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Finite-time Stability, Dissipativity and Passivity Analysis of Discrete-time Neural Networks Time-varying Delays

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Abstract

The neural network time-varying delay was described as the dynamic properties of a neural cell, including neural functional and neural delay differential equations. The differential expression explains the derivative term of current and past state. The objective of this paper obtained the neural network time-varying delay. A delay-dependent condition is provided to ensure the considered discrete-time neural networks with time-varying delays to be finite-time stability, dissipativity, and passivity. This paper using a new Lyapunov-Krasovskii functional as well as the free-weighting matrix approach and a linear matrix inequality analysis (LMI) technique constructing to a novel sufficient criterion on finite-time stability, dissipativity, and passivity of the discrete-time neural networks with time-varying delays for improving. We propose sufficient conditions for discrete-time neural networks with time-varying delays. An effective LMI approach derives by base the appropriate type of Lyapunov functional. Finally, we present the effectiveness of novel criteria of finite-time stability, dissipativity, and passivity condition of discrete-time neural networks with time-varying delays in the form of linear matrix inequality (LMI).

Keywords:

Finite-time Stability;
Dissipativity and Passivity Analysis;
Lyapunov-Krasovskii Functional.

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1- Introduction

Current years we have been attending in researching delay neural networks (NNs), this is mainly to the major feasible applications in many areas, for example, combinatorial optimization, static image processing, pattern recognition, associative memory and signal processing [1]. On the other hand, the time delay is ineluctable in various applied systems, and it is also the principal cause of poor performance, oscillation, and instability of the systems. Therefore, important interest has been considered to delay-dependent conditions of analysis and combination problems of time-delay NNs [2]. Thus, it is significant to learn the stability of discrete-time neural networks (DNNs) with time-varying delay.

First recommended by Popov [3], the idea of passive systems from the beginning occurs in the conditions of electrical circuit theory. In the preliminary 1970s, Willems [4] developed the concept of dissipative systems, and passive systems, by suggestion the symbols of a supply rate and a storage function. Dissipativity theory provides a substructure for the analysis and design of control systems applying an input-output feature used as a basis energy-related judgment.

Neural networks (NNs) have been successfully practiced in a diversity of fields such as signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization, and these a applications contingent on seriously on their dynamic department. The dynamical conduct of NNs are the key to the above-said applications, and a necessary step for the practical design of NNs. Up to now, there have been fruitful research results available in the literature about the dynamic deportment of NNs [1-6]. Many results have been investigated by the dynamic behaviour of continuous-time NNs [7-10]. However, compared with continuous-time NNs, discrete-time neural networks (DNNs) equally have a strong engineering application background for the sake of computer-based simulation

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and the dynamics of continuous-time NNs cannot be preserved by discretization as mentioned in [11]. Hence, it is essential to study the dynamical behaviour of DNNs.

Moreover, in many physical and biological phenomena, the rate of variation in the system state depends on the past states. This characteristic is called a delay (or a time delay) and therefore a system with a time delay is called a time-delay system. Time delay phenomena were first discovered in biological systems and were later found in many engineering systems, such as mechanical transmissions, fluid transmissions, metallurgical processes, and networked control systems. They are often a source of instability, periodic oscillatory, chaos, and poor control performance. Time-delay systems have attracted the attention of many researchers [12-14] because of their importance and widespread occurrence.

It is well known that dissipativeness was initially introduced by *"Willems"* in terms of an inequality involving the storage function and supply rate. Dissipativity hypothesis needs to assume a discriminating some piece in the dissection Also control outline for straight Also nonlinear systems, particularly to high-order framework [15], since from those useful requisition purpose about the view, a significant number frameworks have to make dissipative for accomplishing viable commotion weakening. [16-19]. That provides a strong connection between Physics, system theory, and control engineering. The dissipated theory has proven to be an essential and very useful tool for control applications like robotics, active vibration damping, electromechanical systems, combustion engines, circuit theory, and for control techniques like adaptive control, and inverse optimal control problems. The dissipative theory being a framework for the design and analysis of control systems using an input-output description based on energy-related consideration is applicable in characterizing important system behaviours, such as passivity, and has close connections with passivity theorem, bounded real lemma, Kalman--Yakubovich lemma, and the circle criterion [20-21]. On the other hand, passivity is part of a broader and general theory of dissipativeness. The main idea of passivity theory is that the passive properties of a system can keep the system internally stable. In recent years, dissipated and passivity results for NNs are established in [22-26].

Since Zhang et al. [27] has pointed out that the relaxed passivity conditions for NNs with time-varying delays. New delay-dependent passive criterion is obtained in terms of linear matrix inequalities, which guarantees that the input and output of the considered NNs. Also, Wei et al. [28] shown the passivity problem by using value-map and suitable for Lyaponov-Krasovskii function. Recently, Zeng et al. [29] derived new passivity conditions for NNs with time-varying delays and norm-bounded parameter uncertainties using with the complete delay-decomposing approach. Also, the problem of robust passivity analysis of uncertain NNs with discrete and distributed time-varying delays has been reported, by constructing an augmented Lyapunov functional and combining a new integral inequality with the reciprocally convex approaches respectively. In addition, Park M.J. et al. [30] and Li et al. [31] developed generalized free-matrix-base integral inequality for enhanced passivity condition and derive in form of linear matrix inequalities (LMIs) as complex value. In [32-34], proposed the global asymptotic stability problem for recurrent NNs with multiple time-varying delays. Using the free-weighting matrix technique and incorporating the interconnected information between the upper bounds of multiple time-varying delays.

Recall the past several years. However, there are fewer works have been done on the dynamics of stability, dissipativity, and passivity analysis of DNNs with time-varying delays. Motivated by these earlier efforts, in this paper, we are concerned with the problem of stability, dissipativity, and passivity analysis of discrete-time neural networks with time-varying delays. Based on the newly established integral inequality, a class of new Lyapunov functional including is proposed, and some less conservative delay range-dependent stability, dissipativity and passivity criteria are derived in terms of LMIs.

This paper is organized as follows. Preliminaries is formulates the problem under consideration. Stability, Dissipativity, and passivity conditions for stability, dissipativity, and passivity analysis of DNNs with time-varying delays are derived in main results. Finally, conclusions are drawn in last section.

1-1- Preliminaries

The The following notation will be used in this paper. R^+ denotes the set of all real non-negative numbers; $\mathbb{R}^n$ denotes the n −dimensional space with the scalar product x, y or $x^T y$ of two vectors x, y, and the vector norm $\|.\|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)$ −dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; Matrix A is called semi-positive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. The notation diag{...} stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$ First point.

Lemma 1.1 ([38]) For a positive definite matrix $R > 0$ and any sequence of discrete-time variables $y: [-h, 0] \cap Z \to R^n$, the following inequality holds:

$$\sum_{i=-h+1}^{0} \Delta x(i)^T R \Delta x(i) \geq \frac{1}{h}\Theta_0^T R \Theta_0 + \frac{3}{h}\frac{h+1}{h-1}\Omega_1^T R \Omega_1 + \frac{5(h+1)(h+2)}{(h-2)(h-1)}\Omega_2^T R \Omega_2,$$

where $\Theta_0 = x(0) - x(-h), \Omega_1 = x(0) + x(-h) - \frac{2}{h+1}\sum_{k=-h}^{0} x(k), \Omega_2 = x(0) - x(-h) + \frac{6h}{(h+1)(h+2)}\sum_{i=-h}^{0} x(i) - \frac{12}{(h+1)(h+2)}\sum_{i=-h+1}^{0}\sum_{k=i}^{0} x(k).$

Lemma 1.2 ([38]) For a positive definite matrix $R > 0$ and any sequence of discrete-time variables $y: [-h, 0] \cap Z \to R^n$, the following inequality holds:

$$\sum_{i=-h+1}^{0}\sum_{k=i}^{0} \Delta x(k)^T R \Delta x(k) \geq \frac{2(h+1)}{h}\left[x(0) - \frac{1}{(h+1)}\sum_{i=-h}^{0} x(i)\right]^T R\left[x(0) - \frac{1}{(h+1)}\sum_{i=-h}^{0} x(i)\right]$$
$$+\frac{4(h+1)(h+2)}{h(h-1)}\Omega_4^T R \Omega_4,$$

Where $\Omega_4 = \left[x(0) + \frac{2}{(h+1)}\sum_{i=-h}^{0} x(i) - \frac{6}{(h+1)(h+2)}\sum_{i=-h}^{0}\sum_{k=i}^{0} x(k)\right].$

Definition 1.3 ([37]) The neural network (1) is said to be (Q, S, R) −dissipative, if the following dissipation inequality

$$\sum_{k_0}^{k_p} r(u(k), y(k)) \geq 0, \quad \forall k_p \geq 0,$$

Holds under zero initial condition for any nonzero input $u \in l_2[0, +\infty)$. Furthermore, if for some scalar ?$\gamma > 0$, the dissipation inequality

$$\sum_{k_0}^{k_p} r(u(k), y(k)) \geq \gamma \sum_{k_0}^{k_p} u^T(k)u(k), \quad \forall k_p \geq 0,$$

Holds under zero initial condition for any nonzero input $u \in l_2[0, +\infty)$, then the neural network (1) is said to be strictly $(Q, S, R) - \gamma$ −dissipative. In this paper, we define a quadratic supply rate $r(u, y)$ associated with neural network (1) as follows:

$$r(u, y) = y^T Q y + 2y^T S u + u^T R u,$$

Where $Q \leq 0, S$, and R are real symmetric matrices of appropriate dimensions.

2- Results and Discussion

In this section, we will establish a new criterion on dissipativity analysis of DNNs with time-varying delays;

$$\begin{aligned} &x(k+1) = Cx(k) + D_0 f(x(k)) + D_1 g(x(k-h(k)) + u(k), \quad k \geq 0, \\ &y(k) = f(x(k)), \\ &x(k) = \varphi(k), k \in [-h_2, 0]. \end{aligned} \tag{1}$$

Where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T \in R^n$ is the state of the neural, $u(k) \in R^n$ is the input belonging to l_2, φ is the initial value, n is the number of neurals,

$f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \ldots, f_n(x_n(k))]^T, \quad g(x(k-h(k)) = [g_1(x_1(k-h(k)),$ $g_2(x_2(k-h(k)), \ldots, g_n(x_n(k-h(k))]^T$, are the activation functions;

$C = \text{diag}(\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_n), \bar{a}_i > 0$ represents the self-feedback term, D_0, D_1 denote the connection weights, the discretely delayed connection weights and the distributively delayed connection weight. For every $k \geq 0$, the variable delay $h(k)$ is defined to be a positive integer and $h(k) \in [h_1, h_2], \forall k \geq 0$ for some integers $h_2 \geq h_1 > 1$.

Assumption 2.1 ([35]) For any $s_1, s_2 \in R$, $s_1 \neq s_2$, the continuous and bounded activation functions;

$\hat{f}_i(\cdot)$ and $\hat{g}_i(\cdot)$ satisfy

$$F_i^- \leq \frac{\hat{f}_i(s_1) - \hat{f}_i(s_2)}{s_1 - s_2} \leq F_i^+,$$
$$G_i^- \leq \frac{\hat{g}_i(s_1) - \hat{g}_i(s_2)}{s_1 - s_2} \leq G_i^+, \quad i = 1,2, \ldots, n,$$ where F_i^-, F_i^+, G_i^-, and G_i^+ are known constants.

The following notations are needed;

$$h_{12} = h_2 - h_1, e_i = \Big[\underbrace{0,0 \ldots, \overset{i}{\widehat{I}}, \ldots 0}_{10}\Big]_{10n\times n}^T, i = 1,2, \ldots, 10, y(k) = x(k) - x(k-1),$$

$$\xi(k) = [x^T(k), x^T(k-h_1), x^T(k-h(k)), x^T(k-h_2), f(x(k)), g(x(k-h(k)), \frac{1}{h_1+1}\sum_{i=k-h_1}^{k} x^T(i),$$

$$\frac{1}{h(k)-h_1+1}\sum_{i=k-h(k)}^{k-h_1} x^T(i), \frac{1}{h_2-h(k)+1}\sum_{i=k-h_2}^{k-h(k)} x^T(i), \sum_{i=-h_1+1}^{0} \sum_{j=k+i}^{k} x^T(j), u^T(k)]^T,$$

$$\alpha(k) = \left[x^T(k), \sum_{i=k-h_1}^{k-1} x^T(i), \sum_{i=k-h_2}^{k-h_1-1} x^T(i), \sum_{i=-h_1+1}^{0} \sum_{j=k+i}^{k} x^T(j)\right]^T,$$

$$Z_{10} = diag\left\{Z_1, \frac{3(h_1+1)}{h_1-1}Z_1, \frac{5(h_1+1)(h_1+2)}{(h_1-2)(h_1-1)}Z_1\right\}, Z_2^* = \begin{bmatrix} Z_2 & 0 \\ 0 & 3Z_2 \end{bmatrix}, Z_{20} = \begin{bmatrix} Z_2^* & X \\ & Z_2^* \end{bmatrix},$$

$$\Pi_0 = [C, 0,0,0, D_0, D_1, 0,0,0, I]^T,$$

$$\Pi_1 = [\Pi_0, (h_1+1)e_7 - e_2, (h(k)-h_1+1)e_8 + (h_2-h(k)+1)e_9 - e_3 - e_4, e_{10} + h_1\Pi_0 - (h_1+1)e_7 + e_2],$$

$$\Pi_2 = [e_1, (h_1+1)e_7 - e_1, (h(k)-h_1+1)e_8 + (h_2-h(k)+1)e_9 - e_3 - e_2, e_{10}],$$

$$\Pi_3 = [C-I, 0,0,0, D_0, D_1, 0,0,0, I]^T, \Pi_4 = \left[e_1 - e_2, e_1 + e_2 - 2e_7, e_1 - e_2 + \frac{6h_1}{h_1+2}e_7 - \frac{12}{(h_1+1)(h_1+2)}e_{10})\right],$$

$$\Pi_5 = [e_3 - e_4, e_3 + e_4 - 2e_9, e_2 - e_3, e_2 + e_3 - 2e_8], \Pi_6 = e_1 - e_7,$$

$$\Pi_7 = e_1 + \left(2 - \frac{6}{(h_1+2)}\right)e_7 - \frac{6}{(h_1+1)(h_1+2)}e_{10}, \Xi_1 = \Pi_1 P \Pi_1^T - \Pi_2 P \Pi_2^T,$$

$$\Xi_2 = e_1 Q_1 e_1^T - e_2 Q_1 e_2^T + e_2 Q_2 e_2^T - e_4 Q_2 e_4^T, \Xi_3 = \Pi_3(h_1^2 Z_1 + h_{12}^2 Z_2)\Pi_3^T - \Pi_4 Z_{10}\Pi_4^T - \Pi_5 Z_{20}\Pi_5^T,$$

$$\Xi_4 = \frac{h_1(h_1+1)}{2}\Pi_3 Z_3 \Pi_3^T - \frac{2(h_1+1)}{h_1}\Pi_6 Z_3 \Pi_6^T - \frac{4(h_1^2-1)}{h_1(h_1+2)}\Pi_7 Z_3 \Pi_7^T, \Xi_5 = -e_1 F_1 \Lambda_1 e_1^T + 2e_1 F_2 \Lambda_1 e_5^T - e_5 \Lambda_1 e_5^T,$$

$$\Xi_6 = -e_3 G_1 \Lambda_2 e_3^T + 2e_3 G_2 \Lambda_2 e_6^T - e_6 \Lambda_2 e_6^T, \Xi_7 = -e_5 \Psi_1 e_5^T - 2e_5 \Psi_2 e_{10}^T - e_{10}\Psi_3 e_{10}^T, \Xi = \sum_{i=1}^{7} \Xi_i.$$

Theorem 2.2 For given integer h_1, h_2 satisfying $1 < h_1 \leq h_2$, matrices Ψ_1, Ψ_2, and Ψ_3 with Ψ_1, Ψ_2, and Ψ_3 being real symmetric, system (1) is stability, passivity, and dissipativity for $h_1 < h(k) \leq h_2$, if there are positive define matrices $P \in R^{4n\times 4n}, Z_1 \in R^{n\times n}, Z_2 \in R^{n\times n}, Z_3 \in R^{n\times n}, Q_1 \in R^{n\times n}, Q_2 \in R^{n\times n}$, and any matrix $X \in R^{2n\times 2n}$, diagonal matrices $\Lambda_1 > 0, \Lambda_2 > 0$ of appropriate dimensions such that the following three LMIs are satisfied.

$$\Xi < 0, \quad Z_{20} \geq 0. \tag{2}$$

Proof. Choose a Lyapunov functional candidate as follows:

$$V(k) = \sum_{j=1}^{4} V_j(k), \tag{3}$$

Where $V_1(k) = \alpha^T(k) P \alpha(k)$,

$$V_2(k) = \sum_{i=k-h_1}^{k-1} x^T(i) Q_1 x(i) + \sum_{i=k-h_2}^{k-h_1-1} x^T(i) Q_2 x(i),$$

$$V_3(k) = h_1 \sum_{i=-h_1+1}^{0} \sum_{j=k+i}^{k} y^T(i) Z_1 y(i) + h_{12} \sum_{i=-h_2+1}^{-h_1} \sum_{j=k+i}^{k} y^T(i) Z_2 y(i), \tag{4}$$

$$V_4(k) = \sum_{i=-h_1+1}^{0} \sum_{j=i}^{0} \sum_{u=k+j}^{k} y^T(u) Z_3 y(u).$$

Next, we calculate the difference of $V(k)$. For $V_1(k)$ and $V_2(k)$, we have:

$$\Delta V_1(k) = \xi^T(k) \Xi_1 \xi(k) \tag{5}$$

and

$$\Delta V_2(k) = \xi^T(k) \Xi_2 \xi(k). \tag{6}$$

Calculating $V_3(k)$ gives;

$$\Delta V_3(k) = h_1^2 y_{k+1}^T Z_1 y_{k+1} + h_{12}^2 y_{k+1}^T Z_2 y_{k+1} - h_1 \sum_{i=k-h_1+1}^{k} y^T(i) Z_1 y(i) - h_{12} - h_{12} \sum_{i=k-h_2+1}^{k-h_1} y^T(i) Z_2 y(i). \tag{7}$$

By Lemma1.1, we get;

$$-h_1 \sum_{i=k-h_1+1}^{k} y^T(i) Z_1 y(i) = -\xi^T(k) \Pi_4 Z_{10} \Pi_4^T \xi(k). \tag{8}$$

Under the condition of $Z_{20} > 0$, by Lemma2.1 and the lower bounded lemma, we get;

$$-h_{12} \sum_{i=k-h_2+1}^{k} y^T(i) Z_2 y(i) = -\xi^T(k) \Pi_5 Z_{20} \Pi_5^T \xi(k). \tag{9}$$

Then we have;

$$\Delta V_3(k) = \xi^T(k)\Xi_3\xi(k). \quad (10)$$

Calculating $\Delta V_4(k)$ gives;

$$\Delta V_4(k) = \frac{h_1(h_1+1)}{2} y_{k+1}^T Z_3 y_{k+1} - \sum_{i=h_1+1}^{0} \sum_{i=k+i}^{k} y^T(j)Z_3y(j). \quad (11)$$

By Lemma1.2, we have;

$$-\sum_{i=h_1+1}^{0} \sum_{i=k+i}^{k} y^T(j)Z_3y(j) \leq \xi^T(k)\left(-\frac{2(h_1+1)}{h_1}\Pi_6 Z_3 \Pi_6^T - \frac{4(h_1+1)(h_2+2)}{h_1(h_1+1)}\Pi_7 Z_3 \Pi_7^T\right)\xi(k). \quad (12)$$

Then we have;

$$\Delta V_4(k) = \xi^T(k)\Xi_4\xi(k). \quad (13)$$

From Assumption 2.1, we have;

$$\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} F_1\Lambda_1 & -F_2\Lambda_1 \\ -F_2\Lambda_1 & \Lambda_1 \end{bmatrix}^T \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \leq 0, \quad (14)$$

$$\begin{bmatrix} x(k-h(k)) \\ g(x(k-h(k))) \end{bmatrix}^T \begin{bmatrix} G_1\Lambda_1 & -G_2\Lambda_1 \\ -G_2\Lambda_1 & \Lambda_1 \end{bmatrix}^T \begin{bmatrix} x(k-h(k)) \\ g(x(k-h(k))) \end{bmatrix} \leq 0, \quad (15)$$

Where

$$\Lambda_1 = diag\{\lambda_{11}, \lambda_{12}, \dots, \lambda_{1n}\}, \Lambda_2 = diag\{\lambda_{21}, \lambda_{22}, \dots, \lambda_{2n}\},$$

$$F_1 = diag\{F_1^- F_1^+, F_2^- F_2^+, \dots, F_n^- F_n^+\},_2 = diag\{\frac{F_1^- + F_1^+}{2}, \frac{F_2^- + F_2^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\},$$

$$G_1 = diag\{G_1^- G_1^+, G_2^- G_2^+, \dots, G_n^- G_n^+\},_2 = diag\{\frac{G_1^- + G_1^+}{2}, \frac{G_2^- + G_2^+}{2}, \dots, \frac{G_n^- + G_n^+}{2}\}.$$

Since $\Delta x_k = x_{k+1} - x_k$, by introducing relaxation matrices W_1, W_2 with appropriate dimensions, we obtain the following zero equation;

$$2(\Delta^T(k)Z_1^T + x^T(k)Z_2^T)[Cx(k) + D_0 f(x(k)) + D_1 g(x(k-h(k)) + u(k) - x(k) - \Delta^T(k)] = 0. \quad (16)$$

Define

$$J(i) = y^T(i)\Psi_1 y(i) + 2y^T(i)\Psi_2 u(i) + u^T(i)\Psi_3 u(i). \quad (17)$$

Adding the equations from (5) to (17), then we can get the upper bound of $\Delta V(k) - J(k)$ as

$$\Delta V(k) - J(k) \leq \xi^T(k) \sum_{i=1}^{7} \Xi_i \xi(k) = \xi^T(k)\Xi\xi(k). \quad (18)$$

If $\Xi < 0$, then $\Delta V(k) - J(k) < 0$.

This completes the proof of Theorem2.2.

Remark 2.3 In the above Theorem, we analyzed the passivity and dissipavity for DNN (1). In the following corollary analyze the asymptotic stability for NN:

$$x(k+1) = Cx(k) + D_0 f(x(k)) + D_1 g(x(k-h(k)), \quad k \geq 0, \quad (19)$$
$$x(k) = \varphi(k), k \in [-h_2, 0].$$

The NN (19) is a special case of (1) when $u(k) = y(k) = 0$.

The following notations are needed.

$$h_{12} = h_2 - h_1, e_i = \Bigg[\underbrace{0,0 \dots, \overset{i}{\widehat{I}}, \dots 0}_{9}\Bigg]_{9n \times n}^T, i = 1,2, \dots, 9, y(k) = x(k) - x(k-1),$$

$$\xi(k) = [x^T(k), x^T(k-h_1), x^T(k-h(k)), x^T(k-h_2), f(x(k)), g(x(k-h(k)), \frac{1}{h_1+1}\sum_{i=k-h_1}^{k} x^T(i),$$

$$\frac{1}{h(k)-h_1+1}\sum_{i=k-h(k)}^{k-h_1} x^T(i), \frac{1}{h_2-h(k)+1}\sum_{i=k-h_2}^{k-h(k)} x^T(i), \sum_{i=-h_1+1}^{0} \sum_{j=k+i}^{k} x^T(j)]^T,$$

$$\alpha(k) = \left[x^T(k), \sum_{i=k-h_1}^{k-1} x^T(i), \sum_{i=k-h_2}^{k-h_1-1} x^T(i), \sum_{i=-h_1+1}^{0} \sum_{j=k+i}^{k} x^T(j)\right]^T,$$

$Z_{10}=diag\left\{Z_1,\frac{3(h_1+1)}{h_1-1}Z_1,\frac{5(h_1+1)(h_1+2)}{(h_1-2)(h_1-1)}Z_1\right\},Z_2^*=\begin{bmatrix}Z_2 & 0\\ 0 & 3Z_2\end{bmatrix},Z_{20}=\begin{bmatrix}Z_2^* & X\\ & Z_2^*\end{bmatrix},$

$\Pi_0=[C,0,0,0,D_0,D_1,0,0,0]^T,$

$\Pi_1=[\Pi_0,(h_1+1)e_7-e_2,(h(k)-h_1+1)e_8+(h_2-h(k)+1)e_9-e_3-e_4,e_{10}+h_1\Pi_0-(h_1+1)e_7+e_2],$

$\Pi_2=[e_1,(h_1+1)e_7-e_1,(h(k)-h_1+1)e_8+(h_2-h(k)+1)e_9-e_3-e_2,e_{10}],$

$\Pi_3=[C-I,0,0,0,D_0,D_1,0,0,0]^T,\Pi_4=\left[e_1-e_2,e_1+e_2-2e_7,e_1-e_2+\frac{6h_1}{h_1+2}e_7-\frac{12}{(h_1+1)(h_1+2)}e_{10})\right],$

$\Pi_5=[e_3-e_4,e_3+e_4-2e_9,e_2-e_3,e_2+e_3-2e_8],\Pi_6=e_1-e_7,$

$\Pi_7=e_1+\left(2-\frac{6}{(h_1+2)}\right)e_7-\frac{6}{(h_1+1)(h_1+2)}e_{10},\Xi_1=\Pi_1P\Pi_1^T-\Pi_2P\Pi_2^T,$

$\Xi_2=e_1Q_1e_1^T-e_2Q_1e_2^T+e_2Q_2e_2^T-e_4Q_2e_4^T,\Xi_3=\Pi_3(h_1^2Z_1+h_{12}^2Z_2)\Pi_3^T-\Pi_4Z_{10}\Pi_4^T-\Pi_5Z_{20}\Pi_5^T,$

$\Xi_4=\frac{h_1(h_1+1)}{2}\Pi_3Z_3\Pi_3^T-\frac{2(h_1+1)}{h_1}\Pi_6Z_3\Pi_6^T-\frac{4(h_1^2-1)}{h_1(h_1+2)}\Pi_7Z_3\Pi_7^T,\Xi_5=-e_1F_1\Lambda_1e_1^T+2e_1F_2\Lambda_1e_5^T-e_5\Lambda_1e_5^T,$

$\Xi_6=-e_3G_1\Lambda_2e_3^T+2e_3G_2\Lambda_2e_6^T-e_6\Lambda_2e_6^T,\Theta=\sum_{i=1}^{6}\Xi_i.$

Corollary 2.4 For given integer h_1,h_2 satisfying $0<h_1<h_2$ system (19) is asymptotically stable for $h_1<h(k)\leq h_2$, if there are positive define matrices $P\in R^{4n\times 4n},Z_1\in R^{n\times n},Z_2\in R^{n\times n},Z_3\in R^{n\times n},Q_1\in R^{n\times n},Q_2\in R^{n\times n}$, and any matrix $X\in R^{2n\times 2n}$, diagonal matrices $\Lambda_1>0,\Lambda_2>0$ of appropriate dimensions such that the following three LMIs are satisfied.

$$\Theta<0,\qquad Z_{20}\geq 0. \tag{20}$$

Summing up the above analysis, some sufficient conditions on finite-time stability for dissipativity and passivity analysis of discrete-time neural networks with time-varying delays (1) with (2), (3) are obtained. In the following, we mainly focus on stabilizing by transforming the sufficient conditions into solvable linear matrix inequalities.

3- Conclusion

In this paper, stability, dissipativity and passivity analysis of discrete-time neural networks with time-varying delays was studied. A delay-dependent condition has been provided to ensure the considered discrete-time neural networks with time-varying delays to be stability, dissipativity and passivity. An effective LMI approach has been proposed to derive the stability, dissipativity and passivity criterion. Based on the appropriate type of Lyapunov functional, a sufficient condition for the solvability of this problem is established for the stability, dissipativity and passivity criterion.

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5- Conflict of Interest

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely observed by the authors.

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