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Synchronization of fractional order fuzzy BAM neural networks with time varying delays and reaction diffusion terms

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ABSTRACT In this paper synchronization of fractional order fuzzy BAM neural networks with time varying delays and reaction diffusion terms is studied. The time varying delays consist of discrete delays and distributed delays are considered. Then, some sufficient conditions black are presented to guarantee the global asymptotic stability of the error system by using Lyapunov-Krasovskii functional having the double integral terms, we utilized Jensens inequality techniques and LMI approach. Accordingly, we accomplished synchronization of master-slave fuzzy BAMNNs. The delay dependent stability conditions are set up in terms of linear matrix inequalities(LMIs), which can be productively understood utilizing Matlab LMI control tool box. At last, illustrative numerical results have been provided to verify the correctness and effectiveness of the obtained results.

INDEX TERMS Synchronization,Time varying delays, Reaction Diffusion.

I. INTRODUCTION

Around 300 years back, the foundation of fractional order calculus, which is an extension of classical integer order calculus, was first off mentioned through German mathematician Leibniz and it failed to attract more attention for a long time since it lack of application background and the complexity. Until now, the research on fractional order calculus becomes a hot research topic because of The reality that many real-international gadgets want to be defined with the aid of using fractional order models. Fractional Order calculus is an area of mathematics that deals with extensions of derivatives and integrals to non integer orders and represents a powerful tool [1]–[3]. As usual, there are two advantages in models of fractional order: One is permitting greater degree of freedom in the models, and the other is describing memory properties in the models. Therefore, during recent times, more focus has been paid to the Riemann- Liouville fractional derivatives and of the Caputo fractional derivatives [4]. However, during the last two decades, the study of fractional differential equations has been widely applicable to many real world problems. Recently, numerous reports have pointed out that fractional calculus has the ability to describe numerous phenomena more accurately in various fields, for instance, biological models, finance, quantum mechanics, material science, fluid mechanics, cardiac tissues, medicine, and viscoelastic systems [5]. On account of this, fractional order calculus turned into added into synthetic neural community in beyond few decades, namely, fractional order neural networks (FNNs), that could describe the neuro dynamics of human brain more effectively and accurately in view of the hereditary and memory possessed by fractional calculus. For example, several problems of fractional-order NNs have been well studied such as global stability, pinning synchronization, Asymptotic stability, global Mittag-Leffler stability. Accordingly, [6] necessary and sufficient conditions of observer-based stabilization for a class of fractional-order descriptor systems studied, based on these developments design conditions are obtained in terms of LMIs.

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Time delays are inherent phenomena in interconnected real systems or processes, including feedback control systems, due to transportation of material, energy, or information. Time delay is frequently a source of instability. However, only under some special circumstances or in certain cases, the practical problems may be regarded as linear systems. Also, stability analysis for fractional order linear singular delay differential systems has been analyzed in [7]. Therefore, stability of nonlinear dynamic systems not only is of great significance but also has important value in application. However, the stability of fractional nonlinear systems, of central importance in control theory, remains an open problem. To improve the control performance nonlinear systems, the numerous nonlinear control techniques are applied to controller design, such as fuzzy control, neural control, sliding mode control, and adaptive back stepping control. A large number of results concerning unknown control for nonlinear systems have been obtained [8]–[10].

Traditional neural networks with fuzzy logic are called fuzzy neural networks, they can be used for broadening the range of application of traditional neural networks. Studies have indicated that fuzzy neural systems are valuable models for investigating human psychological exercises. There are numerous significant reports about the fuzzy neural systems (see, for example, [11]). Fuzzy set theory provides the mathematical strength to capture these uncertainties. In addition to time delay, several other factors such as complexity, uncertainty or vagueness can be considered while modeling the neural network problems and this can be studied by the application of fuzzy set theory [12], [13]. In this context, due to the applications in image processing, pattern recognition etc (see [14]–[17]and the references therein). The BAM neural networks were first introduced by Kosko [18]–[20]. It is a special class of repetitive neural systems that can store bipolar rector pairs. BAM neural networks are made up of two neuron layers, i.e. U-layer and V-layer. The neurons in a single layer are completely interconnected to the neurons in the subsequent layer. There is no interconnection among neurons in the same layer. In real life, BAM neural networks have powerful information processing abilities and some good application fields, such as information associative memory, image processing, artificial intelligence, and so on. In addition, time delays existing in axonal signal transmission between real biological neurons have not been discussed in BAM networks. On the other hand, time delays inevitably happens in electronic neural systems inferable from the unavoidable limited exchanging rate of intensifiers. Be that as it may, a couple of concentrates centered dynamic of BAM delayed neural systems. It is alluring to examine the fuzzy BAM neural systems which has a potential [21]–[23] significance in the design and applications of stable neural circuits for neural networks with delays.

In signal transmission, the signal will become weak due to diffusion in signal transmission, so it is very important to consider that the activation varies in space as well as in time and the reaction diffusion effects cannot be neglected in both biological and man-made neural networks. However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that activations change in space just as in time. In the real world, there are lots of reaction diffusion phenomena in nature and engineering fields. In signal transmission, the signal will become weak due to diffusion in signal transmission, so it is very important to consider that the activation varies in space as well as in time and activations change in space just as in time. the reaction-diffusion impacts can't be dismissed in both organic furthermore, man-made [24], [25]. As electrons transport in a nonuniform electromagnetic field, the diffusion phenomena could not be ignored. Therefore, in factual modeling, only considering the change of time seems to be not comprehensive when electrons are moving in asymmetric and nonuniform electromagnetic fields. Influenced by diffusion, neural networks have rich spatial dynamical behaviors, like various Special dynamics in reaction-diffusion systems was originally proposed by Turing in 1952. This pioneering work of Turing not only came into being a theoretical foundation for understanding diverse patterns occurring in the natural world, but also opened a new research field, namely, pattern dynamics, which has received extensive attention and is still a hot topic in many scientific fields such as species dynamics, medicine, neural networks. However, the research on NNs with both reaction-diffusion and time delay is still in its infancy [26]–[30].

The purpose of this paper is to discuss the global asymptotic stability problem for system (5). Based on the Lyapunov Krasovskii functional, sufficient conditions are established in terms of the LMI. The major advantage of the proposed method is that one may avoid calculating fractional-order derivatives of the Lyapunov Krasovskii functionals. Two illustrative examples are also given to show the validity and feasibility of the theoretical results by using MATLAB LMI toolbox. The main challenges and contributions of this paper are summarized as follows:
(1)In order to overcome the difficulties of calculating the fractional-order derivative of a function, we construct an appropriate Lyapunov Krasovskii functional associated with the Riemann – Liouville fractional integral, definite integral and double integers, and calculate its first-order derivative to derive the global asymptotic stability conditions.
(2)The addressed system includes reaction diffusion terms, discrete delays, distributed delays, and fractional-order derivative of the state. We take into account the impact of these factors on the stability of network system simultaneously. The proposed results of this paper are described by the LMIs, which are computationally feasible. The numerical simulations of two illustrative examples are also presented to show the effectiveness and feasibility of the theoretical results.

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(3)However, to the best of our knowledge, not many creators think about the synchronization of fractional order fuzzy BAM neural networks with time varying delays and reaction diffusion terms, up to now there are few papers published on the existence and stability of equilibrium point and periodic solution of the impulsive fuzzy high-order BAM neural networks with continuously distributed delays. Most of the existing papers mainly focus on the first-order systems with time-varying and/or distributed delays. Compared with the stability results of the neural networks, the ones of this paper are more general and less conservative. Thus, it is important and necessary to consider the fuzzy BAM neural networks with time varying delays with reaction diffusion terms.

The rest of this paper is arranged as follows: in Section 2, the main results of this paper are described; next, in Section 3, numerical simulations are presented to illustrate the effectiveness and correctness of the main results; finally, the conclusion of this paper is given in Section 4.

Notations:The notation used throughout this paper is fairly standard. $\mathcal{R}^n$ *and* $\mathcal{R}^{m\times n}$ *denote the set of n-dimensional real vectors,* $m\times n$ *real matrices, respectively. The superscript* $\mathcal{T}$ *denotes the matrix transposition.* $\mathcal{P}\in\mathcal{R}^{n\times n}\geq 0$ *means that matrix* $\mathcal{P}$ *is symmetric and semi-positive(semi-negative)definite.* $\mathcal{I}_n$ *denotes an* $n\times n$ *real identity matrix.* $\mathcal{A}^{\mathcal{T}}$ *is the transpose of matrix* $\mathcal{A}$. $\mathcal{B}^{-1}$ *means the inverse of matrix* $\mathcal{B}$. *Let* $\mathcal{A}=\mathcal{A}^{\mathcal{T}}\mathcal{A}$ *where* $\mathcal{A}\in\mathcal{R}^{n\times n}$.

A. PRELIMINARIES

It is well known that the most common used fractional definitions are adopted in this paper.

Definition 1. *[31] Riemann-Liouville fractional integral of order* λ *for a function* $f:\mathcal{R}^+\to\mathcal{R}$ *is defined as*

$$ {}^{\mathcal{RL}}_{t_0}\mathcal{I}^{\lambda}_{t}f(t)=\frac{1}{\Gamma(\lambda)}\int_{t_0}^{t}(t-s)^{\lambda-1}f(s)\partial s, $$

where $\Gamma(\cdot)$ *is the Euler's gamma function, which is denominated by*

$$ \Gamma(l)=\int_0^{+\infty}\exp(-w)t^{l-1}dw,(Re(l)>0), $$

where $Re(l)$ *is the real part of l.*

Definition 2. *[32] Riemann-Liouville fractional derivative with order a for a function* $f:\mathcal{R}^+\to\mathcal{R}$ *is defined as*

$$ {}^{\mathcal{RL}}_{t_0}\mathcal{D}^{\lambda}_{t}f(t)=\frac{1}{\Gamma(m-\lambda)}\frac{\partial^m}{\partial t^m}\int_{t_0}^{t}(t-\tau)^{m-\alpha-1}x(s)\partial s, $$

where $0\leq m-1<\lambda<m, m\in\mathcal{Z}^+$, $\mathcal{Z}^+$ *denotes the collection of all positive integers.*

Lemma 1. *[33] Let a vector-value function* $u(t)\subset\mathcal{R}^n$ *is differentiable. Then, for any* $t>0$, *one has*

$$ {}^{\mathcal{RL}}_{0}\mathcal{D}^{\lambda}_{t}(u^{\mathcal{T}}(t)\mathcal{Q}u(t))\leq 2u^{\mathcal{T}}(t)(\mathcal{Q}^{\mathcal{RL}}_{0}\mathcal{D}^{\lambda}_{t}u(t)),\ \ 0<\lambda<1. $$

Lemma 2. *[34] Given constant matrices* ξ_1,ξ_2,ξ_3, *where* $\xi_1=\xi_1^{\mathcal{T}},\xi_2=\xi_2^{\mathcal{T}}$, *and* $\xi_2>0$, *then*

$$ \xi_1+\xi_3^{\mathcal{T}}\xi_2^{-1}\xi_3<0 $$

if and only if

$$ \begin{bmatrix}\xi_1 & \xi_3^{\mathcal{T}}\\ \xi_3 & -\xi_2\end{bmatrix}<0. $$

Lemma 3. *[35] Suppose that* $x,y\in\mathbb{R}^n$ *be the two states system (4), then we have*

$$ \begin{aligned} &|\bigwedge_{j=1}^{m}\gamma_{ij}h_j(\bar{y}_j)-\bigwedge_{j=1}^{m}\gamma_{ij}h_j(y_j)|\leq\sum_{j=1}^{m}|\gamma_{ij}||h_j(\bar{y}_j)-h_j(y_j)|,\\ &|\bigvee_{j=1}^{m}\delta_{ij}h_j(\bar{y}_j)-\bigvee_{j=1}^{m}\delta_{ij}h_j(y_j)|\leq\sum_{j=1}^{m}|\delta_{ij}||h_j(\bar{y}_j)-h_j(y_j)|,\\ &|\bigwedge_{i=1}^{n}\tilde{\gamma}_{ji}g_i(\bar{x}_i)-\bigwedge_{i=1}^{n}\tilde{\gamma}_{ji}g_i(x_i)|\leq\sum_{i=1}^{n}|\tilde{\gamma}_{ji}||g_i(\bar{x}_i)-g_i(x_i)|,\\ &|\bigvee_{i=1}^{n}\tilde{\delta}_{ji}g_i(\bar{x}_i)-\bigvee_{i=1}^{n}\tilde{\delta}_{ji}g_i(x_i)|\leq\sum_{i=1}^{n}|\tilde{\delta}_{ji}||g_i(\bar{x}_i)-g_i(x_i)|. \end{aligned} $$

Lemma 4. *[36] For any constant matrix* $\mathcal{M}\in\mathcal{R}^{n\times n},\mathcal{M}=\mathcal{M}^{\mathcal{T}}>0$, *scalar* $\alpha_1\leq\alpha_2$, *the following inequalities hold:*

$$ \begin{aligned} &-(\alpha_1-\alpha_2)\int_{\alpha_2}^{\alpha_1}\Phi^{\mathcal{T}}(s)\mathcal{M}\Phi(s)\partial s\\ &\leq-\Big(\int_{\alpha_2}^{\alpha_1}\Phi(s)\partial s\Big)^{\mathcal{T}}\mathcal{M}\Big(\int_{\alpha_2}^{\alpha_1}\Phi(s)\partial s\Big). \end{aligned} $$

Lemma 5. *[37] If* $p>q>0$, *then the following equality*

$$ {}^{\mathcal{RL}}_{t_0}\mathcal{D}^{p}_{t}{}^{\mathcal{RL}}_{t_0}\mathcal{I}^{q}_{t}f(t)={}^{\mathcal{RL}}_{t_0}\mathcal{D}^{p-q}_{t}f(t) $$

holds for sufficiently good functions $f(t)$. *In particular, this relation holds if* $f(t)$ *is integrable.*

Lemma 6. *[38] Let* Ω *be a cube* $|x|<l$, *and let* $h(x)$ *be a real-valued function belonging to* $\mathcal{C}^1(\Omega)$, *which vanishes on the boundary* $\partial\Omega$ *of* Ω, *i.e.,* $h(x)|_{d\Omega}=0$. *Then,*

$$ \int_{\Omega}h^2(x)\partial x\leq l^2\int_{\omega}(\frac{\partial h}{\partial x})^2\partial x. $$

Assumption 1. *We assume* $\sigma(t)$ *and* $\tau(t)$ *are time-varying functions satisfying:*

$$ \begin{aligned} &(1).0\leq\sigma(t)\leq\sigma,\dot{\sigma}(t)\leq\eta,\\ &(2).0\leq\tau(t)\leq\tau,\dot{\tau}(t)\leq\mu. \end{aligned} $$

where σ,η,τ *and* μ *are known constants.*

Assumption 2. *Each neuron activation function* $g_i(t)(i=1,2,..,n)$ *in (1) are continuous and bounded, and satisfying the following condition:*

$$ l_i^-\leq\frac{g_i(k_1)-g_i(k_2)}{k_1-k_2}\leq l_i^+,\forall k_1,k_2\in\mathcal{R}, $$

$k_1\neq k_2, i=1,2....,n.$

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Assumption 3. *We assume the delay kernels $\mathcal{K}_{ij}$, $\tilde{\mathcal{K}}_{ij}$: $[0,\infty) \to [0,\infty)(i = 1,2,...,m; j = 1,2,...,n)$ are non negative continuous functions that satisfy the following conditions:*

$$\int_0^\infty \mathcal{K}_{ij}(s)\partial s = \int_0^\infty \tilde{\mathcal{K}}_{ji}(s)\partial s = 1$$

II. MAIN RESULTS

Consider the following fuzzy BAMNNs with time varying delays:

$$\begin{aligned}
\frac{\partial^\lambda u_i(x,t)}{\partial t^\lambda} &= \sum_{k=1}^{q} \frac{\partial}{\partial x_k}\left(m_{ik}\frac{\partial u_i(x,t)}{\partial x_k}\right) - p_i u_i(x,t) \\
&+ \sum_{j=1}^{n} a_{ij} f_j(\eta_j(x,t)) + \sum_{j=1}^{n} c_{ij} v_j(x,t) + \bigwedge_{j=1}^{n} \mathcal{T}_{ij} v_j(x,t) \\
&+ \sum_{j=1}^{n} b_{ij} f_j(\eta_j(x,t-\tau(t))) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(\eta_j(x,t-\tau(t))) \\
&+ \bigvee_{j=1}^{n} \beta_{ij} f_j(\eta_j(x,t-\tau(t))) + \bigvee_{j=1}^{n} \mathcal{S}_{ij} v_j(x,t) \\
&+ \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} \mathcal{K}_{ij}(t-s) f_j(\eta_j(x,s))\partial s \\
&+ \bigwedge_{j=1}^{n} \gamma_{ij} \int_{-\infty}^{t} \mathcal{K}_{ij}(t-s) f_j(\eta_j(x,s))\partial s \\
&+ \bigvee_{j=1}^{n} \delta_{ij} \int_{-\infty}^{t} \mathcal{K}_{ij}(t-s) f_j(\eta_j(x,s))\partial s + \mathcal{I}_i, \\
\frac{\partial^\lambda \eta_j(x,t)}{\partial t^\lambda} &= \sum_{k=1}^{q} \frac{\partial}{\partial x_k}\left(\tilde{m}_{jk}\frac{\partial \eta_j(x,t)}{\partial x_k}\right) - \tilde{p}_j \eta_j(x,t) \\
&+ \sum_{i=1}^{m} \tilde{a}_{ji} g_i(u_i(x,t)) + \sum_{i=1}^{m} \tilde{c}_{ji} v_i(x,t) \\
&+ \bigwedge_{i=1}^{m} \tilde{\mathcal{T}}_{ji} v_i(x,t) + \sum_{i=1}^{m} \tilde{b}_{ji} g_i(u_i(x,t-\sigma(t))) \\
&+ \bigwedge_{i=1}^{m} \tilde{\alpha}_{ji} g_i(u_i(x,t-\sigma(t))) \\
&+ \bigvee_{i=1}^{m} \tilde{\beta}_{ji} g_i(u_i(x,t-\sigma(t))) + \bigvee_{i=1}^{m} \tilde{\mathcal{S}}_{ji} v_i(x,t) \\
&+ \sum_{i=1}^{m} \tilde{d}_{ji} \int_{-\infty}^{t} \tilde{\mathcal{K}}_{ji}(t-s) g_i(u_i(x,s))\partial s \\
&+ \bigwedge_{i=1}^{m} \tilde{\gamma}_{ji} \int_{-\infty}^{t} \tilde{\mathcal{K}}_{ji}(t-s) g_i(u_i(x,s))\partial s \\
&+ \bigvee_{i=1}^{m} \tilde{\delta}_{ji} \int_{-\infty}^{t} \tilde{\mathcal{K}}_{ji}(t-s) g_i(u_i(x,s))\partial s + \mathcal{J}_j, \\
\frac{\partial^\lambda u_i(x,t)}{\partial t^\lambda} &= \varphi_{u_i}(x,t), t \in (-\infty,0], i = 1,2,.....,m, \\
\frac{\partial^\lambda \eta_j(x,t)}{\partial t^\lambda} &= \psi_{\eta_j}(x,t), t \in (-\infty,0], j = 1,2,.....,n,
\end{aligned} \tag{1}$$

where $\varphi_{u_i}(x,s)$ and $\psi_{\eta_j}(x,s)$, $i = 1,2,...,m$, $j = 1,2,...,n$, are continuous bounded functions defined on $(-\infty,0]\times\Omega$, respectively. Where ∂^λ denotes the fractional derivative operator of order $\lambda(0<\lambda<1)$; where m and n correspond to the number of neurons in U-layer and η-layer, respectively. For $i = 1,2,...,m, j = 1,2,...,n$ $x = (x_1,x_2,...,x_q)^{\mathcal{T}} \epsilon \Omega \subset \mathcal{R}^q$,$\Omega$ is a bounded compact set with smooth boundary $\partial\Omega$ and mess $\Omega > 0$ in space $\mathcal{R}^q$. $u = (u_1,u_2,...,u_m)^{\mathcal{T}} \epsilon \mathcal{R}^m$, $\eta = (\eta_1,\eta_2,...,\eta_n)^{\mathcal{T}} \epsilon \mathcal{R}^n$, $u_i(x,t)$ and $\eta_j(x,t)$ are the state of the ith neuron and the jth neurons at time t and in space x, respectively. $p_i > 0$, $\tilde{p}_j > 0$, and they denote the rate with which the ith neuron and jth neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; a_{ij} , b_{ij}, $\tilde{a}_{ij}$, $\tilde{b}_{ij}$ and c_{ij}, $\bar{c}_{ji}$ are denote the connection weights of the feedback template and feed forward template, respectively. α_{ij}, β_{ij},$\mathcal{T}_{ij}$ and $\mathcal{S}_{ij}$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in U-layer, respectively; $\tilde{\alpha}_{ji}$, $\tilde{\beta}_{ji}$, $\tilde{\mathcal{S}}_{ji}$, and $\tilde{\mathcal{T}}_{ji}$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy fee-forward MAX template in η-layer, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; I_i and J_j denote the external inputs on the ith neurons in U-layer and the jth neurons in η-layer at time t, respectively; $\mathcal{K}_{ij}(\cdot)$ and $\tilde{\mathcal{K}}_{ji}(\cdot)$are delay kernels functions. $g_i(\cdot)$ and $f_j(\cdot)$ are signal transmission functions of i th neurons and j th neurons respectively.
The boundary conditions and the initial conditions are given by

$$\frac{\partial u_i}{\partial m} := \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, ..., \frac{\partial u_i}{\partial x_q}\right)^{\mathcal{T}} = 0, i = 1,2,........., m,$$

$$\frac{\partial \eta_j}{\partial n} := \left(\frac{\partial \eta_j}{\partial x_1}, \frac{\partial \eta_j}{\partial x_2}, ..., \frac{\partial \eta_j}{\partial x_q}\right)^{\mathcal{T}} = 0, j = 1,2,........., n.$$

Considering that a fractional order fuzzy BAM neural networks depends extremely on initial values, the initial conditions of the slave system is defined to be different from that of the master system. Therefore, a slave system with the same form and parameters of master system (1) is introduced as follows:

$$\begin{aligned}
\frac{\partial^\lambda \bar{u}_i(x,t)}{\partial t^\lambda} &= \sum_{k=1}^{q} \frac{\partial}{\partial x_k}\left(m_{ik}\frac{\partial \bar{u}_i(x,t)}{\partial x_k}\right) - p_i \bar{u}_i(x,t) \\
&+ \sum_{j=1}^{n} a_{ij} f_j(\bar{\eta}_j(x,t)) + \sum_{j=1}^{n} c_{ij} v_j(x,t) + \bigwedge_{j=1}^{n} \mathcal{T}_{ij} v_j(x,t) \\
&+ \sum_{j=1}^{n} b_{ij} f_j(\bar{\eta}_j(x,t-\tau(t))) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(\bar{\eta}_j(x,t-\tau(t)))
\end{aligned}$$

$$
\begin{aligned}
&+\bigvee_{j=1}^{n}\beta_{ij}\mathfrak{f}_j(\bar{\eta}_j(\mathfrak{x},t-\tau(t)))+\bigvee_{j=1}^{n}\mathcal{S}_{ij}\mathfrak{v}_j(\mathfrak{x},t)\\
&+\sum_{j=1}^{n}\mathfrak{d}_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\eta}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\bigwedge_{j=1}^{n}\gamma_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\eta}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\bigvee_{j=1}^{n}\delta_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\eta}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}+\mathcal{I}_i,\\
\frac{\partial^\lambda\bar{\eta}_j(\mathfrak{x},t)}{\partial t^\lambda}&=\sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\tilde{\mathfrak{m}}_{j\mathfrak{k}}\frac{\partial\bar{\eta}_j(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}})-\tilde{\mathfrak{p}}_j\bar{\eta}_j(\mathfrak{x},t)\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{a}}_{ji}\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},t))+\sum_{i=1}^{m}\tilde{\mathfrak{c}}_{ji}\mathfrak{v}_i(\mathfrak{x},t)+\bigwedge_{i=1}^{m}\tilde{\mathcal{T}}_{ji}\mathfrak{v}_i(\mathfrak{x},t)\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{b}}_{ji}\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},t-\sigma(t)))+\bigwedge_{i=1}^{m}\tilde{\alpha}_{ji}\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},t-\sigma(t)))\\
&+\bigvee_{i=1}^{m}\tilde{\beta}_{ji}\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},t-\sigma(t)))+\bigvee_{i=1}^{m}\tilde{\mathcal{S}}_{ji}\mathfrak{v}_i(\mathfrak{x},t)\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{d}}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},\mathfrak{s}))ds\\
&+\bigwedge_{i=1}^{m}\tilde{\gamma}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\bigvee_{i=1}^{m}\tilde{\delta}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\bar{\mathfrak{u}}_i(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}+\mathcal{J}_j,\\
\frac{\partial^\lambda\bar{\mathfrak{u}}_i(\mathfrak{x},t)}{\partial t^\lambda}&=\theta_{\bar{\mathfrak{u}}_i}(\mathfrak{x},t),t\in(-\infty,0],i=1,2,.....,\mathfrak{m},\\
\frac{\partial^\lambda\bar{\eta}_j(\mathfrak{x},t)}{\partial t^\lambda}&=\vartheta_{\bar{\eta}_j}(\mathfrak{x},t),t\in(-\infty,0],j=1,2,.....,\mathfrak{n}.
\end{aligned}
\tag{2}
$$

Defining the synchronization error signal as $\mathfrak{e}_i(\mathfrak{x},t)$*and* $\bar{\mathfrak{e}}_j(\mathfrak{x},t)$ *can be obtained as follows:*
$\mathfrak{e}_i(\mathfrak{x},t)=\bar{\mathfrak{u}}_i(\mathfrak{x},t)-\mathfrak{u}_i(\mathfrak{x},t)$, $\bar{\mathfrak{e}}_j(\mathfrak{x},t)=\bar{\eta}_j(\mathfrak{x},t)-\eta_j(\mathfrak{x},t)$

$$
\begin{aligned}
\frac{\partial^\lambda\mathfrak{e}_i(\mathfrak{x},t)}{\partial t^\lambda}&=\sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\mathfrak{m}_{i\mathfrak{k}}\frac{\partial\mathfrak{e}_i(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}})-\mathfrak{p}_i\mathfrak{e}_i(\mathfrak{x},t)\\
&+\sum_{j=1}^{n}\mathfrak{a}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t))+\sum_{j=1}^{n}\mathfrak{b}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))\\
&+\sum_{j=1}^{n}|\alpha_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))\\
&+\sum_{j=1}^{n}|\beta_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))\\
&+\sum_{j=1}^{n}\mathfrak{d}_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\sum_{j=1}^{n}|\gamma_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\sum_{j=1}^{n}|\delta_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s},\\
\frac{\partial^\lambda\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial t^\lambda}&=\sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\tilde{\mathfrak{m}}_{j\mathfrak{k}}\frac{\partial\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}})-\tilde{\mathfrak{p}}_j\bar{\mathfrak{e}}_j(\mathfrak{x},t)\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{a}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t))+\sum_{i=1}^{m}\tilde{\mathfrak{b}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))\\
&+\sum_{i=1}^{m}|\tilde{\alpha}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))\\
&+\sum_{i=1}^{m}|\tilde{\beta}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{d}}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\sum_{i=1}^{m}|\tilde{\gamma}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\sum_{i=1}^{m}|\tilde{\delta}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t))\mathfrak{d}\mathfrak{s},\\
\frac{\partial^\lambda\mathfrak{e}_i(\mathfrak{x},t)}{\partial t^\lambda}&=\theta_{\bar{\mathfrak{u}}_i}(\mathfrak{x},t)-\varphi_{\mathfrak{u}_i}(\mathfrak{x},t),t\in(-\infty,0],\\
\frac{\partial^\lambda\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial t^\lambda}&=\vartheta_{\bar{\eta}_j}(\mathfrak{x},t)-\psi_{\eta_j}(\mathfrak{x},t),t\in(-\infty,0].
\end{aligned}
\tag{3}
$$

We can rewrite the error system (3) as follows:

$$
\begin{aligned}
\frac{\partial^\lambda\mathfrak{e}_i(\mathfrak{x},t)}{\partial t^\lambda}&=\sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\mathfrak{m}_{i\mathfrak{k}}\frac{\partial\mathfrak{e}_i(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}})-\mathfrak{p}_i\mathfrak{e}_i(\mathfrak{x},t)\\
&+\sum_{j=1}^{n}\mathfrak{a}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t))+\sum_{j=1}^{n}\mathfrak{b}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))\\
&+\sum_{j=1}^{n}[|\alpha_{ij}|+|\beta_{ij}|]\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))\\
&+\sum_{j=1}^{n}\mathfrak{d}_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}\\
&+\sum_{j=1}^{n}[|\gamma_{ij}|+|\delta_{ij}|]\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s},\\
\frac{\partial^\lambda\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial t^\lambda}&=\sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\tilde{\mathfrak{m}}_{j\mathfrak{k}}\frac{\partial\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}})-\tilde{\mathfrak{p}}_j\bar{\mathfrak{e}}_j(\mathfrak{x},t)\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{a}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t))+\sum_{i=1}^{m}\tilde{\mathfrak{b}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))\\
&+\sum_{i=1}^{m}[|\tilde{\alpha}_{ji}|+|\tilde{\beta}_{ji}|]\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))\\
&+\sum_{i=1}^{m}\tilde{\mathfrak{d}}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\mathfrak{d}\mathfrak{s}
\end{aligned}
$$

$$+\sum_{i=1}^{m}[|\tilde{\gamma}_{ji}|+|\tilde{\delta}_{ji}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-s)g_i(e_i(x,s))\partial s. \tag{4}$$

Then, the system can be written in the following more compact form:

$$\begin{aligned}\frac{\partial^\lambda e(x,t)}{\partial t^\lambda} &= \mathcal{M}\Delta e(x,t)-\mathcal{P}e(x,t)+\mathcal{A}f(\bar{e}(x,t))\\&+\mathcal{B}f(\bar{e}(x,t-\tau(t)))+[|\alpha|+|\beta|]f(\bar{e}(x,t-\tau(t)))\\&+\mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-s)f(\bar{e}(x,s))\partial s\\&+[|\gamma|+|\delta|]\int_{-\infty}^{t}\mathcal{K}(t-s)f(\bar{e}(x,s))\partial s,\\\frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda} &= \tilde{\mathcal{M}}\Delta \bar{e}(x,t)-\tilde{\mathcal{P}}\bar{e}(x,t)+\tilde{\mathcal{A}}g(e(x,t))\\&+\tilde{\mathcal{B}}g(e(x,t-\sigma(t)))+[|\tilde{\alpha}|+|\tilde{\beta}|]g(e(x,t-\sigma(t)))\\&+\tilde{\mathcal{D}}\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-s)g(e(x,s))\partial s\\&+[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-s)g(e(x,s))\partial s,\end{aligned} \tag{5}$$

where

$\mathcal{M}=\mathrm{diag}\{m_1,m_2,.....,m_n\}\in\mathcal{R}^n$,
$\tilde{\mathcal{M}}=\mathrm{diag}\{\tilde{m}_1,\tilde{m}_2,.....,\tilde{m}_m\}\in\mathcal{R}^m$,

$$\mathcal{P}=\begin{bmatrix}p_{11}&\cdot&\cdot&\cdot&p_{1n}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\p_{n1}&\cdot&\cdot&\cdot&p_{nn}\end{bmatrix},\quad \mathcal{A}=\begin{bmatrix}a_{11}&\cdot&\cdot&\cdot&a_{1n}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\a_{n1}&\cdot&\cdot&\cdot&a_{nn}\end{bmatrix},$$

$$\mathcal{B}=\begin{bmatrix}b_{11}&\cdot&\cdot&\cdot&b_{1n}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\b_{n1}&\cdot&\cdot&\cdot&b_{nn}\end{bmatrix},\quad \mathcal{D}=\begin{bmatrix}d_{11}&\cdot&\cdot&\cdot&d_{1n}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\d_{n1}&\cdot&\cdot&\cdot&d_{nn}\end{bmatrix},$$

$$\tilde{\mathcal{P}}=\begin{bmatrix}\tilde{p}_{11}&\cdot&\cdot&\cdot&\tilde{p}_{1m}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\tilde{p}_{m1}&\cdot&\cdot&\cdot&\tilde{p}_{mm}\end{bmatrix},\quad \tilde{\mathcal{A}}=\begin{bmatrix}\tilde{a}_{11}&\cdot&\cdot&\cdot&\tilde{a}_{1m}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\tilde{a}_{m1}&\cdot&\cdot&\cdot&\tilde{a}_{mm}\end{bmatrix},$$

$$\tilde{\mathcal{B}}=\begin{bmatrix}\tilde{b}_{11}&\cdot&\cdot&\cdot&\tilde{b}_{1m}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\tilde{b}_{m1}&\cdot&\cdot&\cdot&\tilde{b}_{mm}\end{bmatrix},\quad \tilde{\mathcal{D}}=\begin{bmatrix}\tilde{d}_{11}&\cdot&\cdot&\cdot&\tilde{d}_{1m}\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\cdot&\cdot&\cdot&\cdot&\cdot\\\tilde{d}_{m1}&\cdot&\cdot&\cdot&\tilde{d}_{mm}\end{bmatrix}.$$

Theorem 1. *Assume that for given positive scalars α, β,γ, δ, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, τ_1^2, τ_{1m}^2, τ_{2m}^2,σ_1^2, σ_{1m}^2, σ_{2m}^2, η, μ the system (5) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1>0$, $\mathcal{S}_2>0$,$\mathcal{R}_1>0$,$\mathcal{R}_2>0$, $\mathcal{R}_3>0$,$\mathcal{R}_4>0$,$\mathcal{R}_5>0$, $\mathcal{T}_1>0$,$\mathcal{T}_2>0$, $\mathcal{T}_3>0$,$\mathcal{T}_4>0$,$\mathcal{T}_5>0$, $\mathcal{W}_1>0$,$\mathcal{W}_2>0$,$\mathcal{W}_3>0$, $\mathcal{Z}_1>0$,$\mathcal{Z}_2>0$, $\mathcal{Z}_3>0$,$\mathcal{H}_4>0$, $\mathcal{H}_5>0$, positive diagonal matrices $\mathcal{Q}_1>0$, $\mathcal{Q}_2>0$, $\mathcal{Q}_3>0$, $\mathcal{Q}_4>0$ with appropriate dimension such that the following LMI holds:*

$$(\Omega_{ij})_{24\times 24}<0. \tag{6}$$

where,

$$\begin{aligned}&\Omega_{(1,1)}=\frac{-\mathcal{S}_1}{l_1^2}\mathcal{M}-\mathcal{S}_1\mathcal{P}+\mathcal{R}_2+\tau_1^2\mathcal{W}_1+\tau_{2m}^2\mathcal{W}_3+\mathcal{Q}_1,\\&\Omega_{(1,2)}=\mathcal{S}_1\mathcal{A},\ \Omega_{(1,3)}=\mathcal{S}_1\mathcal{B}+\mathcal{S}_1[|\alpha|+|\beta|],\\&\Omega_{(1,4)}=\mathcal{S}_1\mathcal{D}+\mathcal{S}_1[|\gamma|+|\delta|],\Omega_{(2,2)}=\mathcal{H}_4+\mathcal{R}_5-\mathcal{Q}_3,\\&\Omega_{(3,3)}=-\mathcal{R}_5(1-\mu),\ \Omega_{(4,4)}=-\mathcal{H}_4,\\&\Omega_{(5,5)}=\mathcal{R}_1-\mathcal{R}_2+\mathcal{R}_3+\tau_{1m}^2\mathcal{W}_2,\\&\Omega_{(6,6)}=-\mathcal{R}_1(1-\mu)+\mathcal{Q}_2,\Omega_{(7,7)}=-\mathcal{R}_3+\mathcal{R}_4,\\&\Omega_{(8,8)}=-\mathcal{R}_4,\Omega_{(9,9)}=-\mathcal{W}_1,\\&\Omega_{(10,10)}=-\mathcal{W}_2,\Omega_{(11,11)}=-\mathcal{W}_3,\\&\Omega_{(12,12)}=\frac{-\mathcal{S}_2}{l_2^2}\tilde{\mathcal{M}}-\mathcal{S}_2\tilde{\mathcal{P}}+\mathcal{T}_2+\sigma_1^2\mathcal{Z}_1+\sigma_{2m}^2\mathcal{Z}_3+\mathcal{Q}_3,\\&\Omega_{(12,13)}=\mathcal{S}_2\tilde{\mathcal{A}},\Omega_{(13,13)}=\mathcal{H}_5+\mathcal{T}_5-\mathcal{Q}_1,\\&\Omega_{(12,14)}=\mathcal{S}_2\tilde{\mathcal{B}}+\mathcal{S}_2[|\tilde{\alpha}|+|\tilde{\beta}|],\Omega_{(14,14)}=-\mathcal{T}_5(1-\eta),\\&\Omega_{(12,15)}=\mathcal{S}_2\tilde{\mathcal{D}}+\mathcal{S}_2[|\tilde{\gamma}|+|\tilde{\delta}|],\\&\Omega_{(15,15)}=-\mathcal{H}_5,\Omega_{(16,16)}=\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3+\sigma_{1m}^2\mathcal{Z}_2,\\&\Omega_{(17,17)}=-\mathcal{T}_1(1-\eta)+\mathcal{Q}_4,\Omega_{(18,18)}=-\mathcal{T}_3+\mathcal{T}_4,\\&\Omega_{(19,19)}=-\mathcal{T}_4,\ \Omega_{(20,20)}=-\mathcal{Z}_1,\\&\Omega_{(21,21)}=-\mathcal{Z}_2,\ \Omega_{(22,22)}=-\mathcal{Z}_3,\\&\Omega_{(23,23)}=-\mathcal{Q}_2,\ \Omega_{(24,24)}=-\mathcal{Q}_4.\end{aligned}$$

Proof. : Let us consider the following Lyapunov Krasovskii functional,

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$$\mathcal{V}_1(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\frac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}^{1-\lambda}_{\mathfrak{t}}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t}) + \frac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}^{1-\lambda}_{\mathfrak{t}}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \tag{7}$$

$$\begin{aligned}\mathcal{V}_2(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{ &\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}-\tau_1} \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s}) \\ &\times \mathcal{R}_2\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau_m}^{\mathfrak{t}-\tau_1} \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{s})\mathcal{R}_3\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \\ &+ \int_{\mathfrak{t}-\tau_2}^{\mathfrak{t}-\tau_m} \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{s})\mathcal{R}_4\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}} \mathfrak{f}^{\mathcal{T}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}) \\ &\times \mathcal{R}_5\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \end{aligned} \tag{8}$$

$$\begin{aligned}\mathcal{V}_3(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{ &\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}-\sigma_1} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s}) \\ &\times \mathcal{T}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\sigma_m}^{\mathfrak{t}-\sigma_1} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \\ &+ \int_{\mathfrak{t}-\sigma_2}^{\mathfrak{t}-\sigma_m} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_4\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \\ &+ \int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}} \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{s})\mathcal{T}_5\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{V}_4(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{ &\tau_1 \int_{-\tau_1}^{0}\int_{\mathfrak{t}+\theta}^{\mathfrak{t}} \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\theta \\ &+ \tau_{1m}\int_{-\tau_m}^{-\tau_1}\int_{\mathfrak{t}+\beta}^{\mathfrak{t}-\tau_1} \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{s})\mathcal{W}_2\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\beta \\ &+ \tau_{2m}\int_{-\tau_2}^{-\tau_m}\int_{\mathfrak{t}+\beta}^{\mathfrak{t}} \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_3\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\beta\}\partial\mathfrak{x}, \end{aligned} \tag{10}$$

$$\begin{aligned}\mathcal{V}_5(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{ &\sigma_1 \int_{-\sigma_1}^{0}\int_{\mathfrak{t}+\theta}^{\mathfrak{t}} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\theta \\ &+ \sigma_{1m}\int_{-\sigma_m}^{-\sigma_1}\int_{\mathfrak{t}+\beta}^{\mathfrak{t}-\sigma_1} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\beta \\ &+ \sigma_{2m}\int_{-\sigma_2}^{-\sigma_m}\int_{\mathfrak{t}+\beta}^{\mathfrak{t}} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\beta\}\partial\mathfrak{x}, \end{aligned} \tag{11}$$

$$\mathcal{V}_6(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\sum_{\mathfrak{j}=1}^{\mathfrak{n}}(\mathfrak{h}_4)_{\mathfrak{j}}\int_0^\infty \mathcal{K}_{\mathfrak{j}}(\theta) \times \int_{\mathfrak{t}-\theta}^{\mathfrak{t}} \mathfrak{f}_{\mathfrak{j}}^2(\bar{\mathfrak{e}}_{\mathfrak{j}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}, \tag{12}$$

$$\mathcal{V}_7(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\sum_{\mathfrak{i}=1}^{\mathfrak{m}}(h_5)_i\int_0^\infty \tilde{\mathcal{K}}_{\mathfrak{i}}(\theta) \times \int_{\mathfrak{t}-\theta}^{\mathfrak{t}} \mathfrak{g}_{\mathfrak{i}}^2(\mathfrak{e}_{\mathfrak{i}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}. \tag{13}$$

From Green's formula and the boundary conditions, the following equality holds:

$$\begin{aligned}\int_\Omega \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{M}\Delta\mathfrak{e}(\mathfrak{x},\mathfrak{t})\partial\mathfrak{x} &= \int_\Omega \mathfrak{e}(\mathfrak{x},\mathfrak{t})\mathfrak{m}\Delta\mathfrak{e}(\mathfrak{x},\mathfrak{t})\partial\mathfrak{x} \\ &= \mathfrak{m}\int_\Omega \mathfrak{e}(\mathfrak{x},\mathfrak{t})\Delta\mathfrak{e}(\mathfrak{x},\mathfrak{t})\partial\mathfrak{x} \\ &= \mathfrak{m}[-\int_\Omega(\frac{\partial\mathfrak{e}(\mathfrak{x},\mathfrak{t})}{\partial\mathfrak{x}})^2]\partial\mathfrak{x}. \end{aligned} \tag{14}$$

According to Lemma (6), the following inequality can be obtained:

$$\begin{aligned}\mathfrak{m}[-\int_\Omega(\frac{\partial\mathfrak{e}(\mathfrak{x},\mathfrak{t})}{\partial\mathfrak{x}}^2)]\partial\mathfrak{x} &\le \mathfrak{m}[-\frac{1}{l^2}\int_\Omega \mathfrak{e}^2(\mathfrak{x},\mathfrak{t})]\partial\mathfrak{x} \\ &= -\frac{1}{l^2}\mathfrak{m}\int_\Omega \mathfrak{e}^2(\mathfrak{x},\mathfrak{t})\partial\mathfrak{x} \\ &= -\frac{1}{l^2}\int_\Omega \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{t})\mathcal{M}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\partial\mathfrak{x}. \end{aligned} \tag{15}$$

Taking the time derivative of V(t) along the trajectories of (5), it is evident that $\mathfrak{e}(\mathfrak{x},\mathfrak{t})$,$\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})$ and $\frac{\partial^\lambda e(x,t)}{\partial t^\lambda}$ and $\frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda}$ are continuous. Then, by Lemma (1) and Lemma (5), the following holds:

$$\begin{aligned}\dot{\mathcal{V}}_1(\mathfrak{x},\mathfrak{t}) = \int_\Omega \mathcal{S}_1\{ &\frac{-1}{l_1^2}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{M}\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{P}\mathfrak{e}(\mathfrak{x},\mathfrak{t}) \\ &+ \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{t})\mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\alpha|+|\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \\ &+ \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{t})\mathcal{D}\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\gamma|+|\delta|]\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x} \\ + \int_\Omega \mathcal{S}_2\{ &\frac{-1}{l_2^2}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{M}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{P}}\,\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) \\ &+ \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) \\ &+ \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\tilde{\alpha}|+|\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{D}\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\ &+ \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\mathcal{V}}_2(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{ &\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau_1)[\mathcal{R}_1-\mathcal{R}_2+\mathcal{R}_3]\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau_1) \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))[-\mathcal{R}_1(1-\mu)]\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})) \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[\mathcal{R}_2]\mathfrak{e}(\mathfrak{x},\mathfrak{t}) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau_m)[-\mathcal{R}_3+\mathcal{R}_4] \\ &\times \mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau_m) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau_2)[-\mathcal{R}_4]\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau_2) \\ &+ \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))[\mathcal{R}_5]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) + \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \\ &\times [-\mathcal{R}_5(1-\mu)]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))\}\partial\mathfrak{x}, \end{aligned} \tag{17}$$

$$\dot{\mathcal{V}}_3(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\sigma_1)[\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3]\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma_1)$$

$$+\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma(t))[-\mathcal{T}_1(1-\eta)]\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)) + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)[\mathcal{T}_2]\bar{\mathfrak{e}}(\mathfrak{x},t) + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_m)[-\mathcal{T}_3+\mathcal{T}_4] \times \bar{\mathfrak{e}}(\mathfrak{x},t-\sigma_m) + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_2)[-\mathcal{T}_4]\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma_2) + \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t))[\mathcal{T}_5]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)) + \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\sigma(t))) \times [-\mathcal{T}_5(1-\eta)]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t-\sigma(t)))\}\partial\mathfrak{x}, \quad (18)$$

$$\dot{\mathcal{V}}_4(\mathfrak{x},t) = \int_\Omega \{\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)[\tau_1^2\mathcal{W}_1 + \tau_{2m}^2\mathcal{W}_3]\mathfrak{e}(\mathfrak{x},t) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau_1)[\tau_{1m}^2\mathcal{W}_2]\mathfrak{e}(\mathfrak{x},t-\tau_1) - \tau_1\int_{t-\tau_1}^{t}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} - \tau_{1m}\int_{t-\tau_m}^{t-\tau_1}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_2\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} - \tau_{2m}\int_{t-\tau_2}^{t-\tau m}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_3\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}dx, \quad (19)$$

$$\dot{\mathcal{V}}_5(\mathfrak{x},t) = \int_\Omega \{\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)[\sigma_1^2\mathcal{Z}_1 + \sigma_{2m}^2\mathcal{Z}_3]\bar{\mathfrak{e}}(\mathfrak{x},t) + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_1)[\sigma_{1m}^2\mathcal{Z}_2]\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma_1) - \sigma_1\int_{t-\sigma_1}^{t}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} - \sigma_{1m}\int_{t-\sigma_m}^{t-\sigma_1}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} - \sigma_{2m}\int_{t-\sigma_2}^{t-\sigma m}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})Z_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x}, \quad (20)$$

$$\dot{\mathcal{V}}_6(\mathfrak{x},t) = \int_\Omega \{\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t))\mathcal{H}_4\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t)) - (\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_4 \times (\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}, \quad (21)$$

$$\dot{\mathcal{V}}_7(\mathfrak{x},t) = \int_\Omega \{\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t))\mathcal{H}_5\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)) - (\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_5 \times (\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}, \quad (22)$$

Using Lemma (4), we have

$$-\tau_1\int_{t-\tau_1}^{t}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le -\Big(\int_{t-\tau_1}^{t}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{W}_1\Big(\int_{t-\tau_1}^{t}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \quad (23)$$

$$-\tau_{1m}\int_{t-\tau_m}^{t-\tau_1}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_2\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le -\Big(\int_{t-\tau_m}^{t-\tau_1}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{W}_2\Big(\int_{t-\tau_m}^{t-\tau_1}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \quad (24)$$

$$-\tau_{2m}\int_{t-\tau_2}^{t-\tau m}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_3\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le -\Big(\int_{t-\tau_2}^{t-\tau_m}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\Big)^{\mathcal{T}}\mathcal{W}_3\Big(\int_{t-\tau_2}^{t-\tau_m}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\Big) \quad (25)$$

$$-\sigma_1\int_{t-\sigma_1}^{t}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le \Big(\int_{t-\sigma_1}^{t}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{Z}_1\Big(\int_{t-\sigma_1}^{t}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \quad (26)$$

$$-\sigma_{1m}\int_{t-\sigma_m}^{t-\sigma_1}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le \Big(\int_{t-\sigma_m}^{t-\sigma_1}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{Z}_2\Big(\int_{t-\sigma_m}^{t-\sigma_1}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \quad (27)$$

$$-\sigma_{2m}\int_{t-\sigma_2}^{t-\sigma m}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le \Big(\int_{t-\sigma_2}^{t-\sigma m}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{Z}_3\Big(\int_{t-\sigma_2}^{t-\sigma m}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \quad (28)$$

From the Assumption 2, we have

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{Q}_1\mathfrak{e}(\mathfrak{x},t) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t))\mathcal{Q}_1\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)), \quad (29)$$

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau(t))\mathcal{Q}_2\mathfrak{e}(\mathfrak{x},t-\tau(t)) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\tau(t)))\mathcal{Q}_2\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t-\tau(t))), \quad (30)$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{Q}_3\bar{\mathfrak{e}}(\mathfrak{x},t) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t))\mathcal{Q}_3\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t)), \quad (31)$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma(t))\mathcal{Q}_4\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)))\mathcal{Q}_4\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t))). \quad (32)$$

Combining from (16) to (32), we have

$$\dot{\mathcal{V}}(\mathfrak{x},t) \le \theta_1^{\mathcal{T}}\Omega\theta_1(t). \quad (33)$$

where,

$$\theta_1 = [\mathfrak{e}^T(\mathfrak{x},t), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t)), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\tau(t))), (\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}, \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau_1), \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau(t)), \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau_m), \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau_2), (\int_{t-\tau_1}^{t}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, (\int_{t-\tau_m}^{t-\tau_1}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, (\int_{t-\tau_2}^{t-\tau_m}\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t)), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\sigma(t))), (\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}, \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_1), \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma(t)), \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_m), \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma_2), (\int_{t-\sigma_1}^{t}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}},$$

$$(\int_{t-\sigma_m}^{t-\sigma_1} \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, (\int_{t-\sigma_2}^{t-\sigma_m} \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}},$$
$$\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\tau(t))), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)))]^{\mathcal{T}}. \quad (34)$$

From condition (6), we have

$$\dot{\mathcal{V}}(\mathfrak{x},t) \leq 0 \quad (35)$$

Therefore, $\dot{\mathcal{V}}(\mathfrak{x},t)$ is negative definite from (6). Therefore, we can conclude that the error system (3) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof. □

Remark 1. *Suppose that we will design a suitable feedback controller, under this case, the system (3) is reduce to the following form:*

In the following, we will design a suitable feedback controller, which are

$$\theta_i(t) = \mathcal{K}_i\mathfrak{e}_i(t),$$
$$\theta_j(t) = \bar{\mathcal{K}}_j\bar{\mathfrak{e}}_j(t), \quad (36)$$

where the feedback gains $\mathcal{K}$, $\bar{\mathcal{K}}$ are to be designed.

$$\frac{\partial^\lambda \mathfrak{e}_i(\mathfrak{x},t)}{\partial t^\lambda} = \sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\mathfrak{m}_{i\mathfrak{k}}\frac{\partial\mathfrak{e}_i(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}}) - \mathfrak{p}_i\mathfrak{e}_i(\mathfrak{x},t)$$
$$+\sum_{j=1}^{n}\mathfrak{a}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t)) + \sum_{j=1}^{n}\mathfrak{b}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}|\alpha_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}|\beta_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}\mathfrak{d}_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{j=1}^{n}|\gamma_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{j=1}^{n}|\delta_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\theta_i(t),$$
$$\frac{\partial^\lambda \bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial t^\lambda} = \sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\tilde{\mathfrak{m}}_{j\mathfrak{k}}\frac{\partial\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}}) - \tilde{\mathfrak{p}}_j\bar{\mathfrak{e}}_j(\mathfrak{x},t)$$
$$+\sum_{i=1}^{m}\tilde{\mathfrak{a}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t)) + \sum_{i=1}^{m}\tilde{\mathfrak{b}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}|\tilde{\alpha}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}|\tilde{\beta}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}\tilde{\mathfrak{d}}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{i=1}^{m}|\tilde{\gamma}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{i=1}^{m}|\tilde{\delta}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\theta_j(t).$$

$$\frac{\partial^\lambda \mathfrak{e}_i(\mathfrak{x},t)}{\partial t^\lambda} = \sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\mathfrak{m}_{i\mathfrak{k}}\frac{\partial\mathfrak{e}_i(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}}) - \mathfrak{p}_i\mathfrak{e}_i(\mathfrak{x},t)$$
$$+\sum_{j=1}^{n}\mathfrak{a}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t)) + \sum_{j=1}^{n}\mathfrak{b}_{ij}\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}|\alpha_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}|\beta_{ij}|\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},t-\tau(t)))$$
$$+\sum_{j=1}^{n}\mathfrak{d}_{ij}\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{j=1}^{n}|\gamma_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{j=1}^{n}|\delta_{ij}|\int_{-\infty}^{t}\mathcal{K}_{ij}(t-\mathfrak{s})\mathfrak{f}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\mathcal{K}_i\mathfrak{e}_i(\mathfrak{x},t),$$
$$\frac{\partial^\lambda \bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial t^\lambda} = \sum_{\mathfrak{k}=1}^{q}\frac{\partial}{\partial\mathfrak{x}_\mathfrak{k}}(\tilde{\mathfrak{m}}_{j\mathfrak{k}}\frac{\partial\bar{\mathfrak{e}}_j(\mathfrak{x},t)}{\partial\mathfrak{x}_\mathfrak{k}}) - \tilde{\mathfrak{p}}_j\bar{\mathfrak{e}}_j(\mathfrak{x},t)$$
$$+\sum_{i=1}^{m}\tilde{\mathfrak{a}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t)) + \sum_{i=1}^{m}\tilde{\mathfrak{b}}_{ji}\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}|\tilde{\alpha}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}|\tilde{\beta}_{ji}|\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},t-\sigma(t)))$$
$$+\sum_{i=1}^{m}\tilde{\mathfrak{d}}_{ji}\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{i=1}^{m}|\tilde{\gamma}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\sum_{i=1}^{m}|\tilde{\delta}_{ji}|\int_{-\infty}^{t}\tilde{\mathcal{K}}_{ji}(t-\mathfrak{s})\mathfrak{g}_i(\mathfrak{e}_i(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\bar{\mathcal{K}}_j\bar{\mathfrak{e}}_j(\mathfrak{x},t).$$

Now we can write in compact form as follows:

$$\frac{\partial^\lambda \mathfrak{e}(\mathfrak{x},t)}{\partial t^\lambda} = \mathcal{M}\Delta(\mathfrak{x},t) - \mathcal{P}\mathfrak{e}(\mathfrak{x},t) + \mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t))$$

$$+\mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))+[|\alpha|+|\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))$$
$$+\mathcal{D}\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+[|\gamma|+|\delta|]\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}+\mathcal{K}\mathfrak{e}(\mathfrak{x},\mathfrak{t}),$$
$$\frac{\partial^{\lambda}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})}{\partial\mathfrak{t}^{\lambda}}=\tilde{\mathcal{M}}\Delta\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})-\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})+\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))$$
$$+\tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))+[|\tilde{\alpha}|+|\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))$$
$$+\tilde{\mathcal{D}}\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}+\bar{\mathcal{K}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}),\tag{37}$$

Theorem 2. *Assume that for given positive scalars α, β, γ, δ, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, τ_1^2, σ_1^2, η, μ the system (37) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1>0$, $\mathcal{S}_2>0$, $\mathcal{R}_6>0$, $\mathcal{R}_7>0$, $\mathcal{R}_8>0$ $\mathcal{R}_9>0$, $\mathcal{W}_5>0$, $\mathcal{W}_6>0$, $\mathcal{H}_7>0$, $\mathcal{H}_8>0$, positive diagonal matrices $\mathcal{Q}_1>0$, $\mathcal{Q}_2>0$, $\mathcal{Q}_3>0$, $\mathcal{Q}_4>0$ with appropriate dimension such that the following LMI holds:*

$$(\Psi_{ij})_{14\times14}<0.\tag{38}$$

where,

$$\Psi_{(1,1)}=\frac{-\mathcal{M}\mathcal{S}_1}{l_1^2}-\mathcal{S}_1\mathcal{P}+\mathcal{S}_1\mathcal{K}+\mathcal{R}_6+\tau_1^2\mathcal{W}_5+\mathcal{Q}_1,$$
$$\Psi_{(1,2)}=\mathcal{S}_1\mathcal{A},\ \Psi_{(1,3)}=\mathcal{S}_1\mathcal{B}+\mathcal{S}_1[|\alpha|+|\beta|],$$
$$\Psi_{(1,4)}=\mathcal{S}_1\mathcal{D}+\mathcal{S}_1[|\gamma|+|\delta|],\ \Psi_{(2,2)}=\mathcal{R}_8+\mathcal{H}_7-\mathcal{Q}_3,$$
$$\Psi_{(3,3)}=-\mathcal{R}_8(1-\mu),\ \Psi_{(4,4)}=-\mathcal{H}_7,$$
$$\Psi_{(5,5)}=-\mathcal{R}_6(1-\mu)+\mathcal{Q}_2,\Psi_{(6,6)}=-\mathcal{W}_5,$$
$$\Psi_{(7,7)}=\frac{-\tilde{\mathcal{M}}\mathcal{S}_2}{l_1^2}-\mathcal{S}_2\tilde{\mathcal{P}}+\mathcal{S}_2\bar{\mathcal{K}}+\mathcal{R}_7+\sigma_1^2\mathcal{W}_6+\mathcal{Q}_3,$$
$$\Psi_{(7,8)}=\mathcal{S}_2\tilde{\mathcal{A}},\Psi_{(7,9)}=\tilde{\mathcal{B}}\mathcal{S}_2+\mathcal{S}_2[|\tilde{\alpha}|+|\tilde{\beta}|],$$
$$\Psi_{(7,10)}=\tilde{\mathcal{D}}\mathcal{S}_2+\mathcal{S}_2[|\tilde{\gamma}|+|\tilde{\delta}|],\ \Psi_{(8,8)}=\mathcal{R}_9+\mathcal{H}_8-\mathcal{Q}_1,$$
$$\Psi_{(9,9)}=-\mathcal{R}_9(1-\eta),\Psi_{(10,10)}=-\mathcal{H}_8,$$
$$\Psi_{(11,11)}=-\mathcal{R}_7(1-\eta)+\mathcal{Q}_4,\Psi_{(12,12)}=-\mathcal{W}_6,$$
$$\Psi_{(13,13)}=-\mathcal{Q}_2,\Psi_{(14,14)}=-\mathcal{Q}_4.$$

Proof: *We Construct Lypunov Krasovskii functional as follows:*

$$\mathcal{V}_1(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\frac{1}{2}{}_{\mathfrak{t}_0}^{\mathcal{RL}}\mathcal{I}_{\mathfrak{t}}^{1-\lambda}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x},\tag{39}$$

$$\mathcal{V}_2(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\frac{1}{2}{}_{\mathfrak{t}_0}^{\mathcal{RL}}\mathcal{I}_{\mathfrak{t}}^{1-\lambda}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x},\tag{40}$$

$$\mathcal{V}_3(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x},\tag{41}$$

$$\mathcal{V}_4(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x},\tag{42}$$

$$\mathcal{V}_5(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}}\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\mathcal{R}_8\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x},\tag{43}$$

$$\mathcal{V}_6(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}}\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\mathcal{R}_9\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x},\tag{44}$$

$$\mathcal{V}_7(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\tau_1\int_{-\tau_1}^{0}\int_{\mathfrak{t}+\theta}^{\mathfrak{t}}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_5\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x},\tag{45}$$

$$\mathcal{V}_8(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\sigma_1\int_{-\sigma_1}^{0}\int_{\mathfrak{t}+\theta}^{\mathfrak{t}}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_6\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\partial\theta\}dx,\tag{46}$$

$$\mathcal{V}_9(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\sum_{\mathfrak{j}=1}^{\mathfrak{n}}(\mathfrak{h}_7)_{\mathfrak{j}}\int_0^{\infty}\mathcal{K}_{\mathfrak{j}}(\theta)$$
$$\times\int_{t-\theta}^{t}\mathfrak{f}_{\mathfrak{j}}^2(\bar{e}_j(x,s))dsd\theta\}dx,\tag{47}$$

$$\mathcal{V}_{10}(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\{\sum_{\mathfrak{i}=1}^{\mathfrak{m}}(\mathfrak{h}_8)_{\mathfrak{i}}\int_0^{\infty}\tilde{\mathcal{K}}_{\mathfrak{i}}(\theta)$$
$$\times\int_{\mathfrak{t}-\theta}^{\mathfrak{t}}\mathfrak{g}_{\mathfrak{i}}^2(\mathfrak{e}_{\mathfrak{i}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}.\tag{48}$$

Taking the time derivative of V(t) along the trajectories of (37), it is evident that $\mathfrak{e}(\mathfrak{x},\mathfrak{t})$,$\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})$ and $\frac{\partial^{\lambda}e(x,t)}{\partial t^{\lambda}}$ and $\frac{\partial^{\lambda}\bar{e}(x,t)}{\partial t^{\lambda}}$ are continuous. Then, by Lemma (1) and Lemma (5), the following holds:

$$\dot{\mathcal{V}}_1(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\mathcal{S}_1\{\frac{-1}{l_1^2}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{M}\mathfrak{e}(\mathfrak{x},\mathfrak{t})-\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{P}\mathfrak{e}(\mathfrak{x},\mathfrak{t})$$
$$+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))$$
$$+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\alpha|+|\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})$$
$$\times\mathcal{D}\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})$$
$$\times[|\gamma|+|\delta|]\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\mathcal{K}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x},\tag{49}$$

$$\dot{\mathcal{V}}_2(\mathfrak{x},\mathfrak{t})=\int_{\Omega}\mathcal{S}_2\{\frac{-1}{l_2^2}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{M}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})-\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})$$
$$+\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))+\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))$$
$$+\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\tilde{\alpha}|+|\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))$$
$$+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{D}}\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$
$$+\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}$$

$$+ \bar{\mathcal{K}}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \tag{50}$$

$$\dot{\mathcal{V}}_3(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})) \times \mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))(1-\mu)\}\partial\mathfrak{x}, \tag{51}$$

$$\dot{\mathcal{V}}_4(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})) \times \mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))(1-\eta)\}\partial\mathfrak{x}, \tag{52}$$

$$\dot{\mathcal{V}}_5(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{R}_8\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \times \mathcal{R}_8\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))(1-\mu)\}\partial\mathfrak{x}, \tag{53}$$

$$\dot{\mathcal{V}}_6(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{R}_9\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) \times \mathcal{R}_9\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))(1-\eta)\}\partial\mathfrak{x}, \tag{54}$$

$$\dot{\mathcal{V}}_7(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tau_1^2\mathcal{W}_5\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \tau_1\int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}^{T}(\mathfrak{x},\mathfrak{s}) \times \mathcal{W}_5\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{55}$$

$$\dot{\mathcal{V}}_8(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\sigma_1^2\mathcal{W}_6\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \sigma_1\int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s}) \times \mathcal{W}_6\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{56}$$

$$\dot{\mathcal{V}}_9(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_7\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) - (\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}} H_7 \times (\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}, \tag{57}$$

$$\dot{\mathcal{V}}_{10}(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_8\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) - (\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_8 \times (\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x} \tag{58}$$

Using Lemma 4, we have

$$-\tau_1\int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{W}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le -\Big(\int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{W}_1\Big(\int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \tag{59}$$

$$-\sigma_1\int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{Z}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} \le \Big(\int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big)^{\mathcal{T}}\mathcal{Z}_1\Big(\int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\Big) \tag{60}$$

From the Assumption 2, we have

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_1\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), \tag{61}$$

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))\mathcal{Q}_2\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))\mathcal{Q}_2\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))), \tag{62}$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_3\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})), \tag{63}$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))\mathcal{Q}_4\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))\mathcal{Q}_4\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))). \tag{64}$$

Combining from (49) to (64), we have

$$\dot{\mathcal{V}}(x,\mathfrak{t}) \le \theta_2^{\mathcal{T}}\Psi\theta_2(\mathfrak{t}). \tag{65}$$

where,

$$\theta_2 = [\mathfrak{e}^T(\mathfrak{x},\mathfrak{t}), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))), (\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}, \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})), (\int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}} \mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))), (\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}, \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})), (\int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}} \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s})^{\mathcal{T}}, \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))]^{\mathcal{T}}. \tag{66}$$

Therefore, we can conclude that the error system (37) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof.

Remark 2. *Suppose, the diffusion terms are not taken in system (5), then the system (5) can be remodified as follows:*

$$\begin{aligned}
\frac{\partial^\lambda \mathfrak{e}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} &= -\mathcal{P}\mathfrak{e}(\mathfrak{x},\mathfrak{t}) + \mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) + \mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \\
&+ [|\alpha| + |\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) \\
&+ \mathcal{D}\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\
&+ [|\gamma| + |\delta|]\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}, \\
\frac{\partial^\lambda \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} &= -\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) + \tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) + \tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) \\
&+ [|\tilde{\alpha}| + |\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) \\
&+ \tilde{\mathcal{D}}\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\
&+ [|\tilde{\gamma}| + |\tilde{\delta}|]\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s},
\end{aligned} \tag{67}$$

Theorem 3. *Assume that for given positive scalars α, β,γ, δ, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, τ_1^2, τ_{1m}^2, τ_{2m}^2,σ_1^2, σ_{1m}^2, σ_{2m}^2, η, μ the system (67) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1>0$, $\mathcal{S}_2>0$,$\mathcal{R}_1>0$,$\mathcal{R}_2>0$, $\mathcal{R}_3>0$,$\mathcal{R}_4>0$,$\mathcal{R}_5>0$, $\mathcal{T}_1>0$,$\mathcal{T}_2>0$, $\mathcal{T}_3>0$,$\mathcal{T}_4>0$,$\mathcal{T}_5>0$, $\mathcal{W}_1>0$,$\mathcal{W}_2>0$,$\mathcal{W}_3>0$, $\mathcal{Z}_1>0$,$\mathcal{Z}_2>0$, $\mathcal{Z}_3>0$,$\mathcal{H}_4>0$, $\mathcal{H}_5>0$, positive diagonal matrices $\mathcal{Q}_1>0$, $\mathcal{Q}_2>0$, $\mathcal{Q}_3>0$, $\mathcal{Q}_4>0$ with appropriate dimension such that the following LMI holds:*

$$(\Phi_{ij})_{24\times 24}<0. \tag{68}$$

where,

$$\begin{aligned}
\Phi_{(1,1)}&=-\mathcal{S}_1\mathcal{P}+\mathcal{R}_2+\tau_1^2\mathcal{W}_1+\tau_{2m}^2\mathcal{W}_3+\mathcal{Q}_1,\\
\Phi_{(1,2)}&=\mathcal{S}_1\mathcal{A},\ \Phi_{(1,3)}=\mathcal{S}_1\mathcal{B}+\mathcal{S}_1[|\alpha|+|\beta|],\\
\Phi_{(1,4)}&=\mathcal{S}_1\mathcal{D}+\mathcal{S}_1[|\gamma|+|\delta|],\Phi_{(2,2)}=\mathcal{H}_4+\mathcal{R}_5-\mathcal{Q}_3,\\
\Phi_{(3,3)}&=-\mathcal{R}_5(1-\mu),\ \Phi_{(4,4)}=-\mathcal{H}_4,\\
\Phi_{(5,5)}&=\mathcal{R}_1-\mathcal{R}_2+\mathcal{R}_3+\tau_{1m}^2\mathcal{W}_2,\\
\Phi_{(6,6)}&=-\mathcal{R}_1(1-\mu)+\mathcal{Q}_2,\Phi_{(7,7)}=-\mathcal{R}_3+\mathcal{R}_4,\\
\Phi_{(8,8)}&=-\mathcal{R}_4,\Phi_{(9,9)}=-\mathcal{W}_1,\\
\Phi_{(10,10)}&=-\mathcal{W}_2,\Phi_{(11,11)}=-\mathcal{W}_3,\\
\Phi_{(12,12)}&=-\mathcal{S}_2\tilde{\mathcal{P}}+\mathcal{T}_2+\sigma_1^2\mathcal{Z}_1+\sigma_{2m}^2\mathcal{Z}_3+\mathcal{Q}_3,\\
\Phi_{(12,13)}&=\mathcal{S}_2\tilde{\mathcal{A}},\Phi_{(13,13)}=\mathcal{H}_5+\mathcal{T}_5-\mathcal{Q}_1,\\
\Phi_{(12,14)}&=\mathcal{S}_2\tilde{\mathcal{B}}+\mathcal{S}_2[|\tilde{\alpha}|+|\tilde{\beta}|],\Phi_{(14,14)}=-\mathcal{T}_5(1-\eta),\\
\Phi_{(12,15)}&=\mathcal{S}_2\tilde{\mathcal{D}}+\mathcal{S}_2[|\tilde{\gamma}|+|\tilde{\delta}|],\\
\Phi_{(15,15)}&=-\mathcal{H}_5,\Phi_{(16,16)}=\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3+\sigma_{1m}^2\mathcal{Z}_2,\\
\Phi_{(17,17)}&=-\mathcal{T}_1(1-\eta)+\mathcal{Q}_4,\Phi_{(18,18)}=-\mathcal{T}_3+\mathcal{T}_4,\\
\Phi_{(19,19)}&=-\mathcal{T}_4,\ \Phi_{(20,20)}=-\mathcal{Z}_1,\\
\Phi_{(21,21)}&=-\mathcal{Z}_2,\ \Phi_{(22,22)}=-\mathcal{Z}_3,\\
\Phi_{(23,23)}&=-\mathcal{Q}_2,\Phi_{(24,24)}=-\mathcal{Q}_4.
\end{aligned}$$

From condition (68), we have

$$\dot{\mathcal{V}}(x,t)\le 0 \tag{69}$$

Proof. The proof is similar to that in the proof of Theorem 1 by neglecting diffusion term. □

Remark 3. *Suppose, the discrete delays are not appear in system (5), then the system (5) can be remodified as follows:*

$$\begin{aligned}
\frac{\partial^\lambda \mathfrak{e}(\mathfrak{x},t)}{\partial t^\lambda}&=\mathcal{M}\Delta e(x,t)-\mathcal{P}e(x,t)+\mathcal{A}f(\bar{e}(x,t))\\
&+\mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\\
&+[|\gamma|+|\delta|]\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s},\\
\frac{\partial^\lambda \bar{\mathfrak{e}}(\mathfrak{x},t)}{\partial t^\lambda}&=\tilde{\mathcal{M}}\Delta\bar{\mathfrak{e}}(\mathfrak{x},t)-\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},t)+\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t))\\
&+\tilde{\mathcal{D}}\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\\
&+[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}.
\end{aligned} \tag{70}$$

Theorem 4. *Assume that for given positive scalars γ, δ, $\tilde{\gamma}$, $\tilde{\delta}$, the system (70) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1>0$, $\mathcal{S}_2>0$, $\mathcal{H}_4>0$, $\mathcal{H}_5>0$, positive diagonal matrices $\mathcal{Q}_1>0$, $\mathcal{Q}_3>0$ with appropriate dimension such that the following LMI holds:*

$$(\Upsilon_{i,j})_{6,6}<0. \tag{71}$$

where,

$$\begin{aligned}
\Upsilon_{(1,1)}&=\frac{-\mathcal{S}_1}{l_1^2}\mathcal{M}-\mathcal{S}_1\mathcal{P}+\mathcal{Q}_1,\ \Upsilon_{(1,2)}=\mathcal{S}_1\mathcal{A},\\
\Upsilon_{(1,3)}&=\mathcal{S}_1\mathcal{D}+\mathcal{S}_1[|\gamma|+|\delta|],\Upsilon_{(2,2)}=\mathcal{H}_4-\mathcal{Q}_3,\\
\Upsilon_{(3,3)}&=-\mathcal{H}_4,\Upsilon_{(4,4)}=\frac{-\mathcal{S}_2}{l_2^2}\tilde{\mathcal{M}}-\mathcal{S}_2\tilde{\mathcal{P}}+\mathcal{Q}_3\\
\Upsilon_{(4,5)}&=\tilde{\mathcal{A}}\mathcal{S}_2,\Upsilon_{(4,6)}=\tilde{\mathcal{D}}\mathcal{S}_2+\mathcal{S}_2[|\tilde{\gamma}|+|\tilde{\delta}|],\\
\Upsilon_{(5,5)}&=\mathcal{H}_5-\mathcal{Q}_1,\ \Upsilon_{(6,6)}=-\mathcal{H}_5.
\end{aligned}$$

Proof. : Now let us define Lyapunov Krasovskii functional as follows:

$$\mathcal{V}_1(\mathfrak{x},t)=\int_\Omega\{\frac{1}{2}{}^{\mathcal{RL}}_{t_0}\mathcal{I}_t^{1-\lambda}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{S}_1\mathfrak{e}(\mathfrak{x},t)\}\partial\mathfrak{x}, \tag{72}$$

$$\mathcal{V}_2(\mathfrak{x},t)=\int_\Omega\{\frac{1}{2}{}^{\mathcal{RL}}_{t_0}\mathcal{I}_t^{1-\lambda}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},t)\}\partial\mathfrak{x}, \tag{73}$$

$$\begin{aligned}
\mathcal{V}_3(\mathfrak{x},t)=&\int_\Omega\{\sum_{j=1}^{n}(\mathfrak{h}_4)_j\int_0^\infty\mathcal{K}_j(\theta)\\
&\times\int_{t-\theta}^{t}\mathfrak{f}^2{}_j(\bar{\mathfrak{e}}_j(\mathfrak{x},\mathfrak{s})))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x},
\end{aligned}\tag{74}$$

$$\begin{aligned}
\mathcal{V}_4(\mathfrak{x},t)=&\int_\Omega\{\sum_{i=1}^{m}(\mathfrak{h}_5)_i\int_0^\infty\tilde{\mathcal{K}}_i(\theta)\\
&\times\int_{t-\theta}^{t}\mathfrak{g}_i^2(\mathfrak{e}_i(\mathfrak{g}_i(\mathfrak{x},\mathfrak{s})))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}.
\end{aligned}\tag{75}$$

Taking the time derivative of V(t) along the trajectories of (70), it is evident that $\mathfrak{e}(\mathfrak{x},t)$,$\bar{\mathfrak{e}}(\mathfrak{x},t)$ and $\frac{\partial^\lambda e(x,t)}{\partial t^\lambda}$ and$\frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda}$ are continuous. Then, by Lemma (1) and Lemma (5), the following holds:

$$\begin{aligned}
\dot{\mathcal{V}}_1(\mathfrak{x},t)=&\int_\Omega\mathcal{S}_1\{\frac{-1}{l_1^2}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{M}\mathfrak{e}(\mathfrak{x},t)-\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{P}\mathfrak{e}(\mathfrak{x},t)\\
&+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t))+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\\
&\times\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}+\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)[|\gamma|+|\delta|]\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\\
&\times\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x},
\end{aligned}\tag{76}$$

$$\dot{\mathcal{V}}_2(\mathfrak{x},t)=\int_\Omega\mathcal{S}_2\{\frac{-1}{l_2^2}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{M}}\bar{\mathfrak{e}}(\mathfrak{x},t)-\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},t)$$

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$$+ \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{D}}\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s}) \times \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s}) \times \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \quad (77)$$

$$\dot{\mathcal{V}}_3(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_4\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) - (\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s}) \times \mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_4(\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}, \quad (78)$$

$$\dot{\mathcal{V}}_4(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_5\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) - (\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s}) \times \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_5(\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}, \quad (79)$$

From the Assumption 2, we have

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_1\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), \quad (80)$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_3\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})), \quad (81)$$

$$\dot{\mathcal{V}}(x,\mathfrak{t}) \le \theta_3^{\mathcal{T}}\Upsilon\theta_3(\mathfrak{t}). \quad (82)$$

where,

$$\theta_3 = [\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})), (\int_{-\infty}^{\mathfrak{t}}\mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}, \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t}), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), (\int_{-\infty}^{\mathfrak{t}}\tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}^{\mathcal{T}}]^{\mathcal{T}}.$$

From condition (71), we have

$$\dot{\mathcal{V}}(x,\mathfrak{t}) \le 0.$$

Therefore, we can conclude that the error system (70) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof. □

Remark 4. *Suppose, the distributed delays are not appear in system (5), then the system (5) can be remodified as follows:*

$$\frac{\partial^\lambda \mathfrak{e}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} = \mathcal{M}\Delta\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathcal{P}\mathfrak{e}(\mathfrak{x},\mathfrak{t}) + \mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) + \mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) + [|\alpha|+|\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))),$$
$$\frac{\partial^\lambda \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} = \tilde{\mathcal{M}}\Delta\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) + \tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) + \tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) + [|\tilde{\alpha}|+|\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))). \quad (83)$$

Theorem 5. *Assume that for given positive scalars α, β, $\tilde{\alpha}$, $\tilde{\beta}$, η, μ the system (83) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1 > 0$, $\mathcal{S}_2 > 0$, $\mathcal{R}_1 > 0$, $\mathcal{R}_2 > 0$, $\mathcal{R}_3 > 0$, $\mathcal{R}_4 > 0$, $\mathcal{R}_5 > 0$, $\mathcal{T}_1 > 0$, $\mathcal{T}_2 > 0$, $\mathcal{T}_3 > 0$, $\mathcal{T}_4 > 0$, $\mathcal{T}_5 > 0$, positive diagonal matrices $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{Q}_3 > 0$, $\mathcal{Q}_4 > 0$ with appropriate dimension such that the following LMI holds:*

$$(\Lambda_{i,j})_{16\times16} < 0. \quad (84)$$

where,

$$\Lambda_{(1,1)} = \frac{-\mathcal{S}_1}{\mathfrak{l}_1{}^2}\mathcal{M} - \mathcal{S}_1\mathcal{P} + \mathcal{R}_2 + \mathcal{Q}_1,$$
$$\Lambda_{(1,2)} = \mathcal{S}_1\mathcal{A},\ \Lambda_{(2,2)} = \mathcal{R}_5 - \mathcal{Q}_3,$$
$$\Lambda_{(1,3)} = \mathcal{S}_1\mathcal{B} + \mathcal{S}_1[|\alpha|+|\beta|], \Lambda_{(3,3)} = -\mathcal{R}_5(1-\mu),$$
$$\Lambda_{(4,4)} = \mathcal{R}_1 + \mathcal{R}_3 - \mathcal{R}_2, \Lambda_{(5,5)} = -\mathcal{R}_1(1-\mu) + \mathcal{Q}_2,$$
$$\Lambda_{(6,6)} = -\mathcal{R}_3 + \mathcal{R}_4,\ \Lambda_{(7,7)} = -\mathcal{R}_4,$$
$$\Lambda_{(8,8)} = \frac{-\mathcal{S}_2}{\mathfrak{l}_2^2}\tilde{\mathcal{M}} - \mathcal{S}_2\tilde{\mathcal{P}} + \mathcal{T}_2 + \mathcal{Q}_3,$$
$$\Lambda_{(8,9)} = \tilde{\mathcal{A}}\mathcal{S}_2,\ \Lambda_{(8,10)} = \tilde{\mathcal{B}}\mathcal{S}_2 + \mathcal{S}_2[|\tilde{\alpha}|+|\tilde{\beta}|],$$
$$\Lambda_{(9,9)} = \mathcal{T}_5 - \mathcal{Q}_1,\ \Lambda_{(10,10)} = -\mathcal{T}_5(1-\eta),$$
$$\Lambda_{(11,11)} = \mathcal{T}_1 - \mathcal{T}_2 + \mathcal{T}_3, \Lambda_{(12,12)} = -\mathcal{T}_1(1-\eta) + \mathcal{Q}_4,$$
$$\Lambda_{(13,13)} = -\mathcal{T}_3 + \mathcal{T}_4,\ \Lambda_{(14,14)} = -\mathcal{T}_4,$$
$$\Lambda_{(15,15)} = -\mathcal{Q}_2, \Lambda_{(16,16)} = -\mathcal{Q}_4.$$

Proof. : *Now let us define Lyapunov Krasovskii functional as follows:*

$$\mathcal{V}_1(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\frac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}_{\mathfrak{t}}^{1-\lambda}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \quad (85)$$

$$\mathcal{V}_2(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\frac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}_{\mathfrak{t}}^{1-\lambda}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \quad (86)$$

$$\mathcal{V}_3(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}-\tau_1}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_1\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau_1}^{\mathfrak{t}}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_2\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau_m}^{\mathfrak{t}-\tau_1}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_3\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau_2}^{\mathfrak{t}-\tau_m}e^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_4\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}}\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\mathcal{R}_5\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \quad (87)$$

$$\mathcal{V}_4(\mathfrak{x},\mathfrak{t}) = \int_{\Omega}\{\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}-\sigma_1}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_1\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\sigma_1}^{\mathfrak{t}}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s} + \int_{\mathfrak{t}-\sigma_m}^{\mathfrak{t}-\sigma_1}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{s})\mathcal{T}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}$$

$$+\int_{t-\sigma_2}^{t-\sigma_m} \bar{e}^{\mathcal{T}}(x,s)\mathcal{T}_4\bar{e}(x,s)\partial s + \int_{t-\sigma(t)}^{t} g^{\mathcal{T}}(e(x,s)\mathcal{T}_5 g(e(x,s))\partial s\}dx, \tag{88}$$

Taking the time derivative of V(t) along the trajectories of (83), it is evident that $e(x,t)$,$\bar{e}(x,t)$ *and* $\frac{\partial^\lambda e(x,t)}{\partial t^\lambda}$ *and* $\frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda}$ *are continuous. Then, by Lemma (1) and Lemma (5), the following holds:*

$$\begin{aligned}\dot{\mathcal{V}}_1(x,t) = \int_\Omega \mathcal{S}_1\{\frac{-1}{l_1^2}e^{\mathcal{T}}(x,t)\mathcal{M}e(x,t) - e^{\mathcal{T}}(x,t)\mathcal{P}e(x,t)\\ + e^{\mathcal{T}}(x,t)\mathcal{A}f(\bar{e}(x,t)) + e^{\mathcal{T}}(x,t)\mathcal{B}f(\bar{e}(x,t-\tau(t)))\\ + e^{\mathcal{T}}(x,t)[|\alpha|+|\beta|]f(\bar{e}(x,t-\tau(t)))\}\partial x,\end{aligned} \tag{89}$$

$$\begin{aligned}\dot{\mathcal{V}}_2(x,t) = \int_\Omega \mathcal{S}_2\{\frac{-1}{l_2^2}\bar{e}^{\mathcal{T}}(x,t)\tilde{\mathcal{M}}\bar{e}(x,t) - \bar{e}^{\mathcal{T}}(x,t)\tilde{\mathcal{P}}\bar{e}(x,t)\\ + \bar{e}^{\mathcal{T}}(x,t)\tilde{\mathcal{A}}g(e(x,t)) + \bar{e}^{\mathcal{T}}(x,t)\tilde{\mathcal{B}}g(e(x,t-\sigma(t)))\\ + \bar{e}^{\mathcal{T}}(x,t)[|\tilde{\alpha}|+|\tilde{\beta}|]g(e(x,t-\sigma(t)))\}\partial x,\end{aligned} \tag{90}$$

$$\begin{aligned}\dot{\mathcal{V}}_3(x,t) = \int_\Omega \{e^{\mathcal{T}}(x,t-\tau_1)[\mathcal{R}_1-\mathcal{R}_2+\mathcal{R}_3]e(x,t-\tau_1)\\ + e^{\mathcal{T}}(x,t-\tau(t))[-\mathcal{R}_1(1-\mu)]e(x,t-\tau(t))\\ + e^{\mathcal{T}}(x,t)[\mathcal{R}_2]e(x,t) + e^{\mathcal{T}}(x,t-\tau_m)[-\mathcal{R}_3+\mathcal{R}_4]\\ \times e(x,t-\tau_m) + e^{\mathcal{T}}(x,t-\tau_2)[-\mathcal{R}_4]e(x,t-\tau_2)\\ + f^{\mathcal{T}}(\bar{e}(x,t))[\mathcal{R}_5]f(\bar{e}(x,t)) + f^{\mathcal{T}}(\bar{e}(x,t-\tau(t)))\\ \times[-\mathcal{R}_5(1-\mu)]f(\bar{e}(x,t-\tau(t)))\}\partial x,\end{aligned} \tag{91}$$

$$\begin{aligned}\dot{\mathcal{V}}_4(x,t) = \int_\Omega \{\bar{e}^{\mathcal{T}}(x,t-\sigma_1)[\mathcal{T}_1-\mathcal{T}_2+\mathcal{T}_3]\bar{e}(x,t-\sigma_1)\\ + \bar{e}^{\mathcal{T}}(x,t-\sigma(t))[-\mathcal{T}_1(1-\eta)]\bar{e}(x,t-\sigma(t))\\ + \bar{e}^{\mathcal{T}}(x,t)[\mathcal{T}_2]\bar{e}(x,t) + \bar{e}^{\mathcal{T}}(x,t-\sigma_m)[-\mathcal{T}_3+\mathcal{T}_4]\\ \bar{e}(x,t-\sigma_m) + \bar{e}^{\mathcal{T}}(x,t-\sigma_2)[-\mathcal{T}_4]\bar{e}(x,t-\sigma_2)\\ + g^{\mathcal{T}}(e(x,t))[\mathcal{T}_5]g(e(x,t)) + g^{\mathcal{T}}(e(x,t-\sigma(t)))\\ \times[-\mathcal{T}_5(1-\eta)]g(e(x,t-\sigma(t)))\}\partial x,\end{aligned} \tag{92}$$

From the Assumption 2, we have

$$0 \le e^{\mathcal{T}}(x,t)\mathcal{Q}_1 e(x,t) - g^{\mathcal{T}}(e(x,t))\mathcal{Q}_1 g(e(x,t)), \tag{93}$$

$$\begin{aligned}0 \le e^{\mathcal{T}}(x,t-\tau(t))\mathcal{Q}_2 e(x,t-\tau(t))\\ - g^{\mathcal{T}}(e(x,t-\tau(t)))\mathcal{Q}_2 g(e(x,t-\tau(t))),\end{aligned} \tag{94}$$

$$0 \le \bar{e}^{\mathcal{T}}(x,t)\mathcal{Q}_3\bar{e}(x,t) - f^{\mathcal{T}}(\bar{e}(x,t))\mathcal{Q}_3 f(\bar{e}(x,t)), \tag{95}$$

$$\begin{aligned}0 \le \bar{e}^{\mathcal{T}}(x,t-\sigma(t))\mathcal{Q}_4\bar{e}(x,t-\sigma(t))\\ - f^{\mathcal{T}}(\bar{e}(x,t-\sigma(t)))\mathcal{Q}_4 f(\bar{e}(x,t-\sigma(t))).\end{aligned} \tag{96}$$

$$\dot{\mathcal{V}}(x,t) \le \theta_4^{\mathcal{T}}\Lambda\theta_4(t). \tag{97}$$

where,

$$\begin{aligned}\theta_4 = [&e^{\mathcal{T}}(x,t), f^{\mathcal{T}}(\bar{e}(x,t)), f^{\mathcal{T}}(\bar{e}(x,t-\tau(t))),\\ &e^{\mathcal{T}}(x,t-\tau_1), e^{\mathcal{T}}(x,t-\tau(t)), e^{\mathcal{T}}(x,t-\tau_m), e^{\mathcal{T}}(x,t-\tau_2),\\ &\bar{e}^{\mathcal{T}}(x,t), g^{\mathcal{T}}(e(x,t)), g^{\mathcal{T}}(e(x,t-\sigma(t))), \bar{e}^{\mathcal{T}}(x,t-\sigma_1),\\ &\bar{e}^{\mathcal{T}}(x,t-\sigma(t)), \bar{e}^{\mathcal{T}}(x,t-\sigma_m), \bar{e}^{\mathcal{T}}(x,t-\sigma_2), g^{\mathcal{T}}(e(x,t-\tau(t)))\\ &f^{\mathcal{T}}(\bar{e}(x,t-\sigma(t)))]^{\mathcal{T}}.\end{aligned}$$

From condition (84), we have

$$\dot{\mathcal{V}}(x,t) \le 0. \tag{98}$$

Therefore, we can conclude that the error system (83) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof. □

Remark 5. *Suppose, the diffusion terms are not appear in system (37), then the system (37) can be remodified as follows:*

$$\begin{aligned}\frac{\partial^\lambda e(x,t)}{\partial t^\lambda} &= -\mathcal{P}e(x,t) + \mathcal{A}f(\bar{e}(x,t)) + \mathcal{B}f(\bar{e}(x,t-\tau(t)))\\ &+ [|\alpha|+|\beta|]f(\bar{e}(x,t-\tau(t)))\\ &+ \mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-s)f(\bar{e}(x,s))\partial s\\ &+ [|\gamma|+|\delta|]\int_{-\infty}^{t}\mathcal{K}(t-s)f(\bar{e}(x,s))\partial s + \mathcal{K}e(x,t),\\ \frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda} &= -\tilde{\mathcal{P}}\bar{e}(x,t) + \tilde{\mathcal{A}}g(e(x,t)) + \tilde{\mathcal{B}}g(e(x,t-\sigma(t)))\\ &+ [|\tilde{\alpha}|+|\tilde{\beta}|]g(e(x,t-\sigma(t)))\\ &+ \tilde{\mathcal{D}}\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-s)g(e(x,s))\partial s\\ &+ [|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-s)g(e(x,s))\partial s + \bar{\mathcal{K}}\bar{e}(x,t).\end{aligned} \tag{99}$$

Theorem 6. *Assume that for given positive scalars* α, β,γ, δ, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, $\tilde{\delta}$, τ_1^2, σ_1^2, η, μ *the system (99) is globally asymptotically stable, if there exist positive matrices* $\mathcal{S}_1 > 0$, $\mathcal{S}_2 > 0$, $\mathcal{R}_6 > 0$, $\mathcal{R}_7 > 0$, $\mathcal{R}_8 > 0$, $\mathcal{R}_9 > 0$, $\mathcal{W}_5 > 0$, $\mathcal{W}_6 > 0$, $\mathcal{H}_7 > 0$, $\mathcal{H}_8 > 0$, *positive diagonal matrices* $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{Q}_3 > 0$, $\mathcal{Q}_4 > 0$ *with appropriate dimension such that the following LMI holds:*

$$(\Xi_{i,j})_{14\times 14} < 0. \tag{100}$$

where,

$$\begin{aligned}&\Xi_{(1,1)} = -\mathcal{S}_1\mathcal{P} + \mathcal{S}_1\mathcal{K} + \mathcal{R}_6 + {\tau_1}^2\mathcal{W}_5 + \mathcal{Q}_1,\\ &\Xi_{(1,2)} = \mathcal{S}_1\mathcal{A},\ \Xi_{(1,3)} = \mathcal{S}_1\mathcal{B} + \mathcal{S}_1[|\alpha|+|\beta|],\\ &\Xi_{(1,4)} = \mathcal{S}_1\mathcal{D} + \mathcal{S}_1[|\gamma|+|\delta|],\ \Xi_{(2,2)} = \mathcal{R}_8 + \mathcal{H}_7 - \mathcal{Q}_3,\end{aligned}$$

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$$\Xi_{(3,3)} = -\mathcal{R}_8(1-\mu),\ \Xi_{(4,4)} = -\mathcal{H}_7,$$
$$\Xi_{(5,5)} = -\mathcal{R}_6(1-\mu) + \mathcal{Q}_2, \Xi_{(6,6)} = -\mathcal{W}_5,$$
$$\Xi_{(7,7)} = -\mathcal{S}_2\tilde{\mathcal{P}} + \mathcal{S}_2\bar{\mathcal{K}} + \mathcal{R}_7 + \sigma_1^2\mathcal{W}_6 + \mathcal{Q}_3, \Xi_{(7,8)} = \mathcal{S}_2\tilde{\mathcal{A}},$$
$$\Xi_{(7,9)} = \tilde{\mathcal{B}}\mathcal{S}_2 + \mathcal{S}_2[|\tilde{\alpha}| + |\tilde{\beta}|],$$
$$\Xi_{(7,10)} = \tilde{\mathcal{D}}\mathcal{S}_2 + \mathcal{S}_2[|\tilde{\gamma}| + |\tilde{\delta}|],\ \Xi_{(8,8)} = \mathcal{R}_9 + \mathcal{H}_8 - \mathcal{Q}_1,$$
$$\Xi_{(9,9)} = -\mathcal{R}_9(1-\eta), \Xi_{(10,10)} = -\mathcal{H}_8,$$
$$\Xi_{(11,11)} = -\mathcal{R}_7(1-\eta) + \mathcal{Q}_4, \Xi_{(12,12)} = -\mathcal{W}_6,$$
$$\Xi_{(13,13)} = -\mathcal{Q}_2, \Xi_{(14,14)} = -\mathcal{Q}_4.$$

Proof. : *The proof is similar to that in the proof of Theorem 2 by neglecting the diffusion terms in Theorem 2.* □

Remark 6. *Suppose, the distributed delays are not appear in system (37), then the system (37) can be remodified as follows:*

$$\begin{aligned}\frac{\partial^\lambda \mathfrak{e}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} &= \mathcal{M}\Delta\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - P\mathfrak{e}(\mathfrak{x},\mathfrak{t}) + \mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\\ &+ \mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) + [|\alpha| + |\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))\\ &+ \mathcal{K}\mathfrak{e}(\mathfrak{x},\mathfrak{t}),\\ \frac{\partial^\lambda \bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})}{\partial \mathfrak{t}^\lambda} &= \tilde{\mathcal{M}}\Delta\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \tilde{P}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) + \tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\\ &+ \tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) + [|\tilde{\alpha}| + |\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))\\ &+ \bar{\mathcal{K}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}).\end{aligned} \tag{101}$$

Theorem 7. *Assume that for given positive scalars α, β, $\tilde{\alpha}$, $\tilde{\beta}$, τ_1^2, σ_1^2,η, μ the system (101) is globally asymptotically stable,if there exist positive matrices $\mathcal{S}_1 > 0$, $\mathcal{S}_2 > 0$, $\mathcal{R}_6 > 0$,$\mathcal{R}_7 > 0$, $\mathcal{R}_8 > 0$, $\mathcal{R}_9 > 0$, positive diagonal matrices $\mathcal{Q}_1 > 0$, $\mathcal{Q}_2 > 0$, $\mathcal{Q}_3 > 0$, $\mathcal{Q}_4 > 0$ with appropriate dimension such that the following LMI holds:*

$$(\Gamma_{i,j})_{10\times10} < 0. \tag{102}$$

where

$$\Gamma_{(1,1)} = \frac{-\mathcal{M}\mathcal{S}_1}{l_1^2} - \mathcal{S}_1\mathcal{P} + \mathcal{S}_1\mathcal{K} + \mathcal{R}_6 + \mathcal{Q}_1,\ \Gamma_{(1,2)} = \mathcal{S}_1\mathcal{A},$$
$$\Gamma_{(1,3)} = \mathcal{S}_1\mathcal{B} + \mathcal{S}_1[|\alpha| + |\beta|], \Gamma_{(2,2)} = \mathcal{R}_8 - \mathcal{Q}_3,$$
$$\Gamma_{(3,3)} = -\mathcal{R}_8(1-\mu), \Gamma_{(4,4)} = -\mathcal{R}_6(1-\mu) + \mathcal{Q}_2,$$
$$\Gamma_{(5,5)} = \frac{-\tilde{\mathcal{M}}\mathcal{S}_2}{l_1^2} - \mathcal{S}_2\tilde{\mathcal{P}} + \mathcal{S}_2\bar{\mathcal{K}} + \mathcal{R}_7 + \mathcal{Q}_3, \Gamma_{(5,6)} = \tilde{\mathcal{A}}\mathcal{S}_2,$$
$$\Gamma_{(5,7)} = \tilde{\mathcal{B}}\mathcal{S}_2 + \mathcal{S}_2[|\tilde{\alpha}| + |\tilde{\beta}|], \Gamma_{(6,6)} = \mathcal{R}_9 - \mathcal{Q}_1,$$
$$\Gamma_{(7,7)} = -\mathcal{R}_9(1-\eta),\ \Gamma_{(8,8)} = -\mathcal{R}_7(1-\eta) + \mathcal{Q}_4,$$
$$\Gamma_{(9,9)} = -\mathcal{Q}_2, \Gamma_{(10,10)} = -\mathcal{Q}_4.$$

Proof. Now let us define Lyapunov Krasovskii functional as follows:

$$\mathcal{V}_1(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\tfrac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}_\mathfrak{t}^{1-\lambda}\mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{S}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \tag{103}$$

$$\mathcal{V}_2(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\tfrac{1}{2}{}^{\mathcal{RL}}_{\mathfrak{t}_0}\mathcal{I}_\mathfrak{t}^{1-\lambda}\bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \tag{104}$$

$$\mathcal{V}_3(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}} \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{s})\mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{105}$$

$$\mathcal{V}_4(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}} \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{s})\mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s})\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{106}$$

$$\mathcal{V}_5(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\int_{\mathfrak{t}-\tau(\mathfrak{t})}^{\mathfrak{t}} \mathfrak{f}^\mathcal{T}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\mathcal{R}_8\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{107}$$

$$\mathcal{V}_6(\mathfrak{x},\mathfrak{t}) = \int_\Omega \{\int_{\mathfrak{t}-\sigma(\mathfrak{t})}^{\mathfrak{t}} \mathfrak{g}^\mathcal{T}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\mathcal{R}_9\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}\}\partial\mathfrak{x}, \tag{108}$$

Taking the time derivative of V(t) along the trajectories of (101), it is evident that $\mathfrak{e}(\mathfrak{x},\mathfrak{t})$,$\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})$ and $\frac{\partial^\lambda e(x,t)}{\partial t^\lambda}$ and$\frac{\partial^\lambda \bar{e}(x,t)}{\partial t^\lambda}$ are continuous. Then, by Lemma (1) and Lemma (5), the following holds:

$$\begin{aligned}\dot{\mathcal{V}}_1(\mathfrak{x},\mathfrak{t}) = \int_\Omega \mathcal{S}_1\{&\frac{-1}{\mathfrak{l}_1^2}\mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{M}\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{P}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\\ &+ \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) + \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{B}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))\\ &+ \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})[|\alpha| + |\beta|]\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))) + \mathcal{K}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x},\end{aligned} \tag{109}$$

$$\begin{aligned}\dot{\mathcal{V}}_2(\mathfrak{x},\mathfrak{t}) = \int_\Omega \mathcal{S}_2\{&\frac{-1}{\mathfrak{l}_2^2}\bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{M}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\\ &+ \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) + \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\tilde{\mathcal{B}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))\\ &+ \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})[|\tilde{\alpha}| + |\tilde{\beta}|]\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))) + \bar{\mathcal{K}}\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x},\end{aligned} \tag{110}$$

$$\begin{aligned}\dot{\mathcal{V}}_3(\mathfrak{x},\mathfrak{t}) = \int_\Omega &\{\mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{e}^\mathcal{T}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))\\ &\times \mathcal{R}_6\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t}))(1-\mu)\}\partial\mathfrak{x},\end{aligned} \tag{111}$$

$$\begin{aligned}\dot{\mathcal{V}}_4(\mathfrak{x},\mathfrak{t}) = \int_\Omega &\{\bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t})\mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \bar{\mathfrak{e}}^\mathcal{T}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))\\ &\times \mathcal{R}_7\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t}))(1-\eta)\}\partial\mathfrak{x},\end{aligned} \tag{112}$$

$$\begin{aligned}\dot{\mathcal{V}}_5(\mathfrak{x},\mathfrak{t}) = \int_\Omega &\{\mathfrak{f}^\mathcal{T}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{R}_8\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) - \mathfrak{f}^\mathcal{T}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))\\ &\times \mathcal{R}_8\mathfrak{f}^\mathcal{T}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}-\tau(\mathfrak{t})))(1-\mu)\}\partial\mathfrak{x},\end{aligned} \tag{113}$$

$$\begin{aligned}\dot{\mathcal{V}}_6(\mathfrak{x},\mathfrak{t}) = \int_\Omega &\{\mathfrak{g}^\mathcal{T}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{R}_9\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})) - \mathfrak{g}^\mathcal{T}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))\\ &\times \mathcal{R}_9\mathfrak{g}^\mathcal{T}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}-\sigma(\mathfrak{t})))(1-\eta)\}\partial\mathfrak{x},\end{aligned} \tag{114}$$

From the Assumption 2, we have

$$0 \leq \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{Q}_1\mathfrak{e}(\mathfrak{x},t) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t))\mathcal{Q}_1\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)), \quad (115)$$

$$0 \leq \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau(t))\mathcal{Q}_2\mathfrak{e}(\mathfrak{x},t-\tau(t)) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\tau(t)))\mathcal{Q}_2\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t-\tau(t))), \quad (116)$$

$$0 \leq \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{Q}_3\bar{\mathfrak{e}}(\mathfrak{x},t) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t))\mathcal{Q}_3\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t)), \quad (117)$$

$$0 \leq \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma(t))\mathcal{Q}_4\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)))\mathcal{Q}_4\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t))). \quad (118)$$

$$\dot{\mathcal{V}}(\mathfrak{x},t) \leq \theta_5^{\mathcal{T}}\Gamma\theta_5(t). \quad (119)$$

where,

$$\theta_5 = [\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t)), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\tau(t))), \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t-\tau(t)), \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t)), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\sigma(t))), \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t-\sigma(t)), \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},t-\tau(t))), \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},t-\sigma(t)))]^{\mathcal{T}}.$$

From condition (102), we have

$$\dot{\mathcal{V}}(\mathfrak{x},t) \leq 0. \quad (120)$$

Therefore, we can conclude that the error system (101) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof.

□

Remark 7. *Suppose, the discrete delays are not appear in system (37), then the system (37) can be remodified as follows:*

$$\begin{aligned}\frac{\partial^{\lambda}\mathfrak{e}(\mathfrak{x},t)}{\partial t^{\lambda}} &= \mathcal{M}\Delta\mathfrak{e}(\mathfrak{x},t) - \mathcal{P}\mathfrak{e}(\mathfrak{x},t) + \mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t)) \\ &+ \mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\ &+ [|\gamma|+|\delta|]\int_{-\infty}^{t}\mathcal{K}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \mathcal{K}\mathfrak{e}(\mathfrak{x},t), \\ \frac{\partial^{\lambda}\bar{\mathfrak{e}}(\mathfrak{x},t)}{\partial t^{\lambda}} &= \tilde{\mathcal{M}}\Delta\bar{\mathfrak{e}}(\mathfrak{x},t) - \tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},t) + \tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)) \\ &+ \tilde{\mathcal{D}}\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} \\ &+ [|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \bar{\mathcal{K}}\bar{\mathfrak{e}}(\mathfrak{x},t).\end{aligned} \quad (121)$$

Theorem 8. *Assume that for given positive scalars γ, δ, $\tilde{\gamma}$, $\tilde{\delta}$, the system (121) is globally asymptotically stable, if there exist positive matrices $\mathcal{S}_1 > 0$, $\mathcal{S}_2 > 0$,$\mathcal{H}_7 > 0$, $\mathcal{H}_8 > 0$, positive diagonal matrices $\mathcal{Q}_1 > 0$, $\mathcal{Q}_3 > 0$, with appropriate dimension such that the following LMI holds:*

$$(\xi_{i,j})_{6,6} < 0. \quad (122)$$

where,

$$\begin{aligned}&\xi_{(1,1)} = \frac{-\mathcal{M}\mathcal{S}_1}{\mathfrak{l}_1^2} - \mathcal{S}_1\mathcal{P} + \mathcal{S}_1\mathcal{K} + \mathcal{Q}_1,\ \xi_{(1,2)} = \mathcal{S}_1\mathcal{A}, \\ &\xi_{(1,3)} = \mathcal{S}_1\mathcal{D} + \mathcal{S}_1[|\gamma|+|\delta|],\ \xi_{(2,2)} = \mathcal{H}_7 - \mathcal{Q}_3, \\ &\xi_{(3,3)} = -\mathcal{H}_7, \xi_{(4,4)} = \frac{-\tilde{\mathcal{M}}\mathcal{S}_2}{\mathfrak{l}_2^2} - \mathcal{S}_2\tilde{\mathcal{P}} + \mathcal{S}_2\bar{\mathcal{K}} + \mathcal{Q}_3, \\ &\xi_{(4,5)} = \tilde{\mathcal{A}}\mathcal{S}_2, \xi_{(4,6)} = \tilde{\mathcal{D}}\mathcal{S}_2 + \mathcal{S}_2[|\tilde{\gamma}|+|\tilde{\delta}|], \\ &\xi_{(5,5)} = \mathcal{H}_8 - \mathcal{Q}_1,\ \xi_{(6,6)} = -\mathcal{H}_8.\end{aligned}$$

Proof. : *Now let us define Lyapunov Krasovskii functional as follows:*

$$\mathcal{V}_1(\mathfrak{x},t) = \int_{\Omega}\{\frac{1}{2}{}^{\mathcal{RL}}_{t_0}\mathcal{I}_t^{1-\lambda}\mathcal{S}_1\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathfrak{e}(\mathfrak{x},t)\}\partial\mathfrak{x}, \quad (123)$$

$$\mathcal{V}_2(\mathfrak{x},t) = \int_{\Omega}\{\frac{1}{2}{}^{\mathcal{RL}}_{t_0}\mathcal{I}_t^{1-\lambda}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{S}_2\bar{\mathfrak{e}}(\mathfrak{x},t)\}\partial\mathfrak{x}, \quad (124)$$

$$\mathcal{V}_3(\mathfrak{x},t) = \int_{\Omega}\{\sum_{j=1}^{n}(\mathfrak{h}_7)_j\int_0^{\infty}\mathcal{K}_j(\theta) \times \int_{t-\theta}^{t}\mathfrak{f}_j^2(\bar{\mathfrak{e}}_j(\mathfrak{f}_j(\mathfrak{x},\mathfrak{s})))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}, \quad (125)$$

$$\mathcal{V}_4(\mathfrak{x},t) = \int_{\Omega}\{\sum_{i=1}^{m}(\mathfrak{h}_8)_i\int_0^{\infty}\tilde{\mathcal{K}}_i(\theta) \times \int_{t-\theta}^{t}\mathfrak{g}_i^2(\mathfrak{e}_i(\mathfrak{g}_i(\mathfrak{x},\mathfrak{s})))\partial\mathfrak{s}\partial\theta\}\partial\mathfrak{x}. \quad (126)$$

Taking the time derivative of V(t) along the trajectories of (121), it is evident that $\mathfrak{e}(\mathfrak{x},t)$,$\bar{\mathfrak{e}}(\mathfrak{x},t)$ and $\frac{\partial^{\lambda}e(x,t)}{\partial t^{\lambda}}$ and$\frac{\partial^{\lambda}\bar{e}(x,t)}{\partial t^{\lambda}}$ are continuous. Then, by Lemma (1) and Lemma (5), the following holds:

$$\begin{aligned}\dot{\mathcal{V}}_1(\mathfrak{x},t) = \int_{\Omega}\mathcal{S}_1\{&\frac{-1}{\mathfrak{l}_1^2}\mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{M}\mathfrak{e}(\mathfrak{x},t) - \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{P}\mathfrak{e}(\mathfrak{x},t) \\ &+ \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{A}\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},t)) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\mathcal{D}\int_{-\infty}^{t}\mathcal{K}(t-\mathfrak{s}) \\ &\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)[|\gamma|+|\delta|]\int_{-\infty}^{t} \\ &\times \mathcal{K}(t-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \mathcal{K}\mathfrak{e}(\mathfrak{x},t)\}\partial\mathfrak{x},\end{aligned} \quad (127)$$

$$\begin{aligned}\dot{\mathcal{V}}_2(\mathfrak{x},t) = \int_{\Omega}\mathcal{S}_2\{&\frac{-1}{\mathfrak{l}_2{}^2}\bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{M}}\bar{\mathfrak{e}}(\mathfrak{x},t) - \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{P}}\bar{\mathfrak{e}}(\mathfrak{x},t) \\ &+ \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{A}}\mathfrak{g}(\mathfrak{e}(\mathfrak{x},t)) + \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},t)\tilde{\mathcal{D}}\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s}) \\ &\times \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},t)[|\tilde{\gamma}|+|\tilde{\delta}|]\int_{-\infty}^{t}\tilde{\mathcal{K}}(t-\mathfrak{s})\end{aligned}$$

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$$\times\, \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s} + \bar{\mathcal{K}}\mathfrak{e}(\mathfrak{x},\mathfrak{t})\}\partial\mathfrak{x}, \tag{128}$$

$$\begin{aligned}\dot{\mathcal{V}}_3(\mathfrak{x},\mathfrak{t}) = &\int_\Omega \{\mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_7\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\\ &-(\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_7\\ &\times(\int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x},\end{aligned} \tag{129}$$

$$\begin{aligned}\dot{\mathcal{V}}_4(\mathfrak{x},\mathfrak{t}) = &\int_\Omega \{\mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{H}_8\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\\ &-(\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})^{\mathcal{T}}\mathcal{H}_8\\ &\times(\int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s})\}\partial\mathfrak{x}\end{aligned} \tag{130}$$

From the Assumption 2, we have

$$0 \le \mathfrak{e}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_1\mathfrak{e}(\mathfrak{x},\mathfrak{t}) - \mathfrak{g}^{\mathcal{T}}(\mathfrak{e}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_1\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), \tag{131}$$

$$0 \le \bar{\mathfrak{e}}^{\mathcal{T}}(\mathfrak{x},\mathfrak{t})\mathcal{Q}_3\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}) - \mathfrak{f}^{\mathcal{T}}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}))\mathcal{Q}_3\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})) \le 0. \tag{132}$$

$$\dot{\mathcal{V}}(\mathfrak{t}) \le \theta_6^{\mathcal{T}}\xi\theta_6(\mathfrak{t}). \tag{133}$$

where,

$$\begin{aligned}\theta_6 = [&\mathfrak{e}(\mathfrak{x},\mathfrak{t}), \mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t})), \int_{-\infty}^{\mathfrak{t}} \mathcal{K}(\mathfrak{t}-\mathfrak{s})\mathfrak{f}(\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s},\\ &\bar{\mathfrak{e}}(\mathfrak{x},\mathfrak{t}), \mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{t})), \int_{-\infty}^{\mathfrak{t}} \tilde{\mathcal{K}}(\mathfrak{t}-\mathfrak{s})\mathfrak{g}(\mathfrak{e}(\mathfrak{x},\mathfrak{s}))\partial\mathfrak{s}]^{\mathcal{T}}.\end{aligned}$$

$$\dot{V}(\mathfrak{x},\mathfrak{t}) \le 0. \tag{134}$$

Therefore, we can conclude that the error system (121) has a unique equilibrium point which is globally asymptotically stable. As a result, the response system (2) with various time delays is globally synchronized with and drive system (1).This completes the proof. □

III. NUMERICAL EXAMPLE

In this section, to verify and demonstrate the effectiveness of the derived method, we consider two numerical examples.

Example III.1. *Consider the master system (1) and the slave system (2) of fuzzy BAM NNs with the following parameters:*

$$\mathcal{M} = \begin{bmatrix} 0.79 & 0 & 0 & 0 & 0\\ 0 & 0.79 & 0 & 0 & 0\\ 0 & 0 & 0.79 & 0 & 0\\ 0 & 0 & 0 & 0.79 & 0\\ 0 & 0 & 0 & 0 & 0.79 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} 0.79 & 0.98 & 0.78 & 0.90 & 0.56\\ 0.45 & 0.79 & 0.89 & 0.43 & 0.87\\ 0.87 & 0.45 & 0.79 & 0.98 & 0.67\\ 0.09 & 0.87 & 0.88 & 0.79 & 0.98\\ 0.90 & 0.67 & 0.65 & 0.76 & 0.79 \end{bmatrix}$$

$$\mathcal{A} = \begin{bmatrix} 0.89 & 0.09 & 0.56 & 0.89 & 0.09\\ 0.98 & 0.56 & 0.34 & 0.90 & 0.98\\ 0.67 & 0.77 & 0.23 & 0.45 & 0.34\\ 0.78 & 0.90 & 0.43 & 0.98 & 0.99\\ 0.76 & 0.87 & 0.33 & 0.23 & 0.21 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0.76 & 0.99 & 0.46 & 0.29 & 0.09\\ 0.38 & 0.58 & 0.37 & 0.99 & 0.95\\ 0.69 & 0.79 & 0.28 & 0.46 & 0.35\\ 0.74 & 0.93 & 0.45 & 0.92 & 0.96\\ 0.75 & 0.88 & 0.39 & 0.24 & 0.22 \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} 0.89 & 0.39 & 0.78 & 0.23 & 0.19\\ 0.48 & 0.68 & 0.47 & 0.19 & 0.45\\ 0.64 & 0.59 & 0.18 & 0.36 & 0.45\\ 0.79 & 0.53 & 0.75 & 0.82 & 0.96\\ 0.35 & 0.28 & 0.49 & 0.54 & 0.62 \end{bmatrix}$$

$$\tilde{\mathcal{M}} = \begin{bmatrix} 0.88 & 0 & 0 & 0 & 0\\ 0 & 0.88 & 0 & 0 & 0\\ 0 & 0 & 0.88 & 0 & 0\\ 0 & 0 & 0 & 0.88 & 0\\ 0 & 0 & 0 & 0 & 0.88 \end{bmatrix}$$

$$\tilde{\mathcal{P}} = \begin{bmatrix} 0.88 & 0.87 & 0.45 & 0.34 & 0.54\\ 0.34 & 0.88 & 0.34 & 0.23 & 0.45\\ 0.23 & 0.12 & 0.88 & 0.43 & 0.55\\ 0.23 & 0.45 & 0.23 & 0.88 & 0.22\\ 0.22 & 0.34 & 0.45 & 0.56 & 0.88 \end{bmatrix}$$

$$\tilde{\mathcal{A}} = \begin{bmatrix} 0.99 & 0.29 & 0.16 & 2.89 & 3.09\\ 6.98 & 7.56 & 8.34 & 9.90 & 1.98\\ 9.67 & 8.77 & 1.23 & 2.45 & 3.34\\ 8.78 & 9.90 & 8.43 & 8.98 & 5.99\\ 0.77 & 0.67 & 0.36 & 0.28 & 0.27 \end{bmatrix}$$

$$\tilde{\mathcal{B}} = \begin{bmatrix} 0.78 & 0.98 & 0.09 & 2.76 & 3.89\\ 6.98 & 7.89 & 7.34 & 1.90 & 2.98\\ 5.67 & 6.77 & 2.23 & 3.45 & 4.34\\ 6.78 & 7.90 & 0.43 & 9.98 & 3.99\\ 1.77 & 2.67 & 3.36 & 4.28 & 5.27 \end{bmatrix}$$

$$\tilde{\mathcal{D}} = \begin{bmatrix} 0.78 & 0.98 & 0.09 & 2.76 & 3.89\\ 6.98 & 7.89 & 7.34 & 1.90 & 2.98\\ 5.67 & 6.77 & 2.23 & 3.45 & 4.34\\ 6.78 & 7.90 & 0.43 & 9.98 & 3.99\\ 1.77 & 2.67 & 3.36 & 4.28 & 5.27 \end{bmatrix} \tag{135}$$

Let us consider $\mathfrak{g}_i(\mathfrak{u}_i) = \frac{1}{2}(|\mathfrak{u}_i - 1| - |\mathfrak{u}_i - 1|), i = 1, 2, .., \mathfrak{m}$, $\mathfrak{f}_j(\eta_j) = \frac{1}{2}(|\eta_j - 1| - |\eta_j - 1|), j = 1, 2, .., \mathfrak{n}$, *which satisfy the Assumption 2 ,we get* $\mathcal{F}_j^- = -2, \mathcal{F}_j^+ = 2, \mathcal{G}_i^- = -0.7, \mathcal{G}_i^+ = 0.7, \tau_1 = 0.89, \tau_{1m} = 0.87, \tau_{m2} = 0.88, \mu = 0.67, \alpha = 0.76, \beta = 0.56, \gamma = 0.46, \delta = 0.56, \sigma_1 = 0.98, \sigma_{2m} = 0.96, \tilde{\gamma} = 0.87, \tilde{\delta} = 0.67, \tilde{\sigma}_{1m}^2 = 0.876, \mathfrak{l}_2 = 0.76, \eta = 0.98, \tilde{\alpha} = 0.87, \tilde{\beta} = 0.98.$

By using the Matlab LMI solver to solve the LMIs 6 in Theorem 1, it can be found that the LMIs are feasible and the matrices are

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$$\mathcal{W}_3 = \begin{bmatrix} 13.8329 & 0.0093 & 0.0071 & -0.0130 & 0.0170 \\ 0.0093 & 13.8176 & 0.0181 & 0.0105 & 0.0100 \\ 0.0071 & 0.0181 & 13.7464 & 0.0500 & 0.0062 \\ -0.0130 & 0.0105 & 0.0500 & 13.8024 & 0.0037 \\ 0.0170 & 0.0100 & 0.0062 & 0.0037 & 13.7999 \end{bmatrix}$$

$$\mathcal{R}_2 = \begin{bmatrix} 18.3071 & 0.0033 & 0.0006 & -0.0050 & 0.0042 \\ 0.0033 & 18.3121 & 0.0065 & 0.0031 & 0.0035 \\ 0.0006 & 0.0065 & 18.3359 & 0.0143 & 0.0003 \\ -0.0050 & 0.0031 & 0.0143 & 18.3183 & -0.0005 \\ 0.0042 & 0.0035 & 0.0003 & -0.0005 & 18.3174 \end{bmatrix}$$

$$\mathcal{W}_1 = \begin{bmatrix} 2.6847 & 0.0004 & 0.0001 & -0.0005 & 0.0005 \\ 0.0004 & 2.6852 & 0.0007 & 0.0004 & 0.0004 \\ 0.0001 & 0.0007 & 2.6879 & 0.0016 & 0.0001 \\ -0.0005 & 0.0004 & 0.0016 & 2.6859 & -0.0000 \\ 0.0005 & 0.0004 & 0.0001 & -0.0000 & 2.6858 \end{bmatrix}$$

$$\mathcal{H}_4 = \begin{bmatrix} 2.8628 & -0.0025 & 0.0047 & -0.0076 & 0.0072 \\ -0.0025 & 2.8626 & 0.0042 & 0.0005 & -0.0016 \\ 0.0047 & 0.0042 & 2.8932 & 0.0203 & 0.0103 \\ -0.0076 & 0.0005 & 0.0203 & 2.8623 & 0.0047 \\ 0.0072 & -0.0016 & 0.0103 & 0.0047 & 2.8681 \end{bmatrix}$$

$$\mathcal{R}_1 = \begin{bmatrix} 8.2921 & 0.0019 & 0.0002 & -0.0029 & 0.0023 \\ 0.0019 & 8.2949 & 0.0037 & 0.0017 & 0.0020 \\ 0.0002 & 0.0037 & 8.3084 & 0.0079 & 0.0000 \\ -0.0029 & 0.0017 & 0.0079 & 8.2985 & -0.0004 \\ 0.0023 & 0.0020 & 0.0000 & -0.0004 & 8.2979 \end{bmatrix}$$

$$\mathcal{R}_3 = \begin{bmatrix} 5.1062 & 0.0007 & 0.0002 & -0.0011 & 0.0010 \\ 0.0007 & 5.1074 & 0.0014 & 0.0007 & 0.0008 \\ 0.0002 & 0.0014 & 5.1127 & 0.0033 & 0.0001 \\ -0.0011 & 0.0007 & 0.0033 & 5.1087 & -0.0001 \\ 0.0010 & 0.0008 & 0.0001 & -0.0001 & 5.1086 \end{bmatrix}$$

$$\mathcal{W}_2 = \begin{bmatrix} 2.7063 & 0.0004 & 0.0001 & -0.0005 & 0.0005 \\ 0.0004 & 2.7069 & 0.0007 & 0.0004 & 0.0004 \\ 0.0001 & 0.0007 & 2.7095 & 0.0016 & 0.0001 \\ -0.0005 & 0.0004 & 0.0016 & 2.7075 & -0.0000 \\ 0.0005 & 0.0004 & 0.0001 & -0.0000 & 2.7075 \end{bmatrix}$$

$$\mathcal{R}_4 = \begin{bmatrix} 2.5529 & 0.0004 & 0.0001 & -0.0006 & 0.0005 \\ 0.0004 & 2.5534 & 0.0007 & 0.0004 & 0.0004 \\ 0.0001 & 0.0007 & 2.5561 & 0.0016 & 0.0001 \\ -0.0006 & 0.0004 & 0.0016 & 2.5541 & -0.0000 \\ 0.0005 & 0.0004 & 0.0001 & -0.0000 & 2.5541 \end{bmatrix}$$

$$\mathcal{S}_1 = \begin{bmatrix} 0.1808 & -0.0247 & -0.0973 & -0.0656 & 0.0419 \\ -0.0247 & 0.1726 & -0.0401 & -0.0018 & -0.0293 \\ -0.0973 & -0.0401 & 0.3600 & 0.0439 & -0.0667 \\ -0.0656 & -0.0018 & 0.0439 & 0.1738 & -0.0552 \\ 0.0419 & -0.0293 & -0.0667 & -0.0552 & 0.2014 \end{bmatrix}$$

$$\mathcal{R}_5 = \begin{bmatrix} -4.0984 & -0.0059 & 0.0147 & 0.0119 & -0.0099 \\ -0.0059 & -4.0962 & -0.0045 & -0.0111 & -0.0098 \\ 0.0147 & -0.0045 & -4.1294 & -0.0169 & 0.0095 \\ 0.0119 & -0.0111 & -0.0169 & -4.1031 & 0.0015 \\ -0.0099 & -0.0098 & 0.0095 & 0.0015 & -4.0949 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 1.3682 & 0 & 0 & 0 & 0 \\ 0 & 1.3682 & 0 & 0 & 0 \\ 0 & 0 & 1.3682 & 0 & 0 \\ 0 & 0 & 0 & 1.3682 & 0 \\ 0 & 0 & 0 & 0 & 1.3682 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 0.8072 & 0 & 0 & 0 & 0 \\ 0 & 0.8072 & 0 & 0 & 0 \\ 0 & 0 & 0.8072 & 0 & 0 \\ 0 & 0 & 0 & 0.8072 & 0 \\ 0 & 0 & 0 & 0 & 0.8072 \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} 0.7928 & 0 & 0 & 0 & 0 \\ 0 & 0.7928 & 0 & 0 & 0 \\ 0 & 0 & 0.7928 & 0 & 0 \\ 0 & 0 & 0 & 0.7928 & 0 \\ 0 & 0 & 0 & 0 & 0.7928 \end{bmatrix}$$

$$\mathcal{S}_2 = \begin{bmatrix} 0.0312 & -0.0262 & 0.0054 & -0.0009 & -0.0035 \\ -0.0262 & 0.0282 & -0.0099 & -0.0022 & 0.0032 \\ 0.0054 & -0.0099 & 0.0087 & 0.0026 & -0.0022 \\ -0.0009 & -0.0022 & 0.0026 & 0.0036 & -0.0019 \\ -0.0035 & 0.0032 & -0.0022 & -0.0019 & 0.0052 \end{bmatrix}$$

$$\mathcal{T}_2 = \begin{bmatrix} 13.0830 & -0.0154 & -0.0099 & -0.0087 & -0.0061 \\ -0.0154 & 13.0819 & -0.0081 & 0.0030 & 0.0048 \\ -0.0099 & -0.0081 & 13.0757 & -0.0042 & -0.0122 \\ -0.0087 & 0.0030 & -0.0042 & 13.0731 & -0.0086 \\ -0.0061 & 0.0048 & -0.0122 & -0.0086 & 13.0624 \end{bmatrix}$$

$$\mathcal{Z}_1 = \begin{bmatrix} -9.4849 & -0.0070 & -0.0021 & -0.0027 & -0.0021 \\ -0.0070 & -9.4846 & -0.0035 & 0.0009 & 0.0017 \\ -0.0021 & -0.0035 & -9.4887 & -0.0011 & -0.0035 \\ -0.0027 & 0.0009 & -0.0011 & -9.4901 & -0.0025 \\ -0.0021 & 0.0017 & -0.0035 & -0.0025 & -9.4928 \end{bmatrix}$$

$$\mathcal{Z}_3 = \begin{bmatrix} -9.1113 & -0.0062 & -0.0019 & -0.0025 & -0.0019 \\ -0.0062 & -9.1111 & -0.0031 & 0.0008 & 0.0015 \\ -0.0019 & -0.0031 & -9.1147 & -0.0010 & -0.0031 \\ -0.0025 & 0.0008 & -0.0010 & -9.1159 & -0.0022 \\ -0.0019 & 0.0015 & -0.0031 & -0.0022 & -9.1185 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} -5.4902 & 0 & 0 & 0 & 0 \\ 0 & -5.4902 & 0 & 0 & 0 \\ 0 & 0 & -5.4902 & 0 & 0 \\ 0 & 0 & 0 & -5.4902 & 0 \\ 0 & 0 & 0 & 0 & -5.4902 \end{bmatrix}$$

$$\mathcal{H}_5 = \begin{bmatrix} -1.8291 & 0.0131 & 0.0171 & 0.0270 & 0.0362 \\ 0.0131 & -1.7959 & 0.0357 & 0.0227 & 0.0201 \\ 0.0171 & 0.0357 & -1.6558 & 0.1049 & 0.0157 \\ 0.0270 & 0.0227 & 0.1049 & -1.7725 & 0.0091 \\ 0.0362 & 0.0201 & 0.0157 & 0.0091 & -1.7638 \end{bmatrix}$$

$$\mathcal{T}_1 = \begin{bmatrix} 12.1970 & -0.0082 & -0.0070 & -0.0054 & -0.0037 \\ -0.0082 & 12.1958 & -0.0044 & 0.0019 & 0.0029 \\ -0.0070 & -0.0044 & 12.1934 & -0.0028 & -0.0079 \\ -0.0054 & 0.0019 & -0.0028 & 12.1922 & -0.0056 \\ -0.0037 & 0.0029 & -0.0079 & -0.0056 & 12.1851 \end{bmatrix}$$

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$$\mathcal{T}_5 = \begin{bmatrix} 5.2870 & -0.0036 & 0.0006 & -0.0028 & 0.0011 \\ -0.0036 & 5.2895 & 0.0017 & 0.0017 & 0.0015 \\ 0.0006 & 0.0017 & 5.2943 & 0.0053 & 0.0003 \\ -0.0028 & 0.0017 & 0.0053 & 5.2868 & -0.0004 \\ 0.0011 & 0.0015 & 0.0003 & -0.0004 & 5.2871 \end{bmatrix}$$

$$\mathcal{T}_3 = \begin{bmatrix} 5.6752 & -0.0017 & -0.0010 & -0.0009 & -0.0006 \\ -0.0017 & 5.6751 & -0.0009 & 0.0003 & 0.0005 \\ -0.0010 & -0.0009 & 5.6744 & -0.0004 & -0.0012 \\ -0.0009 & 0.0003 & -0.0004 & 5.6741 & -0.0009 \\ -0.0006 & 0.0005 & -0.0012 & -0.0009 & 5.6730 \end{bmatrix}$$

$$\mathcal{Z}_2 = \begin{bmatrix} -8.7083 & -0.0055 & -0.0017 & -0.0022 & -0.0017 \\ -0.0055 & -8.7081 & -0.0027 & 0.0007 & 0.0014 \\ -0.0017 & -0.0027 & -8.7112 & -0.0009 & -0.0028 \\ -0.0022 & 0.0007 & -0.0009 & -8.7123 & -0.0020 \\ -0.0017 & 0.0014 & -0.0028 & -0.0020 & -8.7146 \end{bmatrix}$$

$$\mathcal{T}_4 = \begin{bmatrix} 2.8373 & -0.0008 & -0.0005 & -0.0004 & -0.0003 \\ -0.0008 & 2.8372 & -0.0004 & 0.0001 & 0.0002 \\ -0.0005 & -0.0004 & 2.8368 & -0.0002 & -0.0006 \\ -0.0004 & 0.0001 & -0.0002 & 2.8367 & -0.0004 \\ -0.0003 & 0.0002 & -0.0006 & -0.0004 & 2.8362 \end{bmatrix}$$

$$\mathcal{R} = \begin{bmatrix} -5.4902 & 0.0039 & 0.0093 & -0.0151 & 0.0187 \\ 0.0039 & -5.4724 & 0.0183 & 0.0120 & 0.0113 \\ 0.0093 & 0.0183 & -5.3989 & 0.0564 & 0.0078 \\ -0.0151 & 0.0120 & 0.0564 & -5.4626 & 0.0042 \\ 0.0187 & 0.0113 & 0.0078 & 0.0042 & -5.4582 \end{bmatrix}$$

$$\mathcal{L} = \begin{bmatrix} 0.8072 & -0.0009 & 0.0107 & 0.0051 & 0.0030 \\ -0.0009 & 0.8086 & -0.0010 & -0.0042 & -0.0019 \\ 0.0107 & -0.0010 & 0.8055 & 0.0047 & 0.0088 \\ 0.0051 & -0.0042 & 0.0047 & 0.8035 & 0.0057 \\ 0.0030 & -0.0019 & 0.0088 & 0.0057 & 0.8132 \end{bmatrix}$$

By Theorem (1), the error system (3), is asymptotically synchronized.

Example III.2. *Consider the master system 1 and the slave system 2 of fuzzy BAM NNs with the following parameters:*

$$\mathcal{M} = \begin{bmatrix} 0.89 & 0 & 0 & 0 & 0 \\ 0 & 0.89 & 0 & 0 & 0 \\ 0 & 0 & 0.89 & 0 & 0 \\ 0 & 0 & 0 & 0.89 & 0 \\ 0 & 0 & 0 & 0 & 0.89 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} 0.76 & 0.89 & 0.98 & 0.56 & 0.98 \\ 0.76 & 9.09 & 0.98 & 0.87 & 0.34 \\ 0.12 & 0.34 & 0.87 & 9.07 & 0.87 \\ 0.77 & 0.23 & 0.34 & 0.45 & 0.67 \\ 0.34 & 0.45 & 0.87 & 0.98 & 0.68 \end{bmatrix}$$

$$\mathcal{A} = \begin{bmatrix} 0.09 & 0.29 & 0.36 & 0.29 & 0.49 \\ 0.78 & 0.86 & 0.94 & 0.10 & 0.28 \\ 0.37 & 0.47 & 0.53 & 0.65 & 0.94 \\ 0.38 & 0.50 & 0.23 & 0.48 & 0.69 \\ 0.26 & 0.37 & 0.53 & 0.29 & 0.11 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0.62 & 0.89 & 0.16 & 0.39 & 0.19 \\ 0.48 & 0.68 & 0.37 & 0.49 & 0.35 \\ 0.61 & 0.79 & 0.28 & 0.26 & 0.25 \\ 0.44 & 0.53 & 0.45 & 0.62 & 0.56 \\ 0.25 & 0.38 & 0.39 & 0.14 & 0.92 \\ 0.57 & 0.66 & 0.77 & 0.89 & 0.92 \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} 0.09 & 0.29 & 0.58 & 0.13 & 0.39 \\ 0.48 & 0.66 & 0.41 & 0.39 & 0.95 \\ 0.24 & 0.59 & 0.88 & 0.26 & 0.55 \\ 0.39 & 0.55 & 0.25 & 0.86 & 0.76 \\ 0.15 & 0.68 & 0.79 & 0.34 & 0.72 \end{bmatrix}$$

$$\tilde{\mathcal{A}}_1 = \begin{bmatrix} 0.99 & 0 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 & 0 \\ 0 & 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 0 & 0.99 \end{bmatrix}$$

$$\mathcal{P}_1 = \begin{bmatrix} 0.66 & 0.09 & 0.98 & 0.89 & 0.45 \\ 0.223 & 9.56 & 0.88 & 0.57 & 0.94 \\ 0.72 & 0.78 & 0.45 & 9.09 & 0.23 \\ 0.12 & 0.63 & 0.24 & 0.65 & 0.17 \\ 0.94 & 0.67 & 0.09 & 0.67 & 0.98 \end{bmatrix}$$

$$\mathcal{A}_1 = \begin{bmatrix} 0.12 & 0.78 & 0.12 & 2.12 & 3.67 \\ 6.87 & 7.56 & 8.59 & 9.90 & 1.98 \\ 9.34 & 8.46 & 1.12 & 2.56 & 3.21 \\ 8.35 & 9.98 & 8.57 & 8.56 & 5.34 \\ 0.56 & 0.47 & 0.34 & 0.78 & 0.45 \end{bmatrix}$$

$$\mathcal{B}_1 = \begin{bmatrix} 0.12 & 0.23 & 0.34 & 2.45 & 3.56 \\ 6.12 & 7.23 & 7.45 & 1.54 & 2.56 \\ 5.34 & 6.55 & 2.66 & 3.23 & 3.55 \\ 6.78 & 9.90 & 9.43 & 8.98 & 4.99 \\ 2.77 & 3.67 & 4.36 & 5.28 & 7.27 \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} 0.23 & 0.44 & 0.56 & 2.67 & 3.87 \\ 6.34 & 7.45 & 7.34 & 1.23 & 2.45 \\ 5.23 & 6.45 & 2.56 & 3.76 & 3.45 \\ 6.56 & 9.45 & 9.56 & 8.56 & 4.45 \\ 2.27 & 3.37 & 4.46 & 5.58 & 7.67 \end{bmatrix}$$

Let us consider $\mathfrak{g}_i(u_i) = \frac{1}{2}(|u_i - 1| - |u_i - 1|), i = 1, 2, .., m,$ $\mathfrak{f}_j(\eta_j) = \frac{1}{2}(|\eta_j - 1| - |\eta_j - 1|), j = 1, 2, .., n,$ *which satisfy the Assumption 2 ,we get* $\mathcal{F}_j^- = -9, \mathcal{F}_j^+ = 7, \mathcal{G}_i^- = -0.79, \mathcal{G}_i^+ = 0.69, \tau_{1I} = 0.89, l_1 = 0.76, \mathfrak{l}_2 = 0.56,$ $\alpha = 0.98,\ \beta = 0.87,\ \mu = 0.98,\ \gamma = 0.34,\ \delta = 0.55,$ $\tilde{\alpha} = 0.78,\ \tilde{\beta} = 0.46,\ \tilde{\gamma} = 0.87,\ \tilde{\delta} = 0.34,\ \eta = 0.87,$ $\sigma_1 = 0.89.$

By using the Matlab LMI solver to solve the LMIs 38 in Theorem 2, it can be found that the LMIs are feasible and the matrices are

$$\mathcal{R}_6 = \begin{bmatrix} -6.1553 & -0.0004 & 0.0000 & 0.0001 & 0.0001 \\ -0.0004 & -6.1515 & -0.0010 & -0.0005 & -0.0004 \\ 0.0000 & -0.0010 & -6.1537 & 0.0004 & 0.0001 \\ 0.0001 & -0.0005 & 0.0004 & -6.1554 & 0.0001 \\ 0.0001 & -0.0004 & 0.0001 & 0.0001 & -6.1554 \end{bmatrix}$$

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$$\mathcal{Q}_1 = \begin{bmatrix} 10.3451 & 0 & 0 & 0 & 0 \\ 0 & 10.3451 & 0 & 0 & 0 \\ 0 & 0 & 10.3451 & 0 & 0 \\ 0 & 0 & 0 & 10.3451 & 0 \\ 0 & 0 & 0 & 0 & 10.3451 \end{bmatrix}$$

$$\mathcal{Q}_3 = \begin{bmatrix} 29.0385 & 0 & 0 & 0 & 0 \\ 0 & 29.0385 & 0 & 0 & 0 \\ 0 & 0 & 29.0385 & 0 & 0 \\ 0 & 0 & 0 & 29.0385 & 0 \\ 0 & 0 & 0 & 0 & 29.0385 \end{bmatrix}$$

$$\mathcal{Q}_4 = \begin{bmatrix} 1.1072 & 0 & 0 & 0 & 0 \\ 0 & 1.1072 & 0 & 0 & 0 \\ 0 & 0 & 1.1072 & 0 & 0 \\ 0 & 0 & 0 & 1.1072 & 0 \\ 0 & 0 & 0 & 0 & 1.1072 \end{bmatrix}$$

$$\mathcal{R}_8 = \begin{bmatrix} 26.8541 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & 26.8541 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 26.8541 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 26.8541 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 26.8541 \end{bmatrix}$$

$$\mathcal{H}_7 = \begin{bmatrix} 1.0921 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & 1.0921 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 1.0921 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 1.0921 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & 1.0921 \end{bmatrix}$$

$$\mathcal{W}_6 = \begin{bmatrix} 1.0979 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0979 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0979 & 0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 1.0979 & 0.0000 \\ -0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0979 \end{bmatrix}$$

$$\mathcal{R}_9 = \begin{bmatrix} 8.1556 & 0.0000 & -0.0000 & 0.0000 & 0.0000 \\ -0.0000 & 8.1556 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 8.1556 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 8.1556 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 8.1556 \end{bmatrix}$$

$$\mathcal{H}_8 = \begin{bmatrix} 1.0946 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & 1.0946 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0946 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0946 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0946 \end{bmatrix}$$

$$\mathcal{W}_5 = \begin{bmatrix} 1.0970 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0970 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0970 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0970 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0970 \end{bmatrix}$$

$$\mathcal{R}_7 = \begin{bmatrix} -0.0709 & -0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0000 & -0.0709 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & -0.0709 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 & -0.0709 & -0.0000 \\ 0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0709 \end{bmatrix}$$

$$\mathcal{V}_1 = \begin{bmatrix} 6.1553 & 0.0004 & -0.0000 & -0.0001 & -0.0001 \\ 0.0004 & 6.1515 & 0.0010 & 0.0005 & 0.0004 \\ -0.0000 & 0.0010 & 6.1537 & -0.0004 & -0.0001 \\ -0.0001 & 0.0005 & -0.0004 & 6.1554 & -0.0001 \\ -0.0001 & 0.0004 & -0.0001 & -0.0001 & 6.1554 \end{bmatrix}$$

$$\mathcal{V}_2 = \begin{bmatrix} -1.5119 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & -1.5119 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -1.5119 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -1.5119 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -1.5119 \end{bmatrix}$$

$$\mathcal{Q}_2 = \begin{bmatrix} -1.0176 & 0 & 0 & 0 & 0 \\ 0 & -1.0176 & 0 & 0 & 0 \\ 0 & 0 & -1.0176 & 0 & 0 \\ 0 & 0 & 0 & -1.0176 & 0 \\ 0 & 0 & 0 & 0 & -1.0176 \end{bmatrix}$$

$$\mathcal{S}_1 = 10^3 * \begin{bmatrix} 0.2139 & -0.1951 & 0.0073 & 0.0047 & 0.0728 \\ -0.1951 & 0.8457 & -0.2344 & -0.1946 & -0.1258 \\ 0.0073 & -0.2344 & 0.1124 & 0.3653 & -0.0061 \\ 0.0047 & -0.1946 & 0.3653 & 0.0478 & -0.0133 \\ 0.0728 & -0.1258 & -0.0061 & -0.0133 & 0.1171 \end{bmatrix}$$

$$\mathcal{S}_2 = 10^6 * \begin{bmatrix} 0.0261 & -0.0169 & 0.0060 & -0.0020 & 0.0064 \\ -0.0169 & 0.1229 & -0.0232 & -0.0191 & 0.0017 \\ 0.0060 & -0.0232 & 0.0033 & 0.0607 & -0.0093 \\ -0.0020 & -0.0191 & 0.0607 & 0.0020 & -0.0036 \\ 0.0064 & 0.0017 & -0.0093 & -0.0036 & 0.0219 \end{bmatrix} \tag{136}$$

Thus, the controller gain matrices $\mathcal{K}_1$ and $\mathcal{K}_2$ can be as follows:

$$\mathcal{K}_1 = 10^4 * \begin{bmatrix} 4.2372 & 0.9191 & 0.3280 & 0.3783 & -1.5878 \\ 0.9195 & 1.4748 & 0.7204 & 0.7259 & 1.1338 \\ 0.3280 & 0.7203 & 0.1329 & 2.1135 & 0.8169 \\ 0.3783 & 0.7258 & 2.1130 & -0.1814 & 0.6349 \\ -1.5878 & 1.1335 & 0.8168 & 0.6348 & 7.5789 \end{bmatrix}$$

$$\mathcal{K}_2 = 10^8 * \begin{bmatrix} -1.1483 & -0.1597 & -0.0726 & 0.1079 & 0.3332 \\ -0.1597 & -0.2595 & -0.0832 & -0.0757 & 0.0187 \\ -0.0726 & -0.0832 & -0.0168 & -0.4563 & -0.0540 \\ 0.1079 & -0.0757 & -0.4563 & -0.0499 & -0.2276 \\ 0.3332 & 0.0187 & -0.0540 & -0.2276 & -1.3353 \end{bmatrix} \tag{137}$$

By Theorem (2), the error system (37), is asymptotically synchronized.

IV. CONCLUSION

In this paper, we have dealt with the synchronization of fractional order fuzzy BAM neural networks with time varying delays and reaction diffusion terms have been investigated. By constructing the novel Lyapunov-Krasovskii functional

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having the double integral terms, we utilized Jensens inequality techniques and LMI approach, we derived sufficient conditions to guarantee the global asymptotical stability of the error dynamics of the considered fuzzy BAMNNs. The obtained results indicate that the synchronization behavior of fuzzy BAMNNs is very sensitive to the initial condition. Moreover, the controller gain matrices can be obtained by solving the LMIs. Finally, illustrative numerical results have been provided to verify the correctness and effectiveness of the obtained results.

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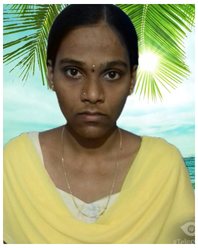

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