the robot arm angle and robot arm angular velocity, respectively. The nonlinear term $\sin(u_1(t))$ can be denoted as: $\sin(u_1(t)) = h_1(u_1(t)).u_1(t) + h_2(u_1(t))\cdot\tilde{\beta}\cdot u_1(t)$, where $h_1(u_1(t)) + h_2(u_1(t)) = 1$, $h_1(u_1(t)), h_2(u_1(t)) \in [0,1]$. The above equations can be solved and we get the following membership function

$$h_1(u_1(t)) = \begin{cases}\dfrac{\sin(u_1(t)) - \tilde{\beta}u_1(t)}{(1-\tilde{\beta})u_1(t)}, & u_1(t) \neq 0,\\ 1, & u_1(t) = 0,\end{cases}$$

$$h_2(u_1(t)) = 1 - h_1(u_1(t)),$$

where $u_1(t) \in (-\pi,\pi)$ and $\tilde{\beta} = 0.01/\pi$.

2) T-S fuzzy modelling:

The SLRA system can be implemented by the subsequent T-S fuzzy model:

Plant rule 1: IF $u_1(t)$ is about 0 rad, THEN

$$\begin{aligned}\dot{u}(t) =&(A_1 + \triangle\mathcal{A}_1(t))u(t)\\ &+ (B_1 + \triangle\mathcal{B}_1(t))u(t-\rho(t)) + \bar{E}_1 w(t).\end{aligned}$$

Plant rule 2: IF $u_1(t)$ is about $\pm$ rad, THEN

$$\begin{aligned}\dot{u}(t) =&(A_2 + \triangle\mathcal{A}_2(t))u(t)\\ &+ (B_2 + \triangle\mathcal{B}_2(t))u(t-\rho(t)) + \bar{E}_2 w(t),\end{aligned}$$

where $u(t) = [u_1^T(t)\ u_2^T(t)]^T$,

$$\mathcal{A}_1 = \begin{bmatrix} 0 & \lambda \\ -\frac{\hat{g}\hat{L}\hat{M}}{\hat{J}} & -\frac{\lambda\hat{R}}{\hat{J}}\end{bmatrix},\ \mathcal{A}_2 = \begin{bmatrix} 0 & \lambda \\ -\frac{\beta\hat{g}\hat{L}\hat{M}}{\hat{J}} & -\frac{\lambda\hat{R}}{\hat{J}}\end{bmatrix},$$

$$\mathcal{B}_1 = \mathcal{B}_2 = \begin{bmatrix} 0 & (1-\lambda) \\ 0 & -\frac{(1-\lambda)\hat{R}}{\hat{J}}\end{bmatrix},\ E_1 = E_2 = \begin{bmatrix} 0 & \frac{1}{\hat{J}}\end{bmatrix}^T,$$

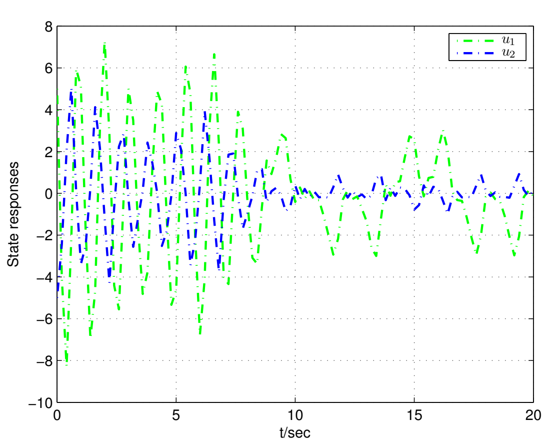

Fig. 5. Evolution of the system state in SLRA model.

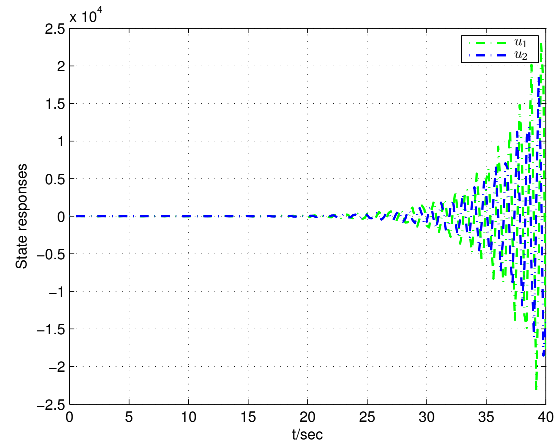

Fig. 6. Evolution of the open-loop system state in SLRA model.

$$
\begin{aligned}
&[\triangle\mathcal{A}_i(t),\ \triangle\mathcal{B}_i(t)] = E_{bi}F_i(t)[\hat{\chi}_{1i},\ \hat{\chi}_{2i}],\\
&E_{b1} = 0.01,\ \ E_{b2} = 0.02,\\
&F_1(t) = 0.5\sin(t),\ \ F_2(t) = -0.5\cos(t),\\
&\hat{\chi}_{11} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix},\ \hat{\chi}_{12} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix},\\
&\hat{\chi}_{21} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix},\ \hat{\chi}_{22} = \begin{bmatrix} -0.01 & 0 \\ 0 & 0.03 \end{bmatrix}.
\end{aligned}
$$

Utilizing the above parameters and setting $\rho_1 = 0.1$, $\rho_2 = 0.2$, $\rho = 0.2$, $d_k = 1.85$ by using the Matlab LMI control toolbox to solve the LMIs in Theorem 2 without neutral-type, i.e., $(D_i + \Delta_i(t)) = Z_5 = 0$, we get the following feedback control gain matrices $\hat{K}_1 = [1.3563\ \ 2.1032]$ and $\hat{K}_2 = [2.3640\ \ 3.5322]$. Fig. 5 represents the state responses of variables $u_1(t)$ and $u_2(t)$ with an initial condition $u(t) = [5,\ -5]^T$, indicating that the evolution of the system converges to an equilibrium point. Fig. 6 depicts the state responses of the SLRA system. It is clear that the system is destabilize, i.e, without controller. Under zero initial condition, Fig. 7 shows the control input of the system. Referring to these figures, it can be observed that the proposed method has been effectively and applicability.

6. A COMPARISON EXAMPLE

In the previous section, we have validated the derived sufficient condition with single-link robot arm system. In order to prove the conservatism of the derived condition, we have considered the system

$$
\begin{aligned}
\dot{u}(t) - \mathcal{D}\dot{u}(t-\mu(t)) =& (\mathcal{A} + \triangle\mathcal{A}(t))u(t)\\
&+ (\mathcal{B} + \triangle\mathcal{B}(t))u(t-\rho(t)),
\end{aligned}
\tag{52}
$$

where the system parameters are given as

$$
\mathcal{A} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},\ \mathcal{B} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\mathcal{D} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},
$$

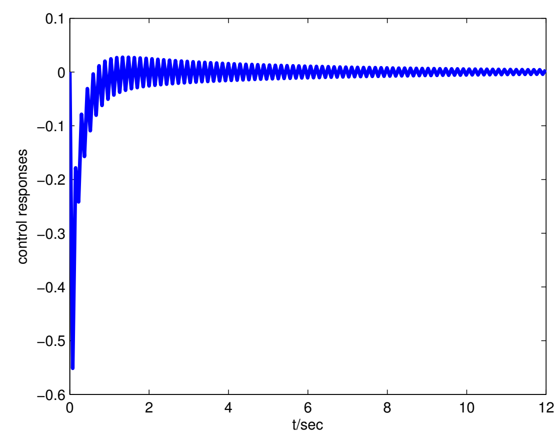

Fig. 7. Evolution of the control response in SLRA model.

where $\triangle\mathcal{A}(t)$ and $\triangle\mathcal{B}(t)$ are of the form in (52) with $\hat{\chi}_1 = diag\{160\ ,1.25\}$, $\hat{\chi}_2 = \hat{\chi}_4 = diag\{10\ ,7.5\}$. In order to compare the conservativeness of Corollary 2 with those of some previous works, the parameters from Example 4.3 of [11] have been used. For given $\mu(t) = \mu$ and $\rho = 0.1$, the maximum allowable delay ρ_2 provided by proposed stability criteria is listed in Table 4. When $c = 0.1$ and different values of μ, Table 5 shows compared results of the upper bound of ρ_2 with the ones in [4,9–11]. By using Corollary 2 and the Matlab LMI Toolbox, Tables 4 and 5 give the obtained maximum allowable ρ_2, such that the considered system is admissible. Form these tables, we can see that the proposed stability criterion for uncertain neutral-type system is less conservative than the existing results. It shows the effectiveness of the methods presented in this paper.

7. CONCLUSION

In this paper, the delayed intermittent control for T-

Table 4. Maximum allowable upper bounds (MAUBs) ρ_2 for different values of c, such that $\rho_1 = 0$ and $\rho = 0.1$.

c	0	0.1	0.2	0.3	0.4	0.5	0.6
[9]	1.166	0.962	0.778	0.616	0.472	0.346	0.235
[10]	1.172	0.966	0.780	0.618	0.473	0.347	0.235
[4]	1.177	0.971	0.786	0.622	0.478	0.349	0.235
[11]	1.275	1.033	0.821	0.638	0.482	0.349	0.235
Corollary 2	1.464	1.286	1.251	0.932	0.876	0.575	0.242

Table 5. Maximum allowable upper bounds (MAUBs) ρ_2 for different values of ρ with c=0.1.

lower bound	ρ	0.1	0.3	0.5	0.7	0.9
$\rho_1 = 0$	[9]	0.962	0.907	0.850	0.789	0.714
	[4]	0.971	0.960	0.960	0.960	0.960
	[11]	1.033	0.984	0.980	0.979	0.979
	Corollary 2	1.162	1.085	1.083	0.986	0.986
$\rho_1 = 0.5$	[9]	0.962	0.907	0.850	0.793	0.793
	[10]	0.966	0.922	0.895	0.891	0.889
	[4]	0.971	0.961	0.961	0.961	0.961
	[11]	1.118	1.043	1.041	1.041	1.041
	Corollary 2	1.264	1.213	1.102	1.062	1.062

S fuzzy neutral-type system is studied with mixed time-varying delays and uncertainties. The physical plant of the system is represented as an average weighted sum of local linear subsystems and the weighting terms are characterized by the membership functions. By constructing an augmented LKF and handling some new integral inequality techniques like SAFBII and DAFBII as well as, some sufficient conditions, we have established GAS criteria for the considered fuzzy neutral-type systems. Moreover, the gains of the delayed intermittent controller are derived by solving a set of LMIs, which can be solved by MATLAB LMI toolbox. Finally, numerical examples are also presented to validate the theoretical results of this study and in addition, the developed methodology has been tested on a practical single-link robot arm model. The limitation of the proposed method is the introduction of relaxation variables, which increases the computational complexity. However, it is necessary to expand the feasible region of the result and obtain more feasible solutions at the same time, we sometimes need to introduce the relaxation variables. If the conditions have a feasible solution, the controller feedback matrices can be calculated according to the feasible solutions of a set of LMIs. It is worth to note that the results of this paper is compared with the existing ones in the literature. Less conservative results can be obtained by using the new method proposed in this paper. The method in this work can be used to deal with more complicated problems such as filtering design, observer design, external disturbances (dissipativity, passivity, H_∞ performance) and distributed event-triggered scheme to save computational efforts. Additionally, more complex systems, including stochastic nonlinear systems based on hidden Markovian model and semi-Markovian model stochastic nonlinear systems are interesting research topics and will be additionally examined in our future works.

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