Mathematical Foundations of Computing

doi:10.3934/mfc.2024006

EXTENDED DISSIPATIVITY-BASED SAMPLED-DATA CONTROLLER DESIGN FOR FUZZY DISTRIBUTED PARAMETER SYSTEMS

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Abstract. This paper presents an extended dissipativity-based sampled-data controller design for Fuzzy distributed parameter systems (DPSs). Then the proposed DPSs solved by integral inequality techniques to construct a Lyapunov-Krasovskii functional (LKF) that demonstrates the stability and stabilization of the dissipativity performance of DPSs. The fuzzy-based sampled-data control (FSDC) scheme is obtained by solving linear matrix inequalities (LMIs), ensuring that the closed-loop system is extended dissipativity. The FSDC can be adjusted to achieve various performance goals, such as $L_2 - L_\infty$, H_∞, passivity, and $(Q, S, R) - \gamma$-dissipative performance for DPSs. The proposed method is verified through simulations using the MATLAB LMI control toolbox.

1. Introduction.

A growing number of engineering applications have focused on the control of distributed parameter systems (DPSs) driven by partial differential equations (PDEs) because of the use of compliant structures, smart materials, and structures, system information networks, multi-scale and multi-physics systems, etc. DPSs have been extensively studied in the past few years, with hyperbolic and parabolic PDE systems receiving particular attention [14, 30, 21]. The authors in [15] discussed robust H_∞ control for semi-linear parabolic DPSs in the presence of external disturbances. Additionally, authors in [12] discussed the control of fuzzy DPSs using an event generator with H_∞ performance, while [5] employed the average dwell time approach to investigate the exponential stability and L_2 gain analysis of DPSs. Despite the abundance of studies focusing on linear PDE systems, questions still persist regarding the regulation of nonlinear DPSs.

2020 *Mathematics Subject Classification.* Primary: 93C57, 93C10; Secondary: 35R13.

Key words and phrases. Distributed parameter system, sampled-data control, linear matrix inequality, T-S fuzzy model, dissipativity.

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Many types of industrial control systems often exhibit nonlinear properties. Studying and designing systems that do not follow a linear path can be difficult due to their often unpredictable behavior. A Takagi-Sugeno (T-S) fuzzy model is a method used to tackle problems related to controlling nonlinear systems. To enhance the understanding of how to effectively manage certain elements of the system, this model combines the concepts of fuzzy theory and linear system theory. To investigate the stability and stabilization concerns related to nonlinear systems, one can employ a fuzzy T-S model. For instance, the following references provide insights on this topic: [20, 27, 23, 9, 22, 8, 24]. This is crucial because these systems occasionally exhibit instability or unsteadiness. For instance, the authors in [20] investigated fuzzy delayed neural networks with random time-varying parameters. In [9], the problem of H_∞ synchronization in T-S fuzzy complex networks with hybrid coupling delays was discussed. Notably, recent results on the stabilization of a switched chaotic system using a T-S fuzzy sampling approach have been reported in [24]. In [22] addresses fuzzy-enhanced robust fault-tolerant control of IFOC motors, considering both matched and mismatched disturbances. It is providing insights into controlling motor systems under various conditions. Authors [8] focuses on output feedback control for interval type-2 T-S fuzzy fractional order systems subjected to actuator saturation. This contributes understanding of control strategies in systems with fractional order dynamics and actuator limitations.

At the same time, sampled-data control (SDC) stands out as the most widely used approach for fuzzy T-S systems due to its practicality and effectiveness [28, 16, 10, 26]. The standard T-S fuzzy model is commonly employed in actual engineering applications to explain various nonlinear dynamic systems for stability analysis and controller design [7, 6, 13]. The sampling controller utilizes periodically observed signals, such as set point signals, drive deviation signals, or signals related to the controlled variable, in order to maintain an influence on the control effect. Utilizing sample control enhances the system's capacity to regulate with precision and mitigate interference, while also optimizing the efficiency and adaptability of the controller. The control signals are exclusively provided at discrete time intervals according to SDC, in contrast to continuous-time control. Consequently, a significant amount of network communication resources can be conserved. The T-S fuzzy model and SDC applications greatly expand the scope of research in nonlinear control theory.

Meanwhile, research on extended dissipative (ED) performance holds crucial significance in various practical control systems and applications, playing a pivotal role in determining the input/output characteristics of a system. The ED concept was initially introduced in [29], presenting a framework that unifies four performance metrics represented using weighting matrices. ED performance incorporates H_∞, $L_2 - L_\infty$, passivity, and the recently established strictly $(Q, S, R) - \gamma-$dissipativity into a cohesive framework. In [4], the authors specifically addressed the fuzzy filtering issue, considering H_∞, $L_2 - L_\infty$, and dissipative performance for delayed nonlinear systems. Compared to strictly $(Q, S, R) - \gamma-$dissipativity, ED demonstrated a more comprehensive nature. The study by [17] explore into the ED problem of generalized neural networks, accounting for random controller gain fluctuations. Authors in [19] investigated the finite-time ED analysis problem of neural networks modeled as delayed TSFS. Furthermore, [1] explored nonfragile synchronization of ED for delayed recurrent neural networks with sampled-data control (SDC).

Inspired by the discussions outlined earlier, this paper focuses on ED analysis for T-S fuzzy DPSs employing a sampled-data controller. The ensuing points highlight the key innovations and significant contributions of our work in relation to existing literature.

i). In this paper, sampled data control design is adopted into the T-S fuzzy DPSs to investigate the ED performance.

ii). Based on the ED theory, a SDC design is considered to realize the famous performances, such as H_∞, L_2-L_∞, passivity and $(Q,S,R)-\gamma$ -dissipativity are achieved by modifying the weighting matrices in a new performance index.

iii). Effective construction of DPSs with SDC has been achieved through the utilization of enhanced inequality approaches to deal with the integral item, which makes the stability conditions. Consequently, the entire analytical approach has been simplified to achieve quadratically stable conclusions.

iv). An improved LKF approach and all the sufficient conditions are expressed in terms of LMIs which can be easily solved by using Matlab software. Subsequently, based on these conditions, the unknown desired SDC gain matrix is determined.

v). Finally, a numerical example is used to show the effectiveness of the T-S fuzzy DPSs with the SDC method proposed in this research.

Notation. The n-dimensional Euclidean space is represented by $\mathbb{R}^n$. A symmetric matrix with positive elements is denoted by P_i, where $P_i > 0$. The symbol diag$\{\cdot\}$ and $*$ is used to represent a diagonal matrix and symmetric matrix. $||\omega(x)||_2 = \sqrt{\int_{l_1}^{l_2}\omega^T(x)\omega(x)dx}$, $H_n^l(l_1,l_2) = W^{l,2}((l_1,l_2)$; $\mathbb{R}^n$ denotes a real sobolev space of absolutely continuous vector functions, $\omega(x) : (l_1,l_2) \to \mathbb{R}^n$ with square-integrable derivatives $d^l\omega(x)/dx^l$ of the order $l \geq 1$ and

$$||\omega(x)||_{H_n^l(l_1,l_2)}=\sqrt{\int_{l_1}^{l_2}\sum_{i=0}^{l}\left(\frac{d^i\omega(x)}{dx^i}\right)^T\left(\frac{d^i\omega(x)}{dx^i}\right)dx}.$$

2. Preliminary system and T-S fuzzy model.

A class of nonlinear delayed DPSs that can be characterized by a collection of parabolic PDEs, which are modeled as follows.

$$\begin{aligned}\xi_t(x,t) &= \Theta\xi_{xx}(x,t) + f_1(\xi(x,t)) + f_d(\xi(x,t-\breve{\tau}(t)))\\ &\quad + g_u(\xi(x,t))u(x,t) + g_\omega(\xi(x,t)\omega(x,t),\\ z(x,t) &= f_2(\xi(x,t)) + f_3(\xi(x,t-\breve{\tau}(t))),\end{aligned} \tag{1}$$

dependent on the Dirichlet boundary condition

$$\xi(l_1,t) = \xi(l_2,t) = 0, \tag{2}$$

and the initial condition

$$\xi(x,t) = \xi_0(x), \tag{3}$$

in which $\xi_t(x,t) = \frac{\partial\xi(x,t)}{\partial t}$, $\xi_{xx}(x,t) = \frac{\partial^2\xi(x,t)}{\partial x^2}$, $\xi(x,t) \in \mathbb{R}^n$; $x \in [l_1,l_2] \subset \mathbb{R}$ and $t \in [0,\infty) \subset \mathbb{R}$ represent the spatial position and time. $u(x,t) \in \mathbb{R}^n$ noted as control input, $z(x,t) \in \mathbb{R}^n$ represents the output, $\omega(x,t) \in \mathbb{R}^n$ is the disturbance satisfying $\int_0^\infty \|\omega(x,t)\|_2^2\ dt < \infty$; $f_1(\cdot)$, $f_d(\cdot)$, $g_u(\cdot)$, $g_w(\cdot)$, $f_2(\cdot)$ and $f_3(\cdot)$ are continuous nonlinear functions; $0 \leq \breve{\tau}(t) \leq \breve{\tau}$, $0 \leq \dot{\breve{\tau}}(t) \leq \mu \leq 1$, where $\breve{\tau}$ and μ are positive constants, and Θ is the given matrix. For our convenience we can define the matrix Θ as follows:

$$\Theta\xi_{xx}(x,t) = \mathcal{A}\xi(x,t). \tag{4}$$

Using the sector non-linearity approach, the nonlinear parabolic PDE system (1) can be shown by T-S fuzzy model described by following **IF-THEN** rules:
Rule $\mathcal{R}^i$: **IF** : $\breve{\theta}_1(x,t)$ is F_1^i, $\breve{\theta}_2(x,t)$ is F_2^i,$\cdots$, and $\breve{\theta}_p(x,t)$ is F_p^i, **THEN**

$$\begin{aligned}\xi_t(x,t) =& \mathcal{A}\xi(x,t) + A_i\xi(x,t) + A_{di}\xi(x,t-\breve{\tau}(t))\\ &+ B_iu(x,t) + C_i\omega(x,t),\\ z(x,t) =& L_i\xi(x,t) + M_i\xi(x,t-\breve{\tau}(t)),\end{aligned} \tag{5}$$

where $\breve{\theta}_j(x,t)$ and F_j^i are premise variables and fuzzy sets ($i = 1,2,\cdots,s, j = 1,2,\cdots,p$), and $A_i, A_{di}, B_i, C_i, L_i, M_i$ are known matrices. We represent the fuzzy controller design as follows:

$$\begin{aligned}\xi_t(x,t) =& \mathcal{A}\xi(x,t) + \sum_{i=1}^{s} h_i(\breve{\theta}(x,t)) \times [A_i\xi(x,t)\\ &+ A_{di}\xi(x,t-\breve{\tau}(t)) + B_iu(x,t) + C_i\omega(x,t)],\\ z(x,t) =& \sum_{i=1}^{s} h_i(\breve{\theta}(x,t)) \times [L_i\xi(x,t) + M_i\xi(x,t-\breve{\tau}(t))],\end{aligned} \tag{6}$$

with

$$h_i(\breve{\theta}(x,t)) = \frac{\beta_i(\breve{\theta}(x,t))}{\sum_{i=1}^{s}\beta_i(\breve{\theta}(x,t))},\ \beta_i(\breve{\theta}(x,t)) = \prod_{j=1}^{p} F_j^i(\breve{\theta}_j(x,t)),$$

$0 < h_i(\breve{\theta}(x,t)) < 1$, and $\sum_{i=1}^{s} \breve{h}_i(\theta(x,t)) = 1$ indicate the membership functions with $\breve{\theta}_j(x,t)$ belonging to the fuzzy set F_j^i.

2.1. **Sampled-data and fuzzy controller design.** For the purpose of designing the controller and ensuring the stability and stabilization of the T-S fuzzy system (6), the parallel distributed compensation (PDC) is utilized with $u(t) = u_j(t),\ t_k \leq t < t_{k+1}$ with sampling instants t_k, $(k = 0,1,2,...)$ satisfying $\lim_{k\to+\infty} = +\infty$. Therefore, the sampled-data fuzzy control laws are designed as
Rule $\mathcal{R}^j$: **IF** : $\breve{\theta}_1(x,t_k)$ is F_1^j, $\breve{\theta}_2(x,t_k)$ is F_2^j,$\cdots$, and $\breve{\theta}_p(x,t_k)$ is F_p^j, **THEN**

$$\begin{aligned}u(x,t) = u_d(x,t_k) = K_j\xi(x,t_k) = K_j\xi(x,t-\hbar(t)),\\ t_k \leq t \leq t_{k+1},\end{aligned} \tag{7}$$

where K_j is the feedback control gain, (x,t_k) is the measurement of state (x,t) at sampling instant t_k, and it is supposed that $0 < t_{k+1} - t_k \leq \hbar$, $\hbar > 0$, $\forall k \geq 0$. Then, by defining $\hbar(t) = t - t_k$ with $\dot{\hbar}(t) = 1$ for $t \neq t_k$.

Then, the control scheme (7), the defuzzified dynamics of the SDC is defined as follows:

$$u(x,t) = \sum_{j=1}^{s} h_j(\breve{\theta}(x,t))K_j\xi(x,t-\hbar(t)). \tag{8}$$

Combining (6)and (8), we get

$$\begin{aligned}\xi_t(x,t) =&\mathcal{A}\xi(x,t) + \hat{A}_i\xi(x,t) + \hat{A}_{di}\xi(x,t-\breve{\tau}(t))\\ &+ \hat{B}_i\hat{K}_j\xi(x,t-h(t)) + \hat{C}_i\omega(x,t)],\\ z(x,t) =&\hat{L}_i\xi(x,t) + \hat{M}_i\xi(x,t-\breve{\tau}(t)).\end{aligned} \tag{9}$$

For convenience, define

$$\begin{aligned}\hat{A}_i &= \sum_{i=1}^{s} h_i(\breve{\theta}(x,t))A_i, \quad \hat{A}_{di} = \sum_{i=1}^{s} h_i(\breve{\theta}(x,t))A_{di},\\ \hat{C}_i &= \sum_{i=1}^{s} h_i(\breve{\theta}(x,t))C_i,\\ \hat{B}_i\hat{K}_j &= \sum_{i=1}^{s}\sum_{j=1}^{s} h_i(\breve{\theta}(x,t))h_j(\breve{\theta}(x,t))B_iK_j,\\ \hat{L}_i &= \sum_{i=1}^{s} h_i(\breve{\theta}(x,t))L_i, \quad \hat{M}_i = \sum_{i=1}^{s} h_i(\breve{\theta}(x,t))M_i.\end{aligned}$$

The key conclusions will be obtained using the following assumption, definition, and lemma.

Assumption 2.1. *For given matrices R_1, R_2, R_3 and R_4 satisfying*

(i). $R_1 = R_1^T \leq 0, R_3 = R_3^T > 0, R_4 = R_4^T \leq 0$
(ii). $(\|R_1\| + \|R_2\|).\,\|R_4\| = 0.$

Definition 2.2. [2] Given matrices R_1, R_2, R_3 and R_4 that meet the requirements of the Assumption 2.1, a system is defined as ED if there exists a scalar $\varsigma > 0$ such that the subsequent inequality is true for all values of $t_f \geq 0$

$$\int_0^{t_f}\int_{l_1}^{l_2} J(x,t)dxdt \geq \sup_{0\leq t\leq t_f}\int_{l_1}^{l_2}\left\{z^T(x,t)R_4z(x,t)dx\right\} + \varsigma, \tag{10}$$

where $J(x,t) = z^T(x,t)R_1z(x,t) + 2z^T(x,t)R_2\omega(x,t) + \omega^T(x,t)R_3\omega(x,t)$.

Definition 2.3. [3] If there is a scalar $\nu > 0$ and it seems to be that the derivative of the Lyapunov function in relation to the time t fulfills, then the system described by (9)

$$\dot{V}(t) \leq -\nu\left|\xi(x,t)\right|^2. \tag{11}$$

Then, (9) with $w(x,t) = 0$ is said to be quadratically stable.

Lemma 2.4. [25] *Let $\xi \in H_n^l(l_1,l_2)$ be a vector function with $\xi(l_1) = \xi(l_2) = 0$, then for any $n\times n$ real matrix $E \geq 0$, we have*

$$\begin{aligned}&\int_{l_1}^{l_2}\xi^T(s)E\xi(s)ds\\ &\leq (l_2-l_1)^2\pi^{-2}\int_{l_1}^{l_2}\left(\frac{d\xi(s)}{ds}\right)^T E\left(\frac{d\xi(s)}{ds}\right)ds.\end{aligned} \tag{12}$$

Moreover, if $\xi(l_1) = 0$ or $\xi(l_2) = 0$, then

$$\begin{aligned}&\int_{l_1}^{l_2} \xi^T(s)E\xi(s)ds \\ &\leq 4(l_2 - l_1)^2\pi^{-2} \int_{l_1}^{l_2} \left(\frac{d\xi(s)}{ds}\right)^T E \left(\frac{d\xi(s)}{ds}\right) ds. \end{aligned} \tag{13}$$

Lemma 2.5. [18] *Suppose that $\hat{f}_1, \hat{f}_2, \cdots, \hat{f}_N : \mathbb{R}^m \longmapsto \mathbb{R}$ takes positive values in an open subset $\mathcal{M}$ of $\mathbb{R}^m$. Then, the following condition is satisfied*

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i \alpha_i=1\}} \sum_i \frac{1}{\alpha_i}\hat{f}_i(t) = \sum_i \hat{f}_i(t) + \max_{\hat{g}_{i,j}(t)} \sum_{i\neq j} \hat{g}_{i,j}(t),$$

subject to

$$\left\{\hat{g}_{i,j}(t) : \mathbb{R}^m \longmapsto \mathbb{R}, \hat{g}_{j,i}(t)\underline{\underline{\Delta}}\hat{g}_{i,j}(t), \begin{bmatrix} \hat{f}_i(t) & \hat{g}_{i,j}(t) \\ \hat{g}_{i,j}(t) & \hat{f}_j(t) \end{bmatrix} \geq 0\right\}.$$

Remark 2.6. This new behavior may have a more general solution, as suggested by the inequality (10), which may be achieved by setting the weighting matrices to have the values $R_l, l = 1, 2, 3, 4$, that is

(i). if $R_1 = 0, R_2 = 0, R_3 = \gamma^2 I, R_4 = I$ and $\varsigma = 0$, then (10) implies $L_2 - L_\infty$ performance.

(ii). if $R_1 = -I, R_2 = 0, R_3 = \gamma^2 I, R_4 = 0$ and $\varsigma = 0$, then (10) implies H_∞ performance.

(iii). if $R_1 = 0, R_2 = I, R_3 = \gamma I, R_4 = 0$ and $\varsigma = 0$, then the expression in (10) degenerates passivity performance.

(iv). if $R_1 = Q, R_2 = S, R_3 = R - \gamma I, R_4 = 0$ and $\varsigma = 0$, then (10) leads to $(Q - S - R) - \gamma$ dissipativity performance.

3. **Main results.**

In this section, we attempt to determine the ED of the systems given in (9). Sufficient conditions which can ensure the quadratically stable of the FSDC under the ED are provided in this section. On this basis, a design method of SDC strategy is presented. For the case that the controller in Theorem 3.1 is given as a known, a dissipativity condition is presented as follows.

Theorem 3.1. *It has been assumed that the fuzzy system (9) is quadratically stable with ED for given positive constants $\check{\tau}$, $\hbar$, μ, $0 < \hat{\epsilon} < 1$, control gain matrices $\hat{K}_j$, and given matrices R_1, R_2, R_3, R_4 satisfying Assumption 2.1. This assumption holds when there exist matrices $P_i > 0 (i = 1, 2, 3, 4, 5, 6), G$, and P_2, P_6 that satisfy $P_2\Theta \geq 0$ and $P_6\Theta \geq 0$, and the following LMIs hold:*

$$\hat{\Pi} = \begin{bmatrix} \hat{\epsilon}P_1 - R_4 & -R_4 \\ * & (1-\hat{\epsilon})P_1 - R_4 \end{bmatrix} > 0. \tag{14}$$

$$\theta_{ij} = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} & \theta_{1,3} & \theta_{1,4} & 0 & 0 & \theta_{1,7} \\ * & \theta_{2,2} & \theta_{2,3} & \theta_{2,4} & \theta_{2,5} & 0 & \theta_{2,7} \\ * & * & \theta_{3,3} & 0 & 0 & 0 & \theta_{3,7} \\ * & * & * & \theta_{4,4} & \theta_{4,5} & 0 & 0 \\ * & * & * & * & \theta_{5,5} & 0 & 0 \\ * & * & * & * & * & \theta_{6,6} & 0 \\ * & * & * & * & * & * & \theta_{7,7} \end{bmatrix} < 0. \tag{15}$$

where

$$\theta_{1,1} = \hbar^2 P_5 - 2P_2,\ \theta_{1,2} = 2P_2\hat{A}_i + (2P_1)^T - (2P_6)^T,$$
$$\theta_{1,3} = 2P_2\hat{A}_{di},\ \theta_{1,4} = 2P_2\hat{B}_i\hat{K}_j,\quad \theta_{1,7} = 2P_2\hat{C}_i,$$
$$\theta_{2,2} = P_3 + P_4 - P_5 + 2P_6\hat{A}_i - \hat{L}_i^T R_1 \hat{L}_i,$$
$$\theta_{2,3} = 2P_6\hat{A}_{di} - \hat{L}_i^T R_1 \hat{M}_i,\quad \theta_{2,4} = 2P_6\hat{B}_i\hat{K}_j + P_5 - G^T,$$
$$\theta_{2,5} = G^T,\ \theta_{2,7} = 2P_6\hat{C}_i - 2\hat{L}_i^T R_2,$$
$$\theta_{3,3} = -(1-\mu)P_3 - \hat{M}_i^T R_1 \hat{M}_i,\ \theta_{3,7} = -2\hat{M}_i^T R_2,$$
$$\theta_{4,4} = -P_5 - P_5 + G + G^T,\quad \theta_{4,5} = P_5 - G^T,$$
$$\theta_{5,5} = -P_4 - P_5,\ \theta_{6,6} = -\frac{\Pi^2}{4(l_2 - l_1)^2}[Sym(P_6\Theta)],$$
$$\theta_{7,7} = -R_3,$$

Proof. Consider the Lyapunov functional candidate:

$$V(t) \quad = \quad \sum_{i=1}^{5} V_i(t), \tag{16}$$

where

$$V_1(t) = \int_{l_1}^{l_2} \xi^T(x,t) P_1 \xi(x,t) dx,$$
$$V_2(t) = \int_{l_1}^{l_2} \left(\frac{\partial \xi^T(x,t)}{\partial x}\right) P_2 \Theta \left(\frac{\partial \xi(x,t)}{\partial x}\right) dx,$$
$$V_3(t) = \int_{l_1}^{l_2} \int_{t-\check{\tau}(t)}^{t} \xi^T(x,s) P_3 \xi(x,s) ds dx,$$
$$V_4(t) = \int_{l_1}^{l_2} \int_{t-\hbar}^{t} \xi^T(x,s) P_4 \xi(x,s) ds dx,$$
$$V_5(t) = \hbar \int_{l_1}^{l_2} \int_{-\hbar}^{0} \int_{t+\theta}^{t} \left(\frac{\partial \xi^T(x,s)}{\partial s}\right) P_5 \left(\frac{\partial \xi(x,s)}{\partial s}\right) ds d\theta dx$$

Then, the time derivative of $V(t)$ is

$$\dot{V}_1(t) = 2\int_{l_1}^{l_2} \xi^T(x,t) P_1 \left(\frac{\partial \xi(x,t)}{\partial t}\right) dx, \tag{17}$$

$$\dot{V}_2(t) = 2\int_{l_1}^{l_2} \left(\frac{\partial \xi^T(x,t)}{\partial x} P_2 \Theta \frac{\partial \xi(x,t)}{\partial x \partial t}\right) dx, \tag{18}$$

$$\dot{V}_3(t) = \int_{l_1}^{l_2} \Big[\xi^T(x,t)P_3\xi(x,t) - (1-\mu)\xi^T(x,t-\breve{\tau}(t))P_3\xi(x,t-\breve{\tau}(t))\Big]dx, \tag{19}$$

$$\dot{V}_4(t) = \int_{l_1}^{l_2} \Big[\xi^T(x,t)P_4\xi(x,t) - \xi^T(x,t-\hbar)P_4\xi(x,t-\hbar)\Big]dx, \tag{20}$$

$$\dot{V}_5(t) = \int_{l_1}^{l_2} \Big[\hbar^2 \frac{\partial\xi^T(x,t)}{\partial t}P_5\frac{\partial\xi(x,t)}{\partial t} - \hbar\int_{t-\hbar}^{t} \frac{\partial\xi^T(x,s)}{\partial s}P_5\frac{\partial\xi(x,s)}{\partial s}ds\Big]dx. \tag{21}$$

Using Dirichlet boundary condition (2), we have

$$\int_{l_1}^{l_2} \frac{\partial\xi^T(x,t)}{\partial x}P_2\Theta\frac{\partial\xi(x,t)}{\partial x\partial t}dx = -\int_{l_1}^{l_2} \frac{\partial^2\xi^T(x,t)}{\partial x^2}P_2\Theta\frac{\partial\xi(x,t)}{\partial t}dx, \tag{22}$$

Furthermore, Jensen's inequality [11] and Lemma 2.5 yields

$$\begin{aligned}
&-\hbar\int_{l_1}^{l_2}\int_{t-\hbar}^{t} \frac{\partial\xi^T(x,s)}{\partial s}P_5\frac{\partial\xi(x,s)}{\partial s}dsdx \\
&= -\hbar\int_{l_1}^{l_2}\left[\int_{t-\hbar}^{t-\hbar(t)} \frac{\partial\xi^T(x,s)}{\partial s}P_5\frac{\partial\xi(x,s)}{\partial s}ds \right.\\
&\quad \left. + \int_{t-\hbar(t)}^{t} \frac{\partial\xi^T(x,s)}{\partial s}P_5\frac{\partial\xi(x,s)}{\partial s}ds\right]dx \\
&\leq -\int_{l_1}^{l_2}\left[\tfrac{\hbar}{\hbar-\hbar(t)}\left(\int_{t-\hbar}^{t-\hbar(t)} \frac{\partial\xi^T(x,s)}{\partial s}ds\right)P_5\left(\int_{t-\hbar}^{t-\hbar(t)} \frac{\partial\xi(x,s)}{\partial s}ds\right)\right.\\
&\quad \left. + \tfrac{\hbar}{\hbar(t)}\left(\int_{t-\hbar(t)}^{t} \frac{\partial\xi^T(x,s)}{\partial s}ds\right)P_5\left(\int_{t-\hbar(t)}^{t} \frac{\partial\xi(x,s)}{\partial s}ds\right)\right]dx \\
&\leq -\int_{l_1}^{l_2}\left[\left(\int_{t-\hbar}^{t-\hbar(t)} \frac{\partial\xi^T(x,s)}{\partial s}ds\right)P_5\left(\int_{t-\hbar}^{t-\hbar(t)} \frac{\partial\xi(x,s)}{\partial s}ds\right)\right.\\
&\quad + \left(\int_{t-\hbar(t)}^{t} \frac{\partial\xi^T(x,s)}{\partial s}ds\right)P_5\left(\int_{t-\hbar(t)}^{t} \frac{\partial\xi(x,s)}{\partial s}ds\right)\\
&\quad \left. + 2\int_{t-\hbar}^{t-\hbar(t)}\left(\frac{\partial\xi^T(x,s)}{\partial s}ds\right)G\left(\int_{t-\hbar(t)}^{t} \frac{\partial\xi(x,s)}{\partial s}ds\right)\right]dx.
\end{aligned} \tag{23}$$

On the other hand, based on equation (9), we have

$$\begin{aligned}
&2\left(\frac{\partial\xi^T(x,t)}{\partial t}P_2 + \xi^T(x,t)P_6\right)\left(-\frac{\partial\xi(x,t)}{\partial t} + \mathcal{A}\xi(x,t)\right.\\
&+ \hat{A}_i\xi(x,t) + \hat{A}_{di}\xi(x,t-\breve{\tau}(t))\\
&\left.+ \hat{B}_i\hat{K}_j\xi(x,t-\hbar(t)) + \hat{C}_i\omega(x,t)\right) = 0.
\end{aligned} \tag{24}$$

Utilizing integrating by parts and combining boundary conditions (2), one gets

$$\int_{l_1}^{l_2} \xi^T(x,t) P_6 \Theta \xi_{xx}(x,t) dx = -\int_{l_1}^{l_2} \xi_x^T(x,t) P_6 \Theta \xi_x(x,t) dx$$

$$\leq -\frac{\pi^2}{4(l_2-l_1)^2} \int_{l_1}^{l_2} (\xi(x,t)-\xi(l_2,t))^T$$

$$P_6 \Theta \left(\xi(x,t)-\xi(l_2,t)\right) dx. \tag{25}$$

Combining (17)-(25) gives

$$\dot{V}(t) - \int_{l_1}^{l_2} J(x,t) dx \leq \eta^T(x,t) \theta \eta(x,t). \tag{26}$$

Define,

$$\eta^T(x,t) = \Big[\xi_t^T(x,t), \xi^T(x,t), \xi^T(x,t-\check{\tau}(t)), \xi^T(x,t-\hbar(t)), \\ \xi^T(x,t-\hbar), (\xi(x,t)-\xi(l_2,t))^T, w^T(x,t)\Big].$$

From the LMI (15), we get the value of $\theta_{ij} < 0$. Since the value of $\theta_{ij} < 0$, and there exist a scalar $\upsilon_1 > 0$ which is sufficiently small such that $\theta < \upsilon_1 I$, then (26) has the following representation:

$$\dot{V}(t) - \int_{l_1}^{l_2} J(x,t) dx \leq -\upsilon_1 \left|\eta(x,t)\right|^2 \leq -\upsilon_1 \left|\xi(x,t)\right|^2,$$

$$\dot{V}(t) \leq \int_{l_1}^{l_2} J(x,t) dx - \upsilon_1 \left|\xi(x,t)\right|^2. \tag{27}$$

When $w(x,t) = 0$ is taken into account, then $J(x,t) = z^T(x,t) R_4 z(x,t)$. Observing that $R_1 \leq 0$ under Assumption 2.1, it produces the results shown below.

$$\dot{V}(t) \leq -\upsilon_1 \left|\xi(x,t)\right|^2. \tag{28}$$

The above result indicates quadratically stability for the system (9). The next step involves proceeding with the ED conditions for the given system. It is simple to conclude that based on the definition of θ.

$$\dot{V}(t) - \int_{l_1}^{l_2} J(x,t) dx \leq 0. \tag{29}$$

When we integrate the aforementioned equation from 0 to t on both sides, we arrive at the following equation.

$$\int_0^t \int_{l_1}^{l_2} J(x,\alpha) dx d\alpha \geq V(t) - V(0) \geq \xi^T(x,t) P_1 \xi(x,t) + \varsigma. \tag{30}$$

To establish the validity of inequality (10), we consider two cases.

Case 1: If $|R_4| = 0$, then (30) implies, for any $t_f \geq 0$:

$$\int_0^{t_f} \int_{l_1}^{l_2} J(x,\alpha) dx d\alpha \geq \xi^T(x,t_f) P_1 \xi(x,t_f) + \varsigma \geq \varsigma. \tag{31}$$

From this, it is evident that (2.2) holds true.

Case 2: If $|R_4| \neq 0$, it follows that the matrices $R_1 = 0$, $R_2 = 0$, and $R_3 > 0$. Therefore, for any $t_f \geq t > 0$, we obtain

$$\int_0^{t_f}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha \geq \int_0^{t}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha \geq \xi^T(x,t)P_1\xi(x,t) + \varsigma, \tag{32}$$

when $t > \breve{\tau}(t)$, it is obvious that $0 < t - \breve{\tau}(t) < t_f$; thus,

$$\int_0^{t_f}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha \geq \xi^T(x,t-\breve{\tau}(t))P_1\xi(x,t-\breve{\tau}(t)) + \varsigma, \tag{33}$$

whereas, if $t \leq \breve{\tau}(t)$, then $-\breve{\tau} \leq t - \breve{\tau}(t) \leq 0$, we ge the following inequality

$$\begin{aligned}
&\varsigma + \xi^T(x,t-\breve{\tau}(t))P_1\xi(x,t-\breve{\tau}(t)) \\
&\leq \varsigma + \|P_1\| \left|\xi(x,t-\breve{\tau}(t))\right|^2 \\
&\leq \|P_1\| \sup_{-\breve{\tau}\leq\kappa\leq 0} \psi\left|\kappa\right|^2 = -V(0) \leq \int_0^{t_f}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha.
\end{aligned} \tag{34}$$

From the above inequality (33) holds for the value of $t_f \geq t \geq 0$. Therefore, according to (32) and (33), we conclude that a scalar exists $0 < \hat{\epsilon} < 1$, such that

$$\begin{aligned}
\int_0^{t_f}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha \geq \varsigma + (1-\hat{\epsilon})\xi^T(x,t-\breve{\tau}(t)) \\
P_1\xi(x,t-\breve{\tau}(t)) + \hat{\epsilon}\xi^T(x,t)P_1\xi(x,t).
\end{aligned}$$

Observing the fact

$$\begin{aligned}
z^T(x,t)R_4z(x,t) = &-\begin{bmatrix} \xi(x,t) \\ \xi(x,t-\breve{\tau}(t)) \end{bmatrix}^T \hat{\Pi} \begin{bmatrix} \xi(x,t) \\ \xi(x,t-\breve{\tau}(t)) \end{bmatrix} \\
&+ (1-\hat{\epsilon})\xi^T(x,t-\breve{\tau}(t))P_1\xi(x,t-\breve{\tau}(t)) \\
&+ \hat{\epsilon}\xi^T(x,t)P_1\xi(x,t),
\end{aligned}$$

for $\hat{\Pi} > 0$, then

$$\begin{aligned}
z^T(x,t)R_4z(x,t) \leq &(1-\hat{\epsilon})\xi^T(x,t-\breve{\tau}(t))P_1\xi(x,t-\breve{\tau}(t)) \\
&+ \hat{\epsilon}\xi^T(x,t)P_1\xi(x,t).
\end{aligned}$$

Known from the above, for any $t \geq 0, t_f \geq 0$ with $t_f \geq t$.

$$\int_0^{t_f}\int_{l_1}^{l_2} J(x,\alpha)dxd\alpha \geq \int_{l_1}^{l_2} z^T(x,t)R_4z(x,t)dx + \varsigma.$$

As a result, (10) is true for any value of $t_f \geq 0$. The system (9) is ED in the sense of Definition 2.2 after discussing the two cases of $\|R_4\| = 0$ and $\|R_4\| \neq 0$, and proof of this theorem is complete. □

Remark 3.2. It is noteworthy that in many industrial processes, the dynamical behaviors are generally complex and non-linear, and their genuine mathematical models are always difficult to obtain. How to model the fuzzy SDC of DPSs concerning extended dissipative performance has become one of the main themes in our research work. Compared with the existing literature [30, 15, 14, 5], underlying extended dissipative with fuzzy SMC (8) in this paper has a wider scope that can cover DPSs. In contrast to previous work on DPSs with various control techniques [30, 15, 14, 5], the model under study is more practical and general, because it considers usual control issues that have been studied with DPSs based on stabilization conditions, but in this paper, we consider an extended dissipative approach with SDC. Thereby, the resulting model concern has a stronger modeling capacity, and our proposed extended dissipative SDC strategy is more suitable for DPSs.

4. **Sampled-data controller design.** This section covers determining appropriate control gains to stabilize the performance of the fuzzy system (9).

Theorem 4.1. *It has been assumed that the fuzzy system (9) is quadratically stable with ED for given positive constants $\check{\tau}, \hbar, \mu, \alpha_1, \alpha_2, 0 < \hat{\epsilon} < 1$, and given matrices $\hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{R}_4$ satisfying Assumption 2.1. This assumption holds when there exist matrices $\hat{P}_i > 0 (i = 1, 3, 4, 5), \hat{G}, \mathcal{X}$ that satisfy $\alpha_1 \Theta \mathcal{X} \geq 0, \alpha_2 \Theta \mathcal{X} \geq 0$ and appropriate dimension matrix Y_j, and the following LMIs hold:*

$$\hat{\Pi} = \begin{bmatrix} \hat{\epsilon}\hat{P}_1 - \hat{R}_4 & -\hat{R}_4 \\ * & (1-\hat{\epsilon})\hat{P}_1 - \hat{R}_4 \end{bmatrix} > 0, \tag{35}$$

$$\Psi_{ij} = \begin{bmatrix} \Psi_{1,1} & \Psi_{1,2} & \Psi_{1,3} & \Psi_{1,4} & 0 & 0 & \Psi_{1,7} \\ * & \Psi_{2,2} & \Psi_{2,3} & \Psi_{2,4} & \Psi_{2,5} & 0 & \Psi_{2,7} \\ * & * & \Psi_{3,3} & 0 & 0 & 0 & \Psi_{3,7} \\ * & * & * & \Psi_{4,4} & \Psi_{4,5} & 0 & 0 \\ * & * & * & * & \Psi_{5,5} & 0 & 0 \\ * & * & * & * & * & \Psi_{6,6} & 0 \\ * & * & * & * & * & * & \Psi_{7,7} \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned}
&\Psi_{1,1} = \hbar^2 \hat{P}_5 - 2\alpha_1 \mathcal{X},\ \Psi_{1,2} = 2\alpha_1 \hat{A}_i \mathcal{X} + (2\hat{P}_1)^T - (2\alpha_2 \mathcal{X})^T, \\
&\Psi_{1,3} = 2\alpha_1 \hat{A}_{di} \mathcal{X},\ \Psi_{1,4} = 2\alpha_1 \hat{B}_i Y_j,\ \Psi_{1,7} = 2\alpha_1 \hat{C}_i \mathcal{X}, \\
&\Psi_{2,2} = \hat{P}_3 + \hat{P}_4 - \hat{P}_5 + 2\alpha_2 \hat{A}_i \mathcal{X} - \hat{L}_i^T \hat{R}_1 \hat{L}_i, \\
&\Psi_{2,3} = 2\alpha_2 \hat{A}_{di} \mathcal{X} - \hat{L}_i^T \hat{R}_1 \hat{M}_i,\ \Psi_{2,4} = 2\alpha_2 \hat{B}_i Y_j + \hat{P}_5 - \hat{G}^T, \\
&\Psi_{2,5} = \hat{G}^T, \quad \Psi_{2,7} = 2\alpha_2 \hat{C}_i \mathcal{X} - 2\hat{L}_i^T \hat{R}_2, \\
&\Psi_{3,3} = -(1-\mu)\hat{P}_3 - \hat{M}_i^T \hat{R}_1 \hat{M}_i,\ \Psi_{3,7} = -2\hat{M}_i^T \hat{R}_2, \\
&\Psi_{4,4} = -\hat{P}_5 - \hat{P}_5 + \hat{G} + \hat{G}^T,\ \Psi_{4,5} = \hat{P}_5 - \hat{G}^T, \\
&\Psi_{5,5} = -\hat{P}_4 - \hat{P}_5,\ \Psi_{6,6} = -\frac{\Pi^2}{4(l_2 - l_1)^2}[Sym(\alpha_2 \Theta \mathcal{X})], \\
&\Psi_{7,7} = -\hat{R}_3.
\end{aligned}$$

Moreover, the SD controller can be built as $\hat{K}_j = Y_j (\mathcal{X}^T)^{-1}$.

Proof. Define $P_2 = \alpha_1 (\mathcal{X}^T)^{-1}, P_6 = \alpha_2 (\mathcal{X}^T)^{-1}, \hat{P}_i = \mathcal{X} P_i \mathcal{X}^T (i = 1, 3, 4, 5), \hat{R}_i = \mathcal{X} R_i \mathcal{X}^T (i = 1, 2, 3, 4)$, $\hat{G} = \mathcal{X} G \mathcal{X}^T$, and $\mathcal{H} = diag\{\mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}\}$. Multiplying (14) and (15) by both sides on $\mathcal{H}$ and $\mathcal{H}^T$, then we can obtain LMI (35) and (36). □

Remark 4.2. If there is no fuzzy rule in (9) then we get

$$\begin{aligned}\xi_t(x,t) =&\mathcal{A}\xi(x,t) + A\xi(x,t) + A_d\xi(x,t-\breve{\tau}(t)) \\ &+ BK\xi(x,t-\hbar(t)) + C\omega(x,t)], \\ z(x,t) =&L\xi(x,t) + M\xi(x,t-\breve{\tau}(t)),\end{aligned} \tag{37}$$

Then it is simple to obtain the subsequent Corollary 4.3.

Corollary 4.3. *It has been assumed that the system (37) is quadratically stable with ED for given positive constants $\breve{\tau}, \hbar, \mu, \alpha_1, \alpha_2, 0 < \hat{\epsilon} < 1$, and given matrices $\hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{R}_4$ satisfying Assumption 2.1. This assumption holds when there exist matrices $\hat{P}_i > 0 (i = 1, 3, 4, 5), \hat{G}, \mathfrak{X}$ that satisfy $\alpha_1\Theta\mathfrak{X} \geq 0, \alpha_2\Theta\mathfrak{X} \geq 0$ and appropriate dimension matrix Y, and the following LMIs hold:*

$$\hat{\Pi} = \begin{bmatrix} \hat{\epsilon}\hat{P}_1 - \hat{R}_4 & -\hat{R}_4 \\ * & (1-\hat{\epsilon})\hat{P}_1 - \hat{R}_4 \end{bmatrix} > 0, \tag{38}$$

$$\Psi = \begin{bmatrix} \Psi_{1,1} & \Psi_{1,2} & \Psi_{1,3} & \Psi_{1,4} & 0 & 0 & \Psi_{1,7} \\ * & \Psi_{2,2} & \Psi_{2,3} & \Psi_{2,4} & \Psi_{2,5} & 0 & \Psi_{2,7} \\ * & * & \Psi_{3,3} & 0 & 0 & 0 & \Psi_{3,7} \\ * & * & * & \Psi_{4,4} & \Psi_{4,5} & 0 & 0 \\ * & * & * & * & \Psi_{5,5} & 0 & 0 \\ * & * & * & * & * & \Psi_{6,6} & 0 \\ * & * & * & * & * & * & \Psi_{7,7} \end{bmatrix} < 0, \tag{39}$$

where

$$\begin{aligned}&\Psi_{1,1} = \hbar^2\hat{P}_5 - 2\alpha_1\mathfrak{X},\ \Psi_{1,2} = 2\alpha_1 A\mathfrak{X} + (2\hat{P}_1)^T - (2\alpha_2\mathfrak{X})^T, \\ &\Psi_{1,3} = 2\alpha_1 A_d\mathfrak{X}, \quad \Psi_{1,4} = 2\alpha_1 BY,\ \Psi_{1,7} = 2\alpha_1 C\mathfrak{X}, \\ &\Psi_{2,2} = \hat{P}_3 + \hat{P}_4 - \hat{P}_5 + 2\alpha_2 A\mathfrak{X} - L^T\hat{R}_1 L, \\ &\Psi_{2,3} = 2\alpha_2 A_d\mathfrak{X} - L^T\hat{R}_1 M,\ \Psi_{2,4} = 2\alpha_2 BY + \hat{P}_5 - \hat{G}^T, \\ &\Psi_{2,5} = \hat{G}^T, \quad \Psi_{2,7} = 2\alpha_2 C\mathfrak{X} - 2L^T\hat{R}_2, \\ &\Psi_{3,3} = -(1-\mu)\hat{P}_3 - M^T\hat{R}_1 M,\ \Psi_{3,7} = -2M^T\hat{R}_2, \\ &\Psi_{4,4} = -\hat{P}_5 - \hat{P}_5 + \hat{G} + \hat{G}^T,\ \Psi_{4,5} = \hat{P}_5 - \hat{G}^T, \\ &\Psi_{5,5} = -\hat{P}_4 - \hat{P}_5, \quad \Psi_{6,6} = -\frac{\Pi^2}{4(l_2 - l_1)^2}[Sym(\alpha_2\Theta\mathfrak{X})], \\ &\Psi_{7,7} = -\hat{R}_3.\end{aligned}$$

Moreover, the SD controller might be built as $K = Y(\mathfrak{X}^T)^{-1}$.

5. **Numerical evaluations.**

In this section, we examined two numerical examples to show the effectiveness of the theoretical results obtained:

Example 5.1. Take into account the system models provided by (9) with the following parameters: $\mathcal{R}^1$: **IF** $\breve{\theta}_1(x,t)$ is F_1^1, **THEN** fuzzy rule 1:

$$\begin{aligned}\xi_t(x,t) =&\Theta\xi_{xx}(x,t) + A_1\xi(x,t) + A_{d1}\xi(x,t-\breve{\tau}(t)) \\ &+ B_1K_1\xi(x,t-\hbar(t)) + C_1\omega(x,t)], \\ z(x,t) =&L_1\xi(x,t) + M_1\xi(x,t-\breve{\tau}(t)).\end{aligned} \tag{40}$$

$\mathcal{R}^2$: **IF** $\breve{\theta}_1(x,t)$ is F_1^2, **THEN** Fuzzy Rule 2:

$$\begin{aligned}\xi_t(x,t) =&\Theta\xi_{xx}(x,t) + A_2\xi(x,t) + A_{d2}\xi(x,t-\breve{\tau}(t))\\ &+ B_2K_2\xi(x,t-\hbar(t)) + C_2\omega(x,t)],\\ z(x,t) =&L_2\xi(x,t) + M_2\xi(x,t-\breve{\tau}(t)).\end{aligned} \tag{41}$$

subject to the Dirichlet boundary conditions

$$\xi_1(l_1,t) = \xi_2(l_1,t) = 0,\ \xi_1(l_2,t) = \xi_2(l_2,t) = 0.$$

and the initial conditions,

$$\xi_1(x,0) = 0.5sin(2x),\ \xi_2(x,0) = 0.3sin(2x).$$

Mode 1:

$$A_1 = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.65 \end{bmatrix},\quad A_{d1} = \begin{bmatrix} 1 & 0.5 \\ -0.3 & 0.6 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.4 & 0.7 \end{bmatrix},\quad C_1 = \begin{bmatrix} 0.4 & 0.5 \\ 0.2 & 0.6 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix},\quad M_1 = \begin{bmatrix} 0.13 & 0 \\ 0 & 0.12 \end{bmatrix}.$$

Mode 2:

$$A_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix},\quad A_{d2} = \begin{bmatrix} 2 & 0.2 \\ -0.4 & 0.8 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & 0.4 \end{bmatrix},\quad C_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0.4 & 0.6 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.18 & 0 \\ 0 & 0.10 \end{bmatrix},\quad M_2 = \begin{bmatrix} 0.61 & 0 \\ 0 & 0.21 \end{bmatrix}$$
$$\Theta = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

The membership functions are chosen as

$$h_1 = \frac{\xi_2(x,t) + 0.1}{0.5},\quad h_2 = 1 - \frac{\xi_2(x,t) + 0.1}{0.5}. \tag{42}$$

Furthermore, the known constants are taken as follows: $\breve{\tau} = 0.3, \mu = 0.5,\ \hbar = 0.15\ \alpha_1 = 0.3,\ \alpha_2 = 0.1,\ l_1 = 0.1,\ l_2 = 0.2$. We will now examine the ED performance from four perspectives, namely, H_∞, $L_2 - L_\infty$, passivity, and dissipative behavior, to see if the control scheme with disturbance input meets it. The equivalent dynamic performance is carried out as follows by selecting R_1,R_2,R_3, and R_4:

1. L_2-L_∞ performance of the system: If $R_1 = 0,\ R_2 = 0,\ R_3 = \gamma^2 I$, and $R_4 = I$ are used, the extended dissipativity corresponds to L_2-L_∞ performance. Utilizing the Theorem 4.1 for LMIs allows for the calculation of the Corresponding Parameters as

$$K_1 = \begin{bmatrix} 0.8117 & -0.0208 \\ -0.0085 & 1.0253 \end{bmatrix},\quad K_2 = \begin{bmatrix} 0.5128 & 0.0035 \\ 0.0287 & 0.8841 \end{bmatrix}.$$

Under the initial condition $[2,-2]^T$, the dynamical behaviors of the state response of the system (40) and (41) are given in Figure 1. Due to the role of the above SDC gain matrix, the state responses of the controller are shown in Figure 2 under the randomized initial condition, which indicates that the trend of the curve is convergent under the effect of the above controller. It clearly indicates that the state trajectories and control inputs of the system (40) and (41) are internally stable and exhibit L_2-L_∞ performance under the given parameters, which means the feasibility of the proposed method.

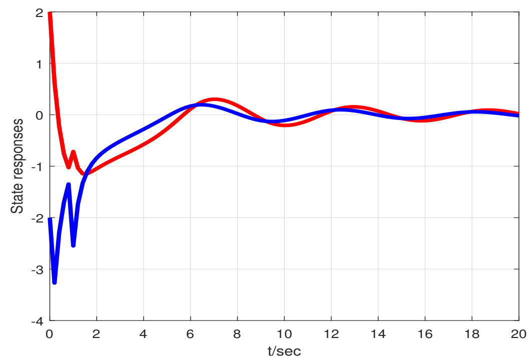

FIGURE 1. Trajectories of states for example 5.1 with L_2-L_∞ performance

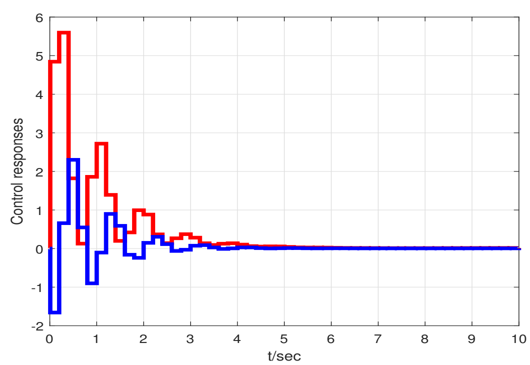

FIGURE 2. Trajectories of control input for the Example 5.1 with L_2-L_∞ performance

2. H_∞ **performance of the system**: If $R_1 = -I$, $R_2 = 0$, $R_3 = \gamma^2 I$, and $R_4 = 0$ are used, the ED equals $Hinfty$ performance. The relevant estimator parameters are derived using the Theorem 4.1 LMIs (35) and (36) with the above-mentioned parameters

$$K_1 = \begin{bmatrix} 0.7928 & -0.0334 \\ -0.0223 & 0.9989 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.5071 & 0.0041 \\ 0.0265 & 0.8654 \end{bmatrix}.$$

Figure 3 shows the numerical simulation of the system (40) and (41) state trajectories under the initial condition $[2, -2]^T$. Figure 4 shows the control input of the system using the designed controller. Consequently, Figures 3 and 4 show that the system (40) and (41) state trajectories $\xi_i(t)(i = 1, 2)$ and control inputs are internally stable. These figures demonstrate the stabilization scenario that exists for both the designed system and the planned controller. It is easy to see that the effectiveness of H_∞ performance within the specified bounds.

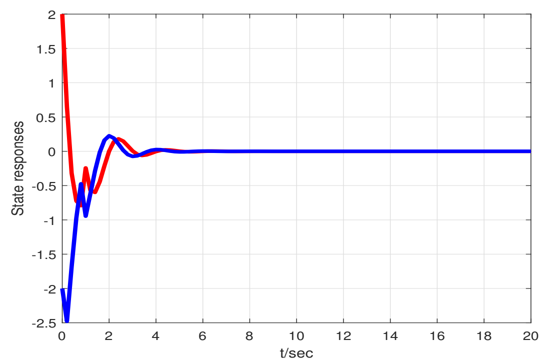

FIGURE 3. Trajectories of states for the Example 5.1 with H_∞ performance

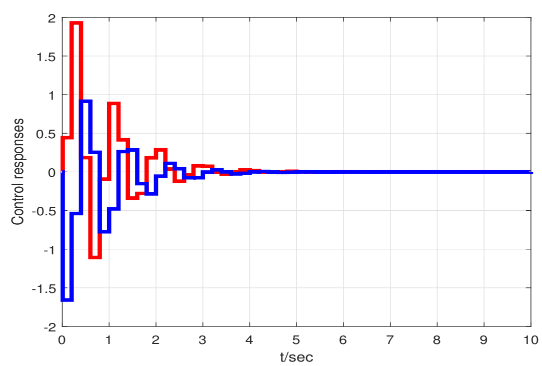

FIGURE 4. Trajectories of control input for the Example 5.1 with H_∞ performance

3. Passivity performance of the system: If taking $R_1 = 0, R_2 = I, R_3 = \gamma I, R_4 = 0$, then the ED means passivity performance. Then, by solving the linear matrix inequality (35) and (36), the controller gains K_1 and K_2 can be obtained as follows:

$$K_1 = \begin{bmatrix} 0.8106 & -0.0205 \\ -0.0076 & 1.0253 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.5115 & 0.0036 \\ 0.0283 & 0.8832 \end{bmatrix}.$$

With the initial condition $[-2, 2]^T$, the state trajectories of the system (40) and (41) are shown in Figure 5, which is quadratically stable under the above gain matrices. In addition, the evolution of SDC $u(t)$ is shown in Figure 6, showing that the system (40) and (41) can achieve passivity performance under random initial conditions. Thus, it could be inferred that the considered system is passive performance with the given parameters. 4. $(Q, S, R)-\gamma$**-dissipativity**: According to the extended dissipation idea, free weight matrices are generated as follows:

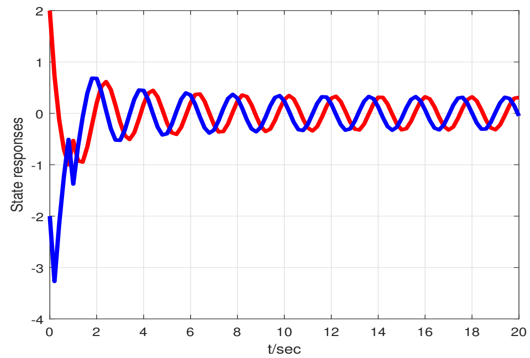

FIGURE 5. Trajectories of states for example 5.1 with passivity performance

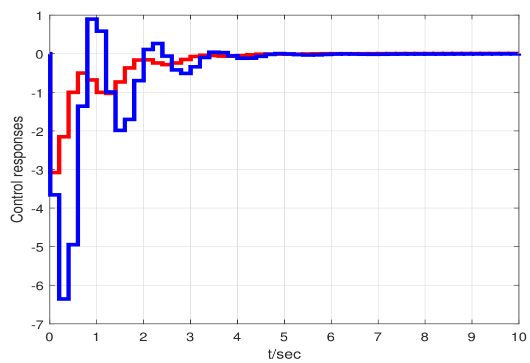

FIGURE 6. Trajectories of control input for the example 5.1 with passivity performance

$R_1 = Q$, $R_2 = S$, $R_3 = R - \gamma I$, and $R_4 = 0$. If $Q = -I$, $S = I$, and $R = 0.2I$ are present, then the ED is equivalent to the $(Q, S, R) - \gamma$-dissipativity. In this case, the value $\gamma = 0.25$ is chosen and the external disturbance is assumed to be $w(t) = 2/(1 + t^2)$. To demonstrate the efficacy of our technique, we use $\hbar = 0.15$ and Matlab to compute the LMIs in Theorem 4.1 to obtain the necessary controller gains matrices.

$$K_1 = \begin{bmatrix} 0.8061 & -0.0264 \\ -0.0153 & 1.0185 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.5103 & 0.0028 \\ 0.0252 & 0.8774 \end{bmatrix}.$$

At the same time, a demonstration of the relevant simulation is carried out to validate the findings presented in Figures 7 and 8. The similar state trajectories under the initial condition have been set at $[2, -2]^T$ and are depicted in Figure 7. To analyze the impact of controller input $u(t)$ on simulation results under randomized initial conditions. Figure 8 displays the control input for the dynamical system (40) and (41). Hence, based on the simulation results, we can infer that both the state curves and the control input would ultimately converge to zero. As a result, the dissipativity performance criteria of $(Q, S, R) - \gamma$ have been met. As a result, Figures. 7 and 8 not only confirm the stability region of (40) and (41), but also highlight the advantages of our planned controller.

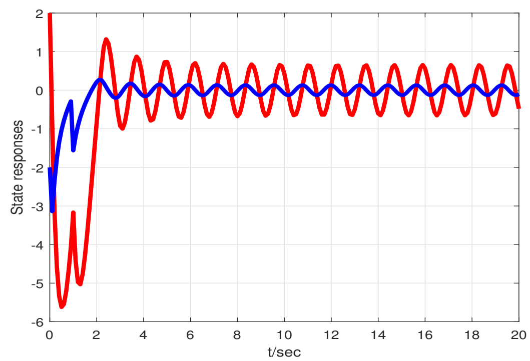

FIGURE 7. Trajectories of states for example 5.1 with $(Q, S, R) - \gamma$-dissipativity performance

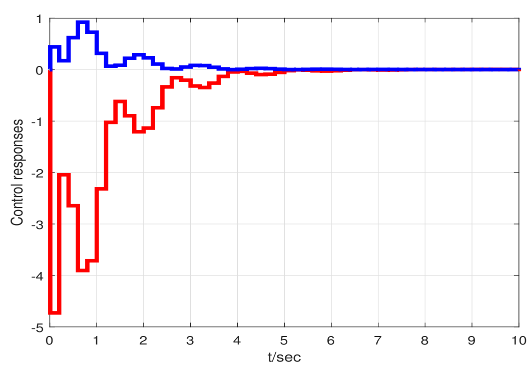

FIGURE 8. Trajectories of control input for the Example 5.1 $(Q, S, R) - \gamma$-dissipativity performance

Remark 5.1. A sufficient condition for the existence of a sampled-data controller is provided by Theorem 4.1 in such a way that the fuzzy system (9) is shown to be extended dissipative. This condition is stated in terms of LMIs, which are equations that are straightforward to solve. You can get the best possible dissipativity performance γ, by configuring the requirements outlined in Remark 2.6.

Example 5.2. Consider the system (37) with these parameters:

$$\begin{aligned} \xi_t(x,t) =& \Theta\xi_{xx}(x,t) + A\xi(x,t) + A_d\xi(x,t-\breve{\tau}(t)) \\ & + B\hat{K}\xi(x,t-\hbar(t)) + C\omega(x,t)], \\ z(x,t) =& L\xi(x,t) + M\xi(x,t-\breve{\tau}(t)), \end{aligned} \tag{43}$$

subject to the Dirichlet boundary conditions

$$\xi_1(l_1,t) = \xi_2(l_1,t) = 0,\ \xi_1(l_2,t) = \xi_2(l_2,t) = 0.$$

TABLE 1. Four Cases of the ED Problems:

Performance/R	R_1	R_2	R_3	R_4
$L_2 - L_\infty$	0	0	$\gamma^2 I$	I
H_∞	-I	0	$\gamma^2 I$	0
Passivity	0	I	γI	0
Dissipativity	-0.5I	I	$0.2I - \gamma I$	0

and the initial conditions: $\xi_1(x,0) = 2{+}0.1\sin(2\pi x)$ and $\xi_2(x,0) = 1.1{+}0.2\sin(2\pi x)$, $x \in [0,1]$

$$A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad A_d = \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1.5 \end{bmatrix},$$
$$L = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

In this example, the values of model parameters are given as follows: $\breve{\tau} = 0.3, \mu = 0.3, \hbar = 0.35,\ \alpha_1 = 0.021,\ \alpha_2 = 0.013,\ \pi = 0.02,\ l_1 = 0.35$, and $l_2 = 0.54$ using these parameters and solving the LMIs in corollary 4.3 using MATLAB software, it is easy to get the feasibility of the system (43). In addition to this, the comprehensive examination of the various performance indices may be easily verified. The ED performance indices have been built as shown in Table 1.

1. $L_2 - L_\infty$ **Performance:** Solving Corollary 4.3 LMIs yields the subsequent control gain matrices.

$$K = \begin{bmatrix} 0.0182 & 0.0118 \\ 0.0112 & 0.0378 \end{bmatrix}.$$

Figure 9 displays the state curves of the system exhibiting $L_2 - L_\infty$ behavior(43) in the simulation. As shown in Figure 9, it can be seen that the state trajectories tend to zero with the given parameters in the $L_2 - L_\infty$ performance, which means that the controller design method proposed in Corollary 4.3 stabilizes the system using the above gain matrices obtained for $L_2 - L_\infty$. Table 2 shows the various values of γ for $\mu = 0.3$. Solve the LMIs in Corollary 4.3 with different values of $\hbar$. It is clear that the optimal value of $\hbar$ is 0.15.

2. H_∞ **Performance:** Solving Corollary 4.3 LMIs yield the subsequent control gain matrices.

$$K = \begin{bmatrix} 1.0909 & -0.0259 \\ -0.0259 & 1.0914 \end{bmatrix}$$

The numerical simulation is depicted to validate the conclusions that were obtained from Figure 10. According to corollary 4.3, the permissible performance of H_∞ may be calculated using a variety of h for the scenario in which $\mu = 0.3$ is used. Table 3 illustrates the connection between γ and $\hbar$. Moreover, in Table 3 shows that for fixed μ, the lowest value of γ decreases as the value of $\hbar$ increases.

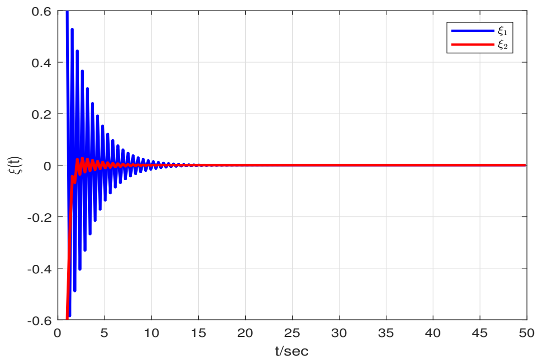

FIGURE 9. State responses of the system (43) with $L_2 - L_\infty$ performances

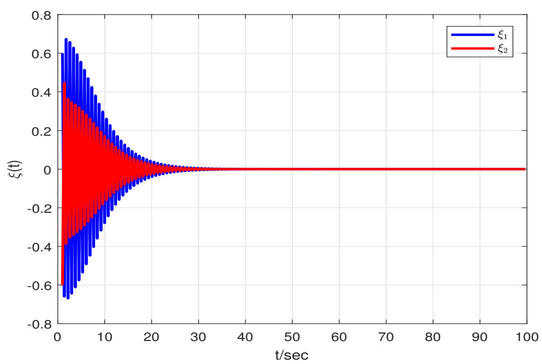

FIGURE 10. State responses of the system (43) with H_∞ performances

3. Passivity Performance: Corollary 4.3 is used to derive the relevant gain matrix parameters, which are

$$K = \begin{bmatrix} 0.4685 & -0.0776 \\ -0.0622 & 0.5031 \end{bmatrix}.$$

Associated with these gain matrices, the corresponding simulation is shown in Figure 11 under random initial conditions. Figure 11 indicates the trajectories of the system (43) with passivity performance. It is simple to see that passive performances start out well in the specified states. By solving the LMIs in Corollary 4.3, the maximum values of $\hbar$ with different μ are achieved for the specified $\gamma = 0.7$ and are shown in Table 4.

4.(Q, S, R)**-** γ **Dissipativity Performance:** Corollary 4.3 is used to derive the relevant gain matrix parameters, which are

$$K = \begin{bmatrix} 0.4711 & -0.0777 \\ -0.0634 & 0.5057 \end{bmatrix}$$

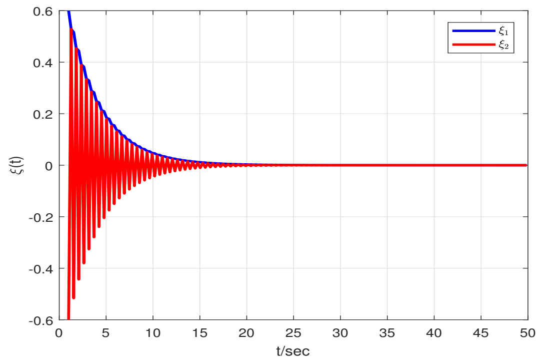

FIGURE 11. State curves of the system (43) with passivity performances

TABLE 2. Different minimums γ for various $\hbar$ and fixed $\mu = 0.3$ in Example 5.2

$\hbar$	0.01	0.075	0.1	0.15
γ	0.6742	0.3245	0.2145	0.0945

TABLE 3. Different minimum γ for various $\hbar$ and fixed $\mu = 0.3$ in Example 5.2

$\hbar$	0.01	0.075	0.1	0.15	0.25
γ	0.9012	0.8745	0.7000	0.5228	0.4512

Figure 12 shows the equivalent simulation with randomized initial values. The system (43) can clearly maintain internal stability despite the (Q, S, R)-γ dissipativity performance. The maximum values of h with different μ are determined by solving the LMI in Corollary 4.3 and are listed in Table 5.

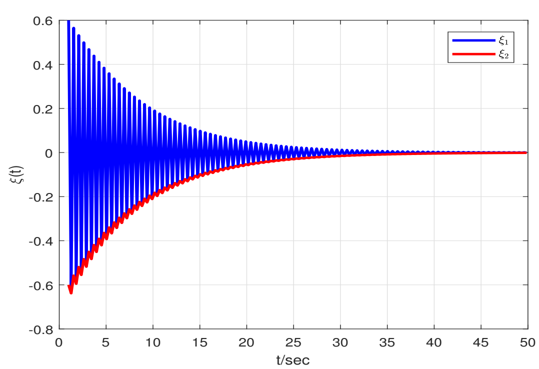

FIGURE 12. State responses of the system with (43) (Q, S, R)-γ Dissipativity performances

TABLE 4. Allowable maximum $\hbar$ for various μ and fixed γ=0.7 in Example 5.2

μ	0.1	0.15	0.2	0.25	0.3
$\hbar$	0.0640	0.0787	0.1513	0.1720	0.2324

TABLE 5. Allowable maximum $\hbar$ for various μ and fixed γ=0.7 in Example 5.2

μ	0.1	0.2	0.3	0.4	0.5
$\hbar$	0.2010	0.1542	0.0840	0.0536	0.0402

Remark 5.2. The extended dissipative analysis was investigated by making use of a number of different system parameters in conjunction with Theorem 4.1. Theorem 4.1 is capable of performing four different kinds of system analysis using the matrices that are stated in Table 1. The results that are shown in Tables 2–4 were acquired by maximizing the value of $\hbar$ while reducing the value of γ. Because the results in Tables 2, 3, and 4 reveal that the optimal values were affected by the system parameters, it was very necessary to locate appropriate values and acquire some conservatism from the extended dissipative analysis. This was demonstrated by the fact that the optimal values were altered.

6. **Conclusion.** This paper describes a way to control a class of T-S fuzzy DPSs using SDC. Using LMIs as a starting point, a Lyapunov functional was used to set up the right conditions for quadratically stable DPSs and ED. This paper suggests that using these techniques can be helpful. Furthermore, it has been demonstrated that the suggested methodologies are supported by simulation studies. This is only a start, but we think it could help improve fuzzy DPSs. Our upcoming research will examine the use of event-triggered control in interval-valued fuzzy DPSs with time-varying delay. Additionally, we will explore state estimation techniques for a specific type of nonlinear DPSs with time-varying delay, specifically those including mobile actuator/sensor pairs. These investigations will be conducted utilizing our own proposed methodology.

Acknowledgments. S. Saravanan gratefully acknowledge this work is funded by the Centre for Nonlinear Systems, Chennai Institute of Technology (CIT), India, vide funding number CIT/CNS/2023/RP-005.

REFERENCES

[1] R. Anbuvithya, S. D. Sri, R. Vadivel, N. Gunasekaran and P. Hammachukiattikul, Extended dissipativity and non-fragile synchronization for recurent neural networks with multiple time-varying delays via sampled-data control, *IEEE Access*, **9** (2021), 31454-31466.

[2] R. Anbuvithya, S. Dheepika Sri, R. Vadivel, P. Hammachukiattikul, C. Park and G. Nallappan, Extended dissipativity synchronization for Markovian jump recurrent neural networks via memory sampled-data control and its application to circuit theory, *International Journal of Nonlinear Analysis and Applications*, **13** (2022), 2801-2820.

[3] M. S. Ali, R. Vadivel, A. Alsaedi and B. Ahmad, Extended dissipativity and event-triggered synchronization for T–S fuzzy Markovian jumping delayed stochastic neural networks with leakage delays via fault-tolerant control, *Soft Computing*, **24** (2020), 3675-3694.

[4] M. S. Aslam, B. Zhang, Y. Zhang and Z. Zhang, Extended dissipative filter design for T–S fuzzy systems with multiple time delays, *ISA transactions*, **80** (2018), 22-34.

[5] L. Bao, P. Huang and S. Fei, Exponential stability and L_2 gain analysis of switched distributed parameter systems with time delay, *Journal of Control and Decision*, **8** (2021), 385-393.

[6] X. Cai, J. Wang, S. Zhong, K. Shi and Y. Tang, Fuzzy quantized sampled-data control for extended dissipative analysis of T–S fuzzy system and its application to WPGSs, *Journal of the Franklin Institute*, **358** (2021), 1350-1375.

[7] H. D. Choi, C. K. Ahn, P. Shi, L. Wu and M. T. Lim, Dynamic output-feedback dissipative control for T–S fuzzy systems with time-varying input delay and output constraints, *IEEE Transactions on Fuzzy Systems*, **25** (2016), 511-526.

[8] T. Deng, X. Zhang and Z. Wang, Output feedback control of interval type-2 T–S fuzzy fractional order systems subject to actuator saturation, *Mathematical Foundations of Computing*, (2023).

[9] G. Fan and Y. Ma, Non-fragile delay-dependent pinning H_∞ synchronization of T–S fuzzy complex networks with hybrid coupling delays, *Information Sciences*, **608** (2022), 1317-1333.

[10] C. Ge, J. H. Park, C. Hua and X. Guan, Dissipativity analysis for T–S fuzzy system under memory sampled-data control, *IEEE Transactions on Cybernetics*, **51** (2019), 961-969.

[11] K. Gu, V. Kharitonov and J. Chen, *Stability of Time Delay Systems*, Birkhäuser Boston, MA, USA, 2003.

[12] H. Ji and B. Cui, Event-generator-based H_∞ control of fuzzy distributed parameter systems, *Fuzzy Sets and Systems*, **432** (2022), 28-49.

[13] H. Li, J. Wang, H. Du and H. R. Karimi, Adaptive sliding mode control for Takagi–Sugeno fuzzy systems and its applications, *IEEE Transactions on Fuzzy Systems*, **26** (2017), 531-542.

[14] D. Liu, R. Han and G. Xu, Controller design for distributed parameter systems with time delays in the boundary feedbacks via the backstepping method, *International Journal of Control*, **93** (2020), 1220-1230.

[15] Y. Liu, J. W. Wang, Z. Wu, Z. Ren and S. Xie, Robust H_∞ control for semilinear parabolic distributed parameter systems with external disturbances via mobile actuators and sensors, *IEEE Transactions on Cybernetics*, **53** (2023), 4880-4893.

[16] W. Ma, X.-C. Jia and X. Chi, Exponential stabilization of sampled-data fuzzy systems via a parameterized fuzzy Lyapunov-Krasovskii functional approach, *Journal of the Franklin Institute*, **358** (2021), 5750-5770.

[17] R. Manivannan, R. Samidurai, J. Cao, A. Alsaedi and F. E. Alsaadi, Non-fragile extended dissipativity control design for generalized neural networks with interval time-delay signals, *Asian Journal of Control*, **21** (2019), 559-580.

[18] P. Park, J. W. Ko and C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, *Automatica J. IFAC*, **47** (2021), 235-238.

[19] S. Shanmugam, S. A. Muhammed and G. M. Lee, Finite-time extended dissipativity of delayed Takagi–Sugeno fuzzy neural networks using a free-matrix-based double integral inequality, *Neural Computing and Applications*, **32** (2020), 8517-8528.

[20] K. Shi, J. Wang, S. Zhong, Y. Tang and J. Cheng, Non-fragile memory filtering of TS fuzzy delayed neural networks based on switched fuzzy sampled-data control, *Fuzzy Sets and Systems*, **394** (2020), 40-64.

[21] X. Song, M. Wang, S. Song and J. Lu, Event-triggered reliable H_∞ control for a class of nonlinear distributed parameter systems within a finite-time interval, *Journal of the Franklin Institute*, **357** (2020), 11368-11393.

[22] J. B. Teguia, A. S. T. Kammogne, S. G. T. Ganmene, M. S. Siewe and G. Kenne, Fuzzy-enhanced robust fault-tolerant control of IFOC motor with matched and mismatched disturbances, *Mathematical Foundations of Computing*, **5** (2022), 295-314.

[23] R. Vadivel, R. Suresh, P. Hammachukiattikul, B. Unyong and N. Gunasekaran, Event-triggered $L_2 - L_\infty$ filtering for network-based neutral systems with time-varying delays via T-S fuzzy approach, *IEEE Access*, **9** (2021), 145133-145147.

[24] R. Vadivel, S. Sabarathinam, Y. Wu, K. Chaisena and N. Gunasekaran, New results on T-S fuzzy sampled-data stabilization for switched chaotic systems with its applications, *Chaos Solitons Fractals*, **164** (2022), 112741, 11 pp.

[25] T. X. Wang, Stability in abstract functional differential equations. Part I. General theorems, *Journal of Mathematical Analysis and Applications*, **186** (1994), 534-558.

[26] T. Wu, L. Xiong, J. Cheng and X. Xie, New results on stabilization analysis for fuzzy semi-Markov jump chaotic systems with state quantized sampled-data controller, *Information Sciences*, **521** (2020), 231-250.

[27] Z. Yin and X. Jiang, Delay-derivative/distribution dependent stability and stabilization criteria for TS fuzzy systems with random time-varying delay, *IEEE Transactions on Fuzzy Systems*, **31** (2023), 1628-1637.

[28] H. B. Zeng, Z. L. Zhai, Y. He, K. L. Teo and W. Wang, New insights on stability of sampled-data systems with time-delay, *Applied Mathematics and Computation*, **374** (2020), 125041, 11 pp.

[29] B. Zhang, W. X. Zheng and S. Xu, Filtering of Markovian jump delay systems based on a new performance index, *IEEE Transactions on Circuits and Systems I: Regular Papers*, **60** (2013), 1250-1263.

[30] X. W. Zhang, H. N. Wu and J. W. Wang, Mixed H_2/H_∞ fuzzy control plus mobile actuator/sensor guidance for semilinear parabolic distributed parameter systems, *IEEE Transactions on Fuzzy Systems*, **29** (2020), 1874-1884.

Received November 2023; revised January 2024; early access February 2024.